Transverse Beam Emittance Measurement by Undulator Radiation Power Noise

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**Fermilab Integrable Optics Test Accelerator (IOTA)**

- First beam Aug 21, 2018

**Particles:** electrons/protons

**Main experiments:**
- Nonlinear beam optics
- Optical stochastic cooling

**Circumference:** 40 m (133 ns)

**Electron energy:** 100 MeV

In our experiments:
- Single electron bunch, up to 0.5 nC
- About 30 cm long (rms)
Layout of the undulator section in IOTA
Parameters of the undulator in IOTA

Many thanks to our collaborators from SLAC for providing the undulator

Undulator:

- Number of periods: $N_u = 10.5$
- Undulator period length: $\lambda_u = 55 \text{ mm}$
- Undulator parameter (peak): $K_u = 1$
- Fundamental of radiation: $1.16 \text{ um}$
- Second harmonic: visible light

$$K_u = \frac{eB\lambda_u}{2\pi m_e c}$$
Spectral-angular radiation distribution

In our experiment:

#1 Detect the fundamental (≈ 1.16 um). InGaAs p-i-n photodiode

#2 Wide band (≈ 0.14 um FWHM). Large acceptance angle > 1/\gamma

Simulated total intensity: 9.1 \times 10^{-3} \text{ photoelectrons/electron}

Measured: 8.8 \times 10^{-3} \text{ photoelectrons/electron}
The initial goal was to systematically study $\text{var}(N)$ as a function of:

- Electron bunch charge (0-0.5 nC)
- Electron bunch transverse emittances, bunch length
- Transmission of optical filters

Then, we realized that we can reverse this procedure and infer the electron bunch parameters from the measured $\text{var}(N)$.
Previous research about statistical properties of synchrotron radiation

Both theoretical and experimental results:


Outline

• Theoretical consideration
• Details about the apparatus and measurement procedure
• Measurements of the fluctuations
• Measurements of electron beam emittances via the fluctuations
Theoretical predictions

\[ \text{var}(\mathcal{N}_{\text{ph}}) = \langle \mathcal{N}_{\text{ph}} \rangle + \frac{1}{M} \langle \mathcal{N}_{\text{ph}} \rangle^2 \]

- Discrete quantum nature of light (Poisson fluctuations)
- Turn-to-turn variations in relative electron positions and directions of motion

\(M\) is conventionally called the number of coherent modes
Origin of the second term

\[
\text{var}(N_{ph}) = \langle N_{ph} \rangle + \frac{1}{M} \langle N_{ph} \rangle^2
\]

- **Simplified 1D model:**
  
  Pulses emitted by the electrons:

  \[
  |E(\omega)|^2 \propto e^{-\frac{(\omega - \omega_0)^2}{2\sigma_\omega^2}}
  \]

  \[
  M = \sqrt{1 + 4\sigma_k^2\sigma_z^2} \sqrt{1 + 4k_0^2\sigma_{\theta x}^2\sigma_{x}^2} \sqrt{1 + 4k_0^2\sigma_{\theta y}^2\sigma_{y}^2}
  \]

  The set of arrival times of the electrons \(\{t_i\}\) is different during every revolution in the ring. Hence, the radiated energy \(W\) fluctuates from turn to turn.

  \[
  \sigma_t = \sqrt{\langle t_i^2 \rangle - \langle t_i \rangle^2}
  \]

  If we also consider transverse electron bunch dimensions and a Gaussian angular radiation profile:

  \[
  W \propto \int dt \left| \sum_{i=1}^{n_e} E(t - t_i) \right|^2 = \int d\omega |E(\omega)|^2 \left| \sum_{i=1}^{n_e} e^{-i\omega t_i} \right|^2
  \]

  \[
  \text{If we also consider transverse electron bunch dimensions and a Gaussian angular radiation profile:}
  \]

  \[
  M = \sqrt{1 + 4\sigma_k^2\sigma_z^2} \sqrt{1 + 4k_0^2\sigma_{\theta x}^2\sigma_{x}^2} \sqrt{1 + 4k_0^2\sigma_{\theta y}^2\sigma_{y}^2}
  \]

In general, $M$ is a function of

- Detector’s angular acceptance
- Detector’s spectral sensitivity, polarization sensitivity
- Spectral-angular properties of the radiation (undulator or bending magnet)
- Electron bunch density distribution over $x, y, z, x', y', \delta_p$

We accounted for this part for the first time.
The obtained expression is very complex and includes a multidimensional integral:

\[
\frac{1}{M} = (1 - 1/n_e) \frac{\sqrt{\pi} \int dkd\phi_1 d\phi_2 d\rho' P_k(r', \phi_1 - \phi_2) I_k(\phi_1, r') I_k(\phi_2, r')}{\langle N_{\text{tot}} \rangle^2},
\]

where

\[
P_k(r', \phi_1 - \phi_2) = \frac{1}{4\pi \sigma_\phi \sigma_\rho} e^{-\frac{(x' - x)^2}{\sigma_\phi^2} - \frac{(y' - y)^2}{\sigma_\rho^2}} e^{-ik\Delta_\phi(\phi_1 - \phi_2) - ik\Delta_\rho(\phi_1 - \phi_2)} e^{-\frac{i^2}{\Delta_\phi^2(\phi_1 - \phi_2)^2 - \Delta_\rho^2(\phi_1 - \phi_2)^2}},
\]

and

\[
I_k(\phi, r') = \sum_{s=1,2} \eta_{s, s}(\phi) E_{k, s}(\phi) E_{k, s}(\phi - r'),
\]

\[
\langle N_{\text{tot}} \rangle = \int \int dkd\phi_1 d\phi_2 E_{k, s}(\phi),
\]

The code for numerical computation is available at https://github.com/IharLobach/fur

Despite the complexity, there is an important conclusion:

- Transversely Gaussian beam
- Arbitrary longitudinal density distribution

If only one parameter of the electron bunch is unknown, it can be inferred from the measured fluctuations.
Details about apparatus

InGaAs PIN photodiode

- Number of detected photons at $i$th revolution:
  \[ N_i = \chi A_i \]
  \[ \chi = 2.08 \times 10^7 \text{ photoelectrons/V} \]
  \[ A_i \in [0, 1.2] \text{ V} \]

*the circuit was built by Greg Saewert

The expected relative fluctuation of $A_i$ was very small $10^{-4} - 10^{-3}$ (rms). It was a big challenge to measure it.

*S many thanks to Mark Obrycki, Peter Prieto, David Johnson, Todd Johnson and Greg Saewert for the equipment for the setup and for their help during our detector tests.

*S comparable to the resolution of our 8-bit scope
Our comb filter had some imperfections:

- Cross-talk (< 1%)
- Small reflected pulse in one of the arms

*they could be taken into account and did not affect final results
Noise filtering algorithm

- The instrumental noise due to oscilloscope’s pre-amp and due to the integrator’s op-amp was about 0.3 mV (rms)
- Therefore, signal-to-noise ratio was about 1

We had to use a special noise filtering algorithm. For each time $t$ within one IOTA revolution, calculate variance of $\Delta$-signal for the 11000 revolutions:

\[ \alpha \text{ var}(N) \]

constant noise level
Measurements and simulations

\[ M = M(\epsilon_x, \epsilon_y, \sigma_p, \sigma_{z}^{\text{eff}}) \]

For the simulation,
- \( \epsilon_x \) and \( \epsilon_y \) were estimated using bending magnet synchrotron radiation monitors and known Twiss functions.
- \( \sigma_{z}^{\text{eff}} \) and \( \sigma_p \) were estimated using the wall-current monitor signal.

Note that the simulation with beam divergence taken into account agrees better.
Measurement of transverse bunch size: 7 synclight stations

Bending magnet radiation (not undulator)

*built by A. Romanov, J. Santucci, G. Stancari, N. Kuklev, ...
Measurement of longitudinal bunch length and shape: Bunch length monitor

- Wall-current monitor → long cable → amplifier → oscilloscope
- The web-server runs on a Raspberry Pi on the Fermilab controls network. It receives the signal from the scope and applies the inverse of the transmission function of the long cable and the amplifier to reconstruct the shape of the electron bunch.

\[ \sigma_z = 20 - 30 \text{ cm} \]

Valeri Lebedev and Kermit Carlson helped with measurement of the transmission function. Dean Edstrom helped with network communication with the oscilloscope.
Neutral density (ND) filters

- ND filter is a filter that has constant attenuation in a wide spectral range
- ND filter does not change the number of coherent modes $M$, however, it does change the average number of detected photons $\langle N \rangle$

The filter wheel was built by Sasha Romanov

Remote controls for the apparatus
Measurements with ND filters (right-hand side)

\[ \epsilon_x, \epsilon_y, \sigma_{\text{eff}}^2, \sigma_p \] change with the beam current due to intrabeam scattering and interaction of the bunch with its environment. Therefore, \( M \) changes too.
We verified our method with a “round” beam, whose emittances could be independently measured by synchrotron radiation monitors, (a) and (c):

Then, we used our fluctuations to **measure the unknown small vertical emittance of a “flat” beam**, (b) and (d):

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Strong coupling

Uncoupled
Measurement of fluctuations with slits or masks would allow measurement of more than one electron bunch parameter.

\[ M = \sqrt{1 + 4\sigma_k^2\sigma_z^2} \sqrt{1 + 4k^2_0\sigma_{\phi_z}^2\sigma_x^2} \sqrt{1 + 4k^2_0\sigma_{\phi_y}^2\sigma_y^2} \]
Limitations

- The fluctuations must not be dominated by the Poisson noise

\[ \langle N \rangle \lesssim \frac{1}{M} \langle N \rangle^2 \quad \Rightarrow \quad \frac{\langle N \rangle}{M} = \alpha \left( \frac{\pi}{2} \right)^{3/2} F_h(K_u) \frac{\gamma^2 N_u^2 n_e}{\sigma_x \sigma_y \sigma_z k_0^3} \gtrsim 1 \]

- \( M \) must be sensitive to changes in \( \sigma_x, \sigma_y \) (\( \epsilon_x, \epsilon_y \))

\[ \sigma_x, \sigma_y \gtrsim \sqrt{2 L_u \lambda_0 / (4\pi)} \]

The sensitivity of this technique improves with shorter wavelength. Therefore, this technique may be particularly beneficial for existing state-of-the-art and next-generation low-emittance high-brightness ultraviolet and x-ray synchrotron light sources. For instance, this technique can measure \( \epsilon_x \approx \epsilon_y \approx 30 \) pm in the Advanced Photon Source Upgrade at Argonne.
Conclusions

• Turn-to-turn undulator radiation power fluctuations have two contributions: (1) quantum due to discrete nature of light and (2) classical due to variations in relative electron positions and directions of motion.

• We derived the second contribution, accounting for electron beam divergence, for the first time.

• We obtained a good agreement for the fluctuations $\text{var}(\mathcal{N})$ between measurements and calculations.

• The process can be reversed, i.e., the measured fluctuations $\text{var}(\mathcal{N})$ can be used to infer the transverse electron beam emittances. This method can be especially useful for low-emittance high-brightness ultraviolet and x-ray synchrotron light sources.
First undulator radiation at Fermilab!

Thank you for your attention!
Additional slides
Origin of the first term

\[ \text{var}(N_{ph}) = \langle N_{ph} \rangle + \frac{1}{M} \langle N_{ph} \rangle^2 \]

- For the case of negligible electron recoil each mode of the radiation field is in a **coherent state**:

\[ |\alpha\rangle = e^{-\frac{1}{2} |\alpha|^2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle \]

\[ \hat{a} |\alpha\rangle = \alpha |\alpha\rangle \]

\[ \langle n \rangle = \langle \alpha | \hat{a}^\dagger \hat{a} | \alpha \rangle = |\alpha|^2 \]

\[ \text{var}(n) = \langle \alpha | (\hat{a}^\dagger \hat{a} - \langle n \rangle)^2 | \alpha \rangle = |\alpha|^2 = \langle n \rangle \]

\[ \sqrt{\frac{\text{var}(n)}{\langle n \rangle}} = \frac{1}{\sqrt{\langle n \rangle}} \]

--- Poisson fluctuations

However, this is correct only for a deterministic classical current, i.e., fixed relative positions of the electrons in the bunch.
Number of coherent modes (in our notation $M$)

Modal decomposition of the transverse radiation coherence function

$$\Gamma_s(r, r') = \int dt E(r, t) E^*(r', t) = \sum_n u_n \mathcal{E}_n(r) \mathcal{E}_n^*(r')$$

$$\int d^2r \mathcal{E}_n(r) \mathcal{E}_m^*(r) = \delta_{n,m}$$

Effective number of terms in the decomposition:

$$N_{eff} = \left(\sum_n u_n\right)^2 / \left(\sum_n u_n^2\right)$$

The value of $N_{eff}$ specifies the ability of the total field to produce interference effects between two arbitrary separate points of the beam cross-section[6, 12] and changes from unity for spatial coherent one-mode wave to infinity for completely incoherent field.
Testing the setup with an independent test light source --- laser diode with a modulated amplifier

- Pulse-to-pulse fluctuations due to pulse generator and amplifier errors are high and can be measured without noise filtering.
- By using ND filters, the fluctuations, in terms of $\text{var}(\mathcal{N})$, can be decreased down to the level, observed in our experiment with undulator radiation.
- At this level, noise filtering is required. However, we can see that the measured $\text{var}(\mathcal{N})$ still agrees with the expected parabola. This proves that the noise filtering algorithm works well.
Bonus plots

Round-beam lifetime

![Graph showing beam lifetime vs. beam current](image)

- Measurement
- Calculation, $\delta_{\text{acc}}^{(\text{eff})} = \delta_{\text{rf}} = 2.8 \times 10^{-3}$
- Calculation, $\delta_{\text{acc}}^{(\text{eff})} = 2.0 \times 10^{-3}$

Round-beam emittance and bunch length

![Graph showing emittance and bunch length vs. beam current](image)

- Round beam (by design $\epsilon_1 = \epsilon_2 = \epsilon$)
- $\epsilon$ via SLMs
- RMS bunch length $\sigma_z$
- Effective bunch length $\sigma_z^{\text{eff}}$