Phase motion: a program for studying longitudinal beam optics in a synchrocyclotron

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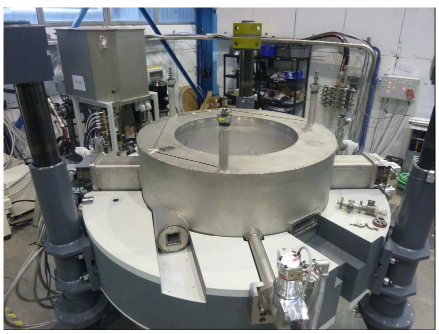
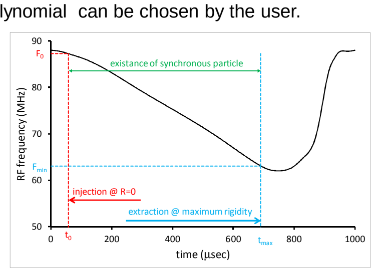
One of the tasks in the development of a synchro-cyclotron concerns the specification of the RF-frequency and voltage tables as a function of time. Two important quantities depend on this: i) the efficiency of capturing injected ions into the RF-bucket and ii) the stability of the RF bucket during the acceleration. A third determining factor is the K-value of the magnetic field (relating the change in revolution frequency to the change of energy). It is possible to study these processes with a full 3-D (transverse and longitudinal) tracking code but such calculations are complex and time-consuming. The purpose of the current study is to develop a simplified tracking code where only the longitudinal motion is calculated. The central region is decoupled from the longitudinal effects and is considered as a 'filter' which excludes certain ranges of RF phase which will in practice be lost horizontally or vertically during the first few turns. This is an approximation, but it serves as a helpful and fast additional calculation tool for study of the RF-frequency and voltage curves and it also allows to follow the particle during the full acceleration cycle. The program phase-motion numerically integrates the equations of longitudinal motion (the phase-equations) in a synchro-cyclotron. As an example the problem of capture in the S2C2 is studied.



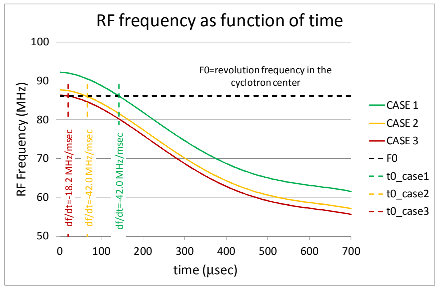
OUTLINE OF THE PROGRAM

- ❑Longitudinal canonical variables are the RF phase ϕ and the energy difference ΔE between particle E and the synchronous particle E_s. Independent variable is time *t*.
- ❑The properties of the synchronous particle are calculated by a separate program .
- ❑An inner time domain ($t_0<t<t_{max}$) is defined for which a synchronous particle exists. This is the case if the RF frequency is smaller than the revolution frequency in the cyclotron center and larger than the revolution frequency at the extraction radius.
- ❑In the outer time domain the synchronous phase and energy are put to zero.
- ❑F_{RF} *(t)* and *V*(*t*) are read from an input file.
- ❑Equations of motion are integrated with a 5[th] order Runge-Kutta with adaptive stepsize
- ❑Polynomial interpolation is used in the frequency table and the synchronous particle table. Order of polynomial can be chosen by the user.



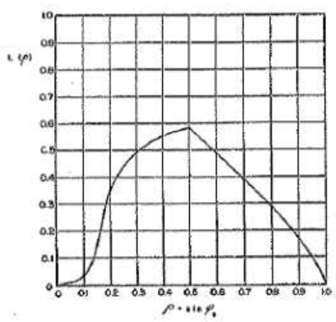
AN EXAMPLE OF APPLICATION

- ❑ Ion capture in the S2C2 RF bucket was studied
- ❑ RF frequency curve based on actual RF-system design
- ❑Maximum RF frequency can be tuned with ± 2 MHz with vertical stubs placed on the RF line
- ❑ Three cases are studied with different maximum RF-frequencies as shown in the table.



case	F_{max}	ΔF	t_0	dF/dt	ϕ_s
	MHz	MHz	μsec	MHz/msec	deg
1	92.234	0.0	142.28	-68.35	168.68
2	87.734	-4.5	66.03	-41.98	173.07
3	86.334	-5.9	20.87	-18.20	177.00

❑ These three cases have different rates of frequency change *dF/dt* at injection in the center (at $t=t_0$) and due to that also different synchronous particle phase ϕ_s. The sine of the synchronous phase (sin ϕ_s) is proportional to dF_{RF}/dt



The Bohm and Foldy theory for ion capture shows that the capture efficiency will decrease for decreasing values of **sinϕ_s** *below 0.5. However, this calculation does not consider the possibility of modifying the value of dF/dt while keeping the rotation frequency of the rotco constant*

EQUATIONS OF MOTION

$$\frac{d\phi}{dt} = 2\pi F_{\rm RF}(t)\left(1 - \frac{hF_{\rm P}}{F_{\rm RF}}\right)$$

$$\frac{d\Delta E}{dt} = F_{\rm RF}\frac{Nq\hat{V}(t)}{h}(\sin\phi - \sin\phi_s)$$

t = time

ϕ = RF-phase

$d\Delta E$ = energy deviation

ϕ_s = synchronous phase

F_{RF} = RF-frequency

F_p = revolution frequency

$\hat{V}$ = dee voltage

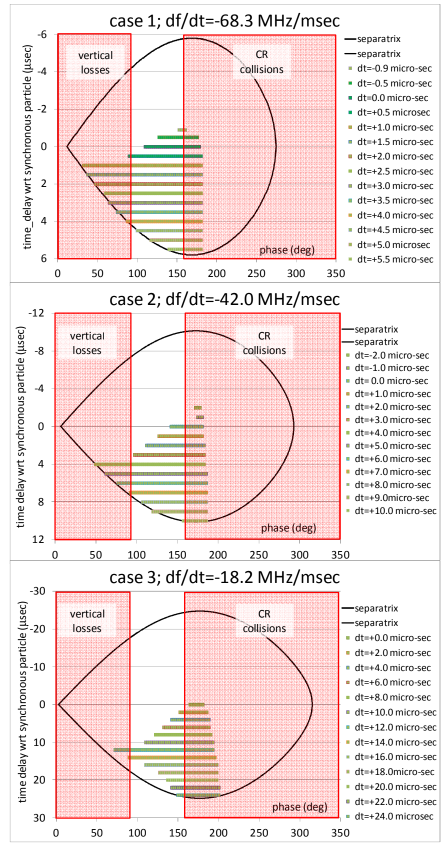
h = harmonic mode

N = number of gaps

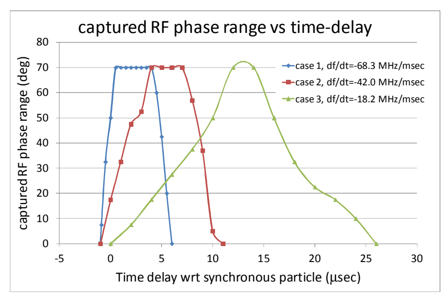
q = particle charge

SYNCHRONOUS PARTICLE

$$\frac{q\hat{V}N\sin\phi_s}{E_s} = -\frac{2\pi}{K\omega_s^2}\frac{d\omega_s}{dt}$$



Longitudinal phase space of the beam injected in the center of the S2C2. Horizontal axis is starting phase. Vertically the starting time delay relative to the synchronous particle ($t=t_0$) is plotted. Black solid lines give the separatrix. The colored dots represent particles that are captured in phase-stable orbits. Each color represents a different starting time. The red zones represent horizontal and vertical losses on the first few turns in the central region (not simulated but educationally estimated)



Capture efficiency is considered as the integral of captured RF phase range as function of the width of the time window during which particles are accepted. Case 3 having the smallest rate of frequency change dF/dt will be the most efficient despite the fact that the width of the RF-frequency window is the smallest for this case.

CONCLUSION

- ❑ the developed program allows studying ion capture and RF bucket stability for varying magnetic field-maps and RF frequency tables
- ❑The studied example confirms the trends that were found in full 3D tracking calculations that were made by AIMA

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