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Electron Transport Parameters in Gases

LTP Seminar, PSI, November 29, 2021





How large is the Lorentz angle α_{L} of drifting electrons in Helium under the following conditions? p = 8 mbarT = 12 KE = 1 kV/cmB = 5 T $\angle(E,B) = 90^{\circ}$







How large is the Lorentz angle α_1 of drifting electrons in Helium under the following conditions? p = 8 mbarT = 12 KE = 1 kV/cmB = 5 T∠(E,B) = 90° $0^{\circ} \le \alpha_{I} \le 90^{\circ}$ Ο





Can somebody use Magboltz^(*) to calculate α_{L} ?

(*)Magboltz: computer programm (CERN) that solves the Boltzmann transport equation (2-term approximation) for electrons in gases under the influence of E and B fields and calculates transport parameters

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Charges in Gaseous Medium

1) Can one calculate (or estimate) α_{L} without 'tensor gymnastics' of exact transport theory?



e.g. using momentum transfer theory: 'approximate formulas' for and relationships between experimentally measurable transport properties

2) Is this possible based only on pure E-field data, e.g. from a measurement or a calculation?



Transport Parameters - 1

volume filled with gas

free charges: • energy loss of charged particle along its track or conversion of γ

electron emission from hot cathode

external E- and B-field (homogenous, constant)

- electrons and positive ions drift towards their corresponding electrode
- after a few collisions: equilibrium of energy loss in collisions and energy gain between collisions

 \rightarrow constant transport properties: $v_{\rm d,e^-}$, $v_{\rm d,ion^+}$, D_T , D_L , $\alpha_{\rm L}$, ε

remark:

- characteristic energy: $\frac{D}{K} \rightarrow \varepsilon = \frac{3}{2} \cdot \frac{D}{K}$
- transport coefficient: mobility $K \leftrightarrow v_d = K \cdot E$ (linking the flow of a property with the "force" which causes it)



Transport Parameter - 2

drift of charge:

- typical values in operation of gaseous detectors :
 - $v_{d,e}$ ~ 1-10 cm/µs
 - [□] v_{d,ion^+} ~ 1000-times slower usually kT-limit → constant mobility K → $v_{d,ion^+} = K \cdot E$
 - □ D_T , D_L ≈ 100-500 µm/√1cm
 - mean energy $\varepsilon \approx \text{few eV}$
 - $^{\rm o}~$ magnetic deflection angle / Lorentz angle $\alpha_{\rm L}~$ \approx up to few tens of degrees

Why are these properties of interest? In particular the electron drift velocity v_d ?



Large Area Gaseous Detectors

- 1908: first wire counter to study natural radioactivity E.Rutherford, H.Geiger, 1908 Geiger, 1913
- 1928: Geiger-Müller counter with single electron sensitivity H.Geiger, W.Müller, 1928 H.Geiger, W.Müller, 1928, 1929
- 1945: proportional tubes H.Raether, 1949
- **1968:** multi-wire proportional chambers G.Charpak, 1968 G.Charpak *et al.*, 1968







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- 1945: proportional tubes H.Raether, 1949
- **1968:** multi-wire proportional chambers G.Charpak, 1968 G.Charpak *et al.*, 1968
- **1992:** Nobel Price in Physics: G.Charpak "...for his invention and development of particle detectors, in particular the multiwire proportional chamber."







multi-wire proportional chamber (MWPC):

- spatial resolution given by granularity of anode wires (several mm)
- field distortions due to (electrostatic) displacements of anode wires









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The drift chamber was "born" in Heidelberg:

- 1971: A. H. Walenta, J. Heintze and B. Schürlein, Nuclear Instruments and Methods 92 (1971) 373-380
- 1972: PhD thesis A. H. Walenta: "Lokalisierung von Teilchenspuren durch Messung der Elektronendriftzeiten in grossflächigen Proportionalzählern"



A.H.Walenta

J.Heintze





Botzmann's Transport Equation

transport of charged particles in neutral gases under the influence of electric and magnetic fields is described by the Boltzmann equation

$$\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{r}} + \vec{a} \frac{\partial f}{\partial \vec{v}} = -J(f)$$

left-hand side:

• changes of *f* due to the independent (collision less) motion of the ions:

$$\frac{\partial f}{\partial t} \rightarrow \text{ with time}$$

$$\vec{v} \frac{\partial f}{\partial \vec{r}} \rightarrow \text{ due to free motion of ions}$$

$$\vec{a} \frac{\partial f}{\partial \vec{v}} \rightarrow \text{ due to external forces: } \vec{a} = \frac{q}{m} (\vec{E} + \vec{v} \times \vec{B})$$

with $f(\vec{r},\vec{v},t)$ velocity distribution function

right-hand side:

- changes due to binary collisions with the neutral particles:
 - $\vec{J}(f) \rightarrow \text{charged-particle} \text{neutral-}$ molecule collision operator required: complete set of cross-sections $\sigma_{ij}(\varepsilon)$



transport parameters (e.g. v_d , ε , tan α_L , D_T , D_L) are calculated from appropriate velocity moments of $f(\vec{r}, \vec{v}, t)$.

$$v_{drift} = \langle \vec{v} \rangle = \frac{1}{n(\vec{r},t)} \cdot \int \vec{v} f(\vec{r},\vec{v},t) d\vec{v}$$

$$\varepsilon_{mean} = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{n(\vec{r},t)} \cdot \int (\frac{1}{2} m v^2) f(\vec{r},\vec{v},t) d\vec{v}$$

with: $n(\vec{r},t) = \int f(\vec{v},\vec{r},t) d\vec{v}$

The basic tasks in kinetic theory:

- "formulate" collision operator J
- solve Boltzmann equation to get $f(\vec{r}, \vec{v}, t)$
- calculate transport parameters



Boltzmann equation can be solved through a decomposition of $f(\vec{r}, \vec{v}, t)$ in spherical harmonics:

$$f(\vec{r}, \vec{v}, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_m^{(l)}(\vec{r}, v, t) Y_m^{[l]}(\hat{v})$$

with:

$$f_{m}^{(l)}(\vec{r},\vec{v},t) = w(\alpha,v) \sum_{\nu=0}^{\infty} F(\nu lm;\alpha,\vec{r},t) R_{\nu l}(\alpha v)$$

$$\alpha^{2} = \frac{m}{kT_{b}}$$

$$w(v;T_{b}) = (\frac{\alpha^{2}}{2\pi})^{3/2} \cdot \exp(\frac{\alpha^{2}v^{2}}{2})$$

$$R_{\nu l}(\alpha v) = N_{\nu l}(\frac{\alpha(t)v}{\sqrt{2}})^{l} S_{l+1/2}^{(\nu)}(\frac{\alpha^{2}v^{2}}{2})$$

$$N_{\nu l}^{2} = \frac{2\pi^{3/2}v!}{\Gamma(\nu+l+3/2)}$$

$$S_{l+1/2}^{(\nu)}$$

- \leftrightarrow decomposition in velocity-space
- \leftrightarrow parameter to optimize convergence T_b = "base temperature"
- \leftrightarrow Maxwellian type weight function

 \leftrightarrow Sonine polynomial



$$f(\vec{r}, \vec{v}, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_m^{(l)}(\vec{r}, v, t) Y_m^{[l]}(\hat{v})$$

indices are a measure for:

deviation from isotropy

m deviation from Maxwellian

 \rightarrow necessary for numerical solution: truncate decomposition at $l = l_{max}$

• set $l_{max} = 1 \rightarrow l = 0, 1$ (2-term approximation) Magboltz 1 (E- and B-field) Bolsig+ (only E-field)

Biagi, NIM A 273 (1988) 533-535 Biagi, NIM A 283 (1989) 716-722 Biagi, NIM A 310 (1991) 133-136

Biagi, NIM A 421 (1999) 234-240

Hagelaar, Pitchford, Plasma Sources Sci. Technol. (2005) 722-722

Robson, Ness, Phys.Rev. A, Vol.33 (1986) 2068 Ness, Robson, Phys.Rev. A, Vol.34 (1986) 2185 Ness, Phys.Rev. A, Vol.47 (1993) 327 White *et al.*, Phys.Rev. E, Vol.60 (1999) 2231

for completeness:

MC → Magboltz 2 (E- and B-field)

• allow $l = l_{max} \rightarrow$ multi-term codes

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Units in Gaseous Electronics

collision operator in Boltzmann transport equation is proportional to N

 \rightarrow gaseous electronics scales with parameter E/N and B/N

 \rightarrow E/N: 1 Townsend = 1 Td = 10⁻²¹ Vm² (Huxley, Crompton, Elford, 1966)



particle number density N
$$N = \frac{n}{V} = \frac{k_B \cdot T}{p}$$

- Sir J.S.E.Townsend 1868-1957
- <u>@ 20°C, 1 bar:</u> 1 Td ≈ 250 V/cm 1 Hx ≈ 0.025 T

→ B/N: 1 Huxley = 1Hx = 10⁻²⁷ Tm³ (Ness, 1991) (Heylen, 1980: 10⁻²³ Tm³)



Sir L.G.H.Huxley 1902-1988





experimental conditions	extrapolation to	extrapolation to		
p = 8 mbar $T = 12 K$ $24.4x$	$p = 195 \text{ mbar} \xrightarrow{5.2x}$ $T = 293 \text{ K}$	p = 1013 mbar T = 293 K		
N = 4.83·10 ²⁴ 1/m ³	$N = 4.83 \cdot 10^{24} \ 1/m^3$	$2x \rightarrow N = 2.5 \cdot 10^{25} 1/m^3$		
E = 1 kV/cm B = 5 T	E = 1 kV/cm $B = 5 T$	2x $E = 5.2 \text{ kV/cm}$ 2x $B = 25.9 \text{ T}$		
E/N = 20.7 Td B/N = 1036 Hx	E/N = 20.7 Td B/N = 1036 Hx	E/N = 20.7 Td B/N = 1036 Hx		



Magnetic Deflection (Lorentz) Angle α_L



E×B-Feld: $E = \begin{pmatrix} 0 \\ 0 \\ -E_z \end{pmatrix} B = \begin{pmatrix} 0 \\ B_y \\ 0 \end{pmatrix} \longrightarrow v = \begin{pmatrix} v_x \\ 0 \\ v_z \end{pmatrix} \longrightarrow \text{Lorentzangle } \alpha_L$ $\tan(\alpha_L) = \frac{v_x}{v_z} = \frac{v_{E \times B}}{v_E}$

 assumption: ratio of drift velocity components equal to ratio of average forces acting on e⁻ (only valid if drifting electrons follow Maxwellian velocity distribution): Grupen 93, Kleinknecht 92

$$\vec{F}_{L} = q \cdot \vec{v}_{0} \times \vec{B} \quad F_{x} = q \cdot v_{0} \cdot B \quad v_{0} = v(E, B = 0)$$

$$\vec{F}_{E} = q \cdot \vec{E} \quad F_{z} = q \cdot E \quad \sum_{k=1}^{N} F_{k} = q \cdot E \quad \sum_{k=1}^{N} F_{k} = q \cdot E$$



Magnetic Deflection (Lorentz) Angle α_L



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• in general:

magnetic deflection coefficient $\psi = \psi(E,B) \longrightarrow$

$$\tan(\alpha_L) = v_0(E, B = 0) \cdot \frac{B}{E} \cdot \psi(E, B)$$

Huxley, Aust. J. Phys. 13 (1960) 718-737 Frost, Phelps, Phys. Rev. 126 (1962) 1621-1633





Hildebrandt, diploma thesis, University Heidelberg, 1995



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Hildebrandt, diploma thesis, University Heidelberg, 1995



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Magnetic Deflection Coefficient



Kunst, PhD thesis, University Heidelberg, 1992 Kunst, Götz, Schmidt, NIM A324 (1993) 127-140



drift velocity \vec{v}_d in case of \vec{E} and \vec{B} fields (derived from equation of motion):

$$\vec{v}_{d} = \frac{K(\varepsilon) \cdot E}{1 + \omega^{2} / v^{2}(\varepsilon)} \left[\hat{E} + \frac{\omega}{v(\varepsilon)} (\hat{E} \times \hat{B}) + \frac{\omega^{2}}{v^{2}(\varepsilon)} (\hat{E} \cdot \hat{B}) \hat{B} \right] \qquad K(\varepsilon) = \frac{e}{\mu \cdot v_{m}(\varepsilon)} \mu = \frac{m \cdot M}{m + M} \omega = \frac{e \cdot B}{m} v_{m}(\varepsilon) = n_{0} \sqrt{\frac{2\varepsilon}{\mu}} \sigma_{m}(\varepsilon)$$



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for $\vec{E} \perp \vec{B}$:
 $\vec{v}_{d} = \frac{K(\varepsilon) \cdot E}{1 + \omega^{2} / v^{2}(\varepsilon)} \left[\hat{E} + \frac{\omega}{v(\varepsilon)} (\hat{E} \times \hat{B}) \right] \qquad \omega = \frac{e \cdot B}{m}$
 $v_{m}(\varepsilon) = n_{0} \sqrt{\frac{2\varepsilon}{\mu}} \sigma_{m}(\varepsilon)$

conclusion 1:
$$\left| \vec{v}_{d_B\hat{f}ield} \right| = \sqrt{v_{d_E\hat{f}}^2 + v_{d_E\hat{f}\times\hat{B}}^2} = \frac{K(\varepsilon) \cdot E}{\sqrt{1 + K^2(\varepsilon) \cdot B^2}}$$

e.g. of interest in view of Tonks' theorem



drift velocity \vec{v}_d in case of \vec{E} and \vec{B} fields (derived from equation of motion):

$$\vec{v}_{d} = \frac{K(\varepsilon) \cdot E}{1 + \omega^{2}/v^{2}(\varepsilon)} \left[\hat{E} + \frac{\omega}{v(\varepsilon)} (\hat{E} \times \hat{B}) + \frac{\omega^{2}}{v^{2}(\varepsilon)} (\hat{E} \cdot \hat{B}) \hat{B} \right] \qquad K(\varepsilon) = \frac{e}{\mu \cdot v_{m}(\varepsilon)}$$
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$$v_{m}(\varepsilon) = n_{0} \sqrt{\frac{2\varepsilon}{\mu}} \sigma_{m}(\varepsilon)$$

conclusion 2:
$$\tan(\alpha_L) = \frac{v_{d_{\hat{E}} \times \hat{B}}}{v_{d_{\hat{E}}}} = \frac{\omega}{\nu(\varepsilon)} = K(\varepsilon) \cdot B$$



arbitrary angle between E and B field

from momentum transfer theory^(*):

$$\tan(\alpha_L) = \frac{K(\varepsilon) \cdot B \cdot \sin \theta}{\sqrt{1 + K^2(\varepsilon) \cdot B^2 \cdot \cos^2 \theta}}$$

for $\theta = 90^{\circ}$:

$$\tan(\alpha_L) = K(\varepsilon) \cdot B$$

(*) momentum transfer theory:

- semi-quantitative alternative to rigorous numerical solution of Boltzmann's equation
- provides formulas for and relationships between experimentally measurable transport properties,

 $\theta = \angle(E,B)$

- e.g. Wannier energy relation, Blanc's law, generalised Einstein relation
- assumes weak energy dependencies of $v_m(\varepsilon)$, Taylor series around some reference energy $\overline{\varepsilon}$ converges rapidly, and assumes $\overline{\varepsilon} = \varepsilon_{CM}$
- based on energy and momentum balance equations
- exact for constant v_m (Boltzmann model), works surprisingly well even if not the case (10% level)



Equivalent Field Concept

concept of equivalent electric field \vec{E}_e :

« \overline{E}_e is the electrical field for B = 0 required to keep the mean energy ε at the same value as in the situation with E and B»

$$\varepsilon(E,B) = \varepsilon(E_e,0)$$

from momentum transfer theory:

$$E_e = E \cdot \cos(\alpha_L)$$

further more: Tonks' theorem $v_d(E,B) = v_{d_{Efield}}(E_e)$

Tonks, Phys. Rev. 51 (1937) 744-747 Tonks, Allis, Phys. Rev. 52 (1937) 710-713 Tonks, Phys. Rev. 97 (1955), 1443-1445

in case of set of measurements, including
$$\alpha_{l}$$
: $\kappa = \frac{v_{d}(E,B)}{v_{d_Efield}(E_{e})} = 1 \pm 10\%$



Equivalent Field Concept

concept of equivalent electric field \vec{E}_e :

approach 1:
$$E_e = \frac{E}{\sqrt{1 + K^2(\varepsilon) \cdot B^2}}$$

approach 2:
$$E_e = \frac{1}{K(\varepsilon)} \sqrt{\frac{2}{M} \left(\varepsilon + \Omega(\varepsilon) - \frac{3}{2} k_B T\right)}$$

$$k_B = Boltzmann's constant$$

$$\Omega(\varepsilon) = \frac{M}{2m} \cdot \varepsilon^* \cdot \frac{\sigma_i(\varepsilon)}{\sigma_m(\varepsilon)} \cdot S(\varepsilon)$$
$$S(\varepsilon) = e^{-\frac{3}{2}\frac{\varepsilon^*}{\varepsilon}}$$
$$\varepsilon^* = \text{excitation energy}$$



How can one estimate the mean energy \mathcal{E} ?

1) drift velocity
$$v_d$$
 $\varepsilon = \frac{3}{2}k_BT + \frac{1}{2}Mv_d^2 - \Omega(\varepsilon)$

Wannier relation

Wannier, Bell System Technical Journal, 32(1) (1953), 170-254 Wannier, Phys. Rev. 83 (1951) 281-289

2) characteristic energy D/K
$$\varepsilon = \frac{3}{2} \frac{D}{K}$$

possible sources for v_d, K or D/K:
 data bases with experimental results
 Bolsig+^(*) (E-field only calculation)

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^(*)BOLSIG+: computer programm (LAPLACE Laboratory, Toulouse, France) that solves the Boltzmann transport equation (2-term approximation) for electrons in gases under the influence of E field only and calculates transport parameters





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Data Bases - LXCat

O A https://fr.lxcat.net/home/

HOME HOW TO USE CONTRIBUTORS DATA CENTER ONLINE CALCULATIONS DOCS AND LINKS DISCUSSION BOARD about the project » news and events » statistics and geography » the lxcat team

About the project

The **Plasma Data Exchange Project** is a community-based project which was initiated as a result of a public discussion held at the 2010 Gaseous Electronics Conference (GEC), a leading international meeting for the **Low-Temperature Plasma** community. This project aims to address, at least in part, the well-recognized needs for the community to organize the means of collecting, evaluating and sharing data both for modeling and for interpretation of experiments.

At the heart of the Plasma Data Exchange Project is LXCat (pronounced "elecscat"), an open-access website for collecting, displaying, and downloading electron and ion scattering cross sections, swarm parameters (*mobility, diffusion coefficient, etc.*), reaction rates, energy distribution functions, etc. and other data required for modeling low temperature plasmas. The available data bases have been contributed by members of the community and are indicated by the contributor's chosen title.



Data Bases - LXCat

0773-0	Budapest (Budapest Drift Tube Database)	Bolsig+ (only pure gases, no mixtures)					
Ном	CDAP (State-to-state electron-impact excitation rate coefficients)	CENTER ONL	INE CA	LCULATIONS	DOCS AND LINKS	DISCUSSION BOARD	
	Christophorou database	about the projec	t » news	s and events » statistics and geography » the lxcat team			
Abo	Dutton database			SCATTERING CROSS SECTIONS			
	eMol-LeHavre (eMol group LeHavre)			DIFFERENTIAL	SCATTERING CROSS SEC	TIONS	
The Plasma Da	ETHZ (ETH Zurich, High Voltage Laboratory)	ject which was i	nitiat			ls	
Electronics Cor	Heidelberg database	or the Low-Tem	pera	INTERACTION	N POTENTIALS		
in part, the well interpretation of	IST-Lisbon database	the means of co	ollec	OSCILLATOR STRENGTHS			
At the heart of t	LAPLACE (measurements after 1975)	nounced "elecso	cat"),	SWARM / TRA	NSPORT DATA		
downloading el	Phelos database	arameters (mot	oility,	A CONTRACTOR OFFICE			
functions, etc. a		ture plasmas. Ti	he av			•	
community and	UNAM database			GLOBAL SPE	CIES FILTERING		
	UT (University of Tartu)			ELECTRONS			
	Viehland database	i l		IONS			
	BOLSIG+ solver						



Electrons in Helium – Experimental Data

E-field only configuration









Electrons in Helium – Bolsig+ Calculation

E-field only configuration





Electron-Helium Scattering Cross Sections $\sigma(\epsilon)$

• e⁻ in He



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Lorentz Angle α_L - Results

«How large is the Lorentz angle α_L of drifting electrons in Helium under the conditions N = 4.8310²⁴ 1/m³, E = 1 kV/cm, B = 5 T, \angle (E,B) = 90°?»

- → calculation with Bolsig+ of ε , K(E,B=0) and D/K(E,B=0) to estimate ε and K (experimental values from measurements agree more or less...)
- ightarrow LXCat database to extract values for $\sigma_{\rm m_{-He}}(\varepsilon)$, $\sigma_{\rm i_{-He}}(\varepsilon)$

$$\begin{aligned} \tan(\alpha_L) &= K(\varepsilon) \cdot B \text{ , } \cos(\alpha_L) = \frac{E_e}{E} \\ \text{Magboltz calculation} \end{aligned} \qquad \begin{aligned} 64.7^\circ &\leq \alpha_L \leq 71.9 \\ \alpha_L &= 66.0^\circ \end{aligned} \qquad \pm 10\% \end{aligned}$$

$$\tan(\alpha_L) = v_0(E, B = 0) \cdot \frac{B}{E} \cdot \psi(E, B) \qquad 0.92 \le \psi \le 1.33$$



Wir schaffen Wissen – heute für morgen

Summary

- Solving Boltzmann's equation and exact transport theory is not a task for the backside of an envelope
- Magboltz allows to calculate transport parameters and is common tool in gaseous detector community.
- Momentum transfer theory provides certain "approximate formulas" to estimate parameters or relations.
- LXCat offers a lot of valuable information!





Wir schaffen Wissen – heute für morgen

Acknowledgement



