

Lepton pair production at NNLO in QED with EW effects

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τ physics at the high-intensity frontier

Low-energy precision experiments with leptons require a precise knowledge of the SM background.

The Belle II experiment at the SuperKEKB facility will provide a world-leading precision for measurements of τ properties. Together with the proposed beam polarisation upgrade it will even be possible to probe the SM prediction of $(g-2)_\tau$. At Belle II's CMS energy of $\sqrt{s} \approx 10.5$ GeV QED effects still dominate even though EW effects start to become relevant.

What is MCMULE?

MCMULE is a framework for fully-differential higher-order calculations for low-energy processes dominated by QED. We use modern methods adapted from QCD:

- dimensional regularisation for UV- and IR-divergences
- subtraction for phase-space integration
- master integrals, automated tools, EFT methods

In particular, MCMULE is based on:

- FKS^ℓ – infrared structure
- massification – multi-scale integrals
- next-to-soft stabilisation – numerics

$\mu \rightarrow e\bar{\nu}\nu$	NNLO + resumm.
$\mu \rightarrow e\bar{\nu}\nu\gamma$	NLO
$\mu \rightarrow e\bar{\nu}\nu e e$	NLO
$e\mu \rightarrow e\mu$	NNLO
$\ell p \rightarrow \ell p$	dom. NNLO
$ee \rightarrow ee$	NNLO
$e^+e^- \rightarrow e^+e^-$	NNLO
$e^+e^- \rightarrow \gamma\gamma$	NNLO
$e^+e^- \rightarrow \ell^+\ell^-$	dom. NNLO + NLO EW



The FKS^ℓ subtraction scheme

We use the simple structure of soft singularities in QED

- ① universal (leading) soft limit

$$\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$$

- ② universal pole structure

$$e^{\hat{\epsilon}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \text{finite}$$

and combine them into an all-order subtraction scheme FKS^ℓ

$$\begin{aligned} \sigma^{(1)} &= \int_n \underbrace{\text{divergent}} + \int_n \int_1 \underbrace{\text{divergent and complicated}} \\ &= \left[\int_n \left(\text{divergent} + \hat{\mathcal{E}} \text{divergent} \right) \right] + \left[\int_n \int_1 \left(\text{divergent and complicated} - \mathcal{E} \text{divergent} \right) \right] \\ &\quad \text{② finite} \qquad \qquad \qquad \text{① complicated but finite} \end{aligned}$$

Further information: mule-tools.gitlab.io

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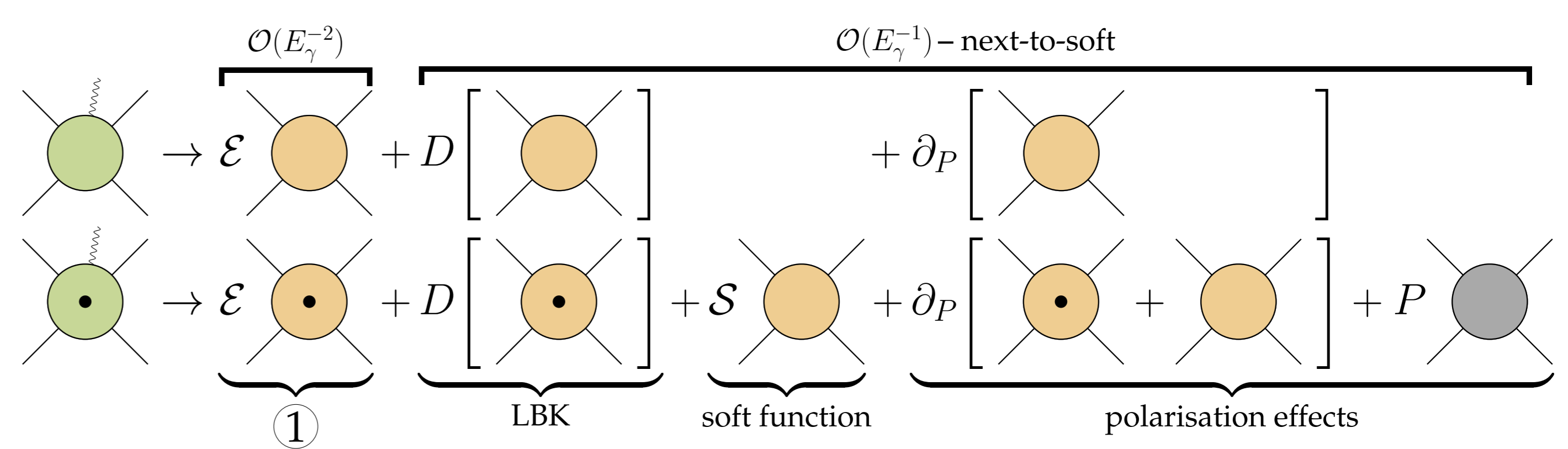
Next-to-soft stabilisation

Numerical stability during the phase-space integration of the emitted photon is one of the main challenges.

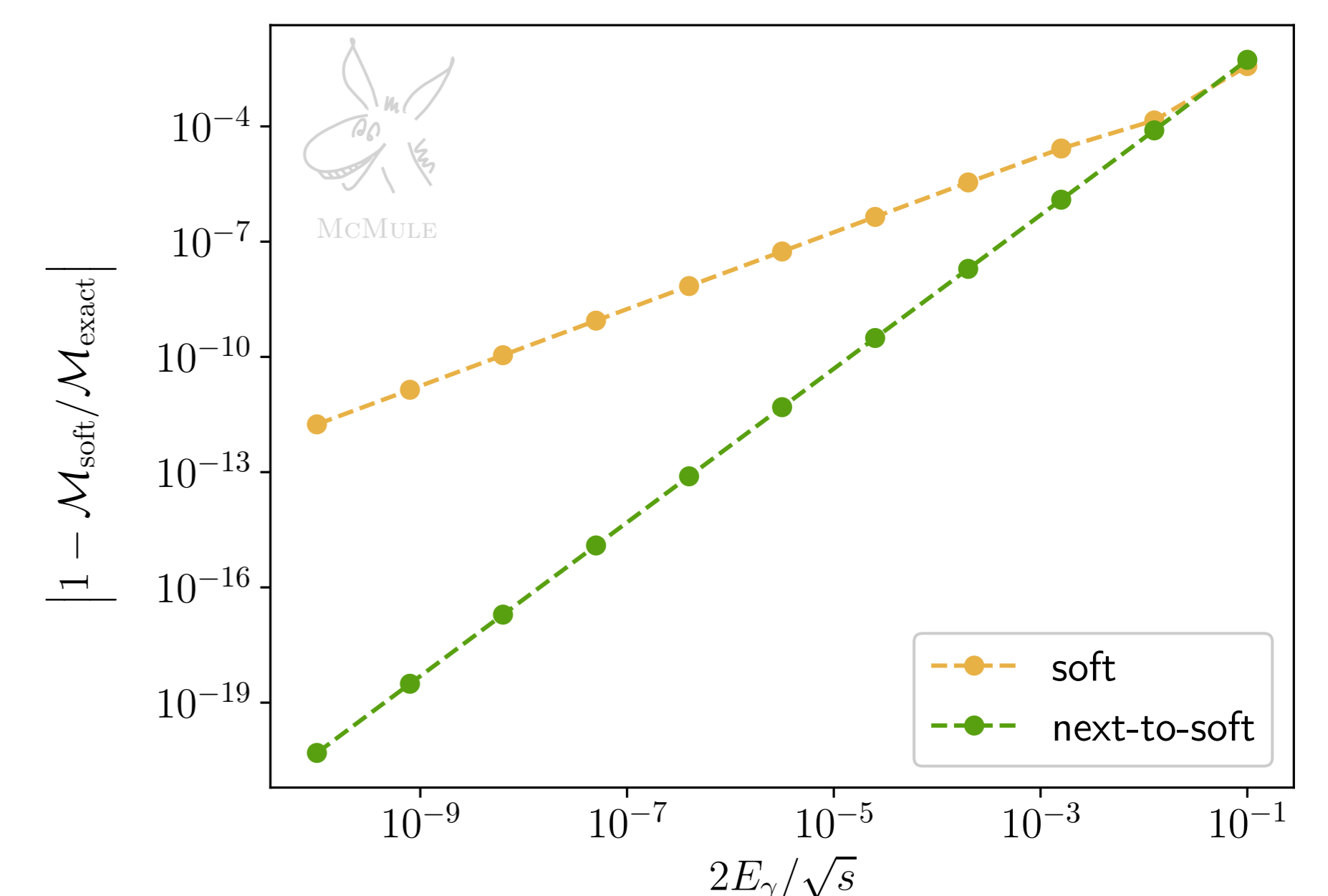
Idea: Use expanded radiative matrix element including next-to-soft contribution for small E_γ .

Subleading term is related to non-radiative matrix elements through:

- Realisation
- LBK operator $D \rightarrow \partial/\partial p_i$
 - universal soft function \mathcal{S}
 - polarisation effects $P \rightarrow$ new object

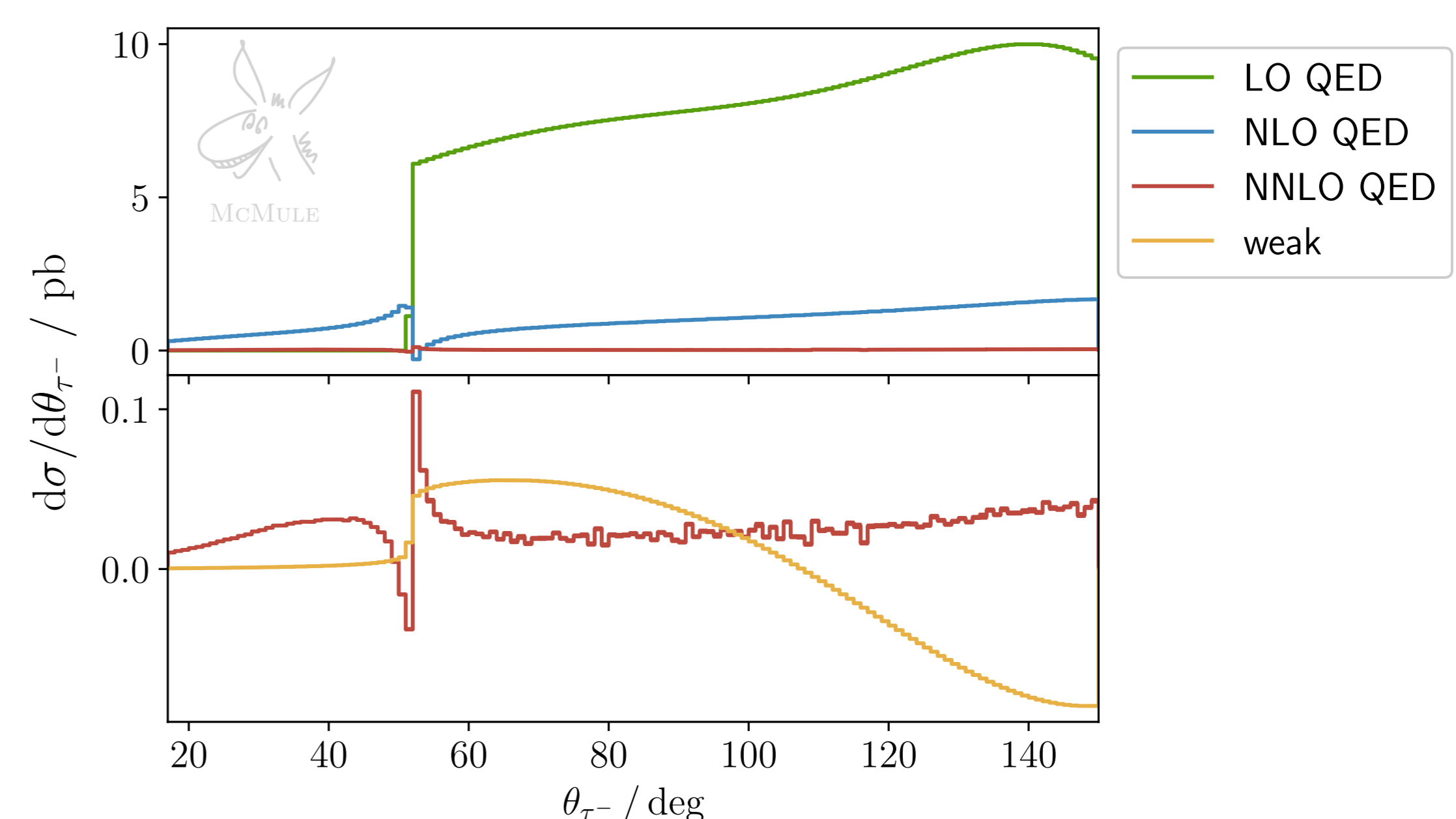
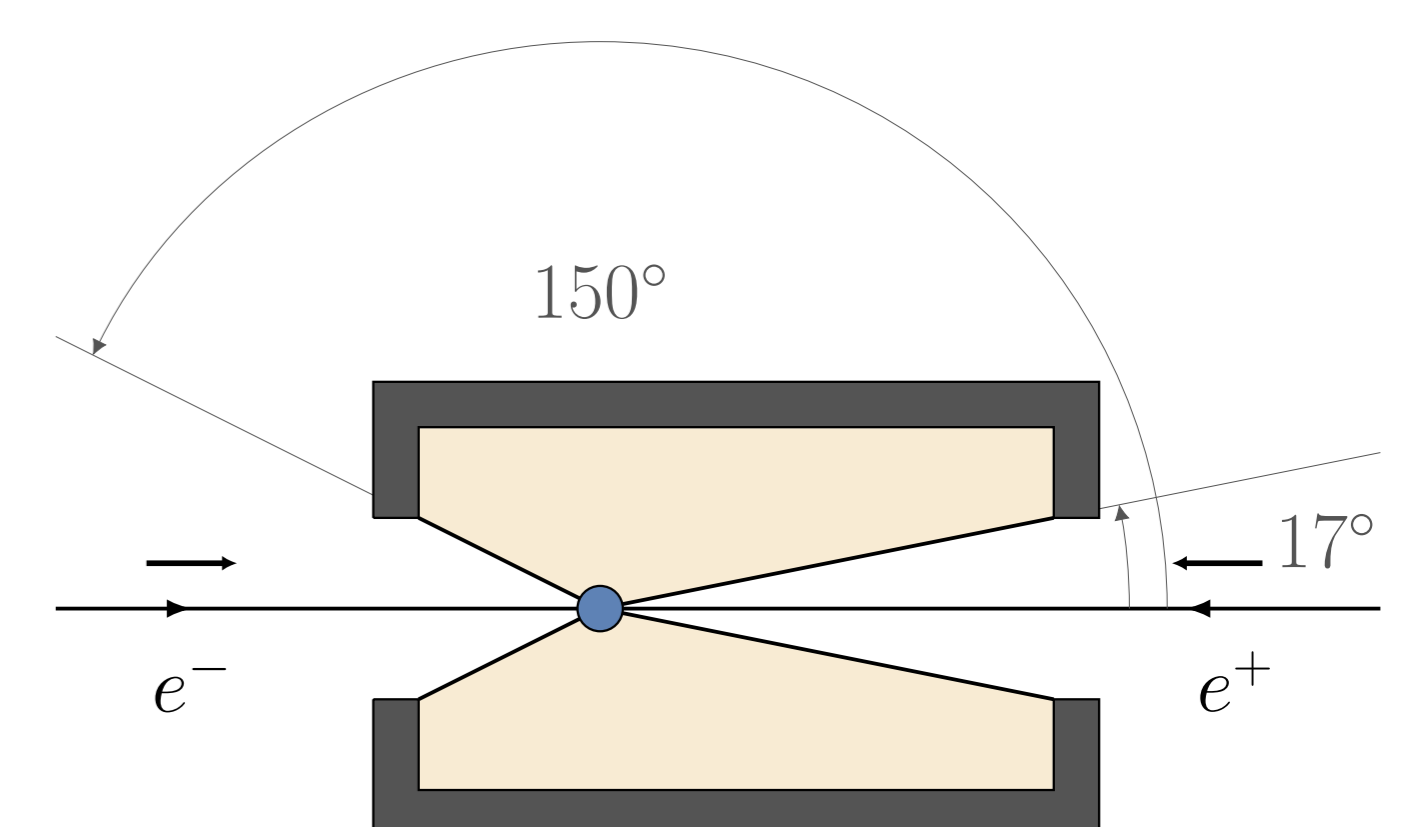


Numerical verification: dominant one-loop corrections to $ee \rightarrow \ell\ell\gamma$ against $\mathcal{M}_{\text{exact}}$ calculated with arbitrary precision in Mathematica



First Results

We present results for the process $e^+e^- \rightarrow \gamma/Z \rightarrow \tau^+\tau^-\{\gamma\gamma\}$ including dominant NNLO QED and NLO EW effects tailored to the Belle II detector with 70% longitudinal initial beam polarisation ($E_{e^+}^{(\text{in})} = 4$ GeV and $E_{e^-}^{(\text{in})} = 7$ GeV).



Cross sections & forward-backward asym. (w/o cuts on θ_i & unpolarised)

	$\sigma_{\text{QED}}^{(0)}$	$\sigma_{\text{QED}}^{(1)}$	$\sigma_{\text{QED}}^{(2)}$	$\sigma_{\text{weak}}^{(0+1)}$
σ/pb	771.641(3)	139.28(1)	2.393(6)	0.14013(6)
$\delta K^i/\%$		18.05(1)	0.4567(7)	0.015314(6)
$A_{\text{FB}}(\theta_{\tau^-})$	0	0.07573(8)	n.a.	72.94(3)