# Fields extrapolation in the axial injection and central region of cyclotrons

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The Cyclone 30XP is a multi-particle, multiport cyclotron capable of accelerating alpha particles up to 30 MeV (electrostatic extraction), deuteron (D-) beams between 7.5 and 15 MeV and proton (H-) beams between 15 and 30 MeV (stripping extraction). The cyclotron injection line, based on the Cyclone 70 design, has different external ion sources connected to a recombination magnet. The beam is axially injected in the cyclotron median plane through a spiral inflector.
This poster highlights the method of magnetic field extrapolation in the inflector volume from measurements of the magnetic field in the median plane of the cyclotron and magnetic field along the injection axis.

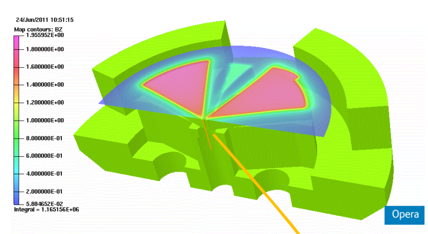
## INTRODUCTION

Tracking particles in the inflector of a cyclotron requires 3D magnetic field data in the whole injection volume. However, we are experimentally, limited to the measurement of magnetic field in the median plane and along the injection axis. Extrapolation of these two sets of data is therefore required to obtain the magnetic field in the injection volume.

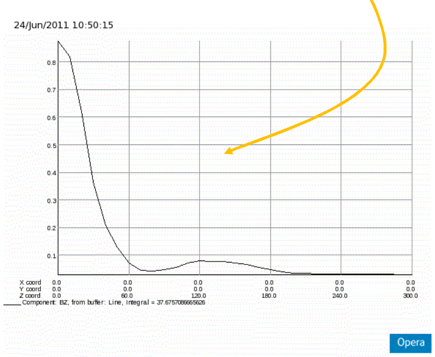
We first developed both sets of magnetic field measurement using a Taylor expansion and then we merged the data to obtain magnetic field value in the whole injection volume.

The comparison between the full OPERA-3D magnetic field data and the extrapolated data was done to validate the method.

## MAGNETIC FIELD DATA




*Magnetic field data in the median plane every degrees from 0 to 360° and every radius from 0 to 950mm every 10mm.*




*Magnetic field data along the injection axis: every 10mm from 0 to 300mm*

## TAYLOR EXPANSION

- Magnetic field in the **median plane**:
  - Assumed median plane symmetry
  - Solving Maxwell equation for:

$$B_r(r,\theta,z)=\sum_{n=0}^{\infty}\alpha_n(r,\theta)\frac{z^n}{n!}$$

$$B_\theta(r,\theta,z)=\sum_{n=0}^{\infty}\beta_n(r,\theta)\frac{z^n}{n!}$$

$$B_z(r,\theta,z)=\sum_{n=0}^{\infty}\gamma_n(r,\theta)\frac{z^n}{n!}$$

  - leads to 1st order to:

$$B_r(r,\theta,z)=z\left.\frac{\partial B_z}{\partial r}\right|_{z=0}$$

$$B_\theta(r,\theta,z)=\frac{z}{r}\left.\frac{\partial B_z}{\partial \theta}\right|_{z=0}$$

$$B_z(r,\theta,z)=\left.B_z\right|_{z=0}$$

- Magnetic field along the **injection axis**:
  - Assumed rotational symmetry
  - Solving Maxwell equation for:

$$B_r(r,\theta,z)=\sum_{n=0}^{\infty}\alpha_n(z)\frac{r^n}{n!}$$

$$B_\theta(r,\theta,z)=0$$

$$B_z(r,\theta,z)=\sum_{n=0}^{\infty}\gamma_n(z)\frac{r^n}{n!}$$

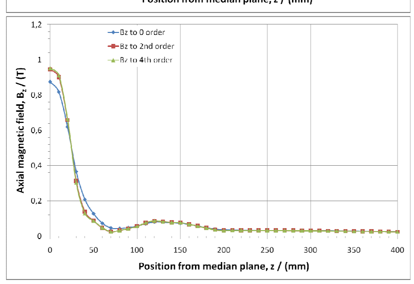
  leads to 3rd order to:

$$B_r(r,\theta,z)=-\frac{r}{2}\left.\frac{\partial B_z}{\partial z}\right|_{r=0}+\frac{r^3}{16}\left.\frac{\partial^3 B_z}{\partial z^3}\right|_{r=0}-\frac{r^5}{576}\left.\frac{\partial^5 B_z}{\partial z^5}\right|_{r=0}$$

$$B_\theta(r,\theta,z)=0$$

$$B_z(r,\theta,z)=\left.B_z\right|_{r=0}-\frac{r^2}{4}\left.\frac{\partial^2 B_z}{\partial z^2}\right|_{r=0}+\frac{r^4}{64}\left.\frac{\partial^4 B_z}{\partial z^4}\right|_{r=0}$$

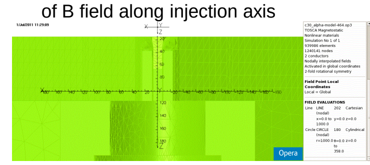
  - @ r=20mm from axis: 3rd order is sufficient


*Taylor expansion at r=20mm from injection axis of radial (top) and axial (bottom) magnetic field up to different orders.*

- **Merging** both sets of data:
  - None of the two field extrapolation provide a good solution for the complete inflector volume
  - Different combinations were tried unsuccessfully:
    - $z<z_{lim}$: mid-plane field, otherwise: injection axis field
    - At each point, look at the distance up to z axis ($d_z$) versus distance up to median plane ($d_{xy}$):
      - For $d_z > d_{xy}$ : use mid-plane field
      - For $d_z < d_{xy}$ : use injection axis field
  - BUT both combinations give rise to singularities at the transition between the two field maps

- **Merging** both sets of data (cont'd):
  - Solution:
    - for $z \le$ *pole half-gap*: use Taylor expansion of mid-plane field normalized by $B_{z\text{-}inj}(r=0)/B_{z\text{-}mid}(r=0)$
    - for $z >$ *pole half-gap* : use Taylor expansion of B field along injection axis

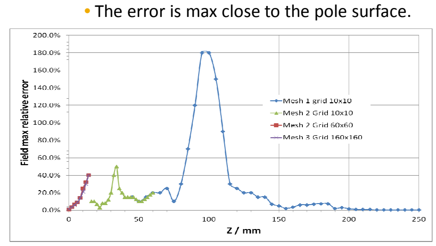



## RESULTS

- To evaluate the error introduced by the method, we compared the magnetic field value provided by the extrapolation method described here with the magnetic field from the equivalent TOSCA 3D model. This was done for different mesh grids.

| Mesh | X range (mid plane) | Y range (mid plane) | Z range (inject. axis) | Field data |
|---|---|---|---|---|
| 1 | 0-32mm | 0-32mm | 45-250mm | Inject. axis |
| 2 | 0-60mm | 0-60mm | 0-60mm | Merged/inj. |
| 3 | 0-160mm | 0-160mm | 0-16mm | Merged |

- The figure below show the maximal relative error on the magnetic field over each mesh $(B_{taylor}-B_{3D})/B_{3D}$
  - **Large errors** can be observed but they occur at distance **far away from the injection axis or mid-plane**, where only few particles venture.
  - The error is nearly zero in regions where most particles are located (injection axis and mid-plane).
  - Mesh 1:
    - The error is large when passing from the 4-fold symmetry to the rotational symmetry.
  - Mesh 2:
    - For *z>pole half-gap*: space available for the beam is limited to a 10mm radius channel.
    - The peak at *z*=34mm corresponds to the position of the pole tips
  - Mesh 3:
    - The error is max close to the pole surface.




*Maximal relative error on the extrapolated field over a X-Y grid as a function of the vertical position along the injection axis.*

## CONCLUSIONS

- We developed the Taylor expansion of the magnetic field along the injection axis and in the median plane.
- We found a method to merge both magnetic field datasets to provide a new, smooth, magnetic field dataset that can be used for tracking particles in the 3D volumes of the inflector and central region of a cyclotron
- This method was successfully used to track particles in the measured magnetic field of the C30XP.