The (Z,A) Dependence of $\mu \rightarrow e$ Conversion

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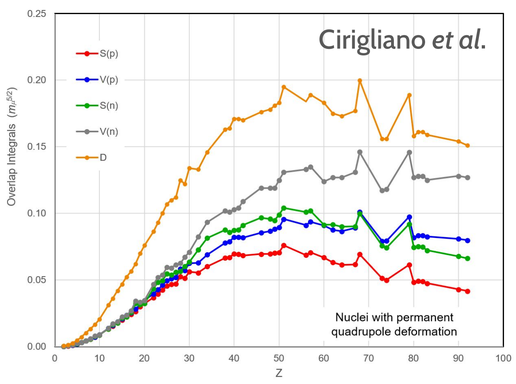
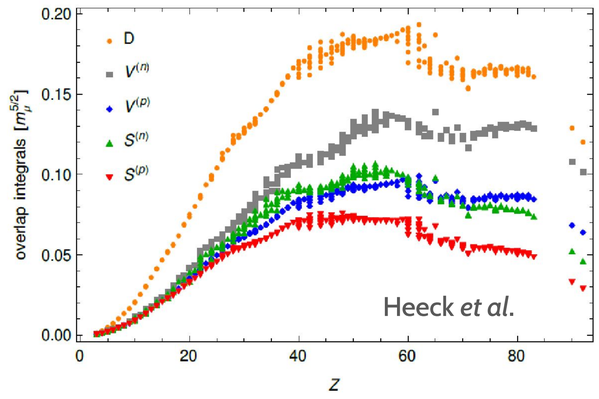
When $\mu \rightarrow e$ conversion is found, the question of the Lorentz structure of the new CLFV (Charged Lepton Flavor Violation) coupling will come to the fore. The different types of CLFV couplings (dipole, vector, scalar) produce a different (Z,A) dependence of the conversion rate. Previous studies of the (Z,A) dependence are extended by:

- Inclusion of muonic X-ray data on nuclear charge distributions
- Treatment of the effect of permanent quadrupole deformations
- Inclusion of neutron distributions using a Hartree-Bogoliubov model
- A revised "normalization" proposal

Motivation

- The study of Z,A dependence of $\mu \rightarrow e$ conversion by Cirigliano *et al.*[1] (left) has recently been updated by Heeck *et al.*[2] (right). We have undertaken a new calculation.

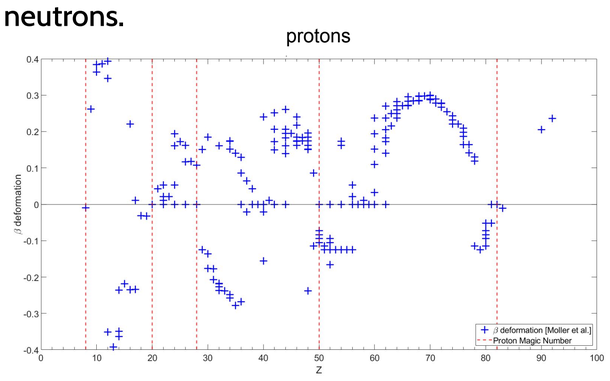


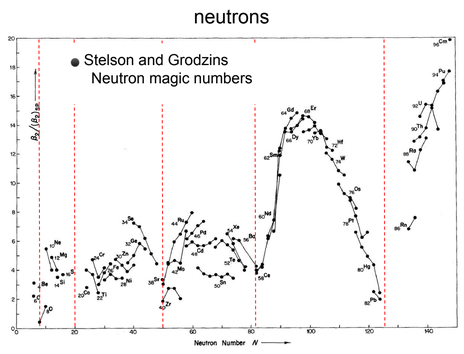


- Previous studies use electron scattering determinations of the nuclear charge distribution, assume spherical symmetry, and use charge distributions scaled by N/Z for the neutron distributions.
- We have:
  1) included muonic X-ray determinations of the nuclear charge distributions
  2) explicitly accounted for permanent quadrupole moments
  3) used a collective model for neutron distributions (which can be as much as 0.3fm larger than proton distributions, since they are typically in higher shells)
  4) propose a new sensitivity metric.

Observations

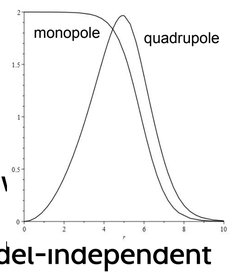
- Many nuclei, including ^{27}Al, the target used in the Mu2e and COMET experiments, have large quadrupole deformations[3,4] due to the shell model structure for protons and neutrons.





- Despite the conversion process occurring largely from the $1S$ state of the muonic atom, the deformation effects the radial nucleon distribution, particularly at the edge of the nucleus.
- We use a deformed Fermi model to represent the p and n distributions:

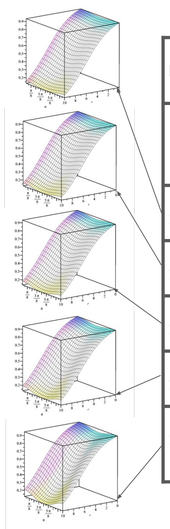
$$\rho(r) = \frac{1}{1 + exp\left[\frac{r\left(1+\frac{\beta}{4}\sqrt{\frac{5}{\pi}}(3cos^2\theta - 1)\right) - c}{t}\right]}$$



- The weighted quadrupole deformation β peaks at the edge of the nucleus, effectively modifying the skin thickness t
- We include the large amount of muonic X-ray data, particularly for nuclei with quadrupole deformations, and for many individual isotopes
- Electron scattering[5] and muonic X-ray[6] data can be combined using model-independent Barrett Moments[7] $\langle r^k e^{-\alpha r} \rangle = \frac{4\pi}{Ze}\int_0^\infty \rho_N r^k e^{-\alpha r} r^2 dr$ where the value of k and α are determined by fitting $2P_{3/2}$ - $1S$ transition energies over a wide range of nuclei.
- The error in the Barrett radii deduced from muonic atom measurements is typically smaller than the error on the same quantity calculated from electron scattering experiments.
- Rather than scale neutron distributions by N/Z, we employ the Zhang *et al.*[8] calculation that uses a relativistic Hartree-Bogoliubov model for even-even nuclei.

The onset of deformation - $_{60}$Nd Isotopes

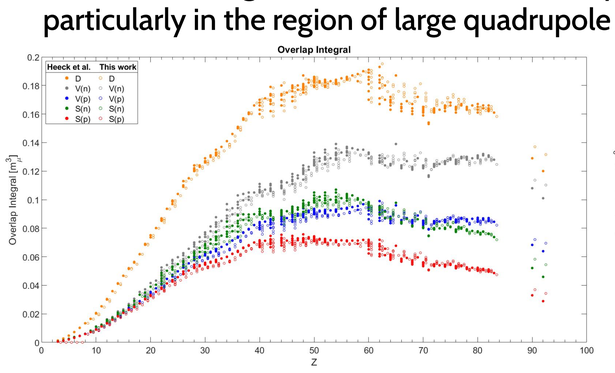
- The extensive muonic X-ray and electron scattering measurements with Nd isotopes allows comparison the effect of the onset of quadrupole deformation and the use of Barrett Moments.

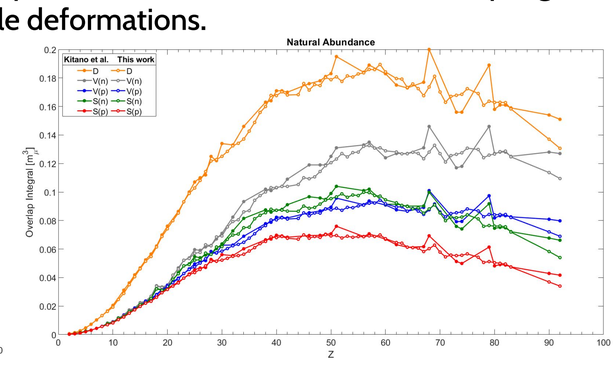


Isotope	Muonic X-rays[9]					Electron Scattering[10]				
	c	t	β	rms radius	Barrett Moment	c	t	β	rms radius	Barrett Moment
142	5.80±0.03	2.32±0.08	0.104	4.9147	15.9865	5.7045	2.539±0.013	0.104	4.9129	15.9299
144	5.85±0.03	2.27±0.08	0.123	4.9387	16.1136	5.6634	2.696±0.013	0.123	4.9444	16.0424
146	5.82±0.03	2.42±0.08	0.151	4.9767	16.2602	5.66	2.760±0.013	0.151	4.9670	16.1359
148	5.84±0.03	2.40±0.08	0.197	5.0023	16.3804	5.6871	2.798±0.022	0.197	5.0007	16.2914
150	5.86±0.03	2.35±0.08	0.279 ±0.003	5.0454	16.5754	5.7185	2.861±0.031	0.279 ±0.003	5.0478	16.5047

Results

- Accounting for permanent quadrupole deformations, using more realistic neutron distributions and the addition of muonic X-ray data through the use of Barrett moments results in changes in the calculated Z dependence of the various CLFV couplings, particularly in the region of large quadrupole deformations.





- The dip at the onset of large quadrupole deformations is reduced, and the excursions in the natural abundance plot are diminished.

Normalization

- If $\mu \rightarrow e$ conversion is observed in ^{27}Al, subsequent experiments will seek to measure the conversion rate in heavier targets. Cirigliano *et al.* calculate the Z dependence by "normalizing" the **coherent** CLFV conversion rate to the **incoherent** process of μ capture,

$$R_{\mu e}(A,Z) = \frac{\Gamma(\mu^- N(A,Z) \rightarrow e^- + N(A,Z))}{\Gamma(\mu^- N(A,Z) \rightarrow \nu_\mu + N'(A,Z-1))}$$

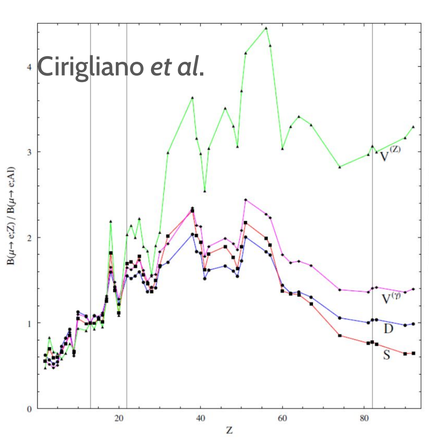
and then plotting the Z dependence divided by the same ratio for ^{27}Al:

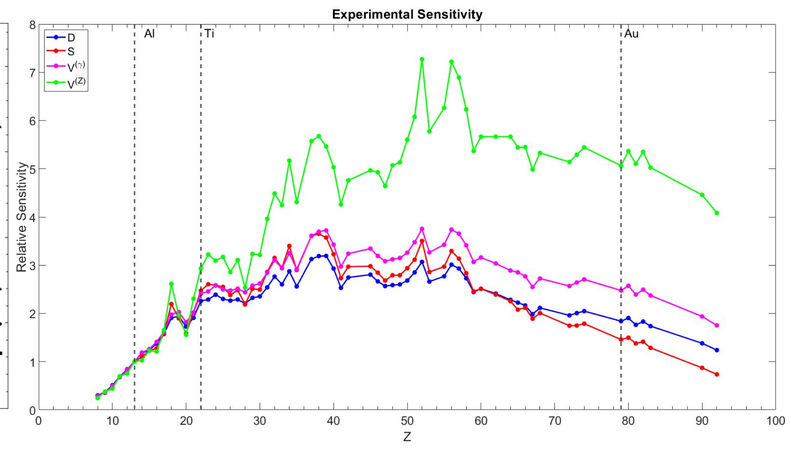
$$R_{\mu e}(27,13) = \frac{\Gamma(\mu^- + {}^{27}_{13}Al \rightarrow e^- + {}^{27}_{13}Al)}{\Gamma(\mu^- + {}^{27}_{13}Al \rightarrow \nu_\mu + {}^{27}_{12}Mg^*)}$$

- BSM theories predict a **rate** of CLFV, not a branching ratio. There's no reason, except a historical one[11], to evoke μ capture.
- **We propose an analogous plot of the relative sensitivity the Z dependence of CLFV experiments with a different normalization.**
- In these experiments, it's necessary to evaluate the experimental live time[12], which depends on the muon lifetime, τ_μ, in that atom, **not** the μ capture lifetime. Thus, all other factors being equal, the relative sensitivity of experiments is

$$R_{\mu e}(A,Z) = \Gamma(\mu^- N(A,Z) \rightarrow e^- + N(A,Z)) * \tau_\mu(Z,A) \;/\; R_{\mu e}(27,13) = \Gamma(\mu^- N(27,13) \rightarrow e^- + N(27,13)) * \tau_\mu(27,13)$$

as shown in the figure below on the right





Discussion

- The revised definition is both more consistent with the actual necessity to determine the experimental live time in calculating the calculation the measured conversion rate, and ameliorates some the (Standard Model) variation with Z due to shell model structure.
- Although the sensitivity ratio suggests titanium might outperform aluminum, in reality many additional experimental factors must be considered:
  - Timing cuts needed to remove pion backgrounds
  - Increased muon decay-in-orbit backgrounds for heavier elements due to the lower momentum of the outgoing electron
- The impact of these must be understood before making the final target choice for future muon conversion experiments.

References

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