--- PROTON STRUCTURE IN & OUT OF MUONIC HYDROGEN ---MUONIC ATOM SPECTROSCOPY THEORY INITIATIVE

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see also poster by Vladyslava Sharkovska

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Satellite Workshop 14. and 15.10.2022

NUCLEAR STRUCTURE EFFECTS



THEORY OF μ H LAMB SHIFT



PORTON CHARGE RADIUS



- Muonic atoms allow for PRECISE extractions of nuclear charge and Zemach radii
- CODATA since 2018 included the μ H result for r_p
- Still open issues: H(2S-8D) and H(IS-3S)
- Question:



FROM PUZZLE TO PRECISION

see talks by A.Antognini, A.Vacchi, P. Strasser, A. Knecht and B. Ohayon

- Several experimental activities ongoing and proposed:
 - IS hyperfine splitting in μ H and μ He (CREMA, FAMU, J-PARC)
 - Improved measurement of Lamb shift in μ H, μ D and μ He⁺ possible (\times 5)
 - Medium- and high-Z muonic atoms
- Theory support is needed



LAMB SHIFT IN MUONIC ATOMS

EXPERIMENT THEORY $\Delta E_{TPE} \pm \delta_{theo} \ (\Delta E_{TPE})$ Ref. Ref. $\delta_{exp}(\Delta_{LS})$ μH $2.3 \ \mu eV$ Antognini et al. (2013) $33 \ \mu eV \pm 2 \ \mu eV$ Antognini et al. (2013) $1710~\mu \mathrm{eV} \pm 15~\mu \mathrm{eV}$ Krauth et al. (2015) $3.4 \ \mu eV$ Pohl et al. (2016) μD $\mu^{3} \mathrm{He^{+}}$ $15.30 \text{ meV} \pm 0.52 \text{ meV}$ 0.05 meVFranke et al. (2017) $\mu^4 \text{He}^+$ $9.34 \text{ meV} \pm 0.25 \text{ meV}$ Diepold et al. (2018)0.05 meVKrauth et al. (2020)Pachucki et al. (2018) $-0.15 \text{ meV} \pm 0.15 \text{ meV}$ (3PE)

present accuracy comparable with experimental precision

present accuracy factor 5-10 worse than experimental precision

μH:

μ**D**, μ³He⁺, μ⁴He⁺:

HYPERFINE SPLITTING IN μ H

TSh USt THEORY INITIATIVE



Theory **compilations**, including **mixed terms** (recoil-finite sizeradiative), hadronic effects, meson contributions.

Aldo Antognini

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THEORY INITIATIVE

- Kick-off meeting to form a dedicated "Muonic Atom Spectroscopy Theory Initiate" similar to the "Muon g-2 Theory Initiate"
- Working groups, possible divisions:
 - I. μH, μD, ..., μX, Mu(?)
 - 2. Lamb shift, fine structure, hyperfine splitting
 - 3. QED, QCD = (EFTs, data-driven dispersive, lattice QCD)
- Main outcome: full SM prediction!
- More workshops foreseen, e.g.:
 - Satellite-workshop to "Nucleon Structure at Low Q", Crete, May 2023?
 - Dedicated MITP workshop in 2024?

HYPERFINE SPLITTING

Theory: QED, ChPT, data-driven **Experiment:** HFS in μ H, μ He⁺, ... dispersion relations, ab-initio few-nucleon theories **Testing the theory** discriminate between theory **Determine** predictions for polarizability Interpreting the exp. fundamental effect Guiding the exp. constants extract E^{TPE} , $E^{\text{pol.}}$ or R_{z} disentangle R_Z & ٠ find narrow 1S HFS polarizability effect by Zemach radius R_Z transitions combining HFS in H & μ H with the help of full ► test HFS theory theory predictions: • combining HFS in H & μ H Input for data-QED, weak, finite with theory prediction for driven evaluations size, polarizability polarizability effect form factors, test nuclear theories structure functions, polarizabilities Spectroscopy of ordinary atoms (H, He⁺) Electron and **Compton Scattering**

COMBINING μ H, H, He, HD+, ...



A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. 72 (2022) 389-418

INTERPLAY WITH THEORY AND OTHER EXPERIMENTS



LIMITING THEORY UNCERTAINTIES

Theoretical approaches to low-energy QCD



STRUCTURE EFFECTS THROUGH 2γ

Proton-structure effects at subleading orders arise through multi-photon processes



"Blob" corresponds to doubly-virtual Compton scattering (VVCS):

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right) \overline{T_1(\nu,Q^2)} + \frac{1}{M^2} \left(p^{\mu} - \frac{p \cdot q}{q^2} q^{\mu}\right) \left(p^{\nu} - \frac{p \cdot q}{q^2} q^{\nu}\right) \overline{T_2(\nu,Q^2)} - \frac{1}{M^2} \left(\gamma^{\mu\nu}q^2 + q^{\mu}\gamma^{\nu\alpha}q_{\alpha} - q^{\nu}\gamma^{\mu\alpha}q_{\alpha}\right) \overline{S_2(\nu,Q^2)}$$

• Proton structure functions: f_1

$$f_1(x,Q^2), f_2(x,Q^2), g_1(x,Q^2), g_2(x,Q^2)$$
Lamb shift
Hyperfine splitting
(HFS)
$$\int_N^{\gamma^*} \int_N^{\chi^*} \int_N^{\chi^*$$

Zemach radius (Hyperfine splitting)

2γ EFFECT IN THE LAMB SHIFT

$$\Delta E(nS) = 8\pi\alpha m \phi_n^2 \frac{1}{i} \int_{-\infty}^{\infty} \frac{\mathrm{d}\nu}{2\pi} \int \frac{\mathrm{d}\mathbf{q}}{(2\pi)^3} \frac{\left(Q^2 - 2\nu^2\right) T_1(\nu, Q^2) - \left(Q^2 + \nu^2\right) T_2(\nu, Q^2)}{Q^4(Q^4 - 4m^2\nu^2)}$$

dispersion relation & optical theorem:

$$T_1(\nu, Q^2) = \overline{T_1(0, Q^2)} + \frac{32\pi Z^2 \alpha M \nu^2}{Q^4} \int_0^1 dx \, \frac{x f_1(x, Q^2)}{1 - x^2 (\nu/\nu_{\rm el})^2 - i0^+}$$
$$T_2(\nu, Q^2) = \frac{16\pi Z^2 \alpha M}{Q^2} \int_0^1 dx \, \frac{f_2(x, Q^2)}{1 - x^2 (\nu/\nu_{\rm el})^2 - i0^+}$$

Caution: in the data-driven dispersive approach the T₁(0,Q²) subtraction function is modelled!

real Compton scattering / polarizabilities: J. McGovern, E. Mornacchi and E. Downie low-energy expansion:

$$\lim_{Q^2 \to 0} \overline{T}_1(0, Q^2) / Q^2 = 4\pi \beta_{M1}$$

modelled Q² behavior: $\overline{T}_1(0,Q^2) = 4\pi\beta_{M1}Q^2/(1+Q^2/\Lambda^2)^4$ determine slope of $\beta_{M1}(Q^2)$ through dilepton production: M.Vanderhaeghen

EUCLIDEAN SUBTRACTION FUNCTION

- Once-subtracted dispersion relation for $\overline{T}_1(\nu, Q^2)$ with subtraction at $\nu_s = iQ$
- Dominant part of polarizability contribution:

$$\Delta E_{nS}^{'(\text{subt})} = \frac{2\alpha m}{\pi} \phi_n^2 \int_0^\infty \frac{\mathrm{d}Q}{Q^3} \frac{2 + v_l}{(1 + v_l)^2} \,\overline{T}_1(iQ, Q^2) \text{ with } v_l = \sqrt{1 + 4m^2/Q^2}$$

- Inelastic contribution for $\nu_s = iQ$ is order of magnitude smaller than for $\nu_s = 0$
- Prospects for future lattice QCD and EFT calculations



FH, V. Pascalutsa, Nucl. Phys. A 1016 (2021) 122323

based on Bosted-Christy parametrization:

$$\Delta E_{2S}^{\text{(inel)}} \left(\nu_s = 0\right) \simeq -12.3 \,\mu\text{eV}$$
$$\Delta E_{2S}^{\text{(inel)}} \left(\nu_s = iQ\right) \simeq 1.6 \,\mu\text{eV}$$

2γ POLARIZABILITY EFFECT IN THE HFS

- Constrained by empirical information: spin structure functions, $g_1(x, Q^2)$ and $g_2(x, Q^2)$, and Pauli form factor $F_2(Q^2)$
- BχPT calculation puts the reliability of dispersive calculations (and BχPT) to the test ?!
- LO BχPT result is compatible with zero
 - Contributions from σ_{LT} and σ_{TT} are sizeable and largely cancel each other





Assuming ChPT is working, it should be best applicable to atomic systems, where the energies are very small !



PROTON ZEMACH RADIUS



TABLE I. Determinations of the proton Zemach radius $R_{\rm Z}$, in units of fm.

ep scattering		$\mu { m H}~2S$ hfs			H 1S hfs	
Lin et al. '21	Borah et al. '20	Antognini et al. '	13	LO B χ PT	Volotka et al. '04	LO $B\chi PT$
$1.054\substack{+0.003\\-0.002}$	1.0227(107)	1.082(37)		1.040(33)	1.045(16)	1.010(9)



- Empirical information on spin structure functions is limited
- Low-Q region is very important (cancelation between $I_1(Q^2)$ and $F_2(Q^2)$)



THEORY OF HYPERFINE SPLITTING

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. 72 (2022) 389-418

The hyperfine splitting of μ H (theory update):

$$E_{1S-\text{hfs}} = \left[\underbrace{182.443}_{E_{\text{F}}} \underbrace{+1.350(7)}_{\text{QED+weak}} \underbrace{+0.004}_{\text{hVP}} \underbrace{-1.30653(17)\left(\frac{r_{\text{Z}p}}{\text{fm}}\right) + E_{\text{F}}\left(1.01656(4)\,\Delta_{\text{recoil}} + 1.00402\,\Delta_{\text{pol}}\right)}_{2\gamma \text{ incl. radiative corr.}}\right] \text{meV}$$

• 2γ + radiative corrections \implies differ for H vs. μ H and IS vs. 2S



The hyperfine splitting of H (theory update):

$$E_{1S-hfs}(H) = \left[\underbrace{1418840.082(9)}_{E_{F}} \underbrace{+1612.673(3)}_{QED+weak} \underbrace{+0.274}_{\mu VP} \underbrace{+0.077}_{h VP} \right]$$

$$-54.430(7) \left(\frac{r_{Zp}}{fm}\right) + E_{F} \left(0.99807(13) \Delta_{recoil} + 1.00002 \Delta_{pol}\right) kHz$$

 2γ incl. radiative corr.

High-precision measurement of the "21 cm line" in H:

$$\delta\left(E_{1S-hfs}^{\text{exp.}}(\mathrm{H})\right) = 10 \times 10^{-13}$$

Hellwig et al., 1970

IMPACT OF H IS HFS



- Leverage radiative corrections $E_{1S-hfs}^{Z+pol}(H) = E_F(H) \left[b_{1S}(H) \Delta_Z(H) + c_{1S}(H) \Delta_{pol}(H) \right] = -54.900(71) \text{ kHz}$ and assume the non-recoil $\mathcal{O}(\alpha^5)$ effects have simple scaling $\frac{\Delta_i(H)}{m_r(H)} = \frac{\Delta_i(\mu H)}{m_r(\mu H)}$, i = Z, pol
 - I. Prediction for μ H HFS from empirical IS HFS in H

$$E_{nS-hfs}^{Z+pol}(\mu H) = \frac{E_{F}(\mu H) m_{r}(\mu H) b_{nS}(\mu H)}{n^{3} E_{F}(H) m_{r}(H) b_{1S}(H)} E_{1S-hfs}^{Z+pol}(H) - \frac{E_{F}(\mu H)}{n^{3}} \Delta_{pol}(\mu H) = \frac{\left[c_{1S}(H) \frac{b_{nS}(\mu H)}{b_{1S}(H)} - c_{nS}(\mu H)\right]}{c_{1S}(H)} = -6 \times 10^{-5} \text{ for } n = 1 = -5 \times 10^{-5} \text{ for } n = 1$$

- 2. Disentangle Zemach radius and polarizability contribution
- 3. Testing the theory

DEUTERON CHARGE RADIUS



- Precise deuteron radius from H-D IS-2S isotope shift and μH Lamb shift
- Higher-order contributions to µD Lamb shift are important:

$$E_{2P-2S}(\mu \text{D}) = \left[228.77408(38) - 6.10801(28) \left(\frac{r_d}{\text{fm}}\right)^2 - \frac{E_{2S}^{2\gamma}}{E_{2S}^{2\gamma}} + 0.00219(92) \right] \text{meV}$$

• Coulomb (non-forward) distortion (starting $\alpha^6 \log \alpha$): $E_{2S}^{\text{Coulomb}} = 0.2625(15) \text{ meV}$

• 2γ incl. eVP and 3γ contributions starting α^6 [Kalinowski, Phys. Rev. A 99 (2019) 030501]

D FORM FACTOR IN PIONLESS EFT

V. Lensky, A. Hiller Blin, V. Pascalutsa, Phys. Rev. C 104 (2021) 054003



- Only one unknown low-energy constant l_1 of a longitudinal photon coupling to two nucleons
- Agreement of chiral EFT and pionless EFT

- Use r_d and r_{Fd} correlation to test low-Q properties of form factor parametrisations
- Abbott parametrisation gives different radii

2γ EFFECT IN μ D LAMB SHIFT



	1.102(20)				
Empirical ($\mu H + iso$)					
Pohl et al. '16 [3]	-1.7638(68)				
This work	-1.7585(56)				

V. Lensky, A. Hiller Blin, FH, V. Pascalutsa, 2203.13030 V. Lensky, FH, V. Pascalutsa, in preparation

N3LO pionless EFT + higher-order single-nucleon effects:

 $E_{2S}^{\text{elastic}} = -0.446(8) \text{ meV}$ $E_{2S}^{\text{inel},L} = -1.509(16) \text{ meV}$ $E_{2S}^{\text{inel},T} = -0.005 \text{ meV}$ $E_{2S}^{\text{hadr}} = -0.032(6) \text{ meV}$ $E_{2S}^{\text{eVP}} = -0.027 \text{ meV}$

- Elastic 2γ several standard deviations
 larger
- Inelastic 2γ consistent with other results
- Agreement with precise empirical value for the 2γ effect extracted with $r_d(\mu H + iso)$

CONCLUSIONS

Theory: QED, ChPT, data-driven dispersion relations, ab-initio few-nucleon theories, LQCD

Experiment: HFS in μ H, μ He⁺, ... LS in μ H, μ D, μ He⁺, ...



Thank you for your attention!