Cyclotron median plane errors due to asymmetric RF cavities

W. Kleeven and M. Abs

IBA is currently investigating asymmetric RF cavities for some of its PET cyclotrons. Those cavities will only occupy two upper pumping holes with dee-stems and outer cavity walls. The corresponding two lower pumping holes will be empty. The advantages of such a configuration are:

1. The four oil diffusion pumps mounted underneath the cyclotron are more efficient,
2. the cavities become considerably shorter due to the almost twice higher effective dee-capacity per cavity
3. the system becomes less expensive and more rigid.

A possible disadvantage could be the vertical asymmetry of the structure which could distort the median plane electric field. For realistic values of the vertical betatron tune and for reasonable values of the RF phase, the median plane excursion remains below 0.1 mm. It is concluded that the vertical asymmetry of the cavity does not pose any problem with respect to the beam optics.

INTRODUCTION AND METHOD

For a beam being accelerated in a cyclotron, the axial shift due to radial magnetic fields or vertical electric field can be deduced from the Hill equation, as shown below (ref).

Starting with this, an equation is derived for the electric median plane error as a function of position and RF phase.

We then used the results for the vertical median plane E-fields obtained with a CST Microwave Studio® cavity model. Typical betatron tunes for an IBA PET cyclotron are also used. Combining all these elements, the distorted median plane at any point in the cyclotron is calculated.

SIMPLIFIED MODEL FOR THE DISTORTED MEDIAN PLANE

A vertically asymmetric RF cavity structure produces a vertical electric field component in the median plane. This field will depend on the position *(r,θ)* and on the time *t*.

Hill equation driven by median plane errors

Electric error | Magnetic Error

$$\frac{d^2z}{d\theta^2}+\nu_z^2 z=\frac{1}{\omega_0 B_0}\left[E_z+vB_{r0}\right]$$

E_z depends on time and RF phase

$$E_z=E_{z0}(\theta)\sin(h\omega_0 t+\phi_{RF})$$

Isochronous cyclotron $\omega_0 t=\theta$

$$E_z=E_{z0}(\theta)\sin(h\theta+\phi_{RF})$$

Fourier analysis of median plane errors

$$\frac{d^2z}{d\theta^2}+\nu_z^2 z=\sum_{n=0}^{\infty} e_n\cos n\theta+f_n\sin n\theta$$

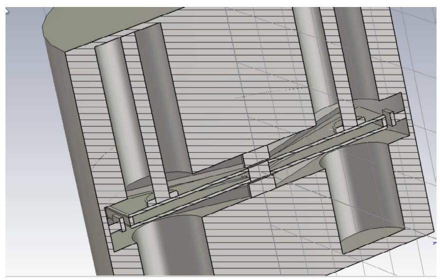
New median plane => periodic solution

$$z(\theta)=z(\theta+2\pi)$$

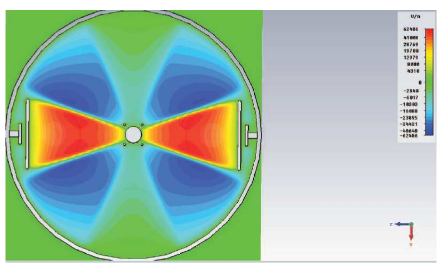
$$z_{mp}=\sum_{n=0}^{\infty}\frac{e_n}{\nu_z^2-n^2}\cos n\theta+\frac{f_n}{\nu_z^2-n^2}\sin n\theta$$

Median plane field errors appear as a driving term on the right-hand-side of the Hill equation describing the vertical betatron oscillations. In the evaluation of this driving term, we ignore the flutter of the magnetic field and approximate the equilibrium orbit by a circle. Furthermore, we assume that the orbits are isochronous. In this case, we can relate the azimuth θ with the time t, using the equation $\theta=\omega_0 t$. In this way, the driving term only is a function of position *(r,θ)* and also depends on the RF phase Φ_{RF} . This function can be written as a Fourier series with period 2π. The new distorted median plane is then found as the periodic solution of the perturbed Hill equation. The result immediately shows the presence of resonances when the vertical betatron tune ν_z approaches an integer. However, for the cyclotron under consideration, this condition does not occur.

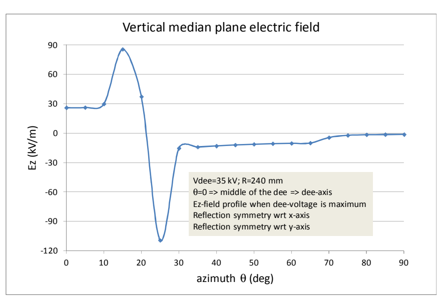
CST MODEL OF THE CAVITY



Model of an asymmetric cyclotron RF cavity as developed in CST. The dee-stems are mounted in the upper pumping holes. The lower ones are empty.

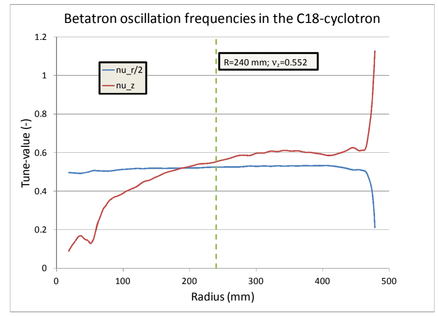


Zone plot of the vertical electric field in the median plane as calculated with the CST model. This is at the moment of maximum dee-voltage



Vertical electric field in the median plane on a circle with R=240 mm and V_{dee}=35 kVolt. This is at the moment where the dee-voltage reaches its maximum.

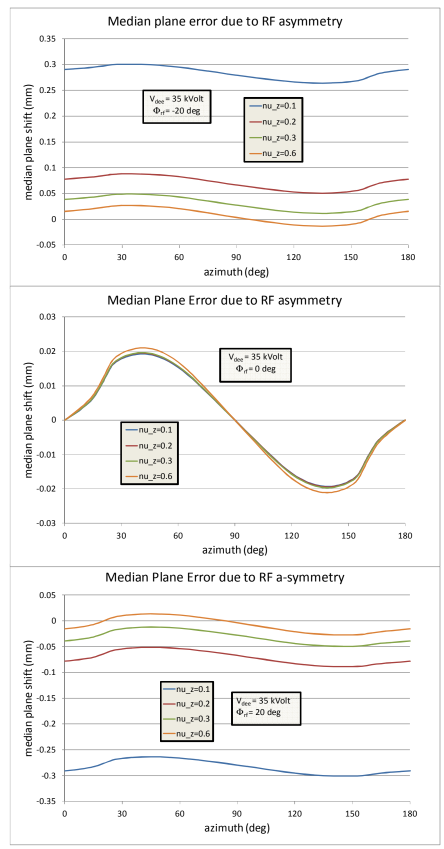
CYCLOTRON TUNE FUNCTIONS



Horizontal and vertical betatron tune functions as a function of radius in the C18-cyclotron

SOME RESULTS

For the given RF cavity geometry, the vertical electric fields are the largest at R=240 mm. The induced median plane errors were calculated at this radius. For a few different values of ν_z.



Vertical position of the distorted median plane in the C18-cyclotron at the radius of 240 mm and as a function of azimuth for a few different values of the vertical betatron function ν_z. This is for a dee-voltage of 35 kVolt and an RF phase of Φ_{RF} = -20 deg (upper), Φ_{RF} = 0 deg (middle), Φ_{RF} = -20 deg (lower),

IBA | Chemin du Cyclotron, 3 | 1348 Louvain-la-Neuve | Belgium | www.iba-worldwide.com