# SwissFEL Praktikum Exercises

## *XFEL Theory*

### Exercise 1: Magnetic undulator

a) Show that the following magnetic field in an undulator is a physical solution:

$$\vec{B} = B_0 \begin{pmatrix} 0 \\ \cosh k_u y \cdot \sin k_u z \\ \sinh k_u y \cdot \cos k_u z \end{pmatrix}$$

b) Show that this field yields a net focusing in the vertical direction.

### Exercise 2: Energy conservation

Show that in the FEL process the total energy is conserved:

$$|A|^2 + \langle \eta \rangle = const$$

### Exercise 3: Dispersion

Calculate numerically the maximum growth rate (maximum real value of the 3 solutions in the dispersion equation $p^3+i\Delta p^2=i$) for a detuning range of $\Delta$ between -2 and 2.

### Exercise 4: Particle tracking

Track numerically particles (about 100) in a radiation field within an undulator. Use as initial condition:

- Particle phases $\theta_j$ are equally distributed between 0 and $2\pi$.
- Same energy for all particles: $\eta_j = 0$
- Radiation field is constant: ($A=0.1$)
- Detuning is zero ($\Delta=0$)

Equation of motion:

$$\theta'_j = \Delta + \eta_j \qquad \eta'_j = 2A \cdot \sin\theta_j$$

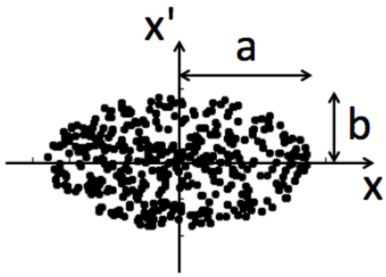
a) Show that the net energy change $<\eta>$ is zero

b) Calculate the energy changes for various detuning values (range between -2 and 2). Find the value, where the electrons have lost the maximum amount of energy.

# *Electron beam dynamics*

## Exercise 5: Phase space

a) Generate a uniform distribution of points within a (x, x') ellipse in phase space, with half-axes a and b:




b) Plot the distribution after propagation through a quadrupole (with k=-1) and a drift (with L=0.5).
c) Calculate the emittance ε before and after the quadrupole/drift, to show that it is constant, with ε = ab/4.

## Exercise 6: Quadrupole scan

Show that a plot of $\langle x^2 \rangle_f$ vs. k is a parabola and that $\langle x^2 \rangle_i$ , $\langle x'^2 \rangle_i$ and $\langle xx' \rangle_i$ (and hence ε) can be determined from $k_{min}$, $\sigma_{min}$ and the curvature $\left. \frac{d^2 \langle x^2 \rangle_f}{dk^2} \right|_{k=k_{\min}}$.