Space-Charge Dominated Beam Transport in High Power Cyclotrons

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Outline

1. "Traditional" understanding of isochronous cyclotrons with space charge (SC).
2. Space charge dominated acceleration.
3. Simplified model and the influence of the phase $\phi(E)$.
4. Conditions for space charge induced "longitudinal focusing".
5. RF considerations.
6. Short intro to OPAL
7. Model versus OPAL simulations.
8. Conclusions.

Consider an isochronous cyclotron with space charge.
"Naive" expectations:

- No longitudinal focussing (isochronism).
- Longitudinal space charge (SC) increases phase width.
- Phase width exceeds acceptance.
- Energy gain depends on phase ⇒ increase energy width (i.e. momentum spread).
- Large momentum spread ⇒ large beam width ⇒ high losses.

Countermeasures:

1. Flattop cavity to increase phase acceptance.
2. Increase cavity voltage: less turns ⇒ lower losses (Joho's N^3-law[1]).

[1] W. Joho, High Intensity Problems in Cyclotrons, Proc. 5th intl. Conf. on Cycl. Appl., Caen 1981.

Counterfacts:

- PSI Injector II with 2.4 mA without flattop and low losses.
- Explanation: Space charge "dominated" acceleration.
- Two bunchers in front of cyclotron (increase SC forces).
- Injector two has high ν_r and ν_z (increase SC forces).
- Works better the higher the beam current.
- Extremely contra-intuitive. And it works.
- But: What is it and how does it work?

A tiny bit of theory.

⇒ Develope simple model:

- Transverse - longitudinal only (⇒ sectors can be omitted.)
- ⇒ Use rotational symmetrie: $\vec{B} = \vec{e}_z\, B_0\, \gamma$.
- ⇒ The (matched) beam sizes are constant.
- ⇒ Space charge forces are constant.
- ⇒ Linear approximation for SC forces.
- ⇒ EQOM should have a simple solution.
- Use TRANSPORT like description in local coordinates: (horiz./vert./long./)=(x,y,z).
- First assume coasting beam, no acceleration.

Single particle dynamics:

- Radial coordinate $x = r(\theta) - r_0$ and x'.
- Longitudinal position $z = r_0\,(\theta - \theta_0)$.
- Momentum deviation $\delta = \frac{\Delta p}{p_0}$.
- Put in state vector $\psi = (x, x', z, \delta)^T$ in local co-moving coordinates.
- Define $h = 1/r_0$ as curvature of orbit.

First-Order Differential Equation

The linearized EQOM including space charge are:

$$\dot{\psi} = \mathbf{F}\,\psi\,,$$

with solution (for "average"[2] force matrix $\mathbf{F} = \frac{1}{L}\log(\mathbf{M})$):

$$\psi(s) = \exp(\mathbf{F}\,s)\,\psi(0)\,,$$

explicitely:

$$\frac{d}{ds}\begin{pmatrix} x \\ x' \\ z \\ \delta \end{pmatrix} = \begin{pmatrix} \cdot & 1 & \cdot & \cdot \\ -k_x + K_x & \cdot & \cdot & h \\ -h & \cdot & \cdot & \frac{1}{\gamma^2} \\ \cdot & \cdot & K_z\,\gamma^2 & \cdot \end{pmatrix}\begin{pmatrix} x \\ x' \\ z \\ \delta \end{pmatrix}\,,$$

Focusing terms and defocusing terms (SC) are colored. Dispersive coupling $h = 1/r_0$. Drift terms in black.

[2] R. Talman: Geometric Mechanics; 2nd Ed., Wiley-VCH Weinheim, Germany, 2007.

K_x and K_z represent horizontal and longitudinal space charge forces[3]:

$$\begin{aligned} K_x &= \frac{K_3\,(1-f)}{(\sigma_x+\sigma_y)\,\sigma_x\,\sigma_z} \\ K_z &= \frac{K_3\,f}{\sigma_x\,\sigma_y\,\sigma_z} \\ K_y &= \frac{K_3\,(1-f)}{(\sigma_x+\sigma_y)\,\sigma_y\,\sigma_z} \end{aligned} \qquad \begin{aligned} K_3 &= \frac{3\,q\,I\,\lambda}{20\,\sqrt{5}\,\pi\,\varepsilon_0\,m\,c^3\,\beta^2\,\gamma^3} \\ f &\approx \frac{\sqrt{\sigma_x\,\sigma_y}}{3\,\gamma\,\sigma_z} \end{aligned}$$

Note that always

$$\begin{aligned} K_x &> 0 \\ K_z &> 0 \end{aligned}$$

and typically

$$K_x \approx K_z \ll k_x\,.$$

[3] Frank Hinterberger, Physik der Teilchenbeschleuniger, 2. Auflage, Springer, Heidelberg 2008.

Focusing of the single-particle motion is given for a matrix **F** with imaginary eigenvalues.
Computing the eigenvalues ($\pm i\,\Omega_+$ and $\pm i\,\Omega_-$):

$$
\begin{aligned}
a &\equiv \frac{k_x - K_x - K_z}{2} \\
b &\equiv K_z\,(K_x + h^2\,\gamma^2 - k_x) \\
\Omega_+ &= \sqrt{a + \sqrt{a^2 - b}} \\
\Omega_- &= \sqrt{a - \sqrt{a^2 - b}}\,.
\end{aligned}
$$

If b is negative $\Rightarrow a < \sqrt{a^2 - b}$, $\Rightarrow \Omega_-$ imaginary, $\Rightarrow$ solution is unstable (divergent).
$\Rightarrow$ a **and** b must be positive to give real–valued frequencies.

With $b \ll a$, $K_x \ll k_x$ and $K_z \ll k_x$ and assumption of perfect isochronism: $k_x = h^2\,\gamma^2 = h^2\,\nu_r^2$, we approximate $a \approx \frac{k_x}{2}$ and $b \approx K_x\,K_z$:

$$\begin{aligned}\Omega_+ &= \sqrt{a+\sqrt{a^2-b}} \approx h\,\nu_r\,\left(1-\frac{K_x\,K_z}{k_x^2}-\dots\right)\\ \Omega_- &= \sqrt{a-\sqrt{a^2-b}} \approx \sqrt{\frac{K_x\,K_z}{2}}\,\left(1+\frac{K_x\,K_z}{2\,k_x^2}+\dots\right)\,.\end{aligned}$$

$\Rightarrow$ Ω_+ is horizontal focusing, reduced by space charge.
$\Rightarrow$ Ω_- is effective longitudinal focusing, induced by space charge and coupling.

Focusing Condition

Focusing requires

$$\begin{aligned} b &= K_z\,(K_x + h^2\,\gamma^2 - k_x) > 0 \\ \Rightarrow & \quad K_x + h^2\,\gamma^2 - k_x > 0 \end{aligned}$$

The focusing force k_x can be calculated by:

$$k_x = h^2\,(1+n) = h^2\,\left(1 + \frac{r}{B}\,\frac{dB}{dr}\right)$$

The isochronous field plus a small but important field error ε can be written as

$$B(r) = B_0\,\gamma\,(1+\varepsilon) = B_0\,\frac{1+\varepsilon}{\sqrt{1-(r/a)^2}}\,,$$

This gives

$$k_x = h^2\,\gamma^2 + \frac{1}{r}\,\frac{d\varepsilon}{dr}\,.$$

Focusing condition:

$$K_x - \frac{1}{r}\,\frac{d\varepsilon}{dr} > 0$$

$\omega_0 = N_h\,\omega_{rf}$ is nominal orbital frequency,
N_h is the harmonic number, ω real orbital frequency and ϕ is phase.
Then:

$$\varepsilon \approx 1 - \frac{\omega_0}{\omega} = -\frac{1}{2\,\pi\,N_h}\,\frac{d\phi}{dE}\,\frac{dE}{dn}\,.$$

With $\frac{dE}{dn} = V\,\cos\phi$ this gives:

$$\begin{aligned} \frac{d\varepsilon}{dr} &= \frac{d\varepsilon}{dE}\,\frac{dE}{dr} \\ &\approx -\frac{V}{2\,\pi\,N_h}\,\frac{dE}{dr}\left(\frac{d^2\phi}{dE^2}\,\cos\phi - \left(\frac{d\phi}{dE}\right)^2\,\sin\phi\right)\,. \end{aligned}$$

If $\phi \approx 0$ then:

$$\frac{1}{r}\,\frac{d\varepsilon}{dr} \approx -C_0 \times \frac{d^2\phi}{dE^2}$$

where

$$C_0 = \frac{E_0\,\gamma^3}{2\,\pi\,N_h\,a^2}\,\frac{dE}{dn}$$

Focusing condition:

$$K_x + C_0\,\frac{d^2\phi}{dE^2} > 0\,.$$

$\Rightarrow$ **Longitudinal focusing depends on phase curve!**

Take RF-effects into account:

$$\frac{d}{ds}\begin{pmatrix} x \\ x' \\ z \\ \delta \end{pmatrix} = \begin{pmatrix} . & 1 & . & . \\ -k_x+K_x & . & . & h \\ -h & . & . & \frac{1}{\gamma^2} \\ . & . & K_z\,\gamma^2+K_{rf} & . \end{pmatrix} \begin{pmatrix} x \\ x' \\ z \\ \delta \end{pmatrix},$$

Focusing terms and defocusing terms (SC) are colored. Dispersive coupling $h = 1/r_0$. Drift terms in black.

- $K_{rf} > 0$: "Debunching" phase.
- $K_{rf} < 0$: "Bunching" phase.

$$K_{rf} = \frac{q\,V_0\,\sin(\phi)}{p\,v}\,\frac{h^2\,N_h}{2\,\pi}\,.$$

- OPAL [4]: **O**bject oriented **P**arallel **A**ccelerator **L**ibrary developed at PSI (`amas.web.psi.ch`).
- Flavor OPAL -cycl dedicated for the simulation of high intensity cyclotrons.
- Space charge solver: Particle in cell (PIC)-method to compute space charge potential.
- FFT-method for solving electrostatic forces.
- Parallel computing allows to track 10^5 or more particles simultaniously in the cyclotron.
- OPAL uses MAD language with extensions.
- Other flavors for beam transport lines / Linacs available.

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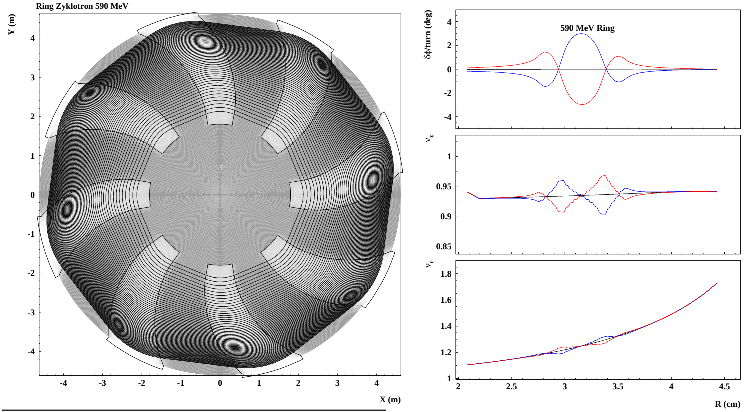
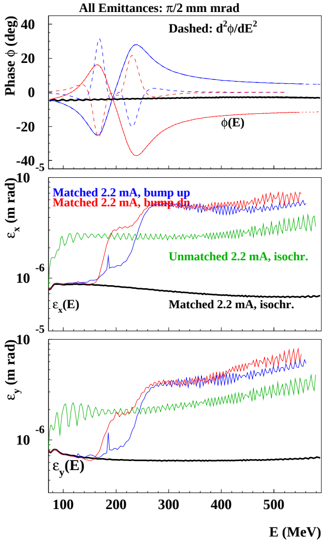
[4] J. J. Yang, A. Adelmann, M. Humbel, M. Seidel, and T. J. Zhang, Phys. Rev. ST Accel. Beams 13, 064201 (2010).
Y. J. Bi, A. Adelmann, R. Dölling, M. Humbel, W. Joho, M. Seidel, and T. J. Zhang, Phys. Rev. ST Accel. Beams 14, 054402 (2011).

1. Create "ideal" ring machine: Geometry similar to ring machine.
2. Adjust perfect or distorted isochronism (see figure).
3. Compute matched beam distribution [5].

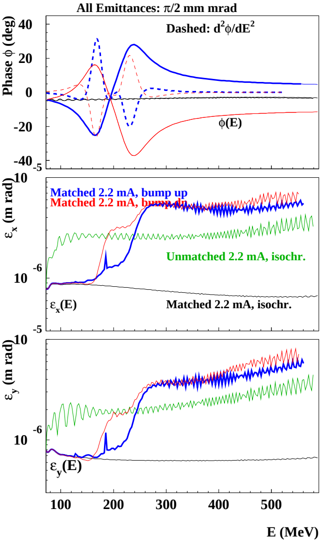


[5] C. Baumgarten; Phys. Rev. ST Accel. Beams 14, 114201 (2011) and 114002 (2011). C. Baumgarten; arXiv:1201.0907 (2012), submitted to Phys. Rev. ST Accel. Beams.



Matched beam, flat phase (black):

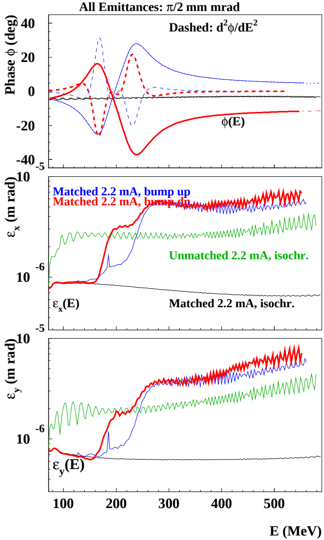
(Matched Beam



Matched beam, blue phase:

(Matched Beam

Matched Beam in Ideal Cyclotron III

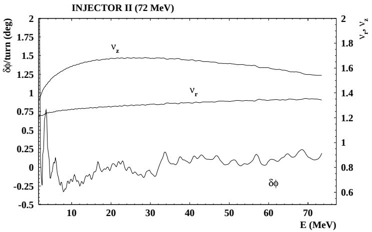


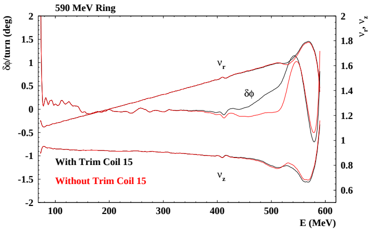
Matched beam, red phase:

(Matched Beam

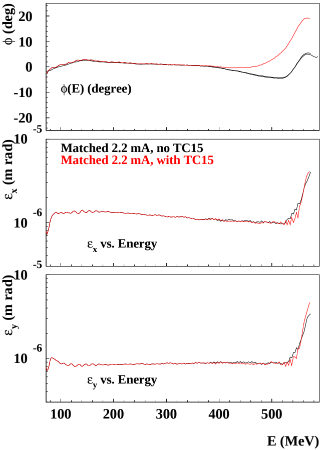
Conditions at PSI Cyclotrons

	PSI Injector II	PSI Ring Cyclotron
Energy	0.86..72 MeV	72..590 MeV
Phase Curve	flat	excursion before extraction
ε at Injection[6]	1.3 $\mu\, m\, rad$	2.5 $\mu\, m\, rad$
ε at Extraction	2.5 $\mu\, m\, rad$	7.5 $\mu\, m\, rad$
Flattop	no	yes
ν_r	$1.2 \ldots 1.35$	$1.1 \ldots 1.8$
ν_z	$1.35 \ldots 1.7$	$0.7 \ldots 0.9$
Buncher	1st and 3rd harm.	10th harmonic





[6] Here: horizontal emittance. Measured by D. Reggiani.



Matched beam, red phase, with TC15:

(Matched Beam in RING)

Effect of Longitudinal Defocusing

What, if the cyclotron is not well-prepared?

1. If the longitudinal focusing frequency is imaginary, the beam expands longitudinally.
2. The horizontal-longitudinal coupling increases also horizontal beam size.
3. The beam expansion reduces space charge forces.
4. The reduced space charge forces reduce focusing.
5. $\Rightarrow$ filamentation $\Rightarrow$ **irreversible** increase of emittance.
6. $\Rightarrow$ increased extraction losses.

Conclusions

First conclusion: "Simple" linear matching model works - even in case of space charge. But: Iteration required for accurate solution[7].

If high power cyclotrons ("dream machines") are supposed to take advantage of longitudinal focusing by space charge, ...

- ...the injected beam should be matched.
- ...the phase curve should be flat over all turns.
- ...a high beam brightness is required (PSI-Ring: $\varepsilon \leq 1.5\,\mu\, m\, rad$ at $2.2\,\mathrm{mA}$).
- ...the focusing frequency ν_z should be as high as possible.
- ...the cyclotron optics should be simulated before the finalization of design.

[7] C. Baumgarten; Phys. Rev. ST Accel. Beams 14, 114201 (2011).

Conclusions

In order to achieve space charge dominated beam transport in the PSI Ring machine...

- ...the emittance of the injected beam must be small enough.
- ...the matching into INJECTOR II should be optimized.
- ...superbuncher has to be commissioned/optimized.
- ...the injected beam must be matched.
- ...the phase curve must be corrected (new Trim Coils/Shimming).

Thank you.

References

[1] J. J. Yang, A. Adelmann, M. Humbel, M. Seidel, and T. J. Zhang, Phys. Rev. ST Accel. Beams 13, 064201 (2010).

[2] Y. J. Bi, A. Adelmann, R. Dölling, M. Humbel, W. Joho, M. Seidel, and T. J. Zhang, Phys. Rev. ST Accel. Beams 14, 054402 (2011).

[3] Frank Hinterberger, Physik der Teilchenbeschleuniger, 2. Auflage, Springer, Heidelberg 2008.

[4] R. Talman: Geometric Mechanics; 2nd Ed., Wiley-VCH Weinheim, Germany, 2007.

[5] C. Baumgarten; Phys. Rev. ST Accel. Beams 14, 114201 (2011).

[6] C. Baumgarten; Phys. Rev. ST Accel. Beams. 14, 114002 (2011).

[7] C. Baumgarten; arXiv:1201.0907 (2012), submitted to Phys. Rev. ST Accel. Beams.