Fermilab U.S. DEPARTMENT OF ENERGY Office of Science

Transverse Beam Emittance Measurement by Undulator Radiation Power Noise

Ihar Lobach (UChicago)

LEAPS WG2 Workshop

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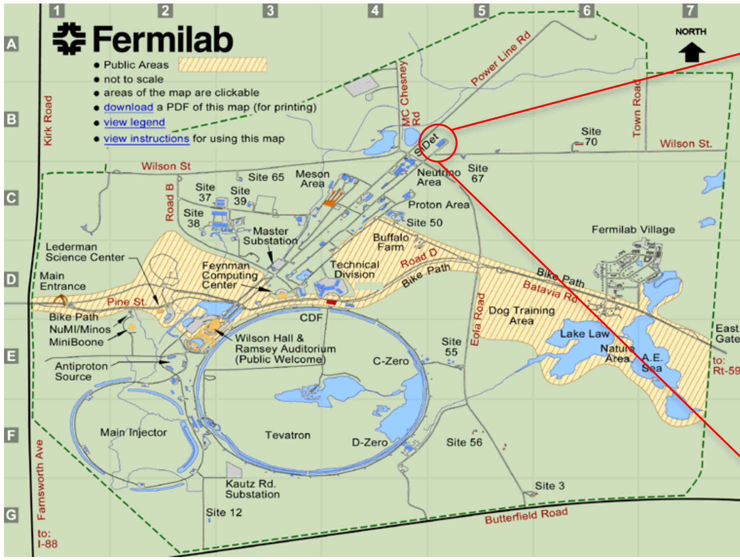
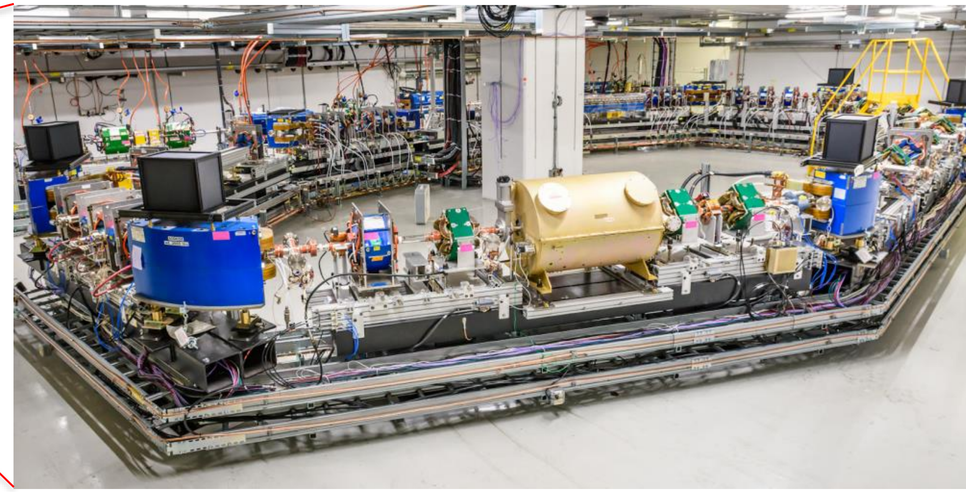
*dissertation advisors

Fermilab Integrable Optics Test Accelerator (IOTA)

- First beam Aug 21, 2018





- Particles: electrons/protons
- Main experiments:
  - Nonlinear beam optics
  - Optical stochastic cooling

Circumference: 40 m (133 ns)
Electron energy: 100 MeV

In our experiments:
Single electron bunch, up to 0.5 nC
About 30 cm long (rms)

Layout of the undulator section in IOTA

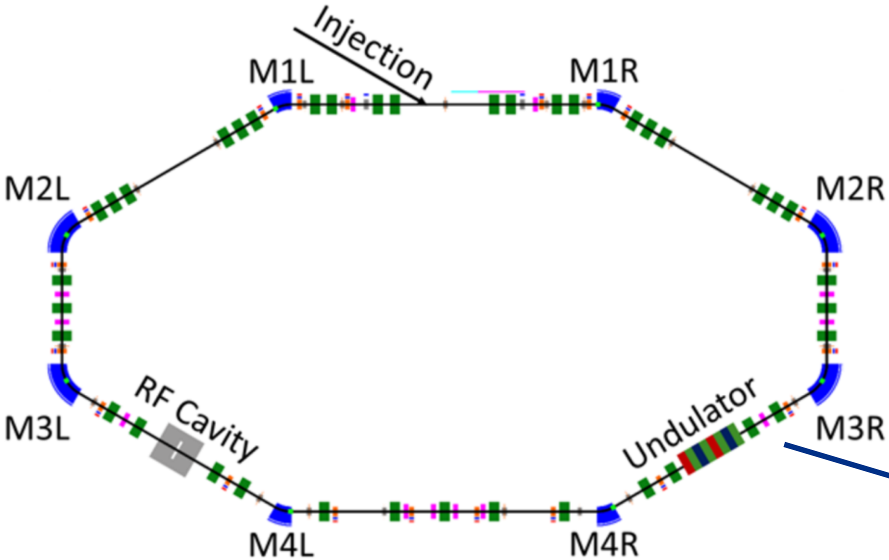
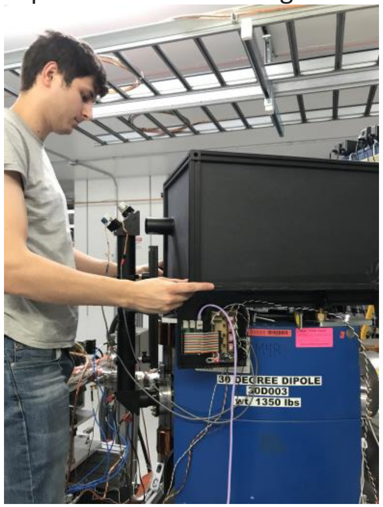
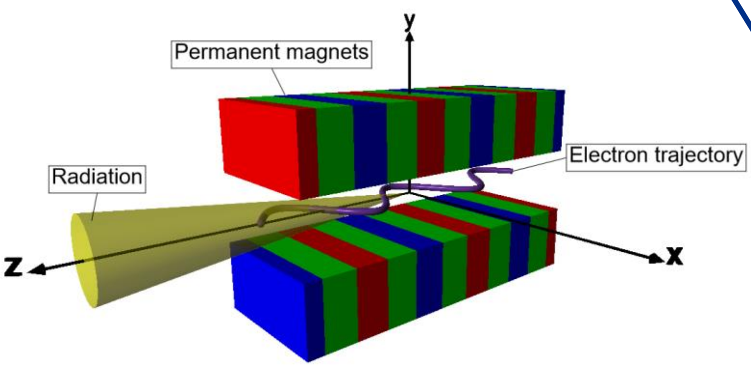
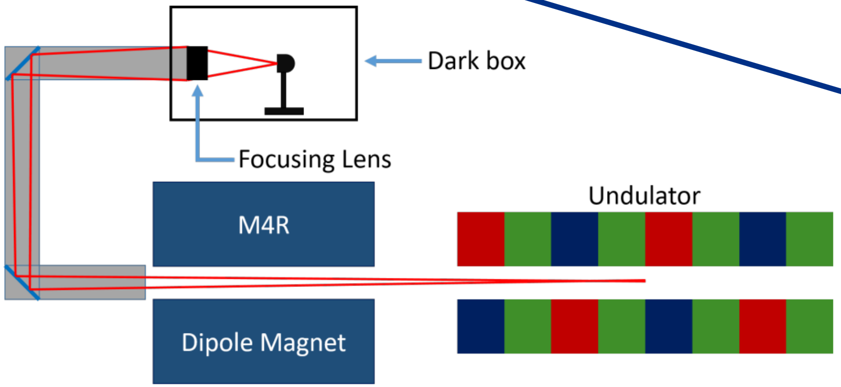


photo credit Evan Angelico

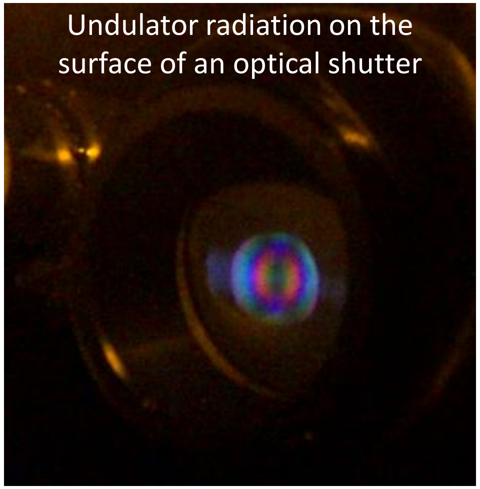






Parameters of the undulator in IOTA

Many thanks to our collaborators from SLAC for providing the undulator

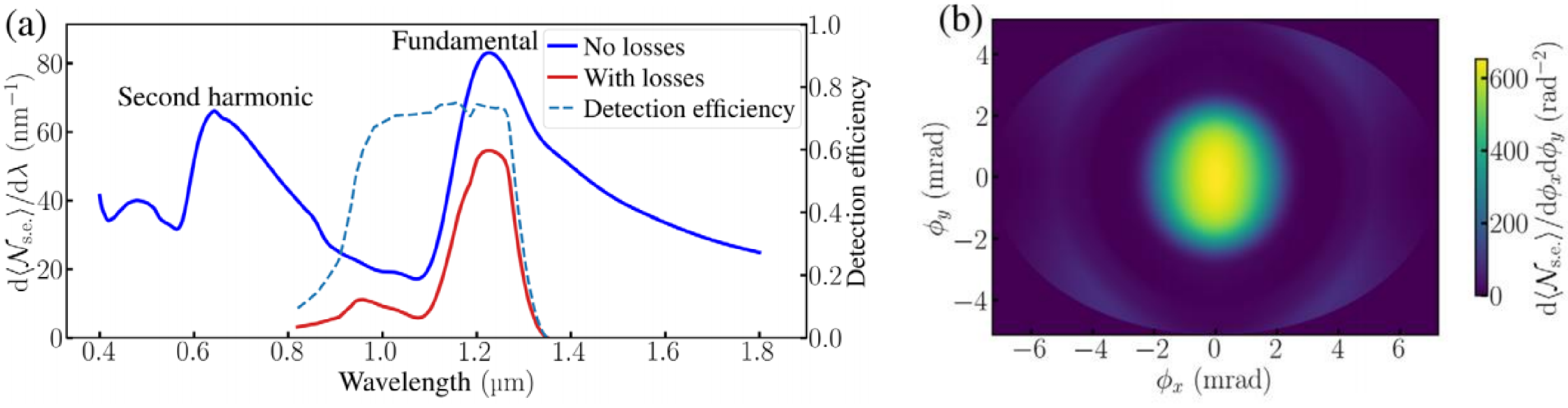


Undulator:

- Number of periods: $N_{\mathrm{u}} = 10.5$
- Undulator period length: $\lambda_{\mathrm{u}} = 55\ \mathrm{mm}$
- Undulator parameter (peak): $K_{\mathrm{u}} = 1$
- Fundamental of radiation: 1.16 um
- Second harmonic: visible light

$$K_{\mathrm{u}} = \frac{eB\lambda_{\mathrm{u}}}{2\pi m_e c}$$

Spectral-angular radiation distribution



In our experiment:

#1	Detect the fundamental (≈ 1.16 um). InGaAs p-i-n photodiode
#2	Wide band (≈ 0.14 um FWHM). Large acceptance angle > $1/\gamma$

Simulated total intensity: 9.1×10^{-3} photoelectrons/electron

Measured: 8.8×10^{-3} photoelectrons/electron

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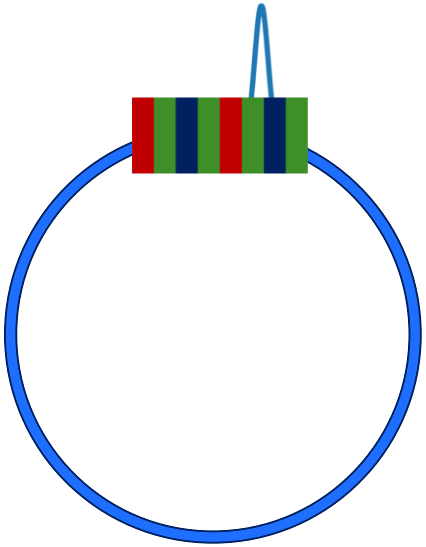
Format of collected data



Revolution number	Number of photocounts, N
0	9994352
1	9997379
2	10002465
3	9999482
4	9996153
...	...
11273 1.5 ms	10000362

$$\mathrm{var}(\mathcal{N}) = \langle \mathcal{N}^2 \rangle - \langle \mathcal{N} \rangle^2$$

The initial goal was to systematically study $\mathrm{var}(\mathcal{N})$ as a function of:

- Electron bunch charge (0-0.5 nC)
- Electron bunch transverse emittances, bunch length
- Transmission of optical filters

Then, we realized that we can reverse this procedure and infer the electron bunch parameters from the measured $\mathrm{var}(\mathcal{N})$

Previous research about statistical properties of synchrotron radiation

Both theoretical and experimental results:

[1] M. C. Teich, T. Tanabe, T. C. Marshall, and J. Galayda, Statistical properties of wiggler and bending-magnet radiation from the Brookhaven Vacuum-Ultraviolet electron storage ring, Phys. Rev. Lett. **65**, 3393 (1990).

[2] V. Sajaev, *Determination of longitudinal bunch profile using spectral fluctuations of incoherent radiation*, Report No ANL/ASD/CP-100935 (Argonne National Laboratory, 2000).

[3] V. Sajaev, Measurement of bunch length using spectral analysis of incoherent radiation fluctuations, in *AIP Conf. Proc.*, Vol. 732 (AIP, 2004) pp. 73–87.

[4] F. Sannibale, G. Stupakov, M. Zolotorev, D. Filippetto, and L. Jägerhofer, Absolute bunch length measurements by incoherent radiation fluctuation analysis, Phys. Rev. ST Accel. Beams **12**, 032801 (2009).

[5] P. Catravas, W. Leemans, J. Wurtele, M. Zolotorev, M. Babzien, I. Ben-Zvi, Z. Segalov, X.-J. Wang, and V. Yakimenko, Measurement of electron-beam bunch length and emittance using shot-noise-driven fluctuations in incoherent radiation, Phys. Rev. Lett. **82**, 5261 (1999).

[6] K.-J. Kim, Start-up noise in 3-D self-amplified spontaneous emission, Nucl. Instrum. Methods Phys. Res., Sect. A **393**, 167 (1997).

[7] S. Benson and J. M. Madey, Shot and quantum noise in free electron lasers, Nucl. Instrum. Methods Phys. Res., Sect. A **237**, 55 (1985).

[8] E. L. Saldin, E. Schneidmiller, and M. V. Yurkov, *The physics of free electron lasers* (Springer Science & Business Media, 2013).

[9] C. Pellegrini, A. Marinelli, and S. Reiche, The physics of x-ray free-electron lasers, Rev. Mod. Phys. **88**, 015006 (2016).

[10] W. Becker and M. S. Zubairy, Photon statistics of a free-electron laser, Phys. Rev. A **25**, 2200 (1982).

[11] W. Becker and J. McIver, Fully quantized many-particle theory of a free-electron laser, Phys. Rev. A **27**, 1030 (1983).

[12] W. Becker and J. McIver, Photon statistics of the free-electron-laser startup, Phys. Rev. A **28**, 1838 (1983).

Outline

- Theoretical consideration
- Details about the apparatus and measurement procedure
- Measurements of the fluctuations
- Measurements of electron beam emittances via the fluctuations

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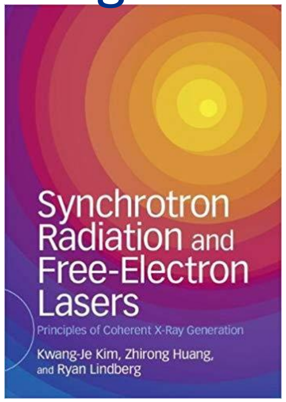
Theoretical predictions

$$\mathrm{var}(\mathcal{N}_{\mathrm{ph}}) = \langle\mathcal{N}_{\mathrm{ph}}\rangle + \frac{1}{M}\langle\mathcal{N}_{\mathrm{ph}}\rangle^2$$

Discrete quantum nature of light (Poisson fluctuations)

Turn-to-turn variations in relative electron positions and directions of motion

Page 28:



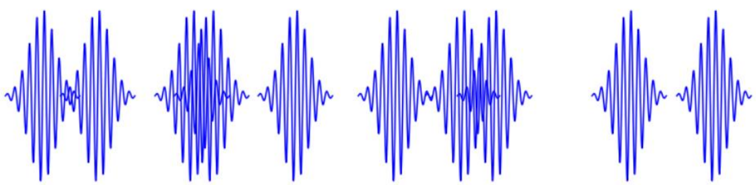
M is conventionally called the number of coherent modes

Origin of the second term

$$\mathrm{var}(\mathcal{N}_{\mathrm{ph}}) = \langle\mathcal{N}_{\mathrm{ph}}\rangle + \frac{1}{M}\langle\mathcal{N}_{\mathrm{ph}}\rangle^2$$

- Simplified 1D model:

Pulses emitted by the electrons:



$$W \propto \int \mathrm{d}t \left|\sum_{i=1}^{n_e} E(t-t_i)\right|^2 = \int \mathrm{d}\omega\, |E(\omega)|^2 \left|\sum_{i=1}^{n_e} e^{-i\omega t_i}\right|^2$$

The set of arrival times of the electrons $\{t_i\}$ is different during every revolution in the ring. Hence, the radiated energy W fluctuates from turn to turn. $\sigma_t = \sqrt{\langle t_i^2\rangle - \langle t_i\rangle^2}$

$$|E(\omega)|^2 \propto e^{-\frac{(\omega-\omega_0)^2}{2\sigma_\omega^2}}$$

$$M = \sqrt{1 + 4\sigma_\omega^2\sigma_t^2}$$

If we also consider transverse electron bunch dimensions and a Gaussian angular radiation profile:

$$M = \sqrt{1+4\sigma_k^2\sigma_z^2}\sqrt{1+4k_0^2\sigma_{\theta_x}^2\sigma_x^2}\sqrt{1+4k_0^2\sigma_{\theta_y}^2\sigma_y^2}$$

- F. Sannibale, et al, *Phys. Rev. ST AB*, 12, 032801 (2009)
- I. Lobach, et al, *Phys. Rev. Accel. Beams*, 23, 090703 (2020)

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Realistic case

In general, M is a function of

- Detector's angular acceptance
- Detector's spectral sensitivity, polarization sensitivity
- Spectral-angular properties of the radiation (undulator or bending magnet)
- Electron bunch density distribution over x, y, z, x', y', δ_p

We accounted for this part for the first time

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Measurements of undulator radiation power noise and comparison with *ab initio* calculations

Ihar Lobach, Sergei Nagaitsev, Valeri Lebedev, Aleksandr Romanov, Giulio Stancari, Alexander Valishev, Aliaksei Halavanau, Zhirong Huang, and Kwang-Je Kim

Phys. Rev. Accel. Beams **24**, 040701 – Published 1 April 2021

Physics See synopsis: Using Fluctuations to Measure Beam Properties

The obtained expression is very complex and includes a multidimensional integral:

$$\frac{1}{M} = (1 - 1/n_e)\frac{\sqrt{\pi}}{\sigma_z^{\mathrm{eff}}}\frac{\int \mathrm{d}k\mathrm{d}^2\boldsymbol{\phi}_1\mathrm{d}^2\boldsymbol{\phi}_2\mathrm{d}^2\boldsymbol{r}'\mathcal{P}_k(\boldsymbol{r}', \boldsymbol{\phi}_1 - \boldsymbol{\phi}_2)\mathcal{I}_k(\boldsymbol{\phi}_1, \boldsymbol{r}')\mathcal{I}_k^*(\boldsymbol{\phi}_2, \boldsymbol{r}')}{\langle \mathcal{N}_{\mathrm{s.e.}}\rangle^2}, \quad (2)$$

with

$$\mathcal{P}_k(\boldsymbol{r}', \boldsymbol{\phi}_1 - \boldsymbol{\phi}_2) = \frac{1}{4\pi\sigma_{x'}\sigma_{y'}} e^{-\frac{(x')^2}{4\sigma_{x'}^2} - \frac{(y')^2}{4\sigma_{y'}^2}} e^{-ik\Delta_x(\phi_{1x}-\phi_{2x})x' - ik\Delta_y(\phi_{1y}-\phi_{2y})y'} e^{-k^2\Sigma_x^2(\phi_{1x}-\phi_{2x})^2 - k^2\Sigma_y^2(\phi_{1y}-\phi_{2y})^2}, \quad (3)$$

$$\mathcal{I}_k(\boldsymbol{\phi}, \boldsymbol{r}') = \sum_{s=1,2} \eta_{k,s}(\boldsymbol{\phi})\mathcal{E}_{k,s}(\boldsymbol{\phi})\mathcal{E}_{k,s}^*(\boldsymbol{\phi} - \boldsymbol{r}'), \quad (4)$$

$$\langle \mathcal{N}_{\mathrm{s.e.}}\rangle = \sum_{s=1,2}\int \mathrm{d}k\mathrm{d}^2\boldsymbol{\phi}\eta_{k,s}(\boldsymbol{\phi})|\mathcal{E}_{k,s}(\boldsymbol{\phi})|^2, \quad (5)$$

where $s = 1, 2$ indicates the polarization component, n_e is the number of electrons in the bunch, $k = 2\pi/\lambda$ is the magnitude of the wave vector; $\boldsymbol{\phi} = (\phi_x, \phi_y)$, $\boldsymbol{\phi}_1 = (\phi_{1x}, \phi_{1y})$ and $\boldsymbol{\phi}_2 = (\phi_{2x}, \phi_{2y})$ represent angles of direction of the radiation in the paraxial approximation. Hereinafter, x and y refer to the horizontal and the vertical axes, respectively, and

$$\sigma_z^{\mathrm{eff}} = 1\Big/\left(2\sqrt{\pi}\int \rho^2(z)\mathrm{d}z\right), \quad (6)$$

where $\rho(z)$ is the electron bunch longitudinal density distribution function, $\int \rho(z)\mathrm{d}z = 1$, and σ_z^{eff} is equal to the rms bunch length σ_z for a Gaussian bunch; $\boldsymbol{r}' = (x', y')$ represents the direction of motion of an electron at the radiator center, relative to a reference electron; $\sigma_{x'}$ and $\sigma_{y'}$ are the rms beam divergences, $\sigma_{x'}^2 = \gamma_x\epsilon_x + D_{x'}^2\sigma_p^2$, $\sigma_{y'}^2 = \gamma_y\epsilon_y$; $\Sigma_x^2 = \epsilon_x/\gamma_x + (\gamma_x D_x + D_{x'}\alpha_x)^2\beta_x\epsilon_x\sigma_p^2/\sigma_{x'}^2$, $\Sigma_y^2 = \epsilon_y/\gamma_y$, $\Delta_x = (\alpha_x\epsilon_x - D_x D_{x'}\sigma_p^2)/\sigma_{x'}^2$, $\Delta_y = \alpha_y/\epsilon_y$, where α_x, β_x, γ_x, α_y, β_y, γ_y are the Twiss parameters of an uncoupled focusing optics in the synchrotron radiation

$$\mathcal{E}_{k,s}(\boldsymbol{\phi}) = \sqrt{\frac{\alpha k}{2(2\pi)^3}}\int \mathrm{d}t\boldsymbol{e}_s(\boldsymbol{k})\cdot\boldsymbol{v}(t)e^{ickt - i\boldsymbol{k}\cdot\boldsymbol{r}(t)}$$

- Transversely Gaussian beam
- Arbitrary longitudinal density distribution

The code for numerical computation is available at https://github.com/IharLobach/fur

Despite the complexity, there is an important conclusion:

$$M = \langle\mathcal{N}\rangle^2/(\mathrm{var}(\mathcal{N}) - \langle\mathcal{N}\rangle) \qquad M_{\mathrm{meas}} = M(\epsilon_x, \epsilon_y, \sigma_p, \sigma_z^{\mathrm{eff}})$$

If only one parameter of the electron bunch is unknown, it can be inferred from the measured fluctuations

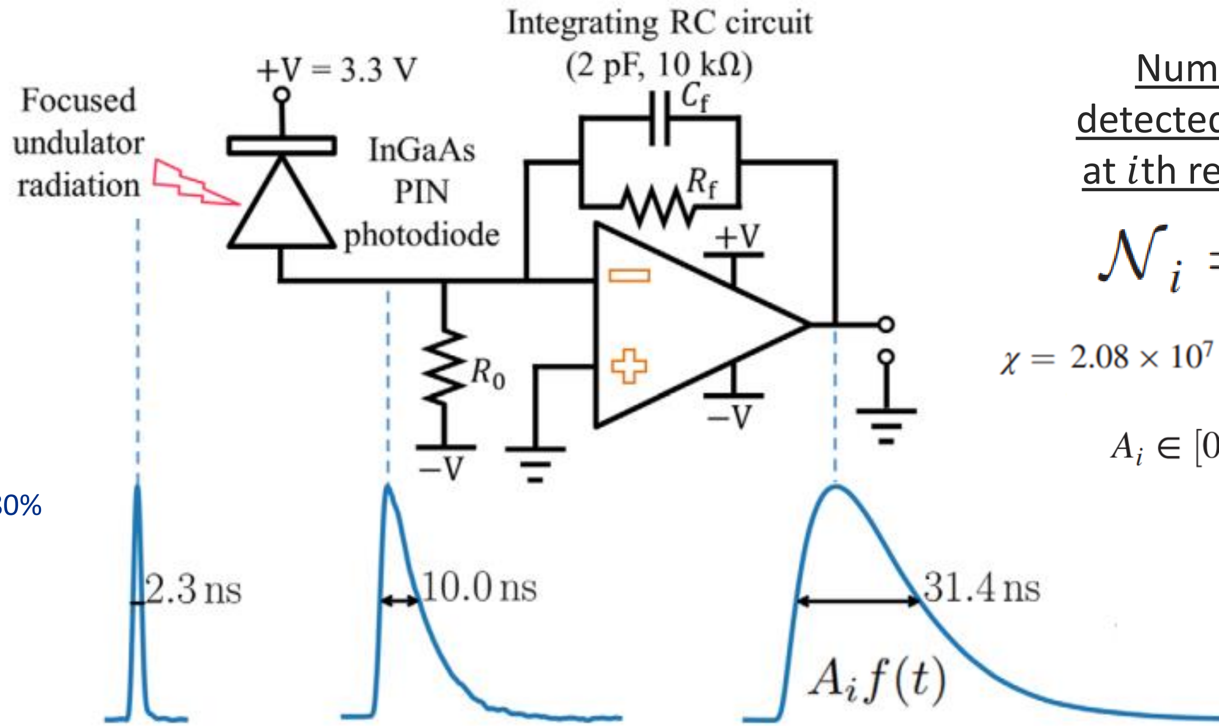
Details about apparatus

InGaAs PIN photodiode

G11193-10R

Sensitive area: Ø1mm
Quantum efficiency at 1.16 µm: 80%

*the circuit was built by Greg Saewert



Number of detected photons at ith revolution:

$$\mathcal{N}_i = \chi A_i$$

$\chi = 2.08 \times 10^7$ photoelectrons/V

$A_i \in [0, 1.2]$ V

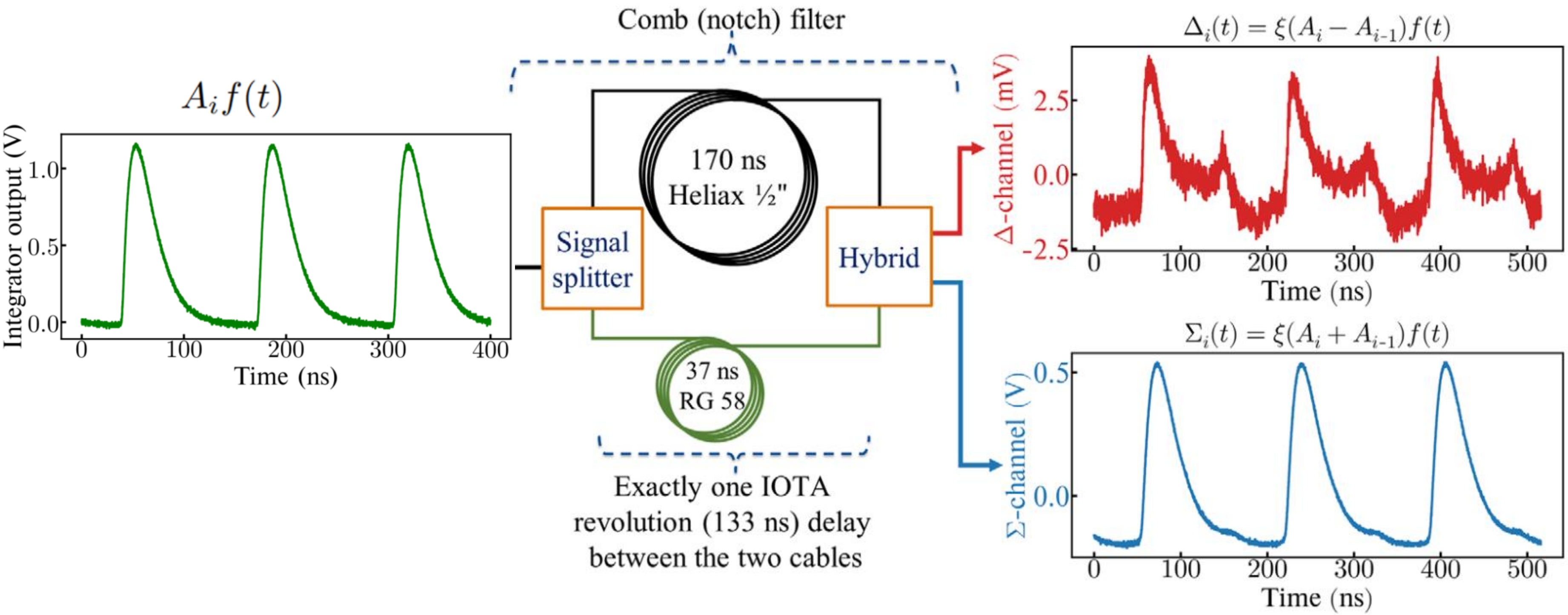
*Many thanks to Mark Obrycki, Peter Prieto, David Johnson, Todd Johnson and Greg Saewert for the equipment for the setup and for their help during our detector tests.

The expected relative fluctuation of A_i was very small $10^{-4} - 10^{-3}$ (rms).
It was a big challenge to measure it.

*comparable to the resolution of our 8-bit scope

Comb (notch) filter

*the idea to use the comb filter was proposed by S. Nagaitsev.
The components were provided by B.J. Fellenz, K. Carlson, and D. Frolov



Our comb filter had some imperfections:

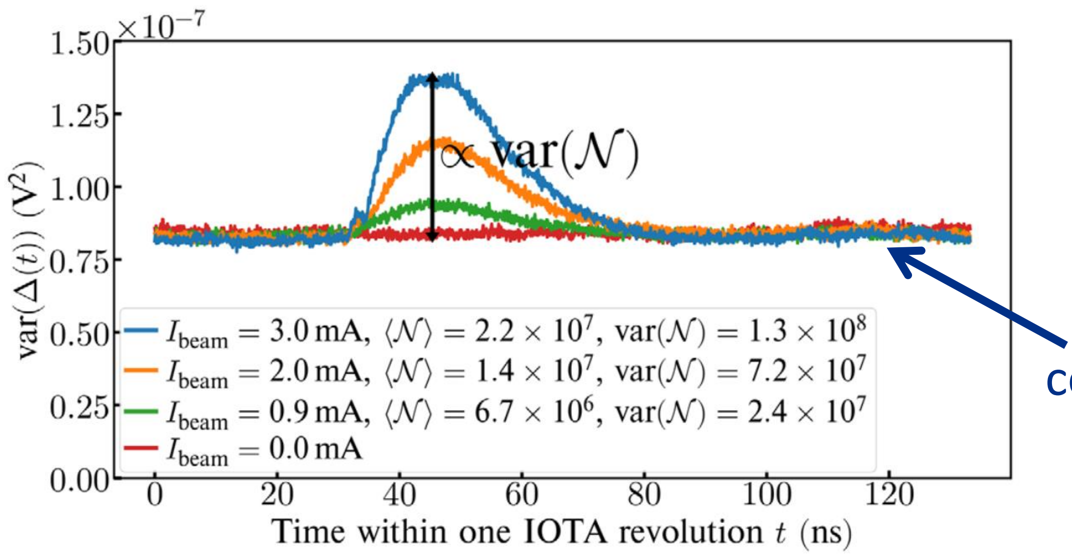
- Cross-talk (< 1%)
- Small reflected pulse in one of the arms

*they could be taken into account and did not affect final results

Noise filtering algorithm

- The instrumental noise due to oscilloscope's pre-amp and due to the integrator's op-amp was about 0.3 mV (rms)
- Therefore, signal-to-noise ratio was about 1

We had to use a special noise filtering algorithm.
For each time t within one IOTA revolution, calculate variance of Δ-signal for the 11000 revolutions:



constant noise level

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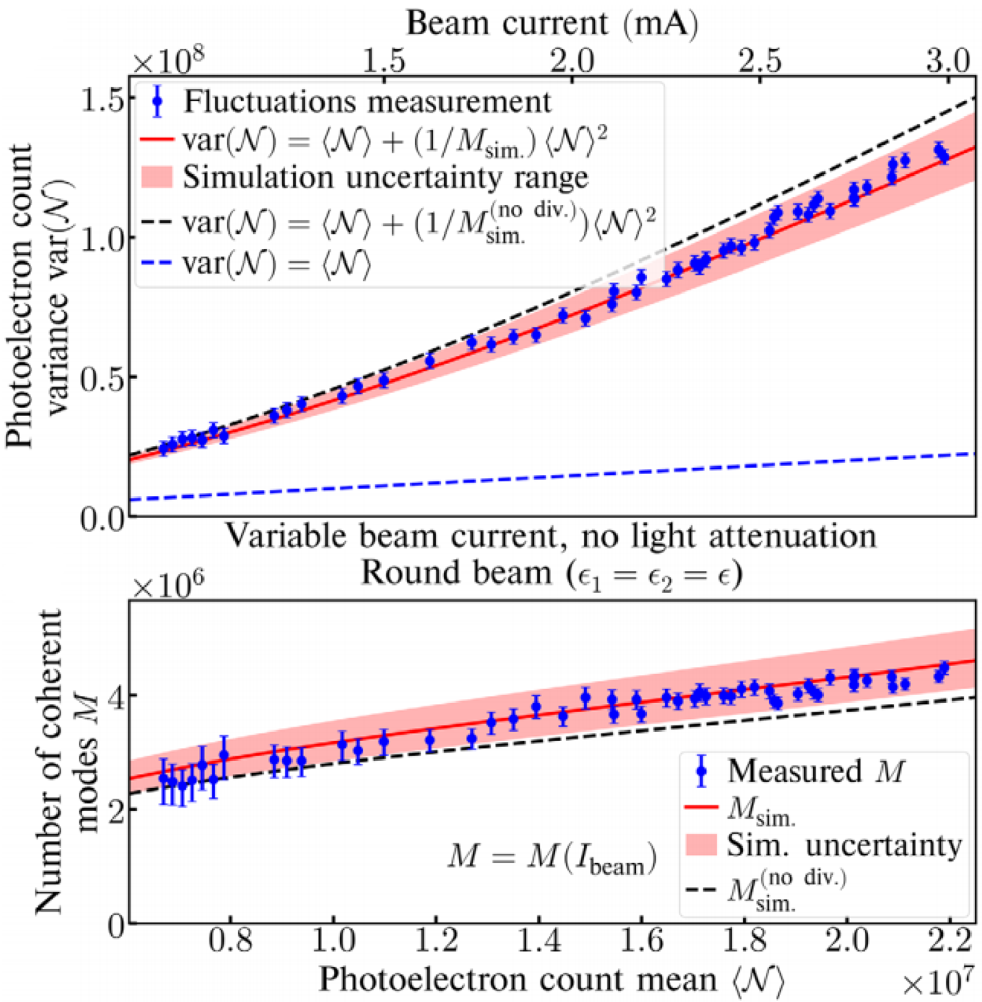
Measurements and simulations



$$M = M(\epsilon_x, \epsilon_y, \sigma_p, \sigma_z^{\text{eff}})$$

For the simulation,

- ϵ_x and ϵ_y were estimated using bending magnet synchrotron radiation monitors and known Twiss functions.
- σ_z^{eff} and σ_p were estimated using the wall-current monitor signal

Note that the simulation with beam divergence taken into account agrees better

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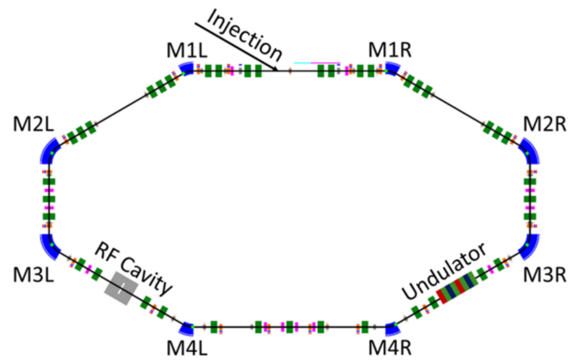
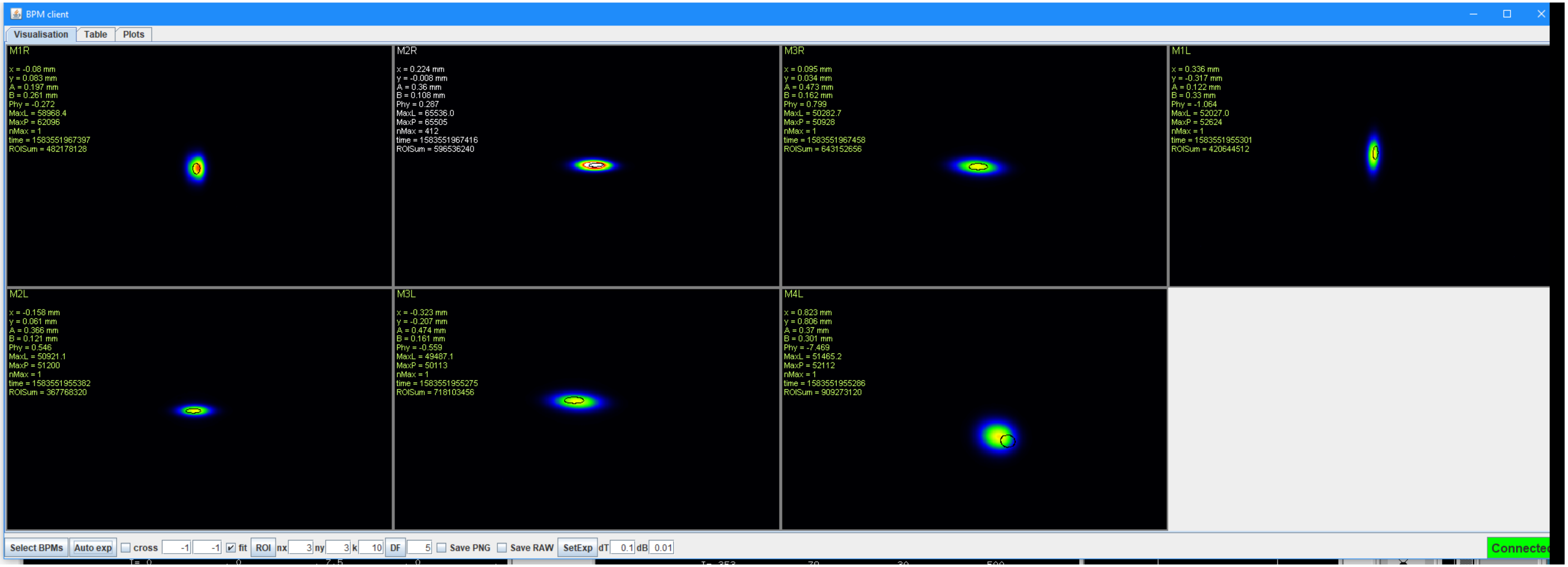
Measurement of transverse bunch size: 7 synclight stations



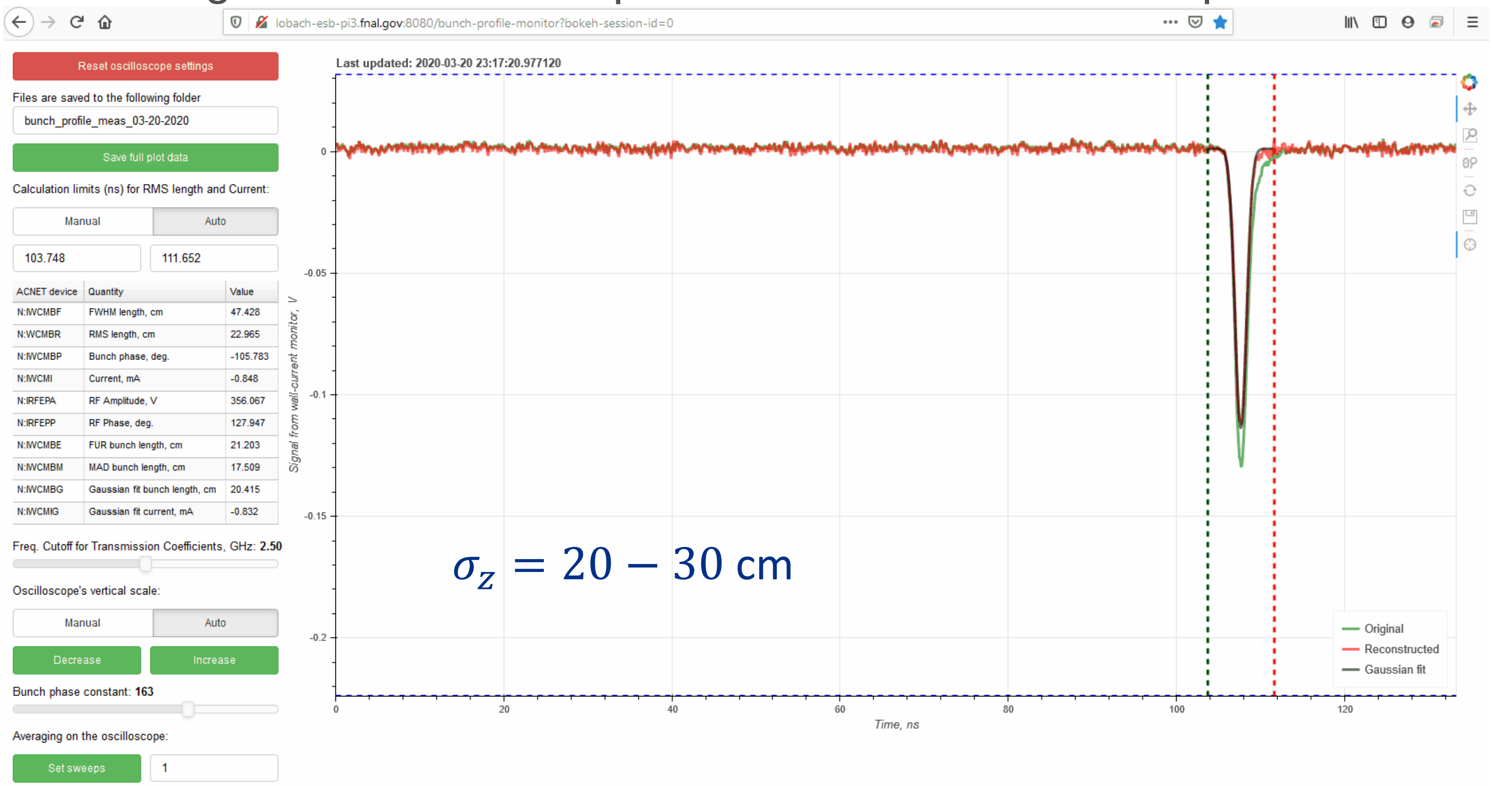
Bending magnet radiation (not undulator)



*built by A. Romanov, J. Santucci, G. Stancari, N. Kuklev, …

Measurement of longitudinal bunch length and shape: Bunch length monitor

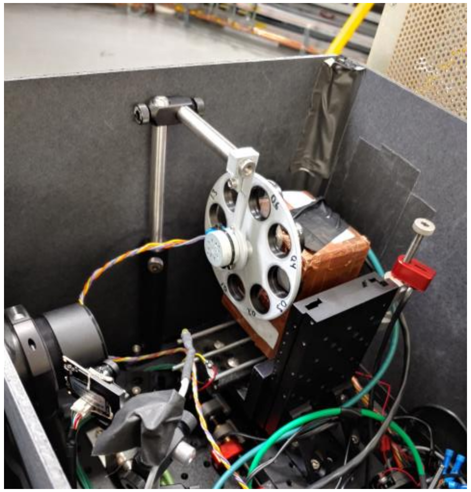
- Wall-current monitor → long cable → amplifier → oscilloscope
- The web-server runs on a Raspberry Pi on the Fermilab controls network. It receives the signal from the scope and applies the inverse of the transmission function of the long cable and the amplifier to reconstruct the shape of the electron bunch



Valeri Lebedev and Kermit Carlson helped with measurement of the transmission function.
Dean Edstrom helped with network communication with the oscilloscope.

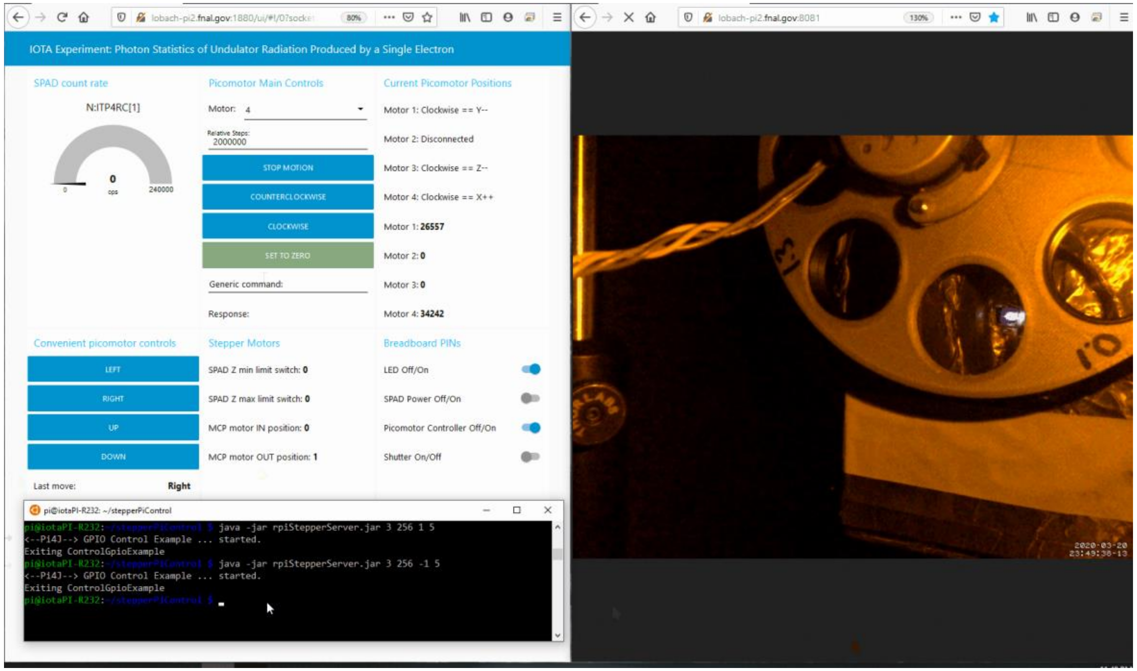
Neutral density (ND) filters

- ND filter is a filter that has constant attenuation in a wide spectral range
- ND filter does not change the number of coherent modes M, however, it does change the average number of detected photons $\langle \mathcal{N} \rangle$

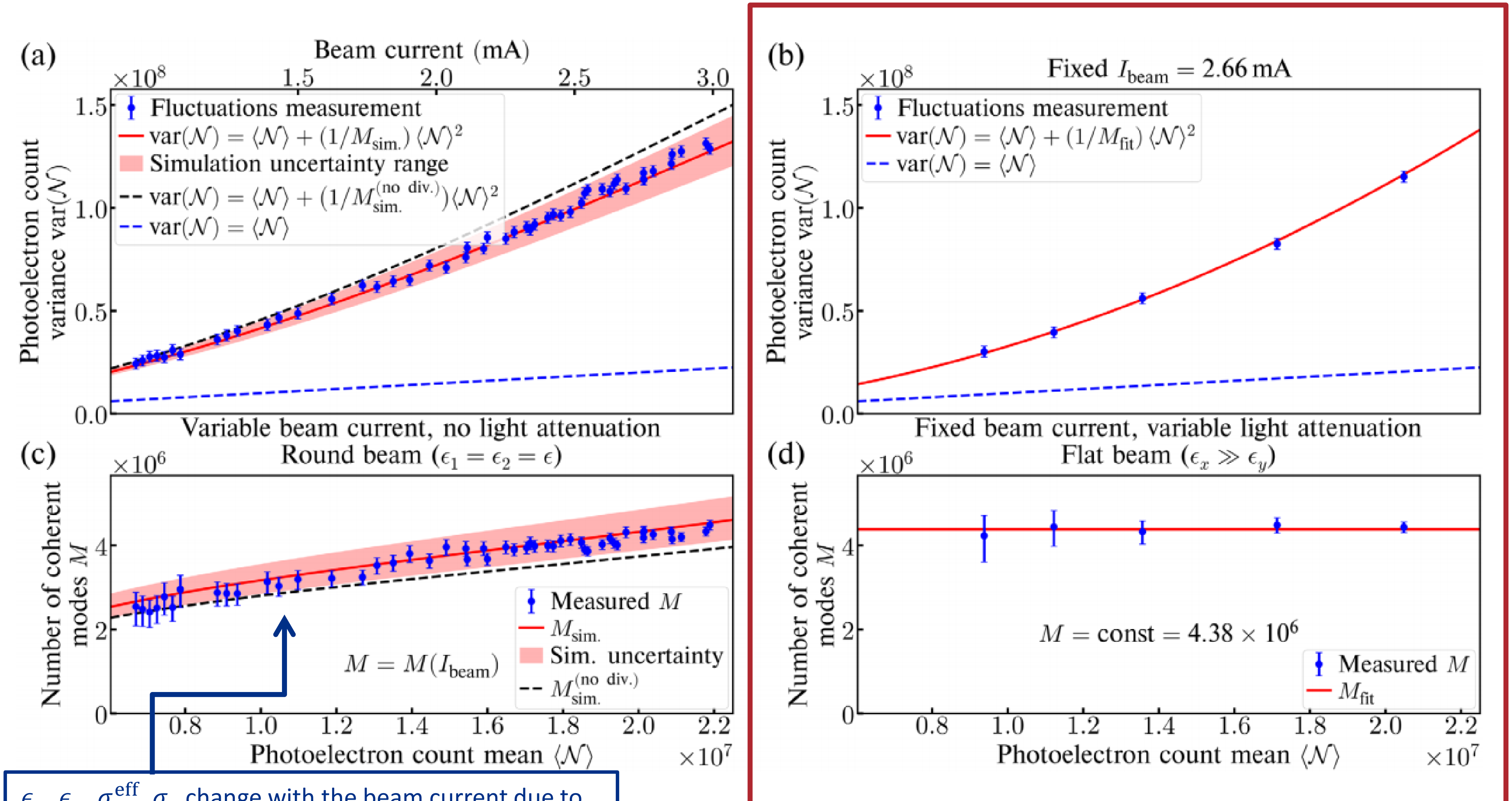


The filter wheel was built by Sasha Romanov

<u>Remote controls for the apparatus</u>



Measurements with ND filters (right-hand side)



ϵ_x, ϵ_y, σ_z^{eff}, σ_p change with the beam current due to intrabeam scattering and interaction of the bunch with its environment. Therefore, M changes too.

Reconstruction of transverse emittances from the measured $\mathrm{var}(\mathcal{N})$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Transverse Beam Emittance Measurement by Undulator Radiation Power Noise

Ihar Lobach, Sergei Nagaitsev, Valeri Lebedev, Aleksandr Romanov, Giulio Stancari, Alexander Valishev, Aliaksei Halavanau, Zhirong Huang, and Kwang-Je Kim
Phys. Rev. Lett. **126**, 134802 – Published 1 April 2021

Physics See synopsis: Using Fluctuations to Measure Beam Properties

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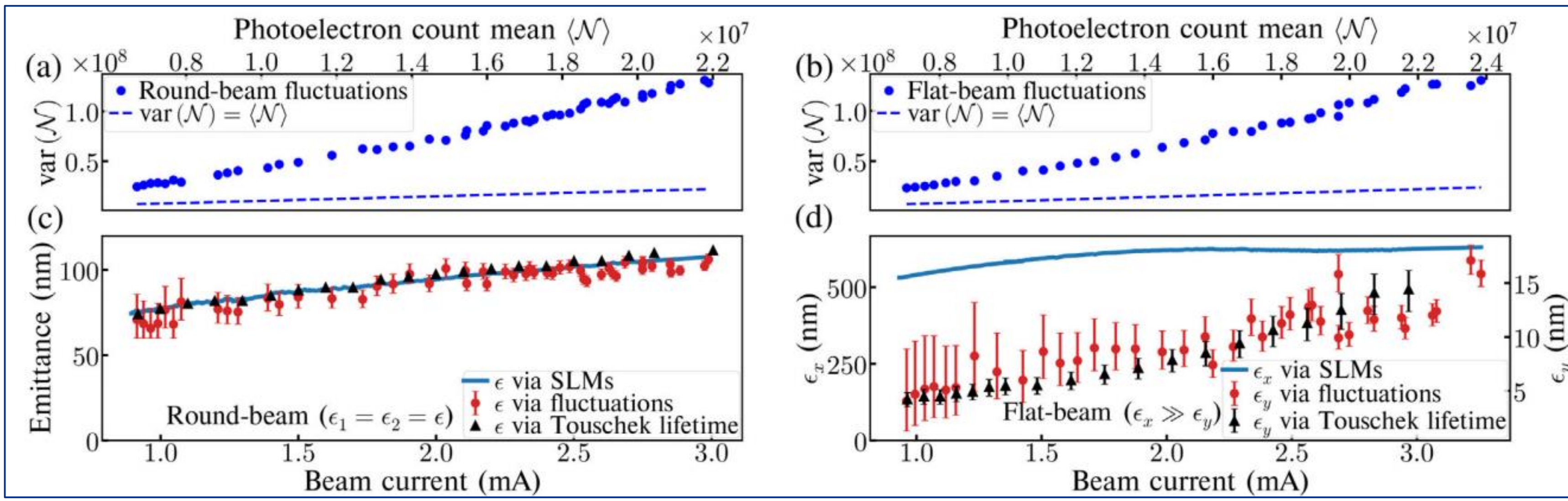
We verified our method with a “round” beam, whose emittances could be independently measured by synchrotron radiation monitors, (a) and (c):

Then, we used our fluctuations to **measure the unknown small vertical emittance of a “flat” beam**, (b) and (d):



Strong coupling

Uncoupled

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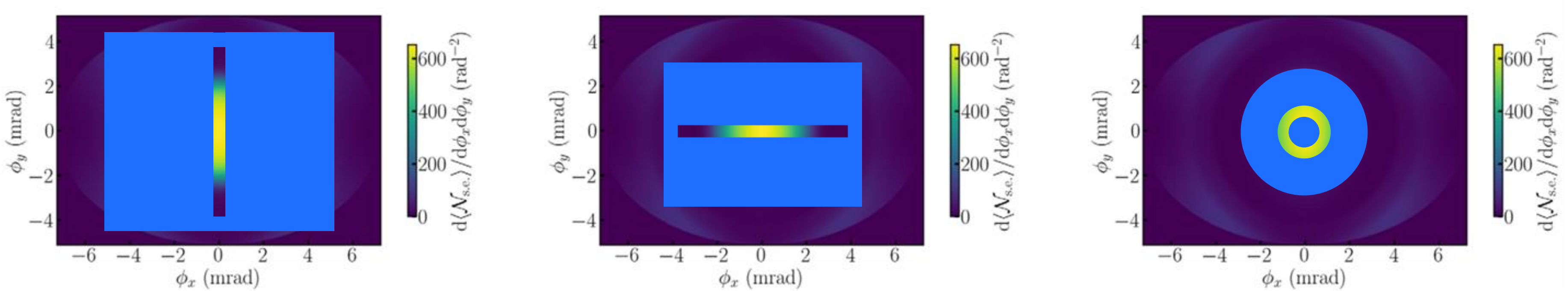
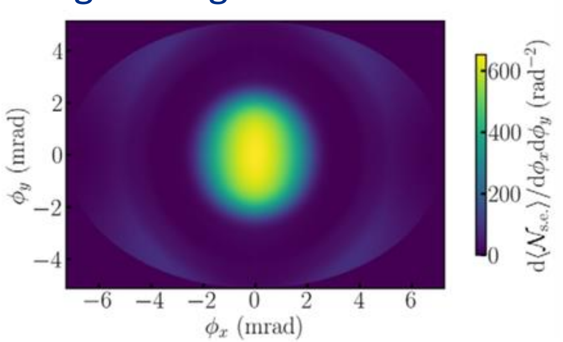
Usage of slits and masks



- Measurement of fluctuations with slits or masks would allow measurement of more than one electron bunch parameter.

Original angular distribution:



$$M = \sqrt{1 + 4\sigma_k^2\sigma_z^2}\sqrt{1 + 4k_0^2\sigma_{\theta_x}^2\sigma_x^2}\sqrt{1 + 4k_0^2\sigma_{\theta_y}^2\sigma_y^2}$$

Limitations

- The fluctuations must not be dominated by the Poisson noise

$$\langle \mathcal{N} \rangle \lesssim \frac{1}{M} \langle \mathcal{N} \rangle^2 \quad \Rightarrow \quad \frac{\langle \mathcal{N} \rangle}{M} = \alpha \left(\frac{\pi}{2}\right)^{\frac{3}{2}} F_h(K_\mathrm{u}) \frac{\gamma^2 N_\mathrm{u}^2 n_e}{\sigma_x \sigma_y \sigma_z k_0^3} \gtrsim 1$$

- M must be sensitive to changes in σ_x, σ_y (ϵ_x, ϵ_y)

$$\sigma_x, \sigma_y \gtrsim \sqrt{2 L_\mathrm{u} \lambda_0}/(4\pi)$$

The sensitivity of this technique improves with shorter wavelength. Therefore, this technique may be particularly beneficial for existing state-of-the-art and next-generation low-emittance high-brightness ultraviolet and x-ray synchrotron light sources. For instance, this technique can measure $\epsilon_x \approx \epsilon_y \approx 30$ pm in the Advanced Photon Source Upgrade at Argonne.

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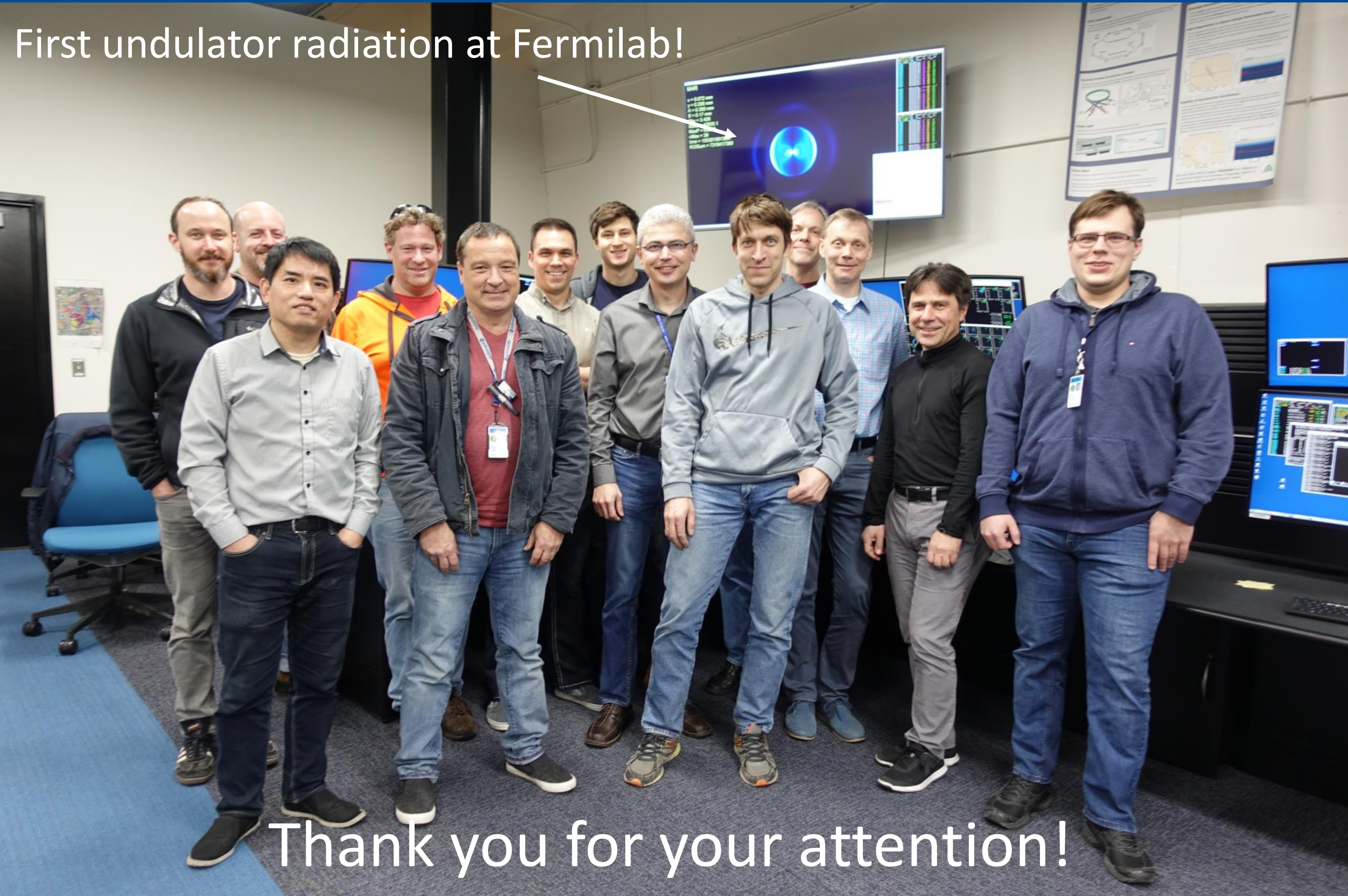
Conclusions

- Turn-to-turn undulator radiation power fluctuations have two contributions: (1) quantum due to discrete nature of light and (2) classical due to variations in relative electron positions and directions of motion.
- We derived the second contribution, accounting for electron beam divergence, for the first time.
- We obtained a good agreement for the fluctuations $\mathrm{var}(\mathcal{N})$ between measurements and calculations.
- The process can be reversed, i.e., the measured fluctuations $\mathrm{var}(\mathcal{N})$ can be used to infer the transverse electron beam emittances. This method can be especially useful for low-emittance high-brightness ultraviolet and x-ray synchrotron light sources.

First undulator radiation at Fermilab!



Thank you for your attention!

Additional slides

Origin of the first term

$$\mathrm{var}(\mathcal{N}_{\mathrm{ph}}) = \langle \mathcal{N}_{\mathrm{ph}} \rangle + \frac{1}{M} \langle \mathcal{N}_{\mathrm{ph}} \rangle^2$$

- For the case of negligible electron recoil each mode of the radiation field is in a **coherent state**:

PHYSICAL REVIEW VOLUME 131, NUMBER 6 15 SEPTEMBER 1963

Coherent and Incoherent States of the Radiation Field*

ROY J. GLAUBER

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$$

$$\langle n \rangle = \langle \alpha | \hat{a}^\dagger \hat{a} | \alpha \rangle = |\alpha|^2$$

$$\mathrm{var}(n) = \langle \alpha | \left(\hat{a}^\dagger \hat{a} - \langle n \rangle \right)^2 | \alpha \rangle = |\alpha|^2 = \langle n \rangle$$

$$\frac{\sqrt{\mathrm{var}(n)}}{\langle n \rangle} = \frac{1}{\sqrt{\langle n \rangle}}$$ -- Poisson fluctuations

However, this is correct only for a deterministic classical current, i.e., fixed relative positions of the electrons in the bunch

Number of coherent modes (in our notation *M*)

Average Number of Coherent Modes for Pulse Random Fields

Alexander M. Lazaruk, Nikolay V. Karelin (Institute of Physics, Belarus Academy of Sciences, Minsk, Belarus)

arXiv:physics/9712011 **[physics.optics]**

- Modal decomposition of the transverse radiation coherence function

$$\Gamma_S(\mathbf{r},\mathbf{r}') = \int dt E(\mathbf{r},t)E^*(\mathbf{r}',t) = \sum_n u_n \mathcal{E}_n(\mathbf{r})\, \mathcal{E}_n^*(\mathbf{r}')$$

$$\int d^2r \mathcal{E}_n(\mathbf{r})\mathcal{E}_m^*(\mathbf{r}) = \delta_{n,m}$$

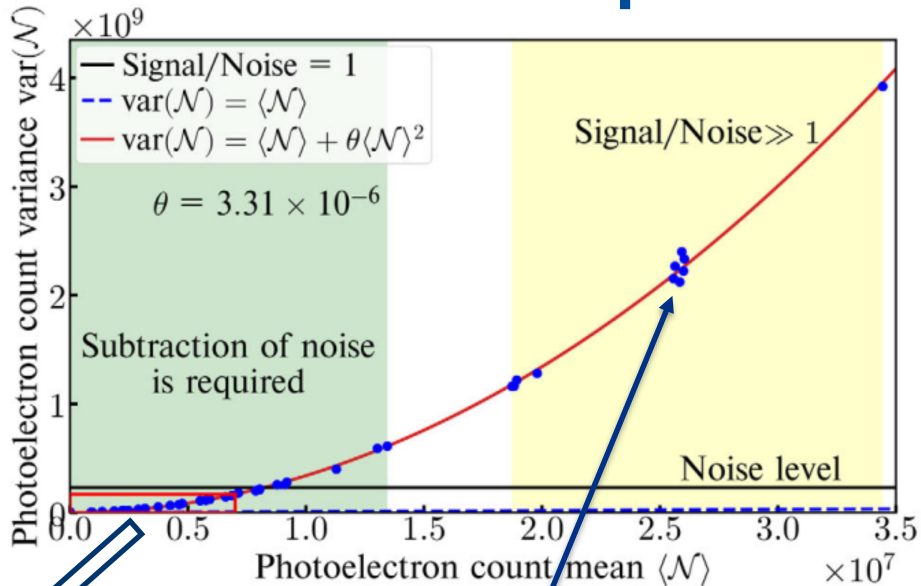
- Effective number of terms in the decomposition:

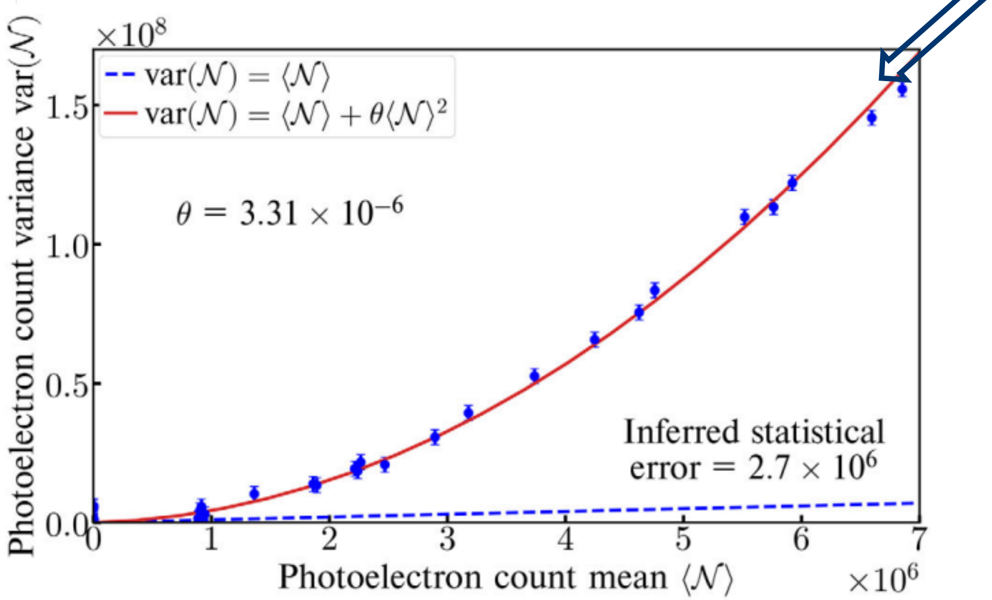
$$N_{\mathit{eff}} = \left(\sum\nolimits_n u_n\right)^2 \Big/ \left(\sum\nolimits_n u_n^2\right)$$

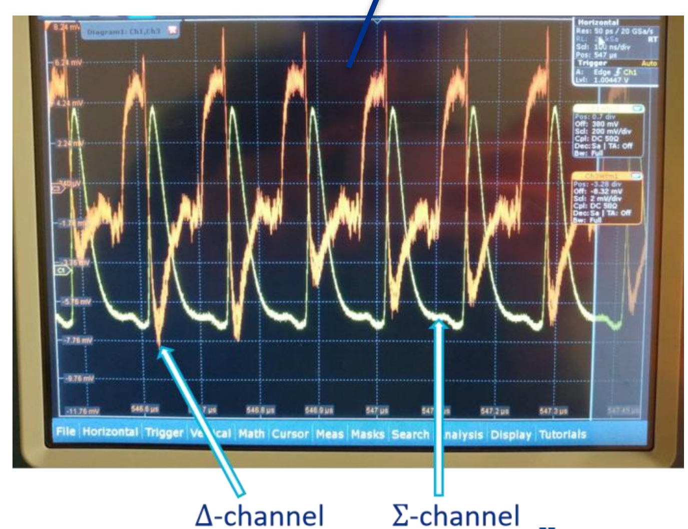
The value of N_{eff} specifies the ability of the total field to produce interference effects between two arbitrary separate points of the beam cross-section[6, 12] and changes from unity for spatial coherent one-mode wave to infinity for completely incoherent field.

Testing the setup with an independent test light source --- laser diode with a modulated amplifier

- Pulse-to-pulse fluctuations due to pulse generator and amplifier errors are high and can be measured without noise filtering.
- By using ND filters, the fluctuations, in terms of $\mathrm{var}(\mathcal{N})$, can be decreased down to the level, observed in our experiment with undulator radiation.
- At this level, noise filtering is required. However, we can see that the measured $\mathrm{var}(\mathcal{N})$ still agrees with the expected parabola. This proves that the noise filtering algorithm works well.



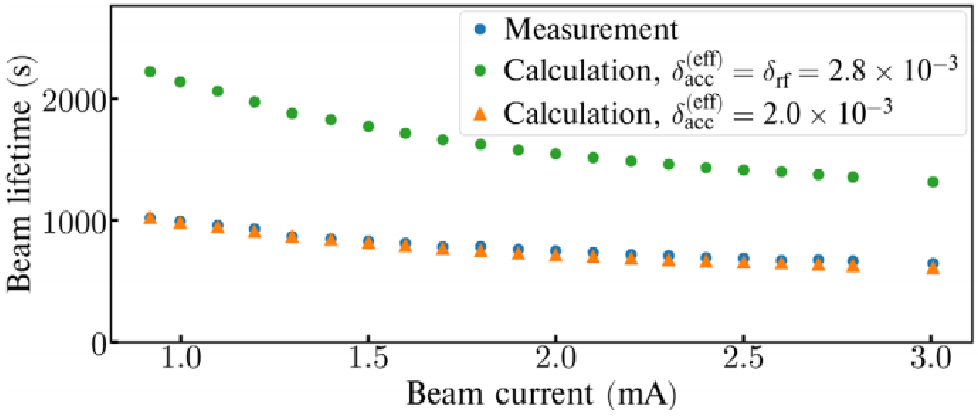




Fermilab

Bonus plots

Round-beam lifetime



Round-beam emittance and bunch length