IBA S2C2: The influence of first harmonic field errors on the beam quality

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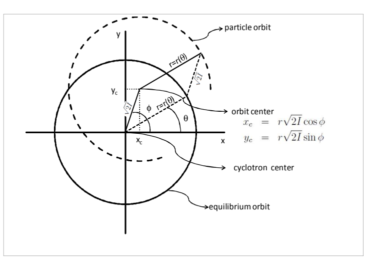
The S2C2 is the new compact proton-therapy accelerator for the IBA ProteusOne range of products. In this communication a study is made of the deteriorating effect of a first harmonic field error due to the n_r=1 resonance in the S2C2, thereby taking into account the acceleration process. It is shown that in the main part of the accelerating region this deteriorating effect is almost non-existing. This is due to the very slow acceleration used in the S2C2. The beam centroid adiabatically follows the magnetic center of the machine and no coherent oscillations are generated. Only the central region and the extraction region require special care in terms of the shimming of the first harmonic error. For the intermediate region the shimming requirement is less strict and can be determined by setting an upper limit for the allowable shift of the magnetic center with respect to the geometrical center. For the purpose of the study, a new tracking program was developed that numerically integrates the equations of orbit center motion in a cyclotron.

INTRODUCTION

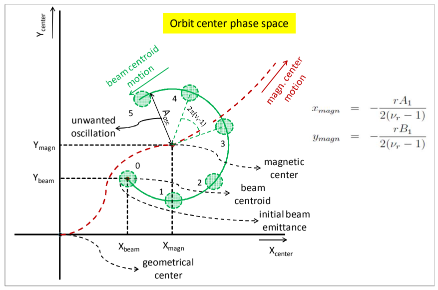
- ❑In the S2C2 large first harmonic field errors are present. This is due to i) the passive extraction system made of soft iron and only partly compensated with correcting bars and ii) the asymmetric yoke penetrations and external equipment
- ❑The first harmonic field error will be shimmed during mapping phase
- ❑In the central region and also in the extraction region allowable errors are small (< 10 Gauss) because of beam centering
- ❑Main question => what level of errors can be allowed in between the centre and the extraction during full region of acceleration

ORBIT CENTRE PICTURE OF THE RADIAL PHASE SPACE

- ❑Betatron oscillations in a cyclotron can be represented by an amplitude and a phase, but also by the coordinates of the orbit centre.
- ❑The latter can be more convenient because the orbit centre oscillates slowly (frequency n_r-1) as compared to the betatron oscillation itself (frequency n_r)
- ❑In the orbit centre representation, the equations of motion can be simplified using approximations that make use of the slowly varying character of the motion and the integration can be done much faster

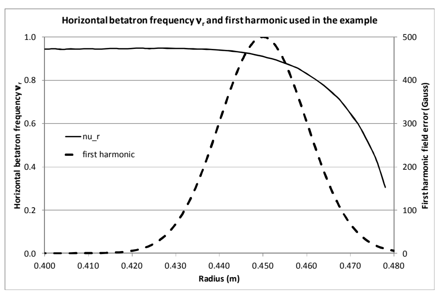


Representation of the radial betatron oscillation around the equilibrium orbit in terms of the coordinates of the orbit center. The real orbit can be re-constructed from the orbit center coordinates., the equilibrium orbit radius r(θ) and the value of the independent variable θ. The canonical variables for the orbit center can be expressed in a Cartesian form (x_c,y_c) or in a polar form (action-angle variables (ϕ,I)). Note that in this illustration an equilibrium orbit with circular shape (synchro-cyclotron case) is shown. For AVF cyclotrons this will be a scalloped orbit.

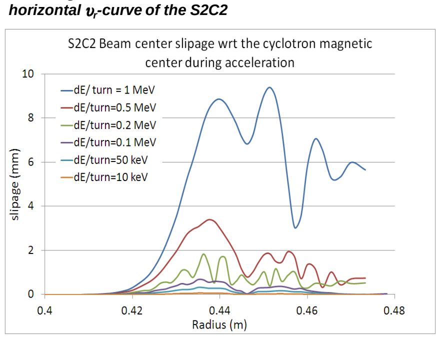


Due to the presence of a first harmonic field error, the magnetic center of the cyclotron is displaced relative to the geometrical center. Particles execute a betatron-oscillation around the magnetic center. When there is acceleration, the magnetic center itself is also moving and the total motion is a complicated superposition of two separate motions: the betatron oscillation and the motion of the magnetic center. The beam quality will degrade when the centroid of the beam is shifting with respect to the magnetic center of the cyclotron. This will occur when the acceleration is very fast and/or the gradients of the first harmonic field error are large. The distance A_{osc} is the amplitude of the betatron oscillation and is a good measure for the harmful effect of the first harmonic field error. Numbers indicate subsequent turns.

ORBIT CENTRE SHIFT DURING ACCELERATION



The dashed line represents the first harmonic field errors that was used in the example. This is a Gaussian profile with an amplitude of 500 Gauss, a σ-value of 1 cm and a centering radius of 45 cm. The solid curve shows the horizontal ν_r-curve of the S2C2



Slippage between the beam centroid and the magnetic center of the S2C2 as calculated numerically with the program orbit_center_motion. The beam is accelerated from 15 MeV up to extraction. Initially the beam is well-centered and the magnetic center coincides with the magnetic center. During acceleration a coherent oscillation A_{osc} is building up. For fast acceleration, this amplitude becomes large and the beam quality is destroyed. For slow acceleration, the beam adiabatically follows the magnetic center and no or very little oscillation amplitude is generated. For the S2C2 the orange curve (dE/turn=10 keV) is applicable and the loss of beam quality is negligible.

MATHEMATICS OF THE MODEL

GENERAL HAMILTONIAN FOR THE ORBIT CENTER

$$H(\phi,I) = (\nu_r - 1)I + \frac{1}{2}(\bar{A}_1\cos\phi + \bar{B}_1\sin\phi)\sqrt{2I}$$
$$+\frac{1}{4}\left(A_0' + \left(A_2 + \tfrac{1}{2}A_2'\right)\cos 2\phi + \left(B_2 + \tfrac{1}{2}B_2'\right)\sin 2\phi\right)(2I)^1$$
$$+\frac{1}{48}(D_1\cos 3\phi + D_2\sin 3\phi + D_3\cos\phi + D_4\sin\phi)\sqrt[3]{2I}$$
$$+\frac{1}{64}(E_0 + E_1\cos 4\phi + E_2\sin 4\phi)(2I)^2$$

MAGNETIC FIELD QUANTITIES

$A_n, B_n \rightarrow$ normalized field harmonics

$$' \rightarrow \frac{d}{dr} \text{ and } '' \rightarrow r^2\frac{d^2}{dr^2} \text{ etc.}$$

$$D_1 = 3A_3 + 5A_3' + A_3'' \text{ and similar for } D_2$$

$$D_3 = 3A_1' + 3A_1'' \text{ and similar for } D4$$

$$E0 = \mu'' + \mu'''$$

$$E_1 = \frac{15A_4' + 9A_4'' + A_4'''}{6}$$

EQUATIONS OF MOTION

$$\frac{d\phi}{d\theta} = \frac{\partial H}{\partial I} \text{ and } \frac{dI}{d\theta} = -\frac{\partial H}{\partial \phi}$$

RELATION WITH ORBIT CENTER COORDINATES

$$x_c = r\sqrt{2I}\cos\phi \text{ and } y_c = r\sqrt{2I}\sin\phi$$

See Hagedoorn and Verster NIM **18,19** (1962) 201-228

SIMPLER CASE FOR THE S2C2

$$H(\phi,I) = (\nu_r - 1)I + \frac{1}{2}(\bar{A}_1\cos\phi + \bar{B}_1\sin\phi)\sqrt{2I}$$

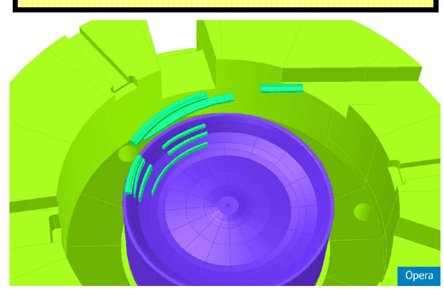
MAGNETIC CENTER OF CYCLOTRON

$$x_{magn} = -\frac{rA_1}{2(\nu_r - 1)}$$

$$y_{magn} = -\frac{rB_1}{2(\nu_r - 1)}$$

ACCELERATION

$$\frac{dr}{d\theta} = ZV_0\frac{\sqrt{r^2B^2c^2 + \frac{A^2E_0^2}{Z^2}}}{2\pi B^2c^2(1+\mu')}\Delta\nu_n(r)$$



The S2C2 extraction system with the regenerator, the extraction channel, the gradient corrector and 5 first harmonic correction bars

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