Prospects for a Muon $g-2$/EDM Measurement Using Re-accelerated Cold Muons from HIMB with muCool at PSI

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We present motivation for a high-precision measurement of the anomalous magnetic moment $g-2$ of the muon and improved searches for its electric dipole moment (EDM). A future muon $g-2$ experiment at PSI, using re-accelerated muons from a new cold source (MuCool) coupled to the future high-intensity muon beam (HIMB), complements ongoing efforts at Fermilab and J-PARC, potentially further increasing the precision. Similarly, we show that the sensitivity of a search for a muon EDM can be improved to better than $1 \times 10^{-23}\, e{\cdot}\mathrm{cm}$. A discovery of a muon EDM would prove the existence of physics beyond the Standard Model, while a null result sets a stringent limit on a currently poorly constrained Wilson coefficient.

I. INTRODUCTION AND MOTIVATION

Ever since Schwinger's famous prediction $a_\ell = (g-2)_\ell/2 = \alpha/(2\pi)$ [1] (and its experimental verification [2]), the anomalous magnetic moments of the electron and muon have been critical precision tests of the Standard Model (SM). Currently, the experimental determination of the muon $g-2$ is dominated by the Brookhaven measurement [3]

$$a_\mu^{\mathrm{exp}} = 116\,592\,089(63) \times 10^{-11}, \tag{1}$$

a precision of 0.54 ppm. Comparison with the current SM prediction [4]

$$a_\mu^{\mathrm{SM}} = 116\,591\,810(43) \times 10^{-11} \tag{2}$$

then reveals a 3.7σ tension. Experimental efforts to corroborate or refute this tension are underway at Fermilab [5] and J-PARC [6], with a precision goal of 0.14 ppm and 0.45 ppm, respectively, and the J-PARC experiment pioneering a new experimental technique that does not rely on the magic momentum in a storage ring, see Ref. [7] for a more detailed comparison of the two methods. Below, we will show that a $g-2$ experiment at PSI could reach a statistical precision around 0.1 ppm, and thus potentially further increase the precision. The SM prediction (2), currently at 0.37 ppm, represents a coherent theory effort organized in the Muon $g-2$ Theory Initiative [4], and is mainly based on the underlying work from Refs. [8–27]. The uncertainty is completely dominated by hadronic contributions, with hadronic vacuum polarization (HVP) and hadronic light-by-light scattering (HLbL) at 0.34 ppm and 0.15 ppm, respectively. Improvements on both HVP and HLbL will continue over the next years, including new $e^+e^- \to$ hadrons data, lattice-QCD calculations at a similar level of precision, and, potentially, direct input on space-like HVP from the proposed MUonE experiment [28, 29].

To explain the current 3.7σ tension with physics beyond the SM (BSM), some form of enhancement mechanism is required given that the absolute value of the difference exceeds the size of the electroweak contribution, but there are well-motivated scenarios that do display such an enhancement. First, new light weakly-coupled models can provide an explanation via the small mass of the new particle [30–33]. Second, solutions with new heavy particles above the electroweak scale are possible if the chirality flip originates from a large coupling to the SM Higgs, instead of the muon Yukawa coupling in the SM, leading to a chiral enhancement that allows for viable solutions for particle masses up to tens of TeV [34–36].

In particular, in the minimal supersymmetric SM (MSSM), this enhancement factor is provided by $\tan\beta$ (see, e.g., Refs. [37–39] for an overview), which for top–bottom Yukawa coupling unification is expected to be ≈ 50 [40, 41] and thus provides a possible enhancement mechanism [42–44]. While in case of a universal supersymmetry breaking mechanisms the LHC bounds on the supersymmetric partners are already so stringent that this enhancement is insufficient to explain a_μ, less minimal scenarios can still account for

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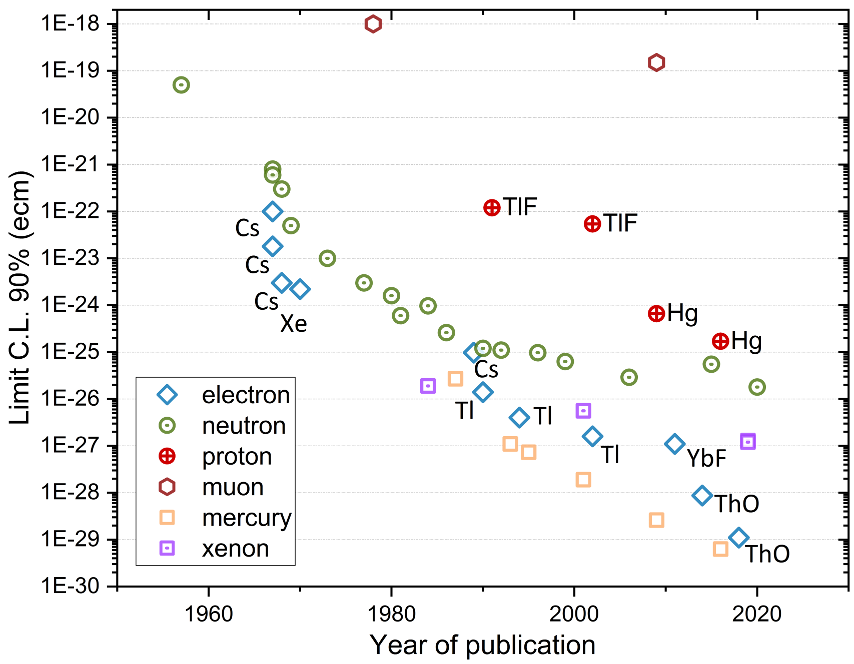


FIG. 1: Historical overview of EDM limits (90% C.L.). The labels in the plot next to the date (Cs, Tl, TlF, ThO, Xe, and YbF) refer to the measured system from which the limit was derived. So far, all EDM measurements were in agreement with a null result and were therefore interpreted as upper limits.

it (see, e.g., Ref. [45]). Furthermore, leptoquarks, which provide a viable solution to the hints for lepton flavor universality violation in semi-leptonic B decays (see below), can even possess a chiral enhancement factor of $m_t/m_\mu \approx 1700$ [46–51], allowing for a TeV scale explanation with perturbative couplings that is not in conflict with direct LHC searches. In addition, there exists a plethora of alternative BSM explanations of the muon $g-2$, including composite or extra-dimensional models [52–54] and models with two Higgs doublets [55–57].

While discovering BSM physics with $g-2$ requires one to identify a departure from the SM prediction, a measurement of an EDM would immediately signal the breakdown of the SM. A non-zero EDM of a fundamental particle violates time-reversal symmetry, and by invoking the CPT theorem of quantum field theories [58] also the combined symmetry of charge conjugation and parity inversion (CP). Many BSM theories have new complex parameters that are sources of CP violation, as these parameters are naturally expected to have a generic phase of order one. In fact, the only complex parameter within the SM (disregarding the QCD θ term), the phase of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [59], is close to maximal [60, 61]. Furthermore, CP violation is also one of three necessary conditions to explain the observed baryon asymmetry in the Universe (BAU) [62]. However, even though the CKM phase is close to maximal, CP violation within the SM is by far not sufficient to explain the observed BAU [63–68]. This strongly motivates theories with additional complex parameters as extensions of the SM, providing additional sources of CP violation. Clearly, such sources of CP violation are expected to generate at some level non-vanishing EDMs of fundamental particles, which can significantly exceed the tiny values within the SM [69].

Therefore, many experiments searching for non-vanishing EDMs have been performed over the last decades, as summarized in Fig. 1, and the current status can be found in Ref. [70]. As we can see, the limits on the muon EDM are particularly weak compared to the other constraints. Therefore, a search for a permanent EDM of the muon gives access to one of the least tested areas of the SM of particle physics and is hence an important piece of this comprehensive and complementary experimental strategy to unveil BSM physics [71].

One reason why in the past the focus of EDM searches was not on the muon EDM is that the impressive limits on the electron EDM from measurements using atoms or molecules, e.g., thorium oxide molecules $d_e < 1.1 \times 10^{-29}\, e{\cdot}\mathrm{cm}$ [72], were commonly rescaled, assuming minimal flavor violation (MFV) [73–76] (by the ratio m_μ/m_e), with a resulting limit $d_\mu < 2.3 \times 10^{-27}\, e{\cdot}\mathrm{cm}$ orders of magnitude below the direct limit $d_\mu < 1.5 \times 10^{-19}\, e{\cdot}\mathrm{cm}$ [77]. However, MFV is, to some extent, an ad-hoc symmetry invented to allow light particle spectra, in particular within the MSSM, where this reduces the degree of fine-tuning in the Higgs sector while respecting at the same time flavor constraints. Since the LHC did not discover any new particles directly [78, 79], the whole concept of naturalness is challenged. Furthermore, LHCb, Belle, and BaBar discovered significant tensions in semi-leptonic B decays [80–87] implying a 5σ-level discrepancy when analyzed together [88–90]. These remarkable hints for BSM physics point towards the violation of lepton flavor universality and are therefore not compatible with MFV in the lepton sector [91].

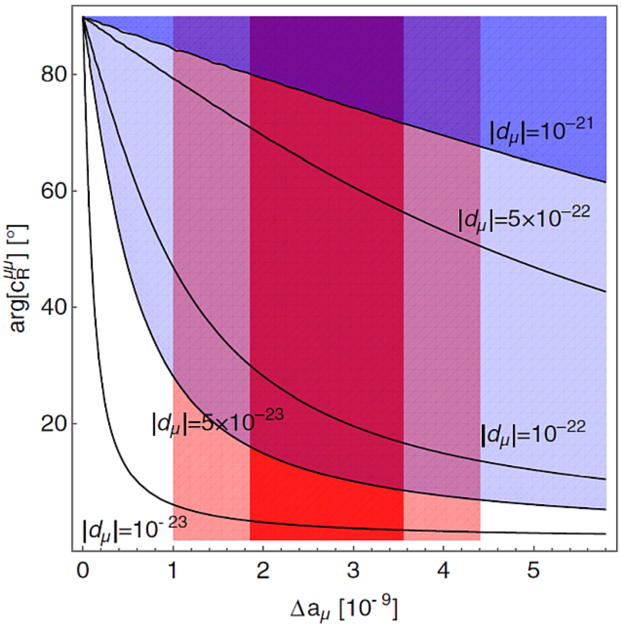


FIG. 2: Contours of d_μ as a function of the tension in the anomalous magnetic moment of the muon, Δa_μ, and the phase of the associated Wilson coefficient [36].

In fact, the 3.7σ tension in the muon $g-2$ sets the scale for the muon EDM if the CP violating phase in the respective Wilson coefficient is sizable. That is, even though the value of $g-2$ is not directly related to the EDM, any BSM contribution would result from the real part of the Wilson coefficient whose imaginary part determines the EDM. Moreover, while $g-2$ by itself does not conflict the MFV paradigm, solutions with chiral enhancement mentioned above violate the MFV scaling [36, 92]. At the same time, such scenarios automatically provide an a priori free phase, see Fig. 2, leading to a large EDM unless the phase happens to be small. A concrete model that implements this mechanism is provided by leptoquarks, which can, as discussed above, explain the $g-2$ tension without any prior constraints on the phase.

Therefore, it is well-motivated that the BSM flavor structure goes beyond MFV, a notion sometimes contested on grounds of naturalness arguments. However, in the limit of vanishing neutrino masses lepton flavor is conserved, and thus it is possible to completely disentangle the muon from the electron EDM via a symmetry, meaning that no fine-tuning is necessary. This could for example be achieved via a $L_\mu - L_\tau$ symmetry [93–95], which can naturally give rise to the observed neutrino mixing matrix [96–98], and, even after its breaking, protects the electron EDM and $g-2$ from BSM contributions [99]. Also from an EFT point of view [36, 100, 101], it is clear that the muon EDM can be large and that a measurement of it is in practice the only way of determining the imaginary part of the associated Wilson coefficient.

For these reasons, both a high-precision measurement of the muon $g-2$ and a dedicated muon EDM experiment are highly motivated, and would be valuable contributions in the search for physics beyond the SM in low-energy precision observables.

II. A $g-2$/EDM MEASUREMENT AT PSI

Currently the most precise measurement of the muon $g-2$ and EDM is being performed at Fermilab, with an expected relative statistical precision for $g-2$ of 0.1 ppm and a systematic error of about the same size [5]. The second experiment proposed at J-PARC [6] will have a precision of about 0.45 ppm including systematic effects. Both experiments also intend to use the same data to search for the muon EDM with a precision of about $1\times10^{-21}\,e\cdot$cm [6, 102].

Independent of the final result of the $g-2$ measurement, a second result with the same precision and vastly different experimental techniques and systematic uncertainties is strongly desirable, while an improvement of the sensitivity on a muon EDM measurement will test uncharted terrain and ideally complements precision measurements with high-energy particle collision and other EDM searches.

Both measurements observe the spin precession $\vec{\omega}$ of a muon in a storage ring with an electric field $\vec{E}$ and magnetic field $\vec{B}$ given by:

$$\vec{\omega} = \frac{q}{m}\left[a\vec{B} - \left(a + \frac{1}{1-\gamma^2}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right] + \frac{q}{m}\frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right). \tag{3}$$

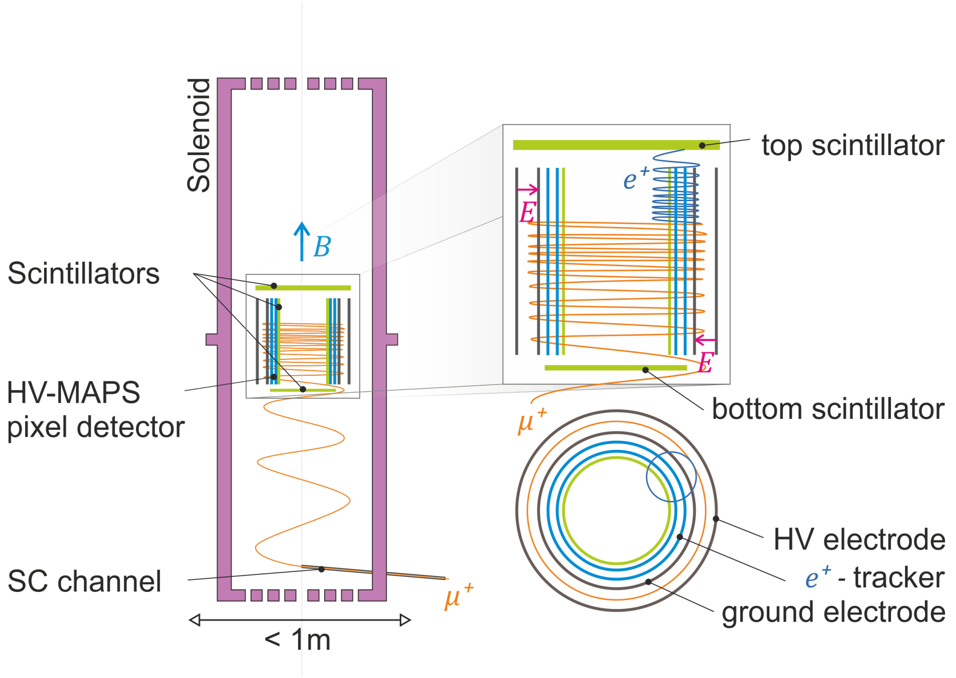


FIG. 3: Sketch of the helix muEDM instrument, not to scale.

Certain choices of the muon momentum and the combination of electric and magnetic fields permit measurements with strongly reduced systematic effects.

The muEDM collaboration at PSI proposes a search for the muEDM with a sensitivity of about $6 \times 10^{-23}\, e{\cdot}\mathrm{cm}$ per year of data-taking using muons with a momentum of $\vec{p} = 125\,\mathrm{MeV}/c$, $|\vec{\beta}| = |\vec{v}|/c = 0.77$, and an average polarization of better than $P = 93\%$ from the μE1 beam line at PSI with a particle flux of up to $2 \times 10^8\, \mu^+/\mathrm{s}$. The concept is based on the frozen-spin technique [103, 104] combined with a spiral injection into a magnetic field of $B = 3\,\mathrm{T}$, similar as in the J-PARC $(g-2)$/muEDM experiment [6, 105]. A sketch of the experiment is shown in Fig. 3.

The search profits from the large electric field in the rest frame of the muon $|E^*| = |\gamma c \vec{\beta} \times \vec{B}| = 1.1\,\mathrm{GV/m}$. A radial electrode system provides an electric field $\vec{E}_\mathrm{f}$ perpendicular to the motion of the muon and the magnetic field $\vec{B}$, hence $\vec{\beta} \cdot \vec{B} = \vec{\beta} \cdot \vec{E}_\mathrm{f} = 0$, and $\vec{B} \cdot \vec{E}_\mathrm{f} = 0$. By adjusting the strength of the electric field $\vec{E}_\mathrm{f}$ such that

$$a\vec{B} = \left(a - \frac{1}{\gamma^2 - 1}\right) \frac{\vec{\beta} \times \vec{E}_\mathrm{f}}{c}, \tag{4}$$

it is possible to cancel the anomalous precession term in Eq. (3), which then simplifies to

$$\vec{\omega} = \frac{q}{m} \frac{\eta}{2} \left(\vec{\beta} \times \vec{B} + \frac{\vec{E}_\mathrm{f}}{c}\right), \tag{5}$$

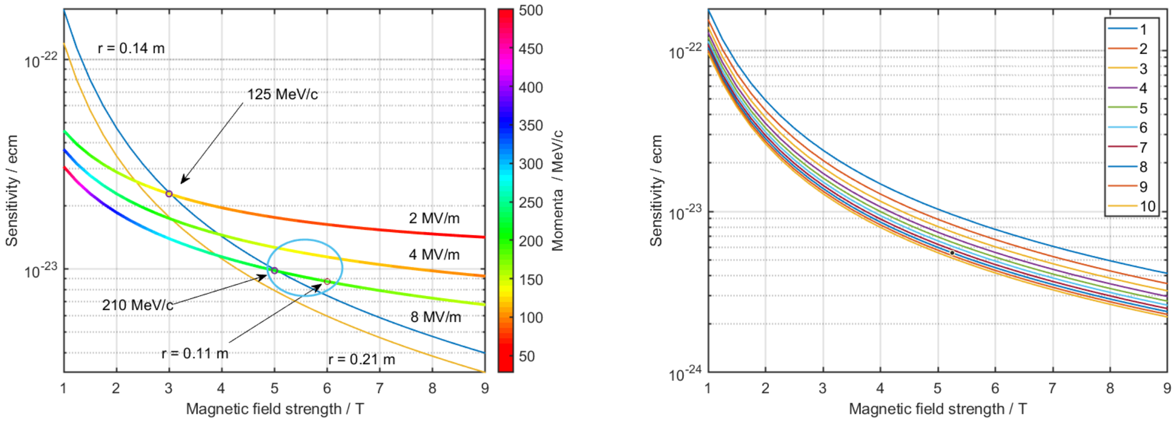
known as the frozen-spin technique.

The statistical sensitivity of this proposal is limited by the maximum muon momentum and the large lateral phase at μE1. As we outline in the following, by using a dedicated beam line with sub millirad divergence and a momentum of $P = 190\,\mathrm{MeV}/c$, this concept has the potential to improve the sensitivity of the muEDM search to better than $1 \times 10^{-23}\, e{\cdot}\mathrm{cm}$. Without the electric field, i.e., with both electrodes grounded, this same setup measures the anomalous magnetic moment and could reach a relative precision of the muon $g-2$ of about 0.1 ppm.

A. Prospects for a search of the muon EDM

The sensitivity for a muon EDM experiment deploying the frozen-spin technique [103] with $E \approx aBc\beta\gamma^2$ is given by

$$\sigma(d_\mu) = \frac{\hbar}{2\beta c P \gamma B \sqrt{N} \alpha \tau}. \tag{6}$$



(a) (b)

FIG. 4: Sensitivity landscape for muon EDM searches. (a) Single muon-on-demand: Sensitivity increases with magnetic field strength keeping the radius constant. Electric fields above 8 MV are very difficult for large electrodes with extremely thin material thickness (positron transmission). Region within circle indicates possible parameter space for a future muEDM search using re-accelerated muons from muCool. (b) Increase in sensitivity with multiplicity of muons per injection for the case $r = 0.14\,\mathrm{m}$, as function of the magnetic field.

	muE1 125 MeV/c	HIMB muCool 125 MeV/c	HIMB muCool 210 MeV/c
E-Field (MV/m)	2	2	8
B-Field (T)	3	3	5
radius (m)	0.14	0.14	0.14
e^+/year (1 muon)	7.3×10^{11}	4×10^{12}	3×10^{12}
Sensitivity/ year ($10^{-23} e{\cdot}$cm) (1 muon)/	6	2.3	1
Sensitivity/ year ($10^{-23} e{\cdot}$cm) (3 muons)	-	1.9	0.8

TABLE I: Comparison of two future scenarios on a re-accelerated cold muon beam with the concept proposed on muE1. A measurement with a sensitivity better than $1 \times 10^{-23}\,e{\cdot}$cm can only be realized with a larger magnetic field and higher momentum, which in turn also requires a higher electric field. Note that injection of more than one muon per measurement also requires a longer time out, which partially compensates for the increase in injected muons.

Figure 4 illustrates expected sensitivities to a muon EDM in the case that a new dedicated beam could deliver muons with a phase space optimally adapted for injection. We assume an overall positron detection efficiency of 80%. As a benchmark case we consider the experimental layout described in the letter of intent [106] with a muon orbit of $r = 0.14\,\mathrm{m}$. Note that the sensitivity at $B = 3\,\mathrm{T}$, the nominal field value for the experiment proposed for the muE1 beamline, will be improved by a factor two if the next muon can be injected on demand, whenever the previous positron decay was confirmed or a time out of five times the laboratory lifetime has been reached. In this single-muon-at-a-time scenario it is essentially impossible to improve the sensitivity to better than $1 \times 10^{-23}\,e{\cdot}$cm. Figure 4b shows that with a mean muon multiplicity above three and a magnetic field above $B = 5\,\mathrm{T}$ a sensitivity of better than $8 \times 10^{-24}\,e{\cdot}$cm, c.f. current limit $d_\mu < 1.5 \times 10^{-19}\,e{\cdot}$cm [77], in a year of measurement can be reached. This could be accomplished with a cold muon rate from muCool of $5 \times 10^5\,\mathrm{s}^{-1}$ and a pulse repetition rate of 100 kHz.

B. Prospects for a high-precision measurement of the muon $g - 2$

The same experimental setup used for a search of the muEDM can be used for a dedicated measurement of the anomalous magnetic moment of the muon. In this case no electric field will be applied and Eq. (3) will reduce to

$$\vec{\omega} = \frac{q}{m}\left(a\vec{B}\right) \tag{7}$$

in the absence of an electric dipole moment. For this purpose the central electrode will be removed, to reduce multiple scattering, while the outer electrode is simply grounded.

The relative sensitivity of the anomalous frequency measurement is given by

$$\sigma(a_\mu)_{\mathrm{rel}} = \frac{\sqrt{2} \quad m_\mu}{P\sqrt{N}\gamma\tau\alpha \ e a_\mu B}. \tag{8}$$

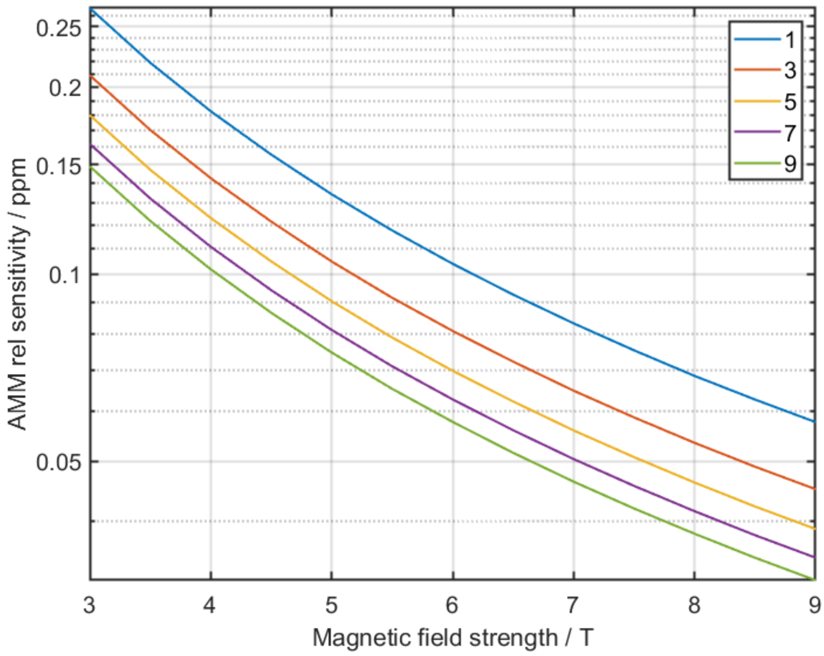


FIG. 5: Statistical sensitivity in ppb for a measurement of the muon $g-2$. For this calculation the radius was fixed to $r = 0.14\,\mathrm{m}$, and $P = qBr$ as usual.

For the sensitivity calculation, shown in Fig. 5, we assume the same values for P and α as in the muEDM scenario and a positron detection efficiency of 70%. It can be seen that for a field strength of $B = 6\,\mathrm{T}$ a statistical sensitivity of 0.1 ppm can be reached, matching the expected final precision of the experiment at Fermilab. An improvement can be seen when the muon multiplicity is increased with potential to achieve about 0.06 ppm, or to match the Fermilab sensitivity with a lower field strength of $B = 4\,\mathrm{T}$.

In terms of the systematic uncertainties, the experimental design benefits from the more compact magnet and hence a high field homogeneity is expected, as demonstrated in MRI technologies. Further investigation needs to be done in order to fully understand the exact levels of these uncertainties, but based on the expectations for both the Fermilab and J-PARC experiments a precision of 0.07 ppm should be achievable.

III. CONCLUSIONS

The search for a muon EDM and a high-precision measurement of the muon $g-2$ are great science opportunities to unveil new sources of CP violation and to cross check the ground-breaking results of the Fermilab and J-PARC measurements.

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