A longitudinal phase space study of H+ recapture in a synchrocyclotron

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INTRODUCTION

One of the important concerns in a synchrocyclotron for clinical use is the predictability and accuracy of the beam pulse. During studies of the longitudinal motion we have observed that protons falling out of the RF bucket but not lost, may regain stable oscillations around a subsequent synchronous particle and hence be recaptured in a later beam pulse. Such a behavior must be quantified and suppressed for a high quality treatment beam. We have extended the 1-D phase_motion code for longitudinal motion to describe the case of recaptured particles in a synchrocyclotron. The strong advantages of this 1-D approximation are i) rapid study of large numbers of particles over many thousands of turns and ii) optimization of the RF-frequency and voltage curves during the full acceleration up to the maximum radius. In particular, we can use the RF frequency and RF voltage tables as tuning parameters to eliminate recapture behaviour.

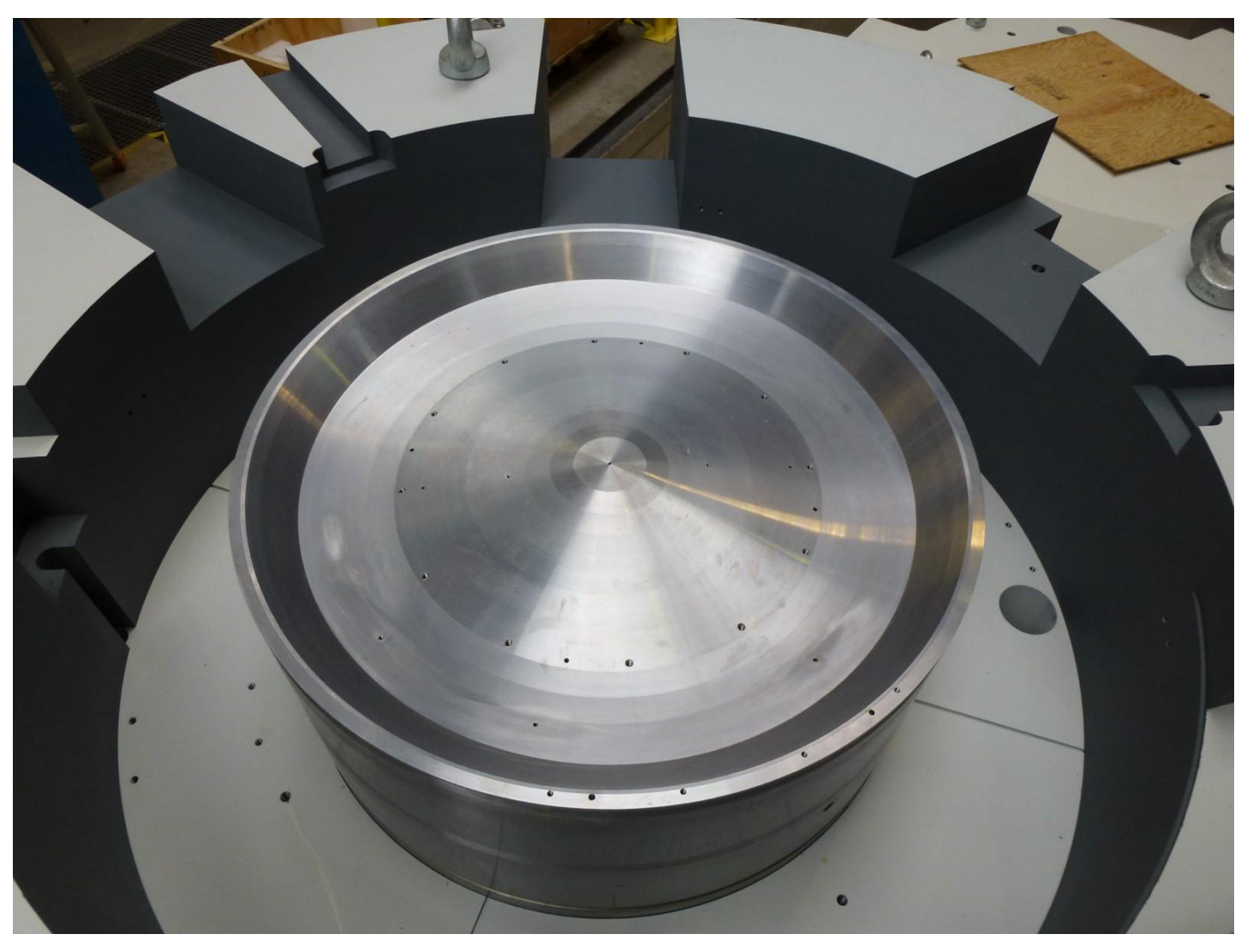

Figure 1: Lower half of the S2C2 magnet iron in the assembly hall.

EXTENSION OF THE PROGRAM

➢ **Requirements of the code:**

- Calculate the longitudinal physics of the S2C2 over multiple frequency cycles in both the acceleration and deceleration regimes
- Show if H+ recapture occurs and to what extent
- Determine how to predict and control recapture

➢ **Expansion of the time regime:**

The code has been expanded to include the full RF frequency cycle (acceleration and deceleration) over continually recurring pulses. We have also included calculations of particle properties outside of the RF bucket and in recapture scenarios.

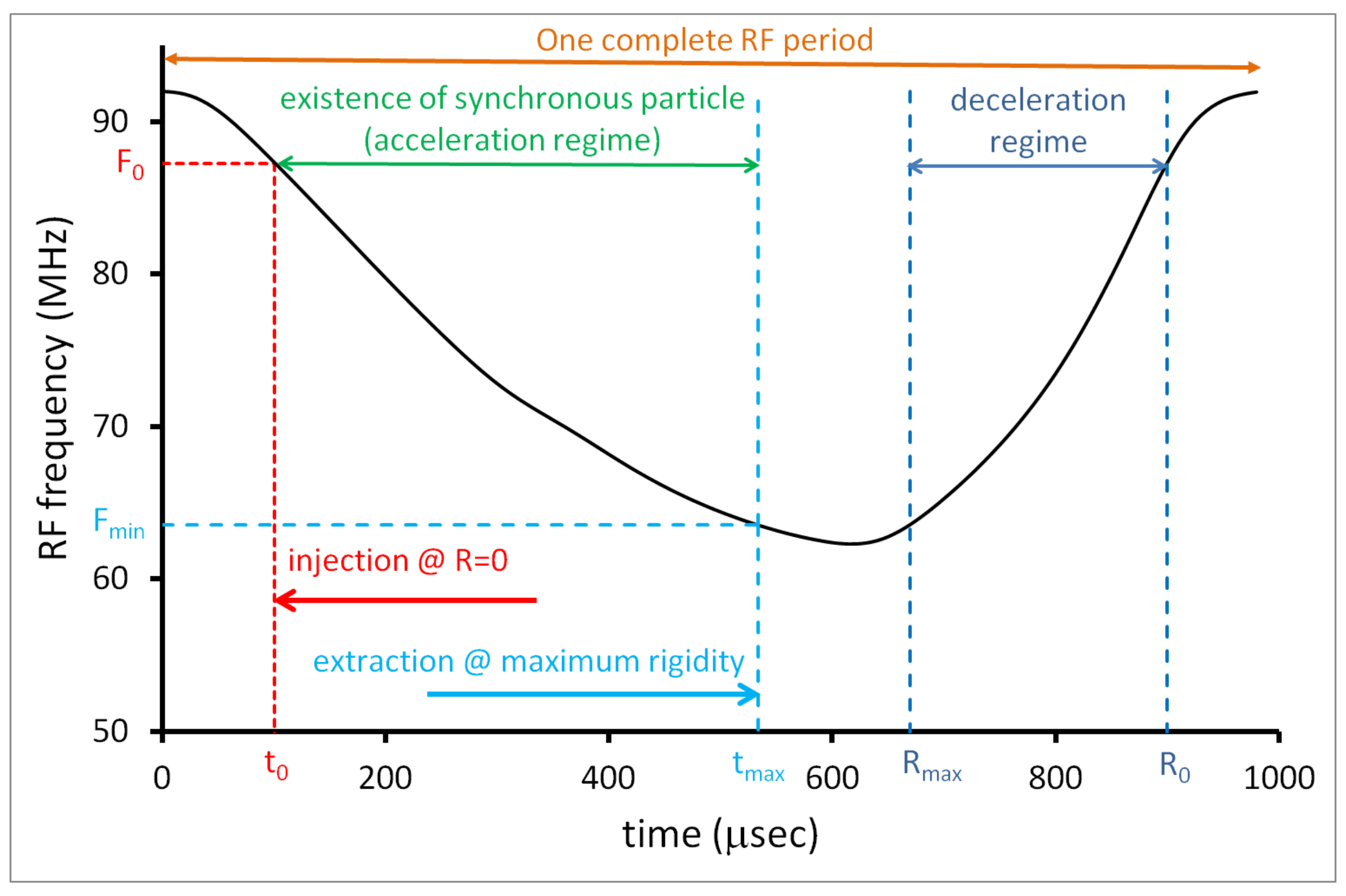

Figure 2: The RF-frequency cycle of the S2C2.

➢ **Validation of the code precision:**

The integration method used by the code must be precise over long time periods (~tens of ms). We have tested the precision of the code by theoretical cross checks derived from the Hamiltonian energy function and found that phase slip is not observed.

OBSERVED BEHAVIOUR

By selecting initial conditions we observe that it is possible for protons to fall out of and back into synchronism with the applied frequency. This resonance can be observed in both the accelerating and decelerating regimes.

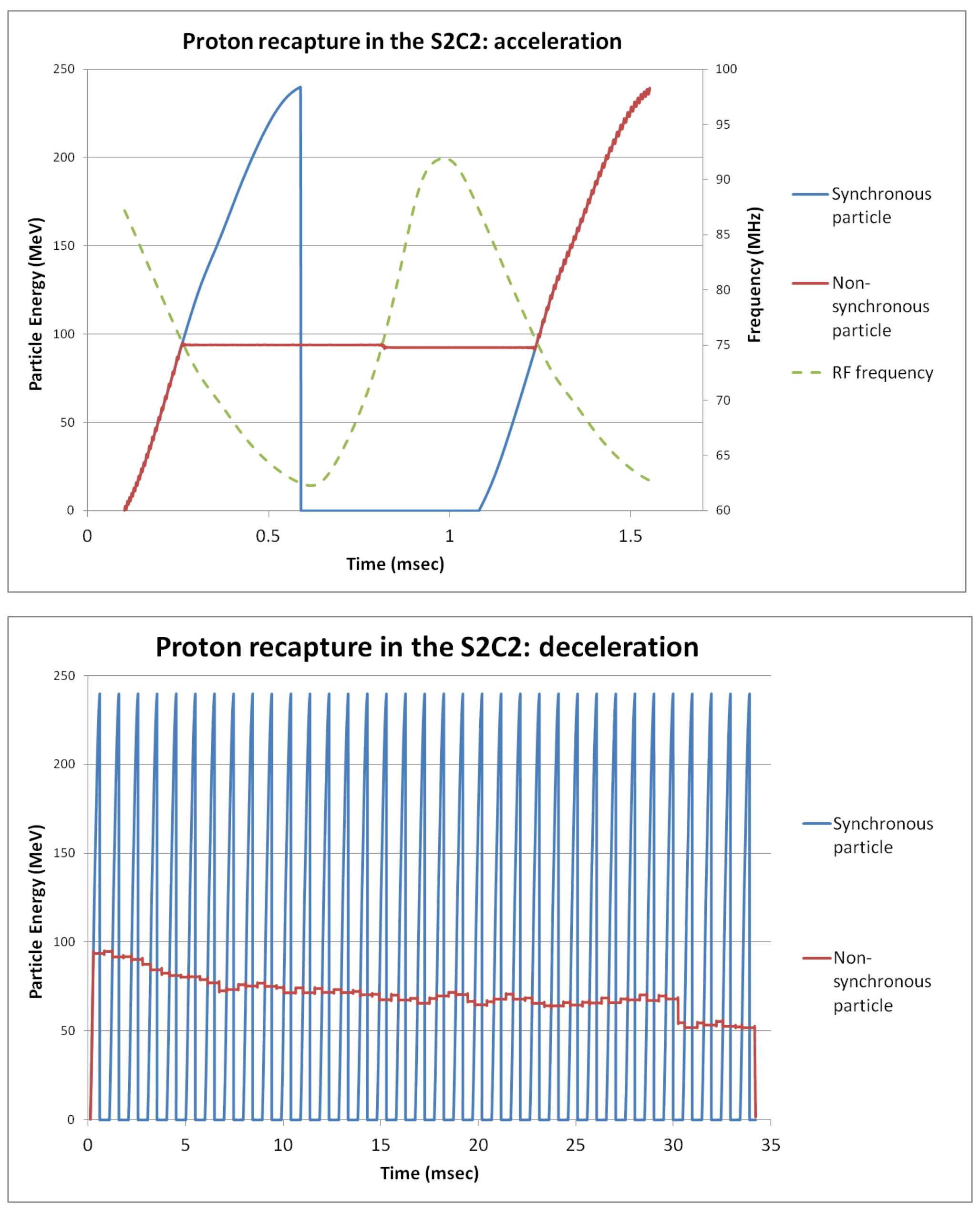

Figure 3: The top plot shows a particle which is re-accelerated after recapture whereas the bottom plot shows a particle recaptured in the deceleration regime.

We understand this behaviour in terms of two distinct separatrices – one for acceleration and one for deceleration:

Equation of acceleration separatrix:

$$\frac{1}{\Omega}\left(\frac{d\phi}{dt}\right) = \pm[(\frac{2}{cos\phi_s})(\cos\phi + \phi sin\phi_s + \cos\phi_s - (\pi - \phi_s)sin\phi_s]^{1/2}$$

Equation of deceleration separatrix:

$$\frac{1}{\Omega}\left(\frac{d\phi}{dt}\right) = \pm[(\frac{2}{cos\phi_s})(\cos\phi + \phi sin\phi_s + \cos\phi_s - (3\pi - \phi_s)sin\phi_s]^{1/2}$$

Over each complete RF period the separatrix describing the system will change from the accelerating regime (negative df/dt) to the decelerating regime (positive df/dt). Particles circulating at the edge of the accelerating separatrix can become "detached" and fall out of resonance with the synchronous particle at a fixed energy value. The separatrix is continually driven to change energy via the RF frequency curve and therefore passes the "lost" particle twice every period. There are therefore two opportunities for recapture during each RF cycle. For recapture to be successful the particle phase and energy must coincide with the phase space area of the relevant separatrix at the time of coincidence.

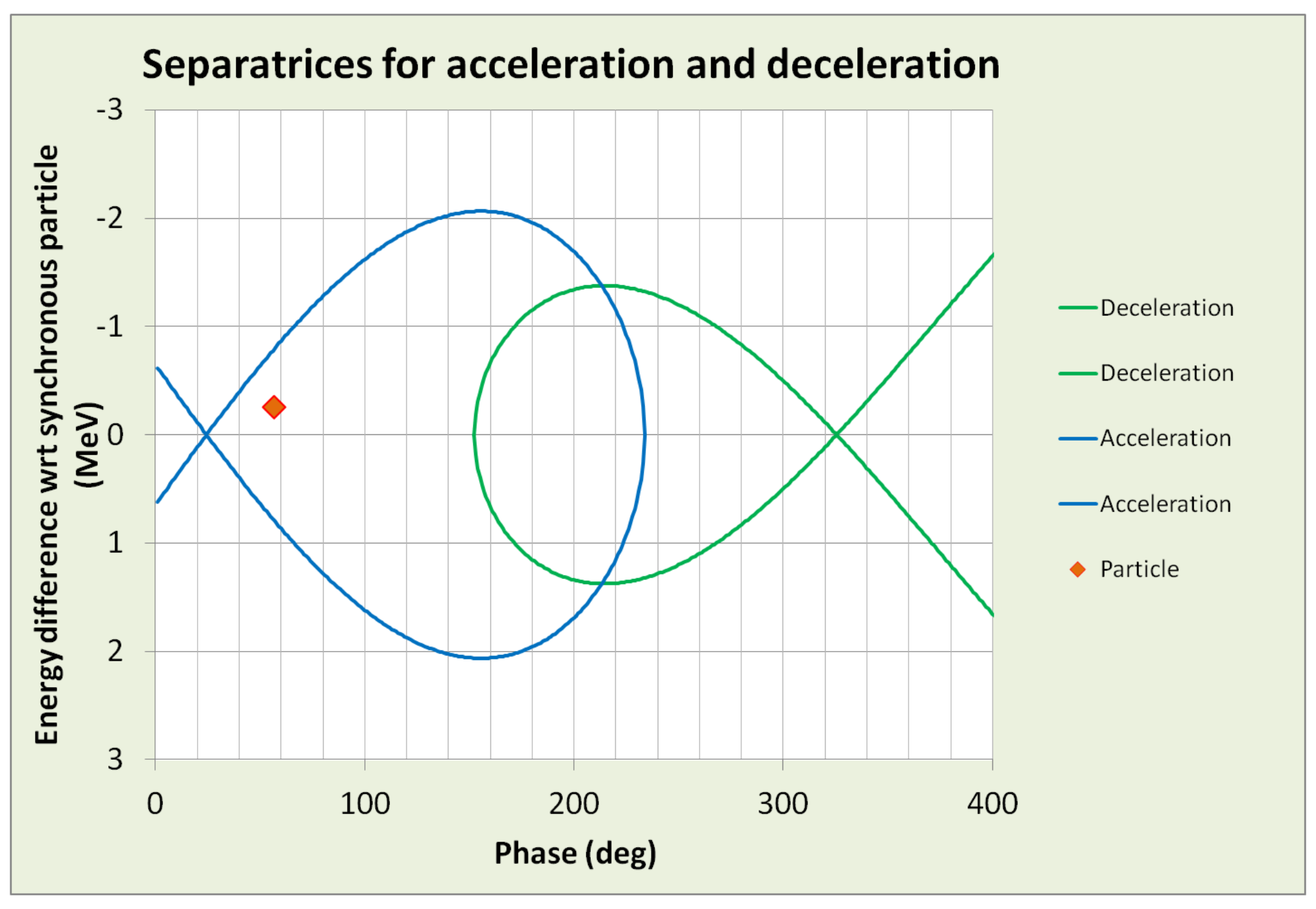

Figure 4: The separatrices for acceleration and deceleration.

APPLICATION TO MANY PARTICLE SIMULATION

We have performed a simulation of 10,000 randomly generated particles. 17% of these particles have fallen out of the RF bucket area, all of which were subsequently recaptured in the deceleration regime and therefore eventually lost by collision with the central region electrodes.
We can understand this clear bias to decelerate by considering the changing size of the bucket area.

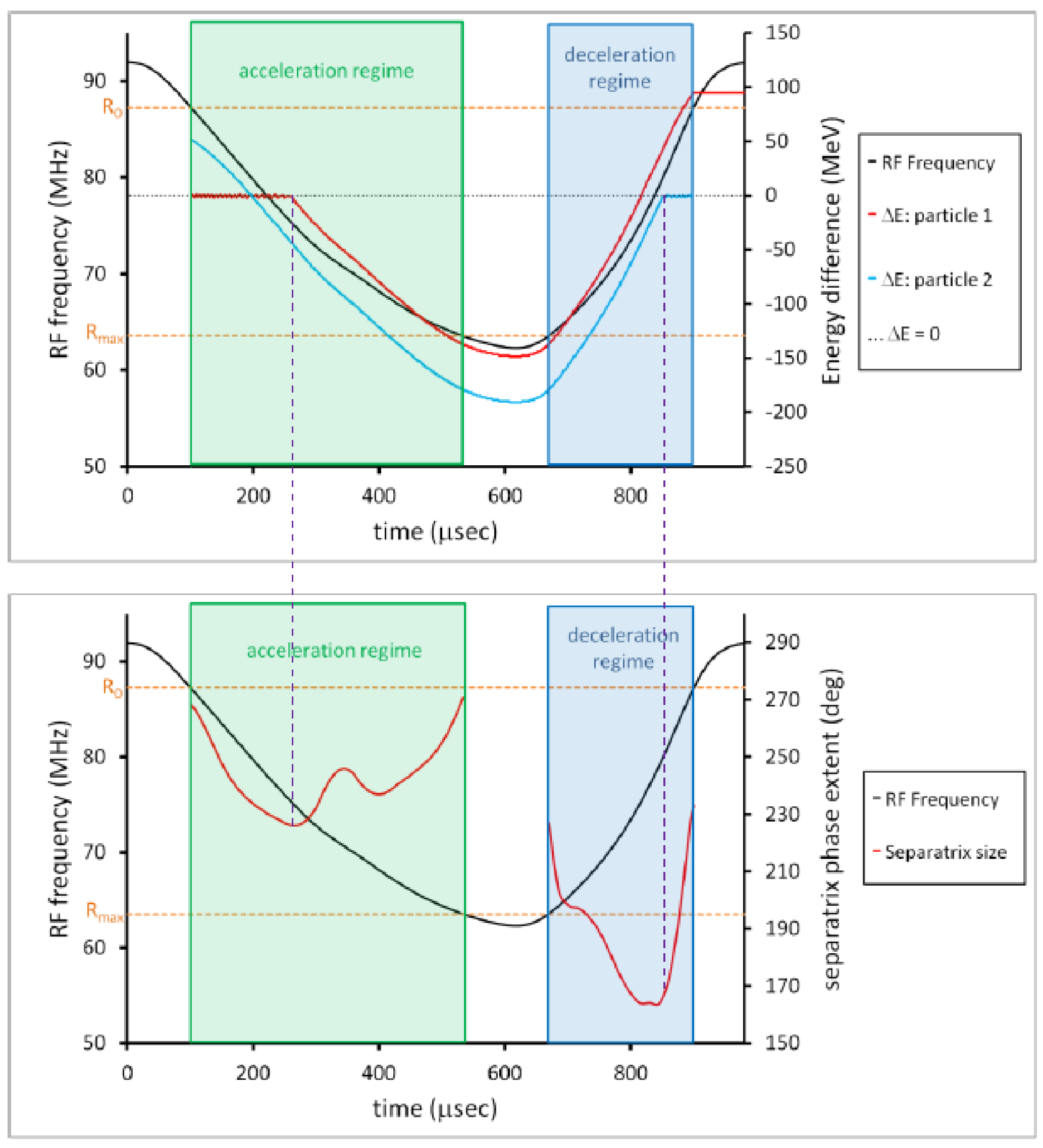

Figure 5: Particle 1 is lost as the bucket area decreases in the acceleration regime. Particle 2 is recaptured in the expanding deceleration separatrix.

Figure 5 shows that with the df/dt curve and voltage profile used here the separatrix is decreasing in size at the beginning of the acceleration regime. For this reason, accelerated particles circulating close to the edge of the separatrix easily fall out of the shrinking bucket area. Lost particles are fixed in energy as they neither accelerate or decelerate. Conversely, in the decelerating part of the RF cycle we see that as the separatrix passes the normal particle (ΔE=0), the size of the deceleration separatrix begins to rapidly increase. Particles close in energy and phase to the deceleration separatix can therefore be easily recaptured in the rapidly expanding bucket area. In this way we can explain why particles are much more likely to be recaptured into deceleration than into acceleration.
In the case where the acceleration separatrix does not decrease, particles do not fall out of the RF bucket and recapture is no longer a concern.

CONCLUSION

We have extended the IBA phase_motion code to describe the longitudinal motion of particles in a synchrocyclotron over continually recurring pulses.
We have seen that in the case of a shrinking acceleration separatrix where a proportion of particles fall out of the RF bucket, there will also exist a corresponding deceleration separatrix which rapidly expands and thus easily captures the "lost particles". These decelerated particles are terminated by collisions in the centre of the machine.
Particle fates are therefore to stay within the bucket area, or to fall out and subsequently be terminated by the deceleration regime.
In both cases the clinical beam will remain unaffected by particle recapture.

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