Study on Systematic Errors of Emittance Measurements

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In this talk the systematic errors of emittance measurements at the 250MeV Injector test stand for the SwissFEL are discussed. Errors in the determination of the beam size and inaccuracies in the knowledge of the transfer matrices, mainly induced by beam energy uncertainties, are analysed using Monte-Carlo-Techniques. The effect of deviations from a perfect matched optics in the diagnostic FODO lattice is an important contribution to the relevant systematic errors.

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- Introduction
- Overview of Optics „Options“
- Systematic Errors
  - Beam Optics Mismatch
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  - Energy Errors

Matrix Inversion

- Beam spot size:

$$\begin{pmatrix} \langle x^2_{(1)} \rangle \\ \langle x^2_{(2)} \rangle \\ \langle x^2_{(3)} \rangle \end{pmatrix} = \underbrace{\begin{pmatrix} R^{(1)}_{11}{}^2 & 2R^{(1)}_{11}R^{(1)}_{12} & R^{(1)}_{12}{}^2 \\ R^{(2)}_{11}{}^2 & 2R^{(2)}_{11}R^{(2)}_{12} & R^{(2)}_{12}{}^2 \\ R^{(3)}_{11}{}^2 & 2R^{(3)}_{11}R^{(3)}_{12} & R^{(3)}_{12}{}^2 \end{pmatrix}}_{\boldsymbol{A}} \begin{pmatrix} \langle x_0^2 \rangle \\ \langle x_0 x_0' \rangle \\ \langle x_0'^2 \rangle \end{pmatrix}$$

- Matrix inversion: (not optimal for measured data with errors)

$$\begin{pmatrix} \langle x_0^2 \rangle \\ \langle x_0 x_0' \rangle \\ \langle x_0'^2 \rangle \end{pmatrix} = \boldsymbol{A}^{-1} \begin{pmatrix} \langle x^2_{(1)} \rangle \\ \langle x^2_{(2)} \rangle \\ \langle x^2_{(3)} \rangle \end{pmatrix}$$

$$\epsilon_{x,\mathrm{rms}} = \sqrt{\langle x_0^2 \rangle \langle x_0'^2 \rangle - \langle x_0 x_0' \rangle^2} \qquad \begin{pmatrix} \beta_{x0} \\ \alpha_{x0} \\ \gamma_{x0} \end{pmatrix} = \begin{pmatrix} \langle x_0^2 \rangle / \epsilon_{x,\mathrm{rms}} \\ -\langle x_0 x_0' \rangle / \epsilon_{x,\mathrm{rms}} \\ \langle x_0'^2 \rangle / \epsilon_{x,\mathrm{rms}} \end{pmatrix}$$

- Least squares optimization to measured data with errors:

$$f_i(\langle x_0^2 \rangle, \langle x_0 x_0' \rangle, \langle x_0'^2 \rangle) = R^{(i)}_{11}{}^2 \langle x_0^2 \rangle + 2R^{(i)}_{11}R^{(i)}_{12} \langle x_0 x_0' \rangle + R^{(i)}_{22}{}^2 \langle x_0'^2 \rangle$$

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{\langle x^2_{(i)} \rangle - f_i(\langle x_0^2 \rangle, \langle x_0 x_0' \rangle, \langle x_0'^2 \rangle)}{\sigma_{\langle x^2_{(i)} \rangle}} \right]^2$$

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Matrix Inversion – “standard tool”

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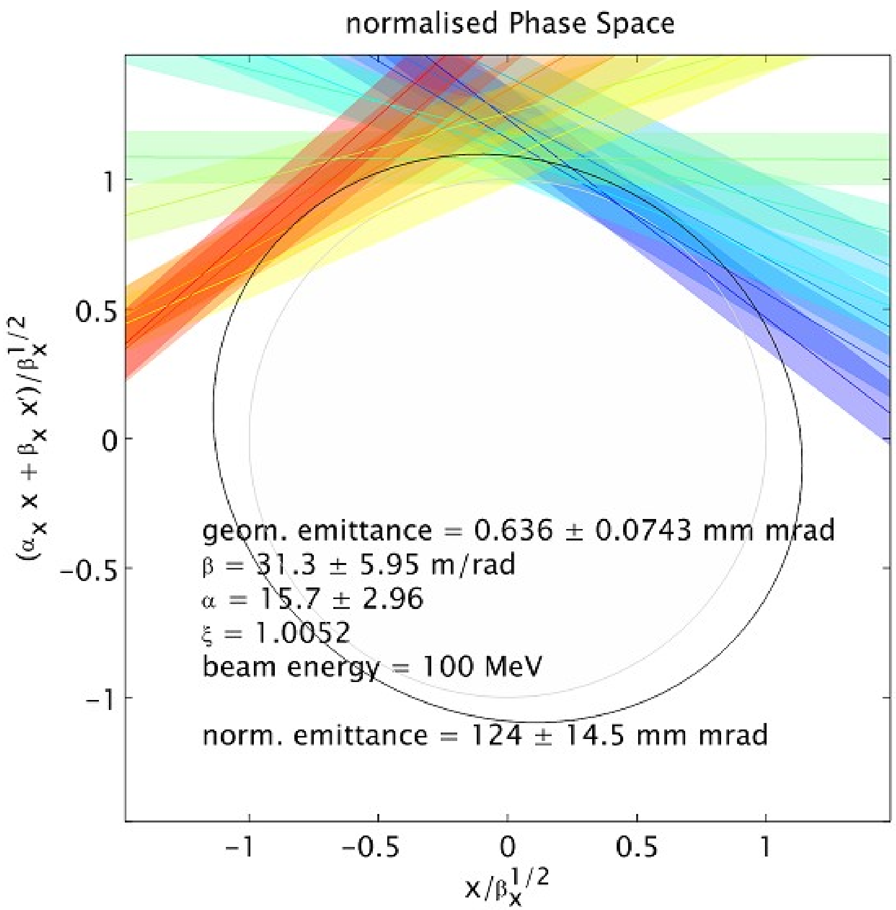

Matlab function: invemit.m
Multi Purpose function which accepts matrices and beam sizes and their errors.
Already used in the solenoid scan tool (Frederic Le Pimpec) for OBLA 4MeV

Diagnostic section

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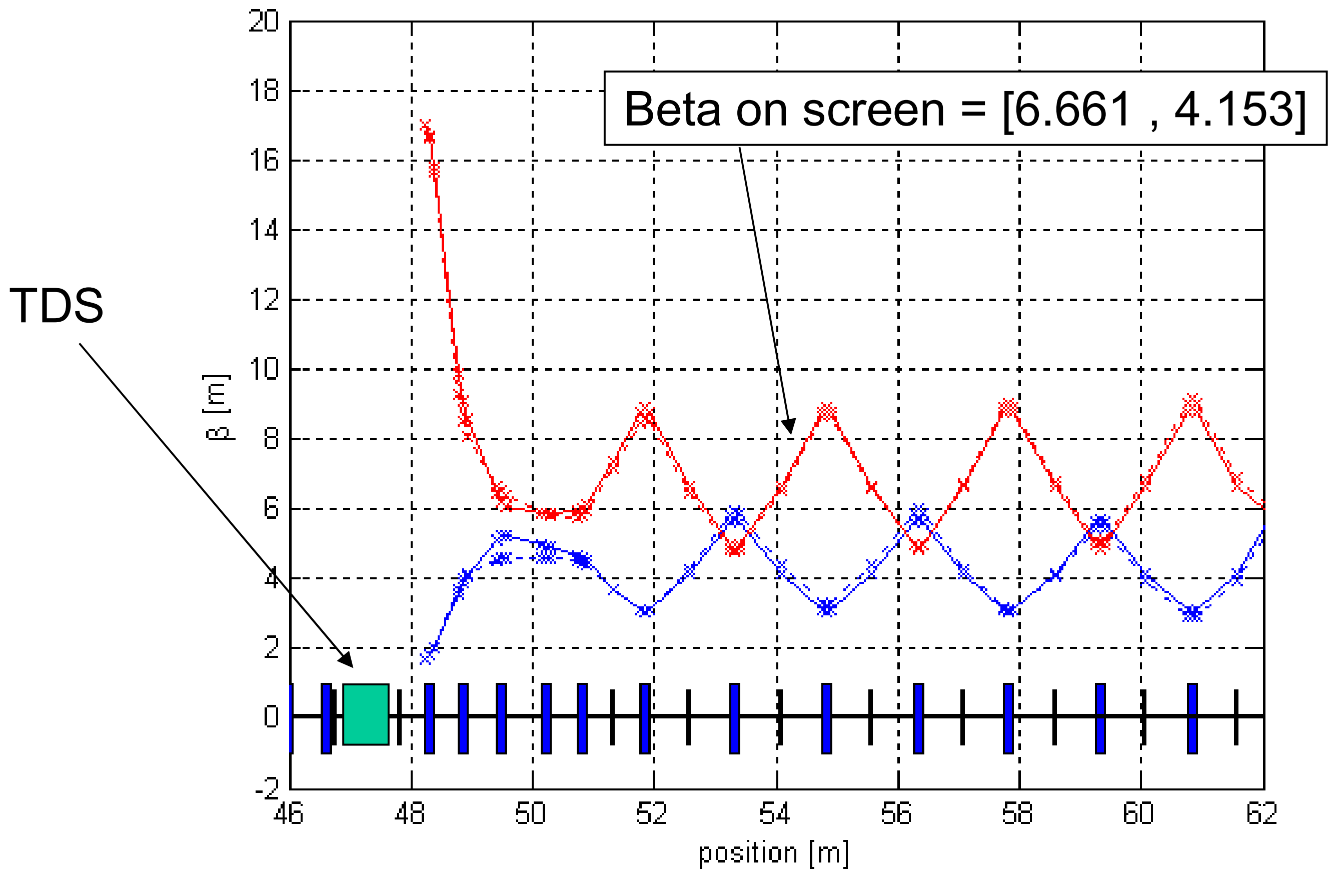

- Asymmetric setup optimised for slice emittance measurement
- Phase advance per cell = [42.0651   26.3405] deg

“Optics Options“

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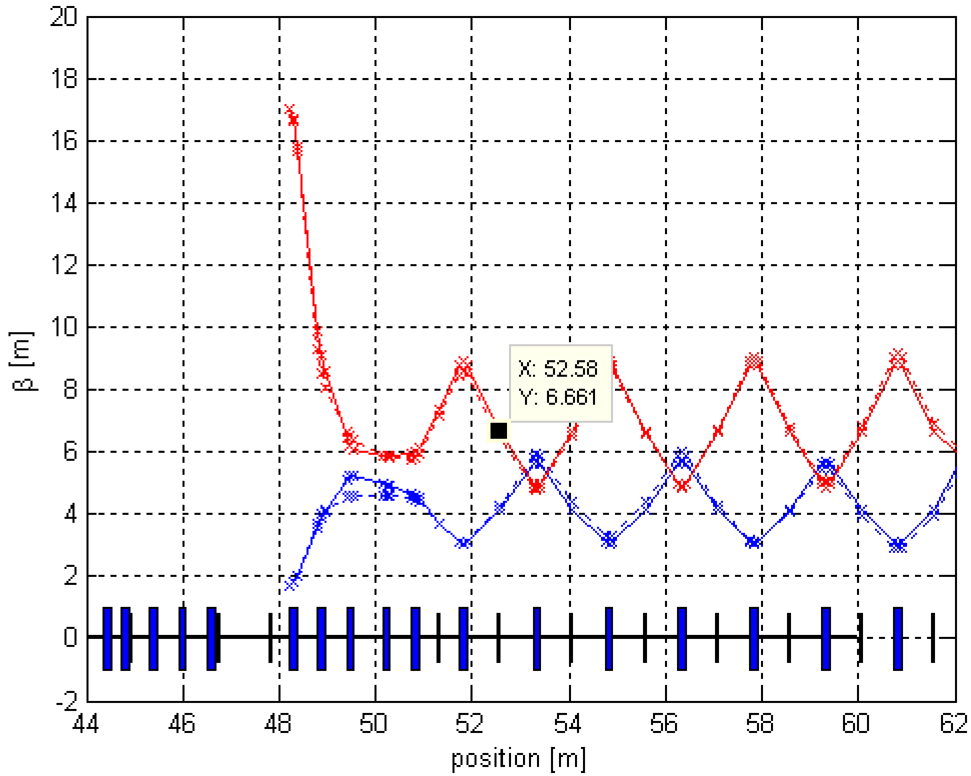
- Different modification options for the diagnostic section were briefly discussed in context with the relocation of the 250MeV Injector.

- (short) original design option 3 FODO cells
- (long) drifts are stretched by 30% => higher beta for same phase advance in the periodic solution as (short)
- (extra) as (short) but 2 extra quads are added to have 4 FODO cells with 9 screens for emittance measurements (7 in short)

short

Beta on screen = [6.661 , 4.153]

Phase advance = [42.0651   26.3405] deg



long

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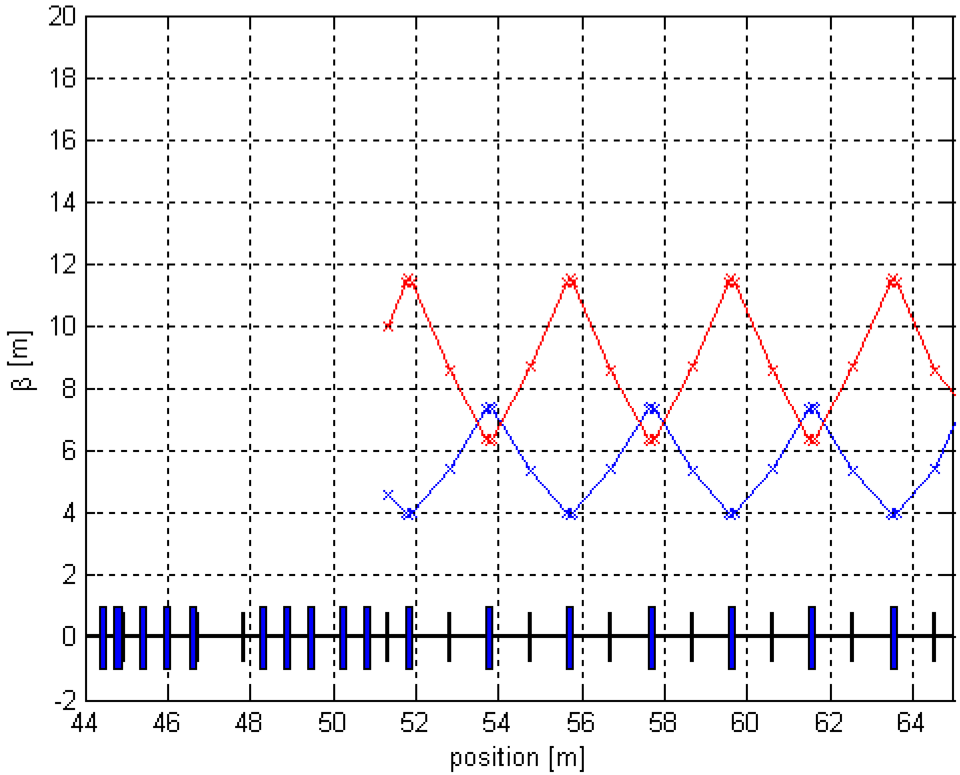
Matched Beta on screen = [8.554 , 5.439]

about 30% bigger beta => 14% bigger beam

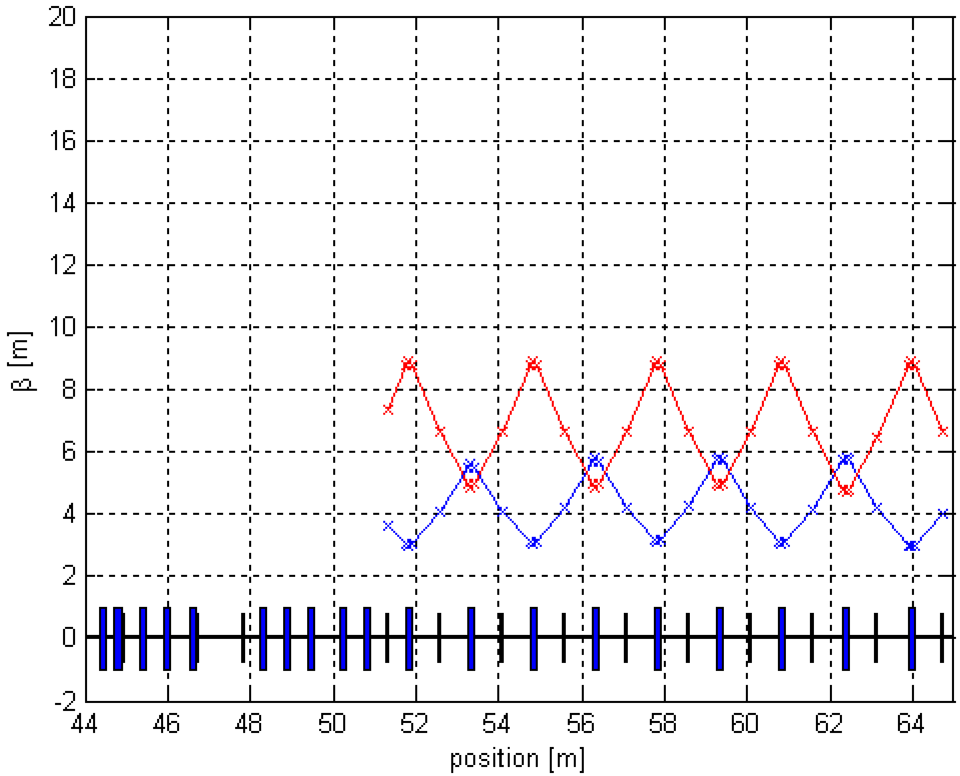
Phase advance = [42.0651   26.3405] deg



extra

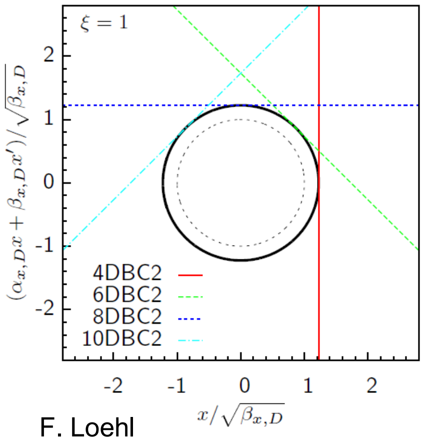
Beta on screen = [6.661 , 4.153]

Phase advance = [42.0651   26.3405] deg



Systematic errors

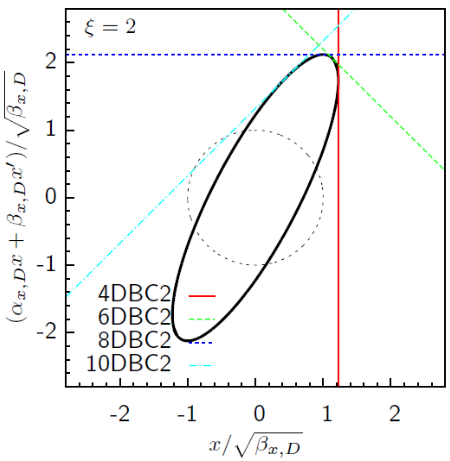
- Monte-Carlo-Study on systematic errors from beam size errors
- Effect of the errors on optics mismatched of the beam



F. Loehl

Mismatch parameter: $\xi = \frac{1}{2}(\beta\gamma_D - 2\alpha\alpha_D + \gamma\beta_D)$

Beam mismatch is induced by varying alpha according to: $\alpha_x = \alpha_{x,D} - \sqrt{2\xi - 2}$
while adjusting gamma to keep the emittance constant.

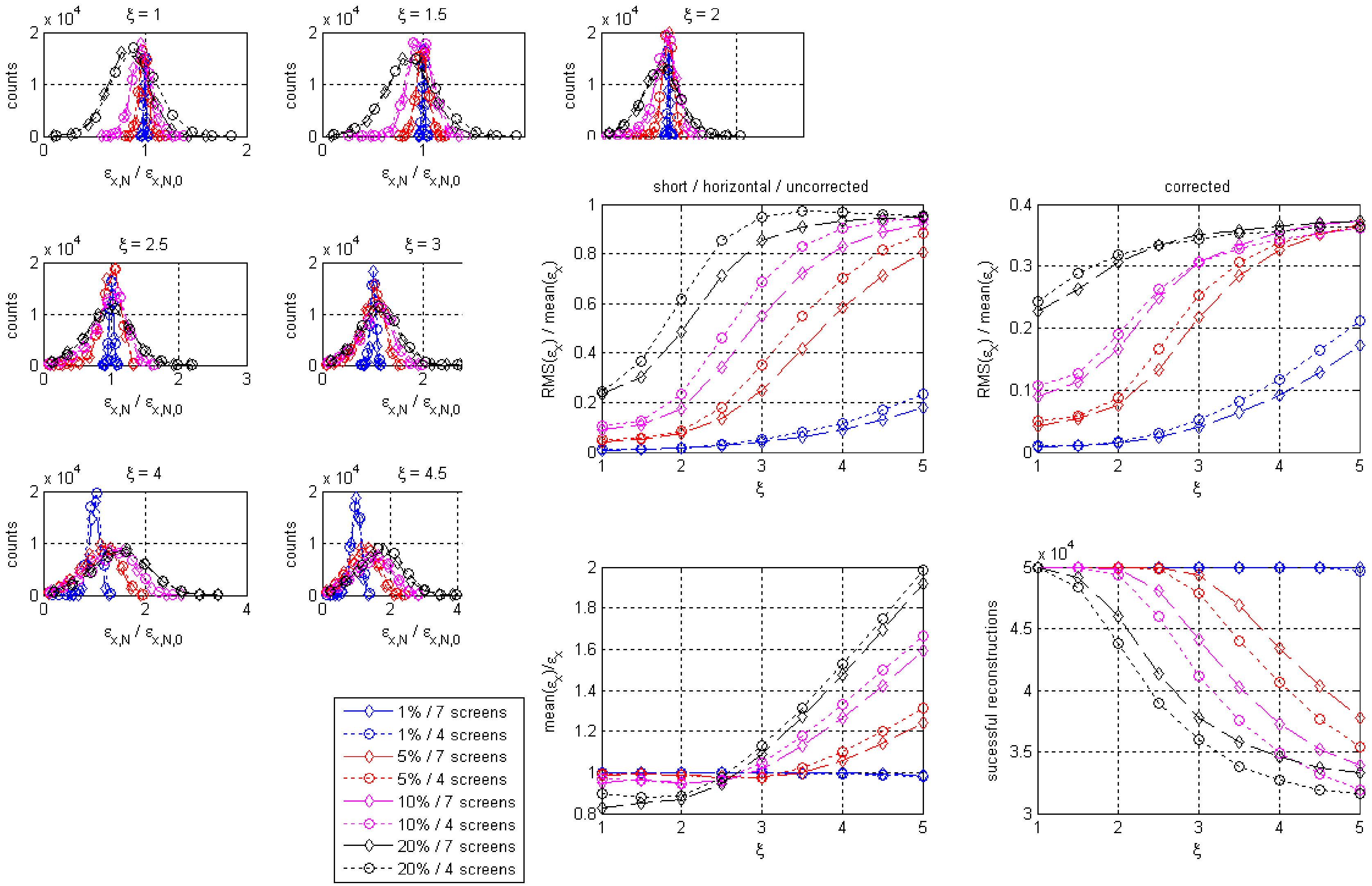


50.000 random beam sizes with different sigmas are generated and analysed with invemit.m.

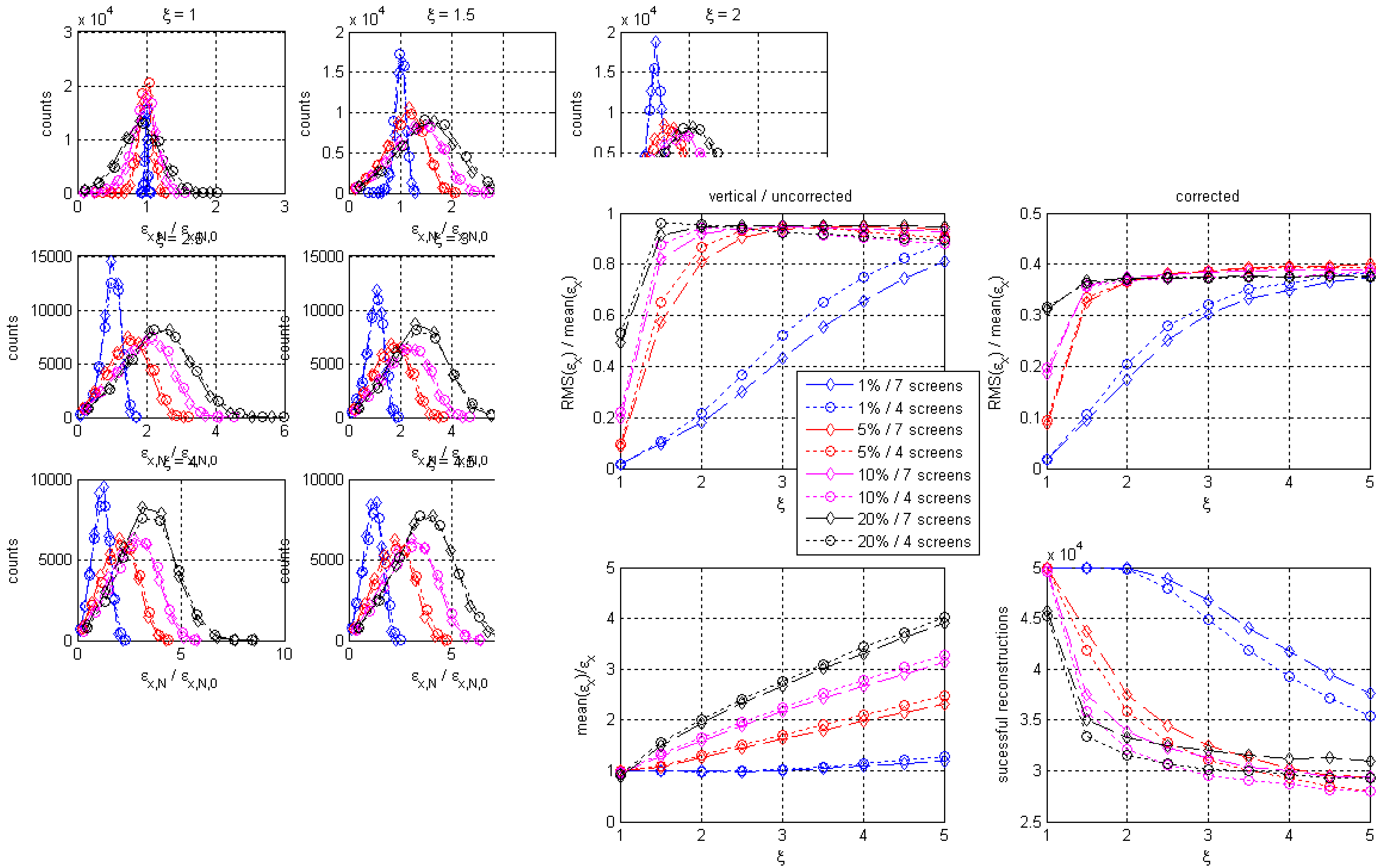
Beam was generated with 0.4 mm mrad emittance.

One screen per cell (4 screens) and one screen per half cell (7 screens) are compared.

short / horizontal



short / vertical



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long / horizontal

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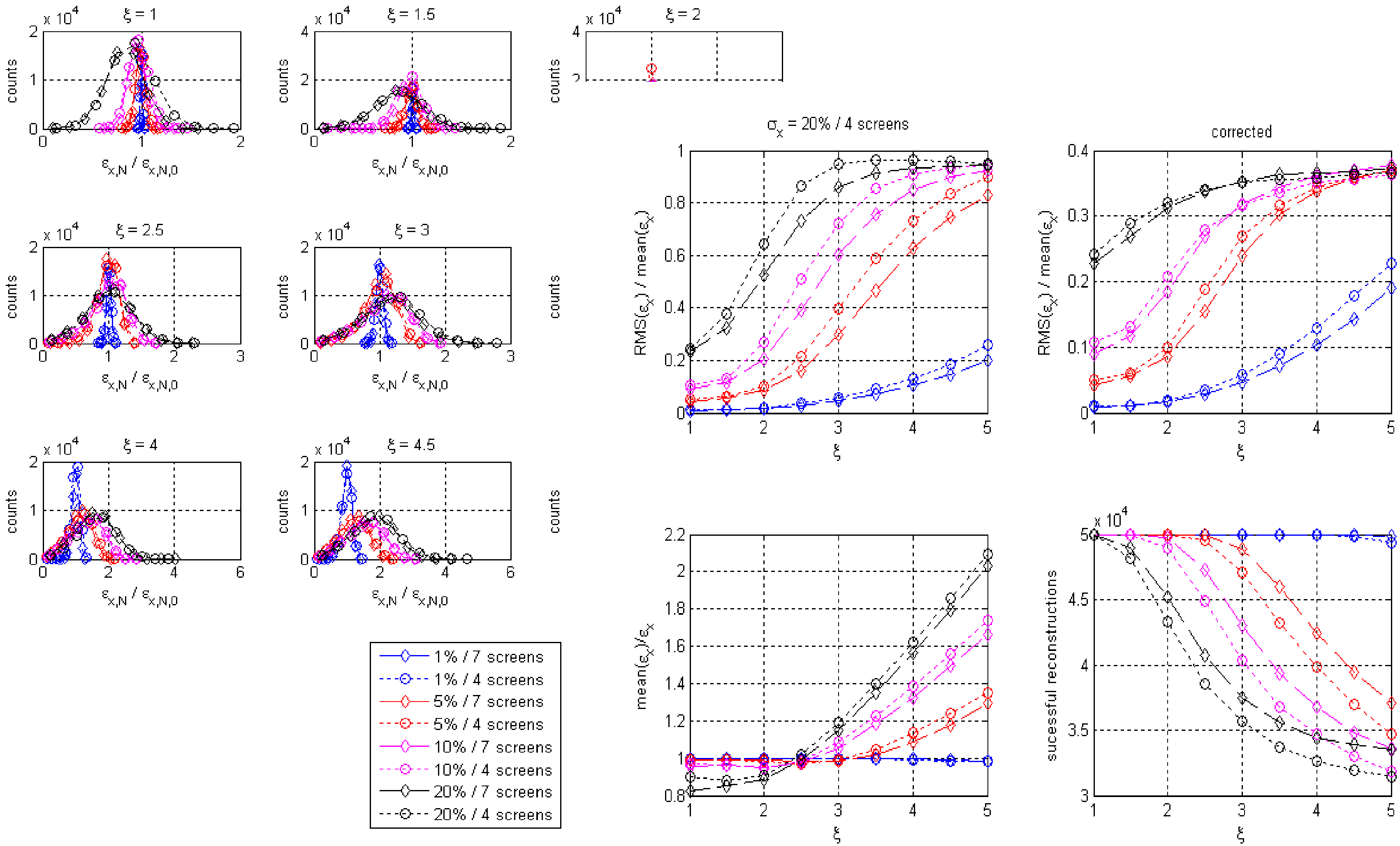
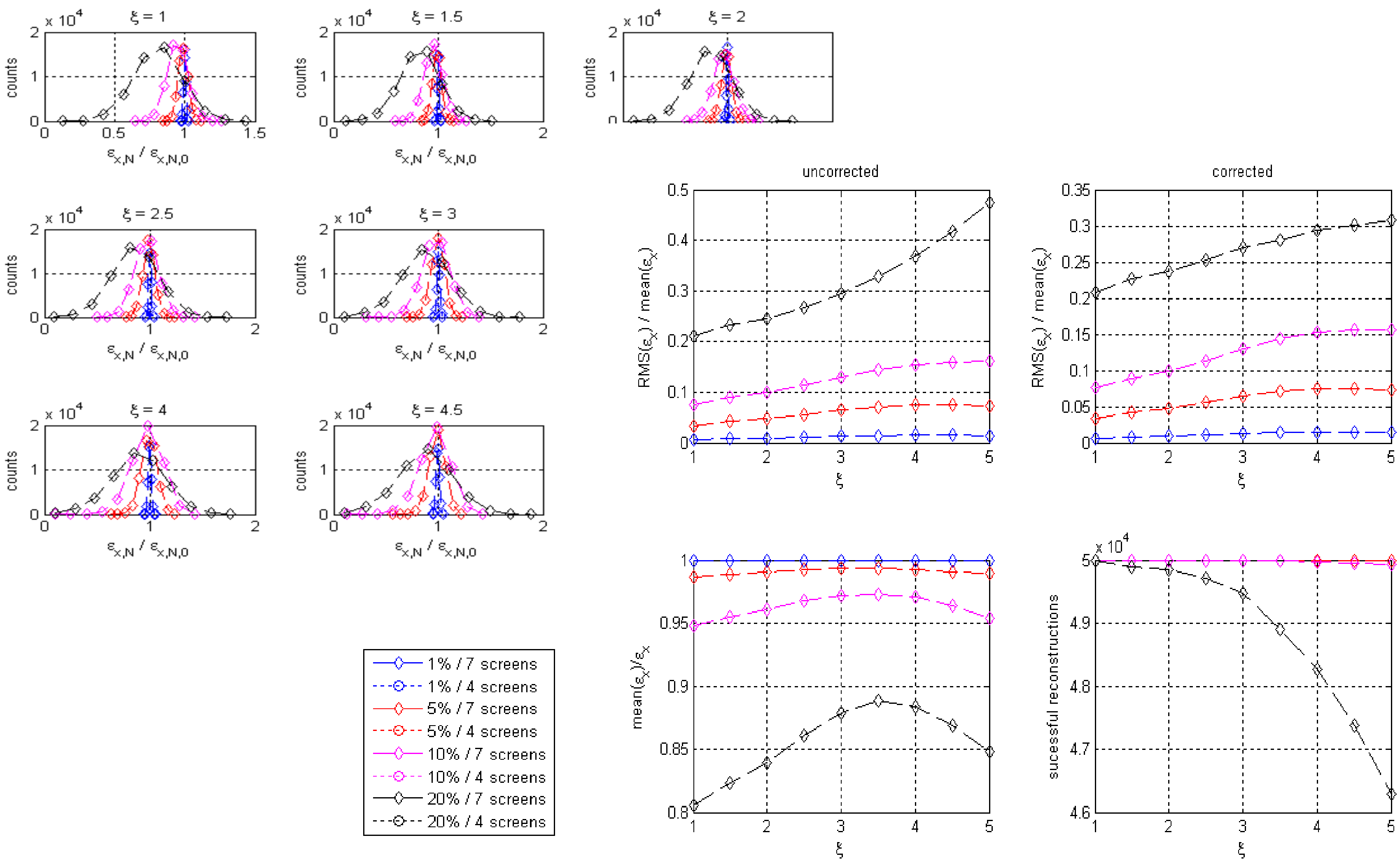


extra / horizontal



extra / vertical

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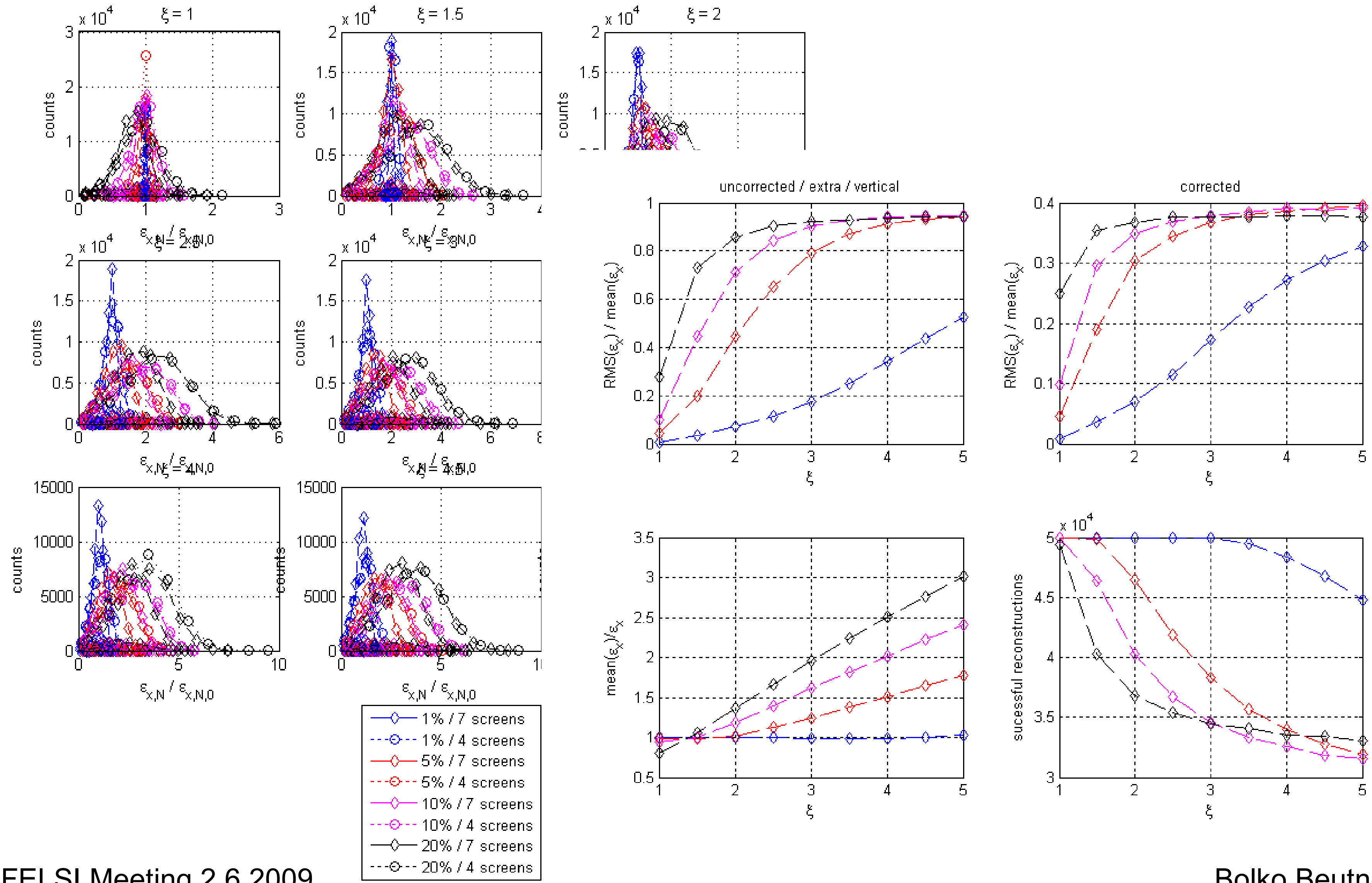

Mismatch Summary

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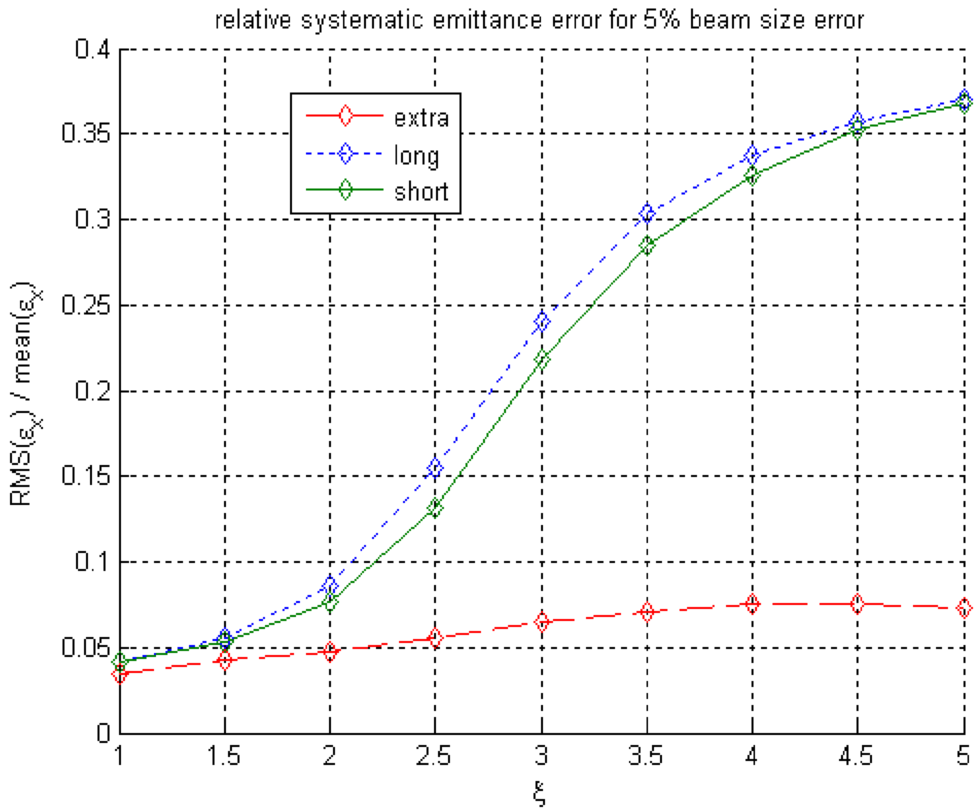
- Beam Mismatch affects the emittance reconstruction
- Additional FODO cells in the diagnostic section reduce the uncertainties of the fit
- Beams with high mismatch will have higher twiss errors as well
  => wrong beam matching
  => iterative procedure



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Optics Scan (Mismatch Map)

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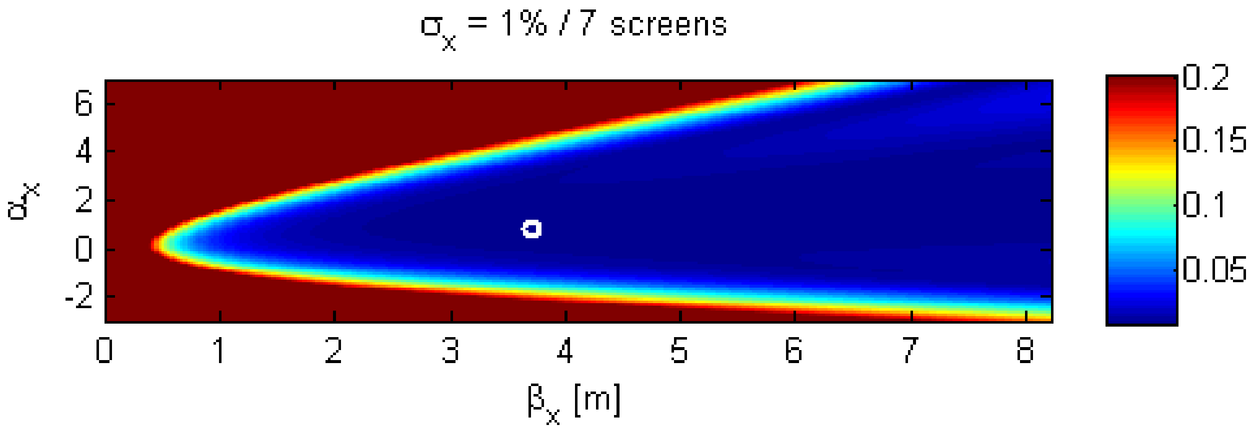
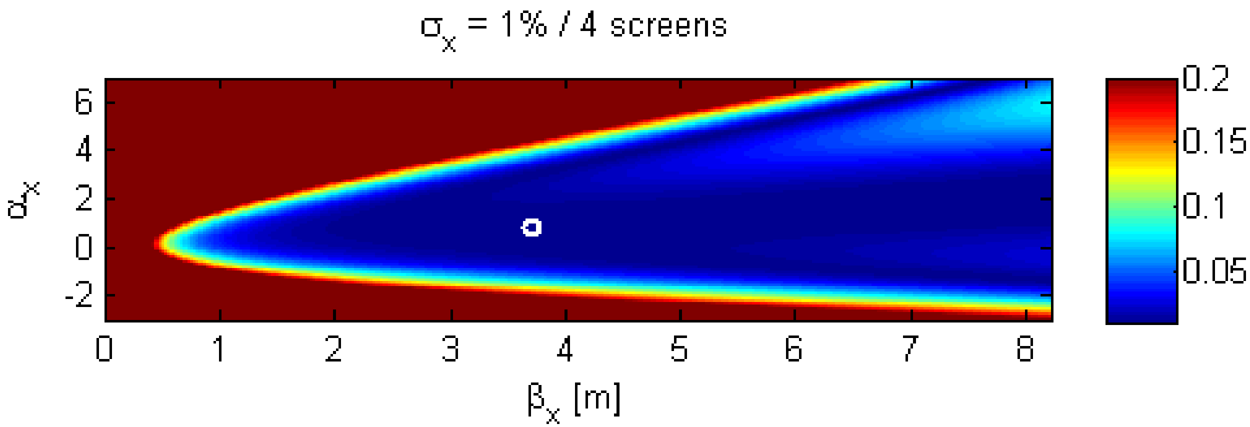
- Scan of an induced mismatch parameter gives a good idea but a more general approach is the scan of a wide range of optics parameters
- In this scan the error from the error propagation of the matrix inversion is plotted for different initial optics in the FODO channel
- Errors are limited to 20% / 50% in order to have the smaller errors still visible

1% Beam Size Error





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1% Beam Size Error

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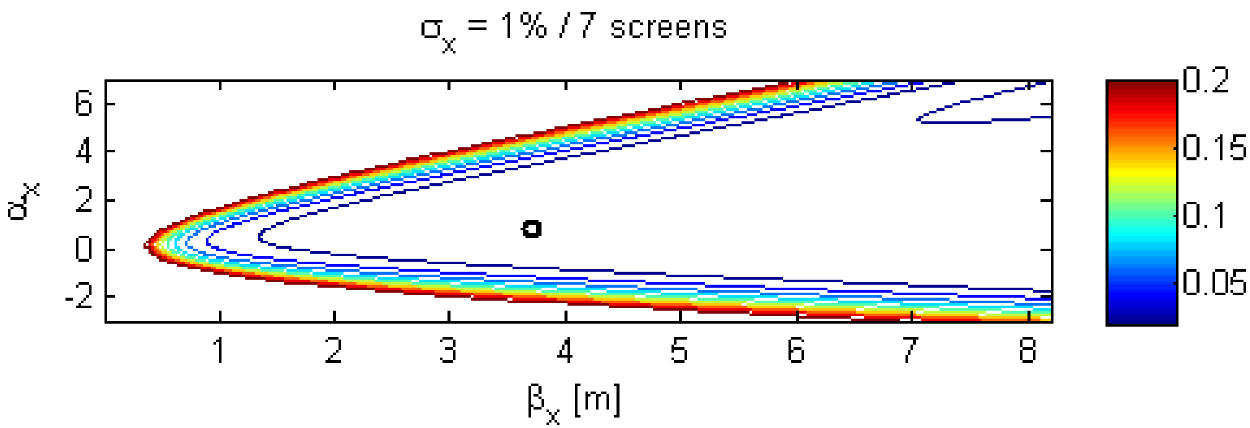

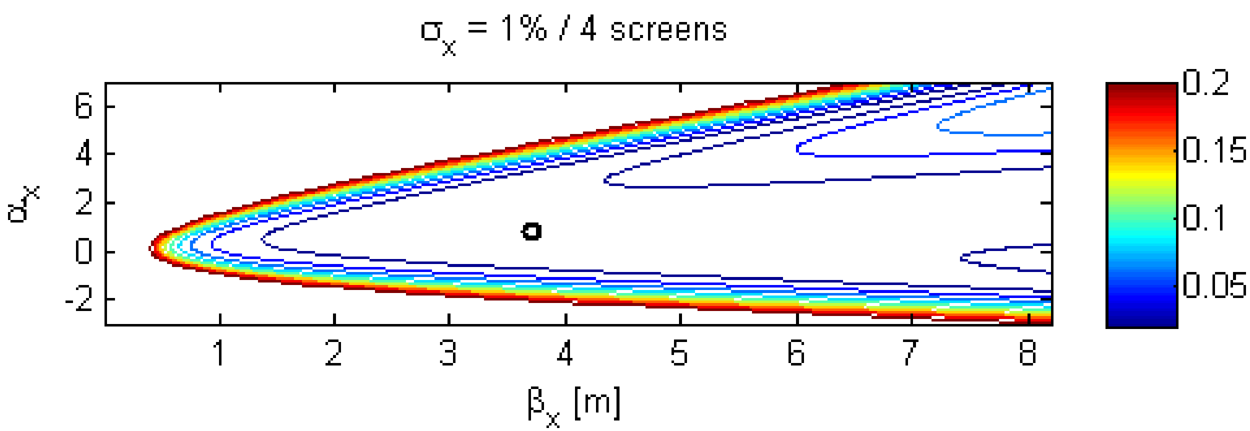

5% Beam Size Error

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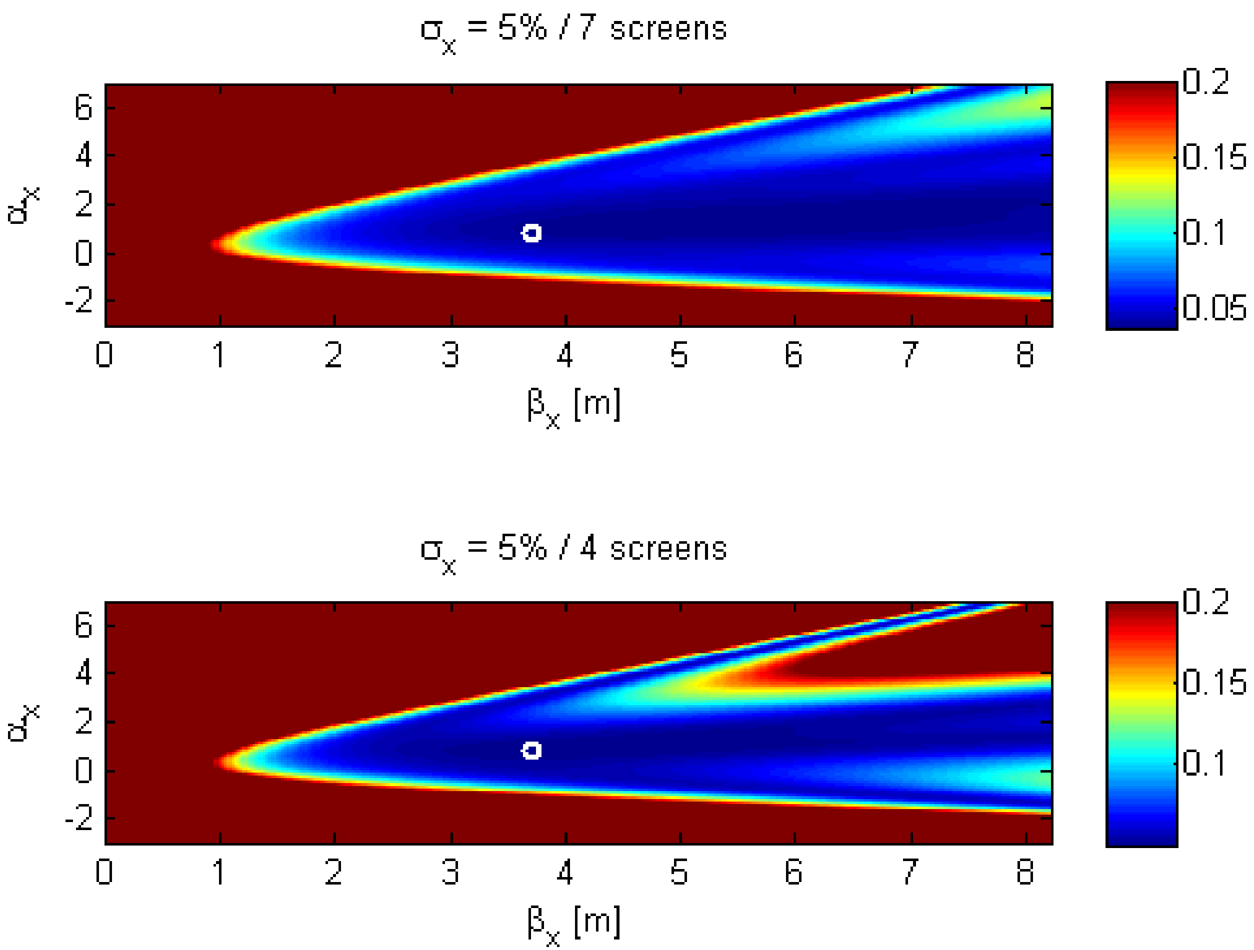

5% Beam Size Error

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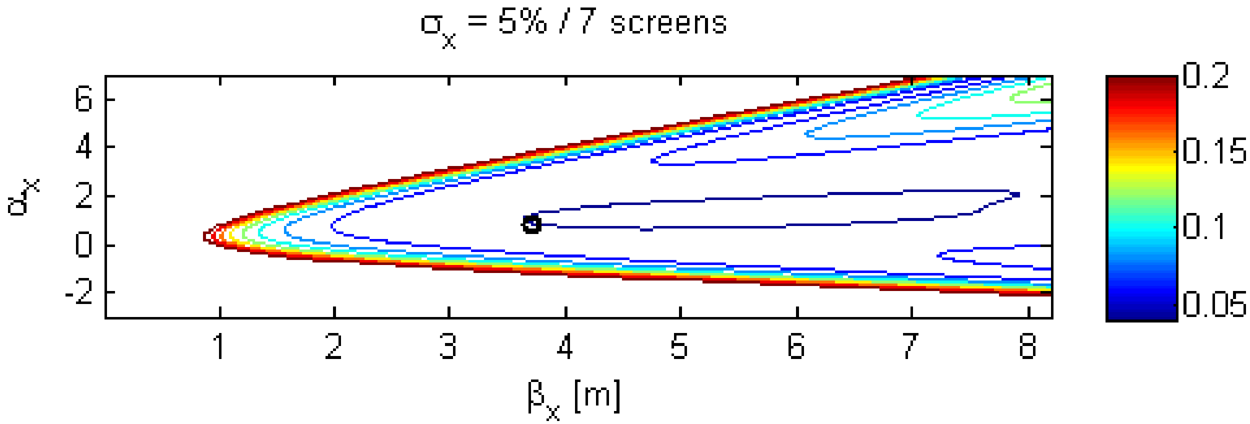
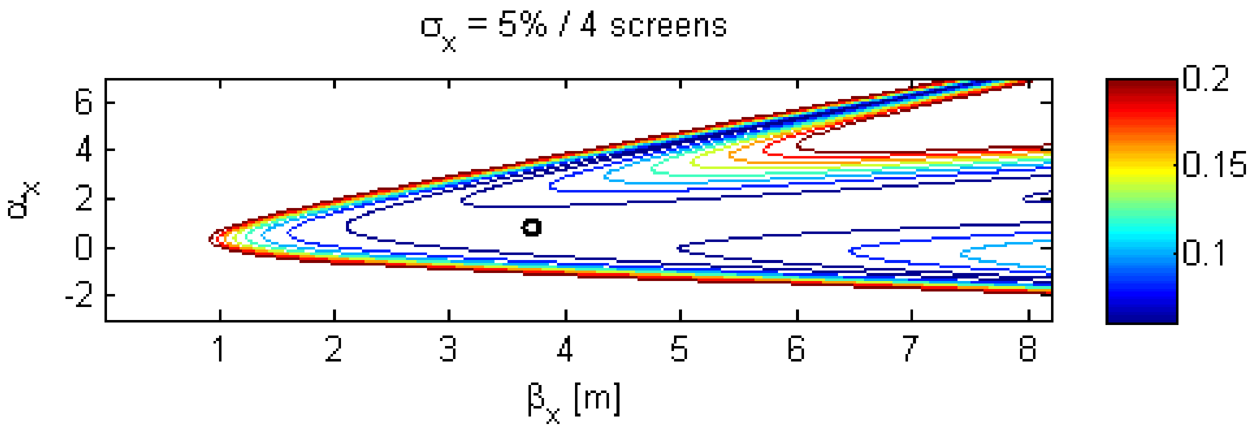
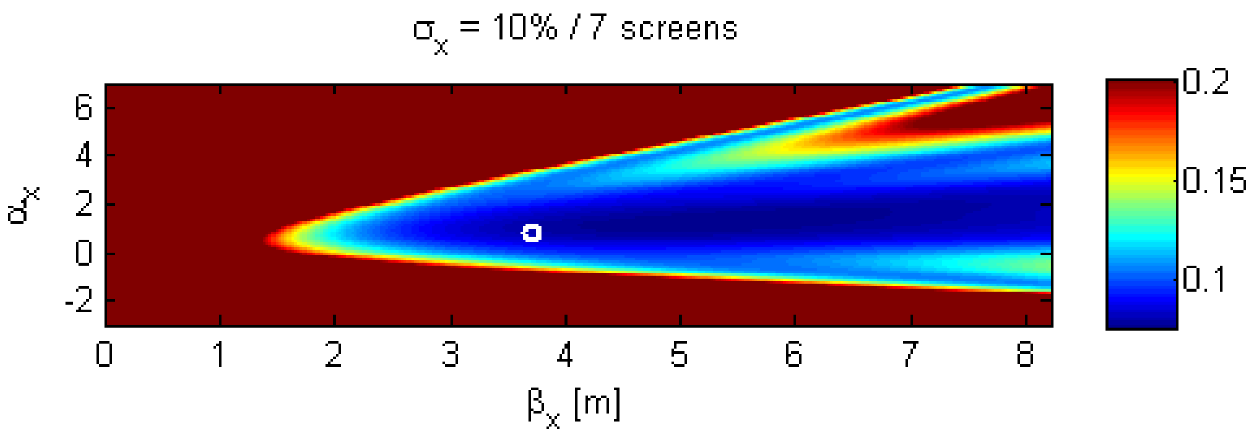
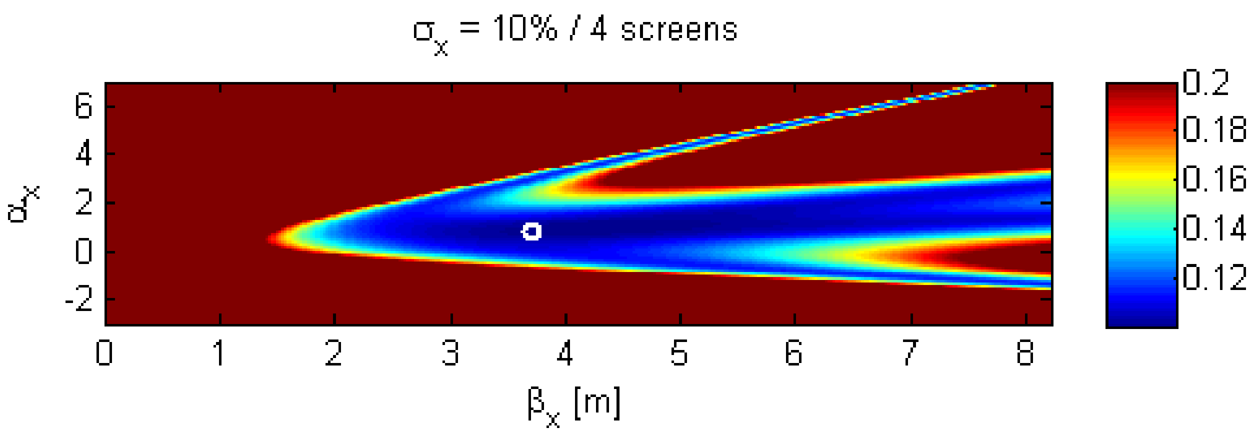




10% Beam Size Error





10% Beam Size Error

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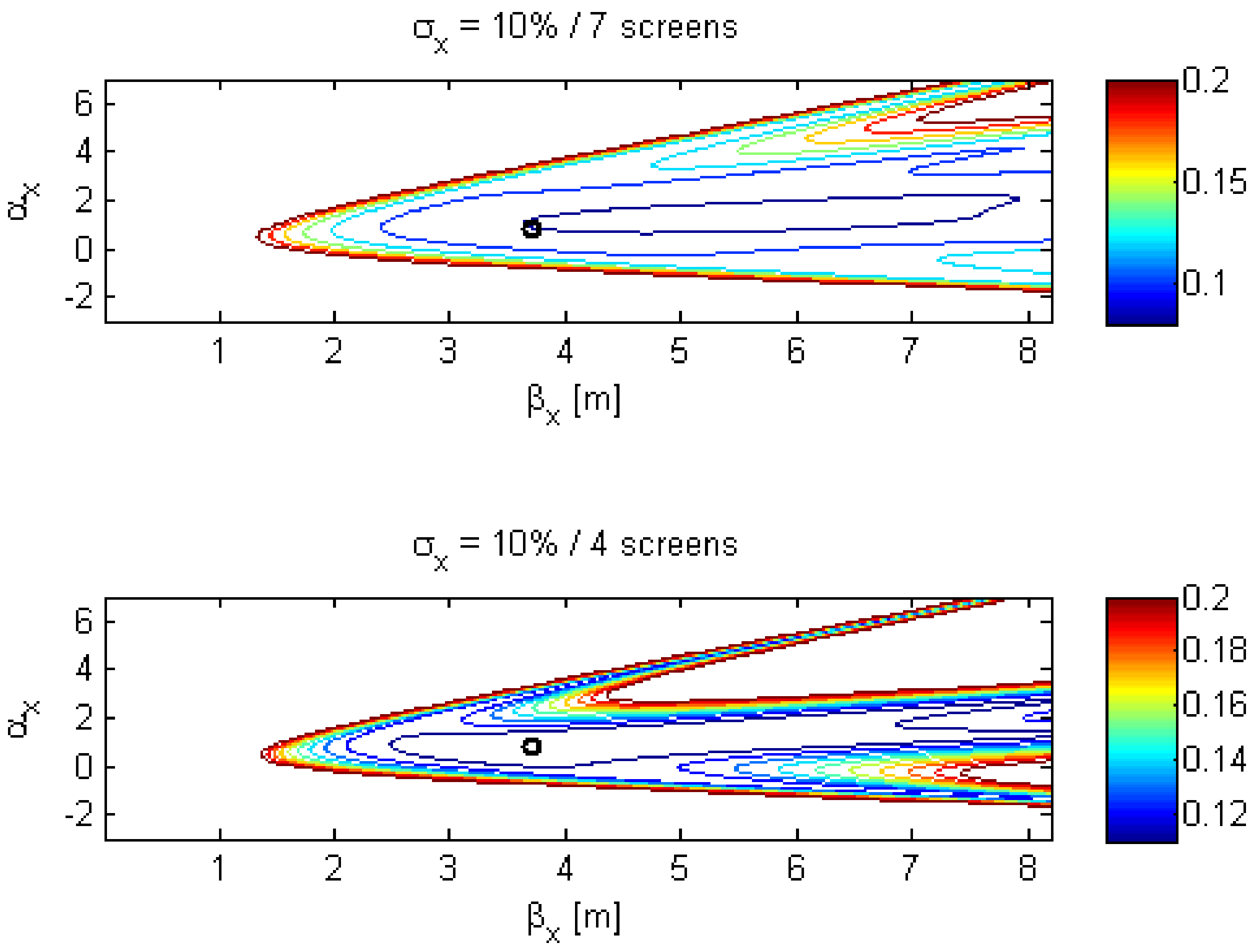

20% Beam Size Error

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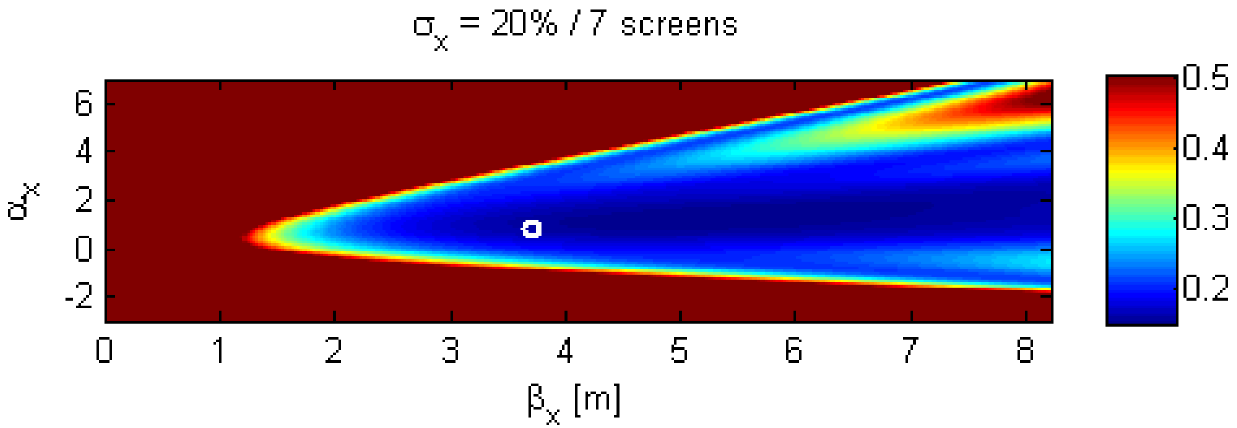

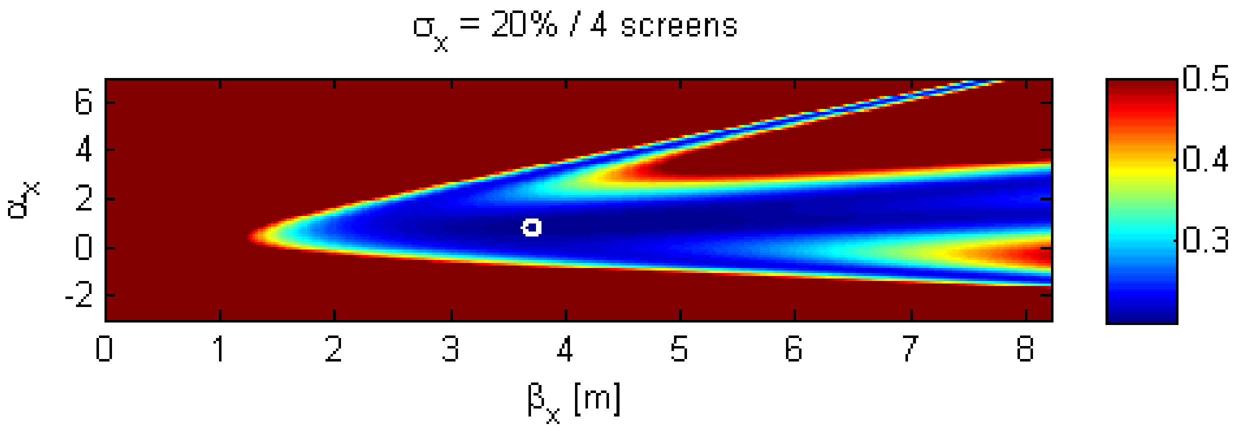

20% Beam Size Error

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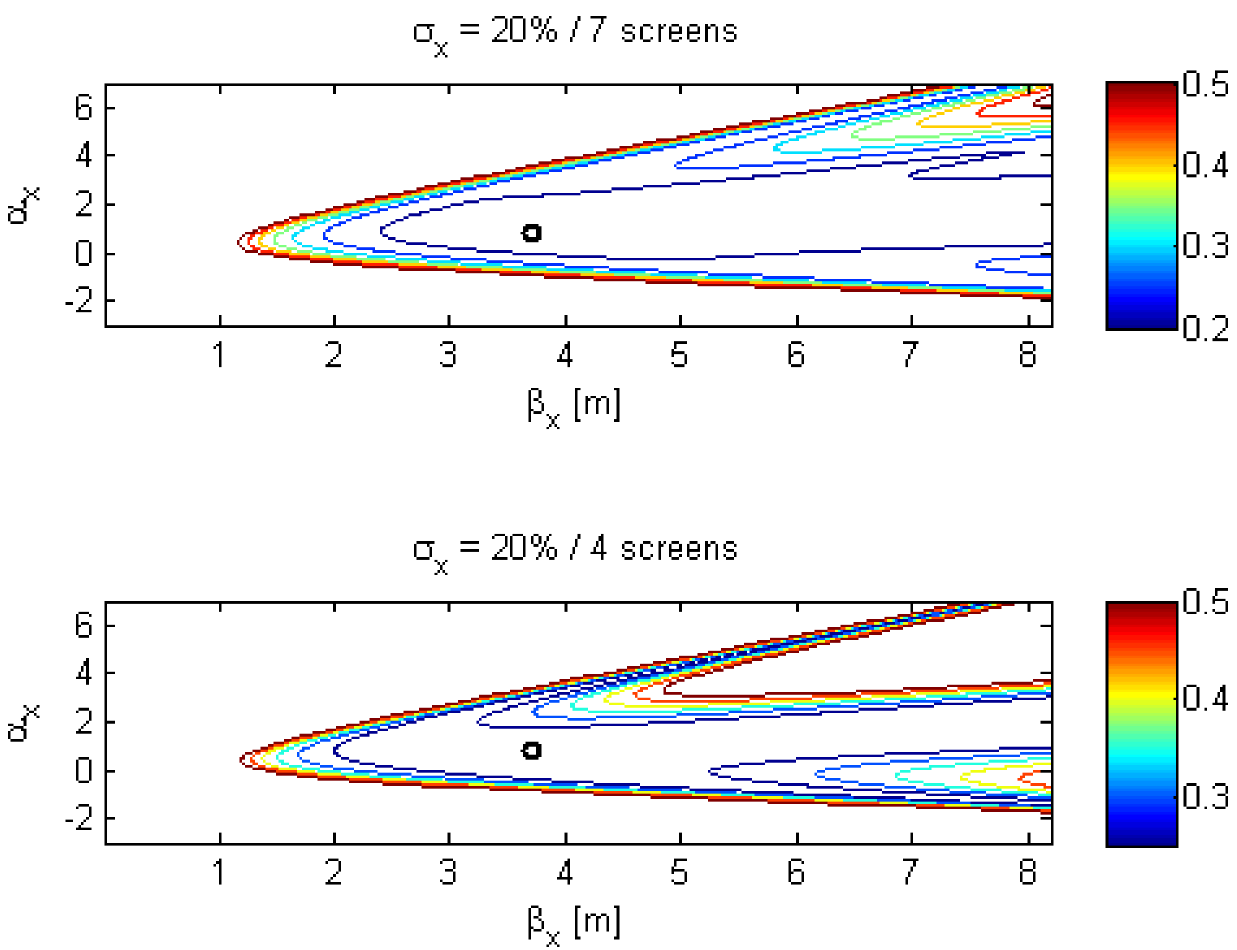

Error Map Summary

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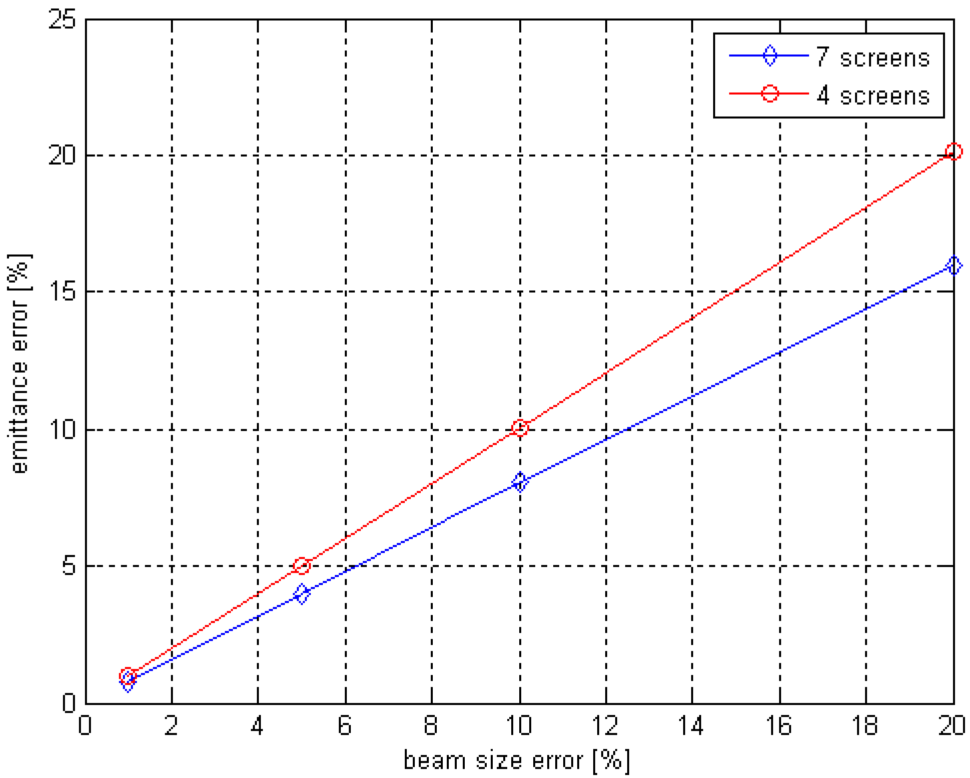

- Emittance error at the working point (matched solution)

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Energy Errors

- Uncertainty in the energy results in the uncertainty of the transport matrix
- In a FODO section the energy errors cancel out for the determination of normalised emittance – provided the beam is matched into the periodic solution
- Monte-Carlo on energy errors for a wide range of beam optics

$$E = E_0(1+\Delta)$$

$$k = \frac{k_0}{1+\Delta}$$

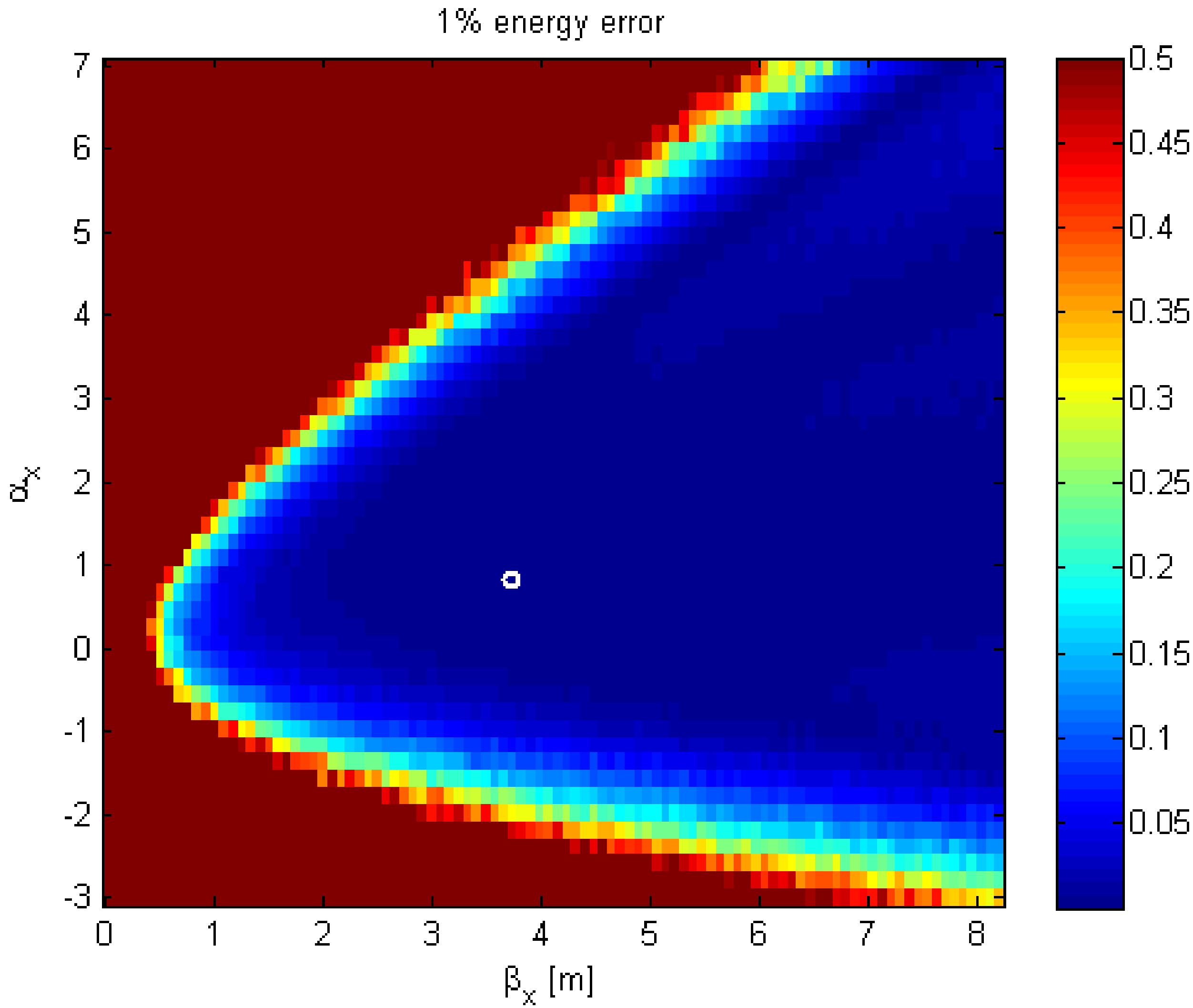
$$\beta = \beta_0(1+\Delta)$$

$$\epsilon_0 = \frac{\langle x^2 \rangle}{\beta_0}$$

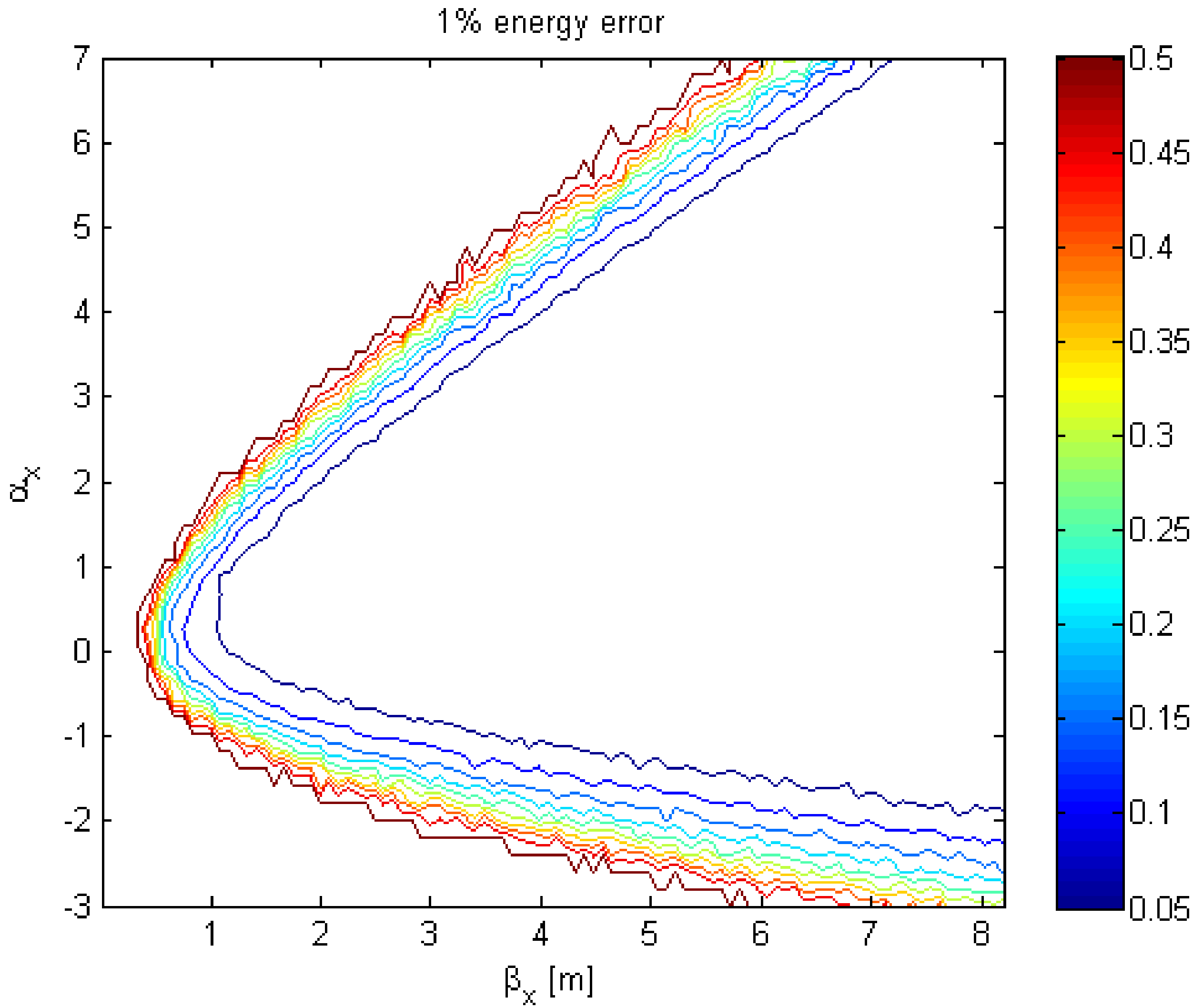
$$\epsilon = \frac{\langle x^2 \rangle}{\beta} = \frac{\langle x^2 \rangle}{\beta_0(1+\Delta)} = \frac{\epsilon_0}{1+\Delta}$$

$$\gamma_{\rm rel}\epsilon = \gamma_{\rm rel,0}(1+\Delta)\frac{\epsilon_0}{1+\Delta} = \gamma_{\rm rel,0}\epsilon_0$$

1% Energy Error



1% Energy Error



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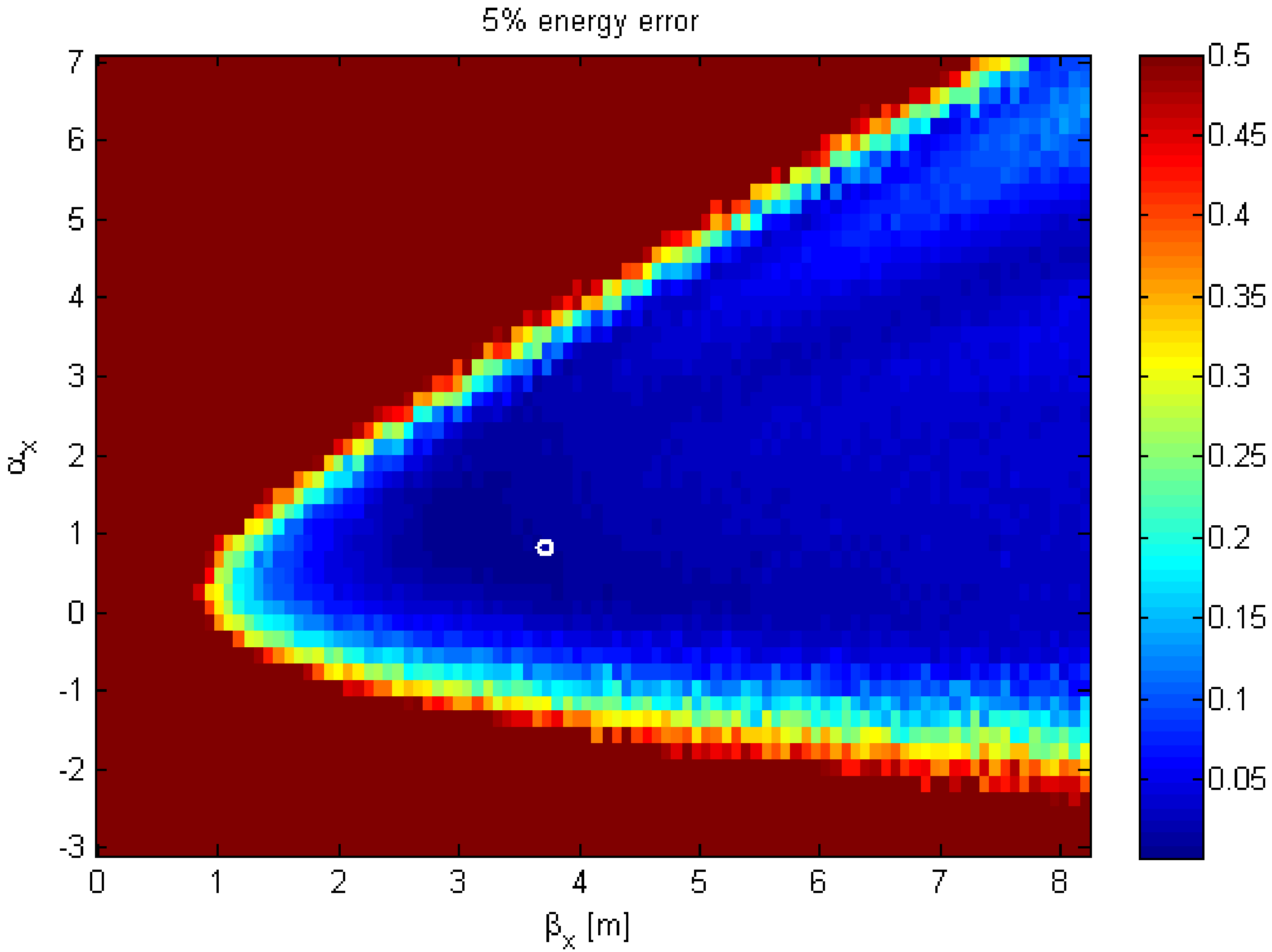
5% Energy Error



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5% Energy Error

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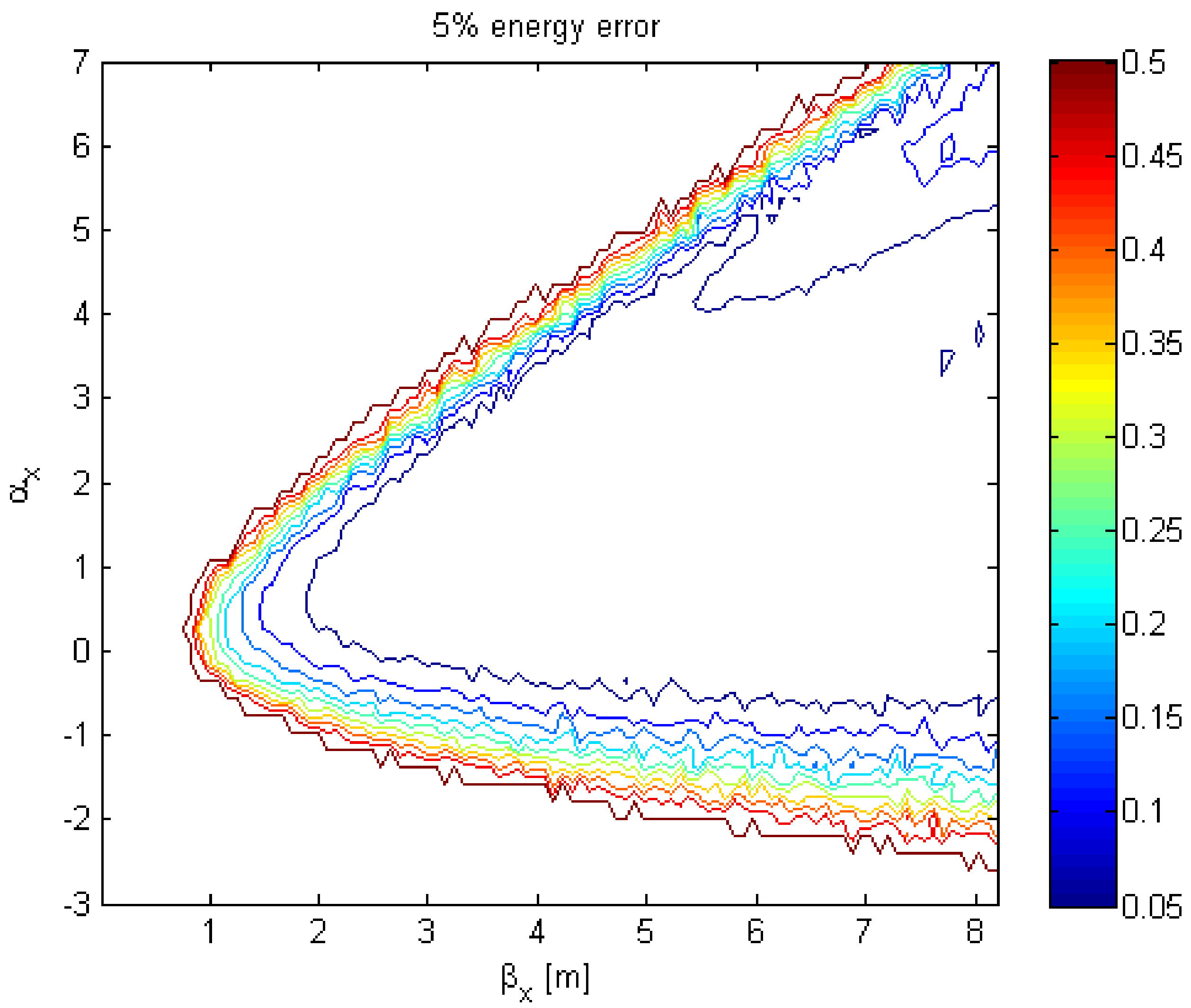

10% Energy Error

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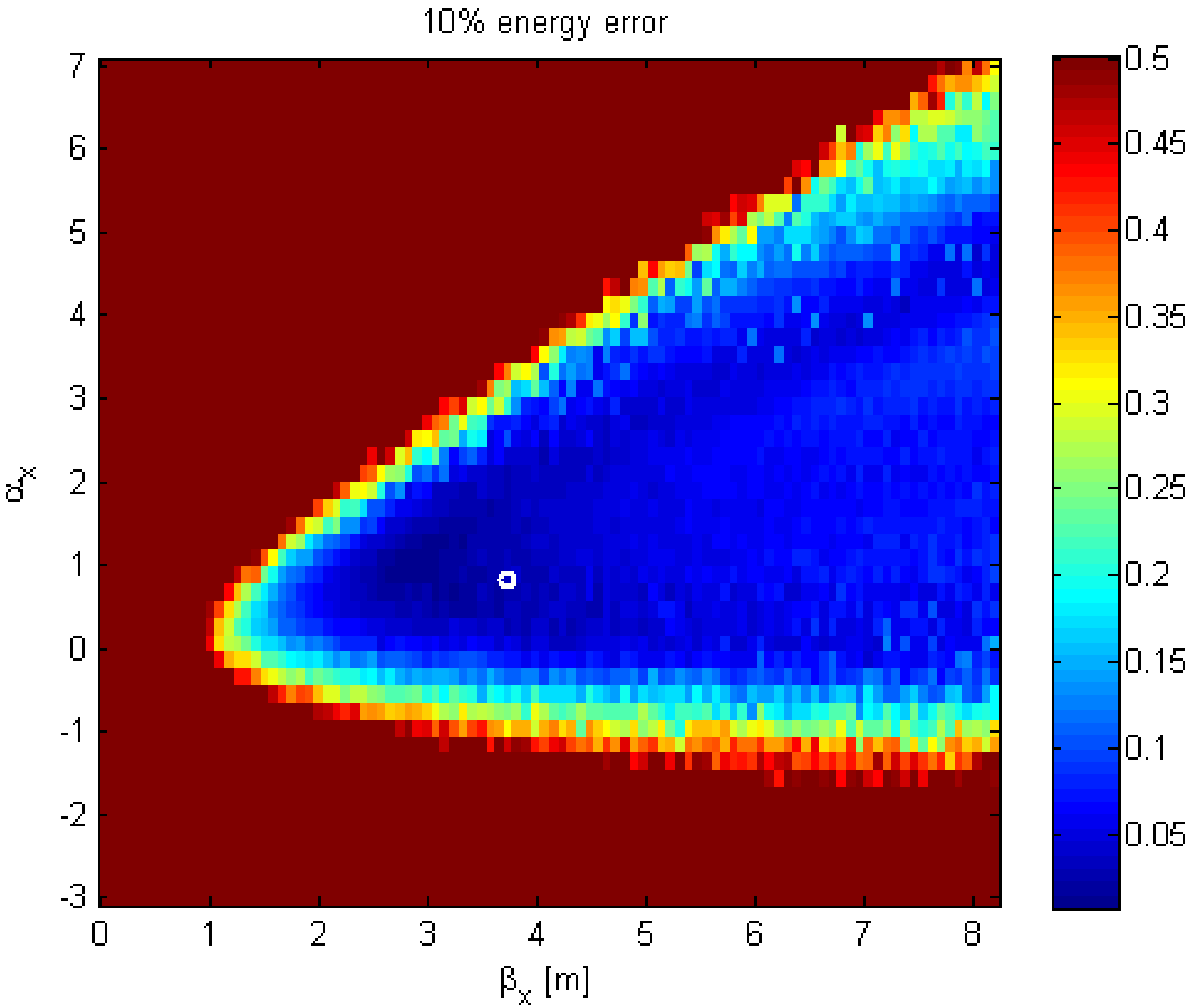
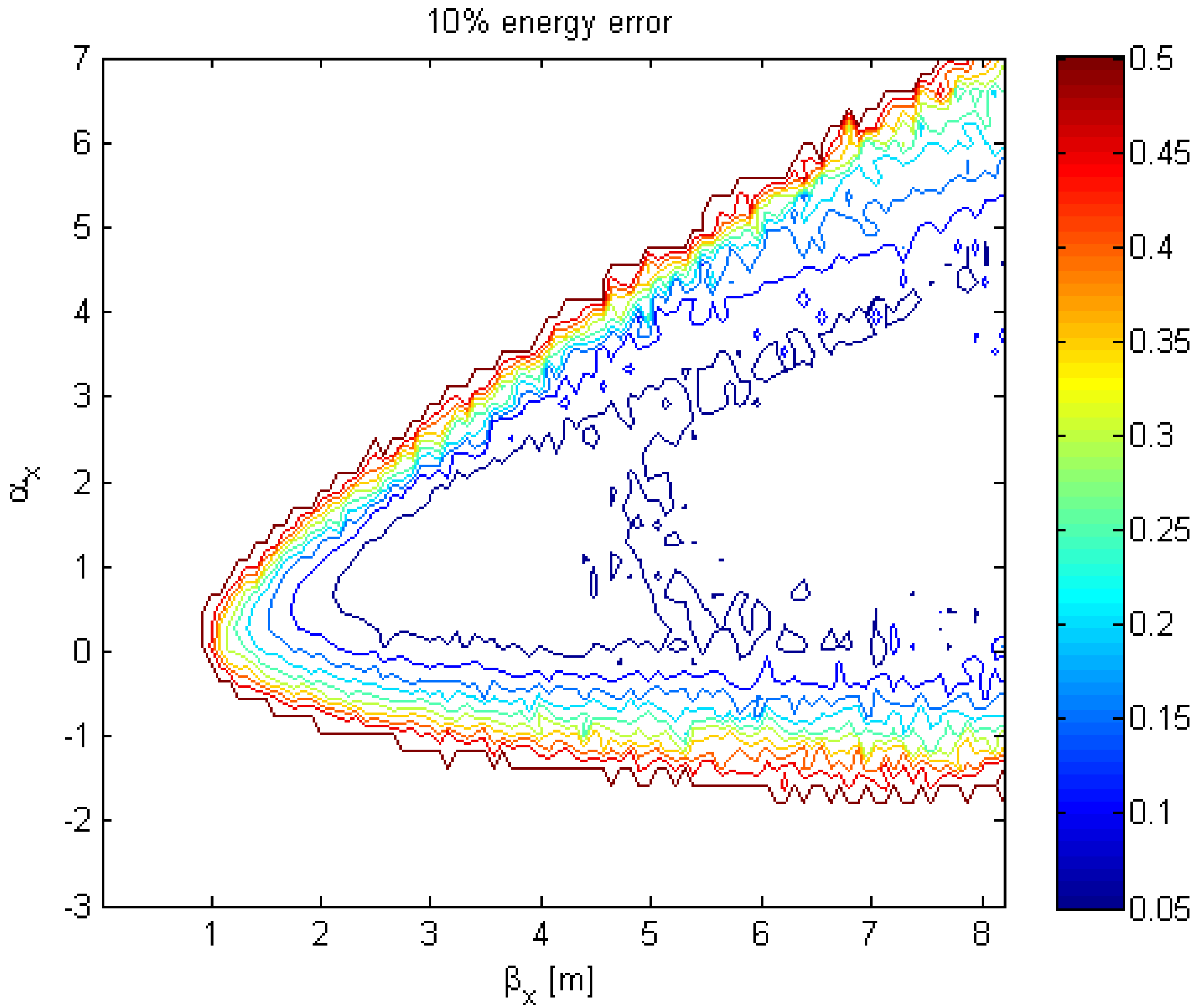


10% Energy Error



Energy Error Summary

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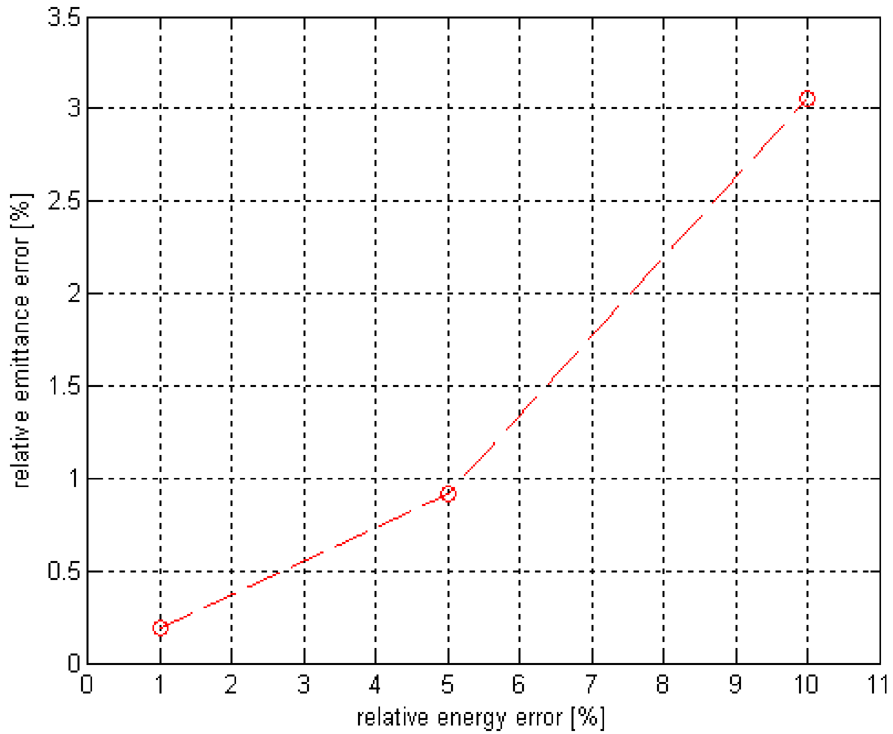

- Emittance error at the working point (matched solution)
- <1% emittance resolution if energy is known with an accuracy of 5% or better

Thanks to R. Ischebeck, F. Löhl, and Y. Kim.

Thank You for Your Attention!