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Proton Structure through the Two-Photon Exchange

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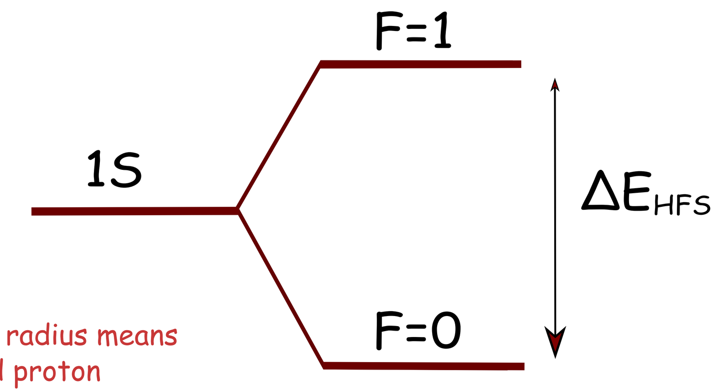
1. Motivation

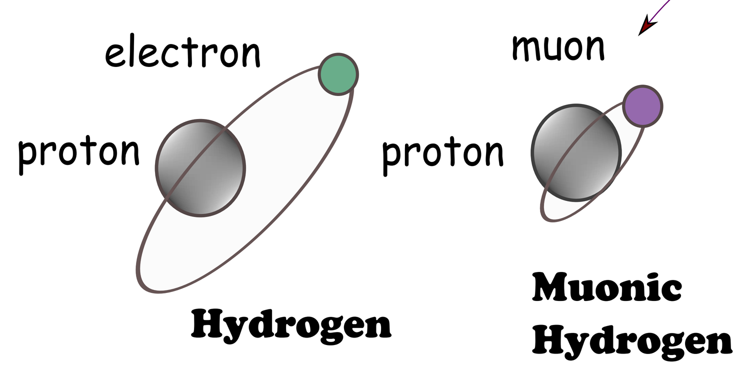
The upcoming measurements of the ground-state hyperfine splitting (**HFS**) in muonic hydrogen (**μH**) by:

- CREMA (@PSI) [1]
- FAMU (@RIKEN-RAL) [2,3]
- J-PARC/RIKEN-RAL (@J-PARC) [4]

will achieve **ppm** precision!



smaller Bohr radius means enhanced proton structure effects



[1] P. Amaro et al., SciPost Phys. 13, 020 (2022).
[2] C. Pizzolotto et al., Phys. Lett. A 403, 127401 (2021).
[3] C. Pizzolotto et al., Eur. Phys. J. A 56, 185 (2020).
[4] M. Sato et al., in Proceedings, 20th International Conference on Particles and Nuclei (PANIC 14), 2014.

2. Proton Structure in HFS

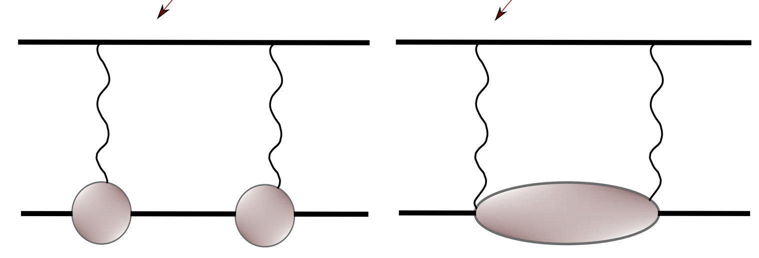
The leading order (LO) in α HFS of the nS-levels is given by the Fermi energy E_F. The subleading contributions can be split into QED, electroweak and strong corrections:

$$\Delta E_\mathrm{HFS}(nS) = \frac{E_\mathrm{F}}{n^3}(1+\Delta_\mathrm{QED}+\Delta_\mathrm{weak}+\Delta_\mathrm{strong}), \quad E_\mathrm{F} = \frac{8\,\alpha}{3a^3}\frac{1+\kappa}{mM}, \tag{1}$$

with M the proton mass, m the lepton mass, κ the proton anomalous magnetic moment, α the fine structure constant and $a = 1/(\alpha m_r)$ the Bohr radius with $m_r = mM/(m+M)$.

The proton structure, i.e. the strong correction, can be divided into three terms: Zemach radius, recoil, and polarizability contributions:

$$\Delta_\mathrm{strong} = \Delta_\mathrm{Z} + \Delta_\mathrm{recoil} + \Delta_\mathrm{pol.} \;!$$



with

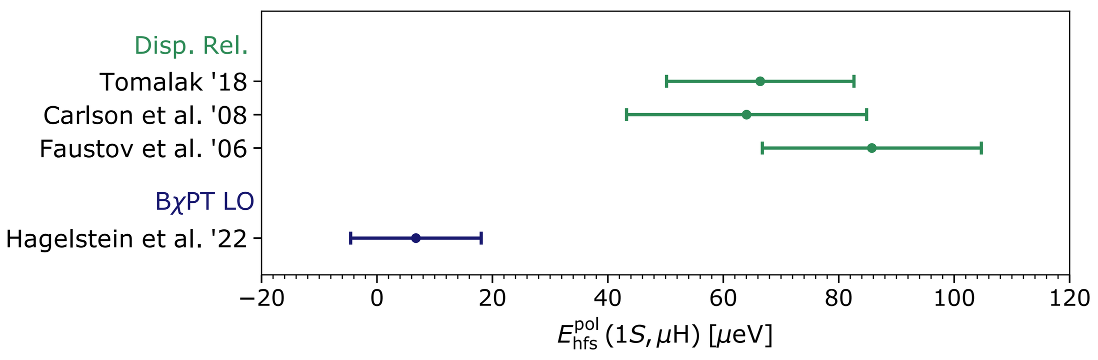
$$R_Z \equiv \int \mathrm{d}r\, r \int \mathrm{d}\mathbf{r}'\, \varrho_E(|\mathbf{r}'-\mathbf{r}|)\,\varrho_M(\mathbf{r}') \tag{2a}$$

$$\equiv -\frac{4}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa}-1\right] \tag{2b}$$

being the Zemach radius defined in terms of the charge and magnetization distributions, ϱ_E and ϱ_M, and the electromagnetic form factors, G_E and G_M.

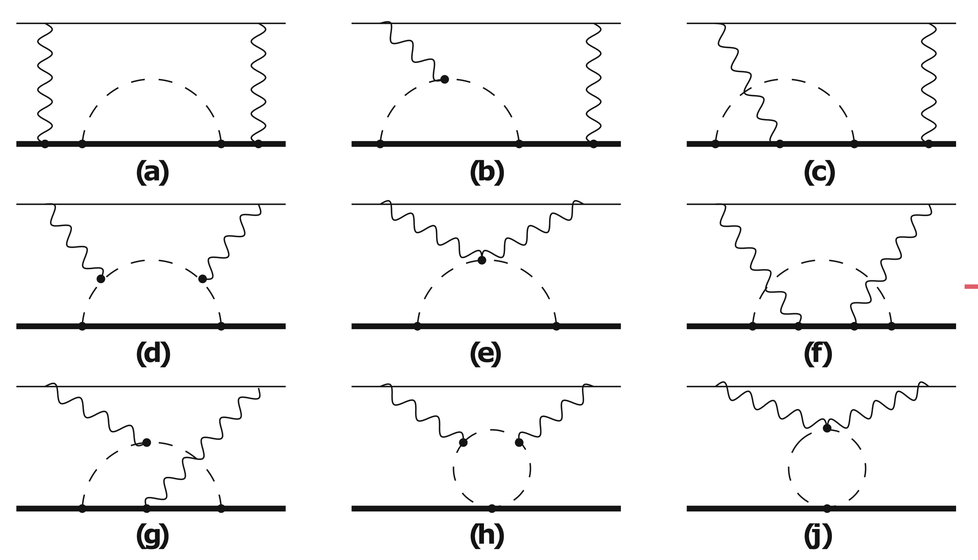
3. Polarizability Contribution as the Main Theoretical Uncertainty

The predictions for $\Delta_\mathrm{pol.}$ from the data-driven dispersive approach and the LO baryon chiral perturbation theory (BχPT) are in tension [5]:



On one hand, the possible cause for the discrepancy might lie in the scarce data for the proton spin structure function g_2, which enter the dispersive method. On the other, the tension between the two approaches might vanish when including the next-to-leading-order (NLO) BχPT.

The LO contribution is given by the following pion-nucleon loop diagrams [6]:



[5] F. Hagelstein, V. Lensky and V. Pascalutsa, in preparation.
[6] J. M. Alarcon, V. Lensky and V. Pascalutsa, Eur. Phys. J. C 74 (2014) 2852.

Usually, the polarizability effect is split into the contributions from the two spin structure functions, g_1 and g_2:

$$\Delta_\mathrm{pol.} = \Delta_1 + \Delta_2 = \frac{\alpha m}{2\pi(1+\kappa)M}(\delta_1+\delta_2), \tag{3a}$$

$$\delta_1 = \int_0^\infty \mathrm{d}Q \left[K_{F_2}(Q^2)\, F_2^2(Q^2) + \int_0^{x_0} \mathrm{d}x\, K_1(x,Q^2)\, g_1(x,Q^2)\right], \tag{3b}$$

$$\delta_2 = \int_0^\infty \mathrm{d}Q \int_0^{x_0} \mathrm{d}x\, K_2(x,Q^2)\, g_2(x,Q^2), \tag{3c}$$

with $K_1(x,Q^2)$, $K_2(x,Q^2)$, $K_{F_2}(x,Q^2)$ being the kernel functions, Q^2 the photon virtuality, $x = Q^2/(2M\nu)$ the Bjorken variable with ν the lab-frame photon energy, x_0 the pion-production threshold and $F_2(Q^2)$ the Pauli form factor.

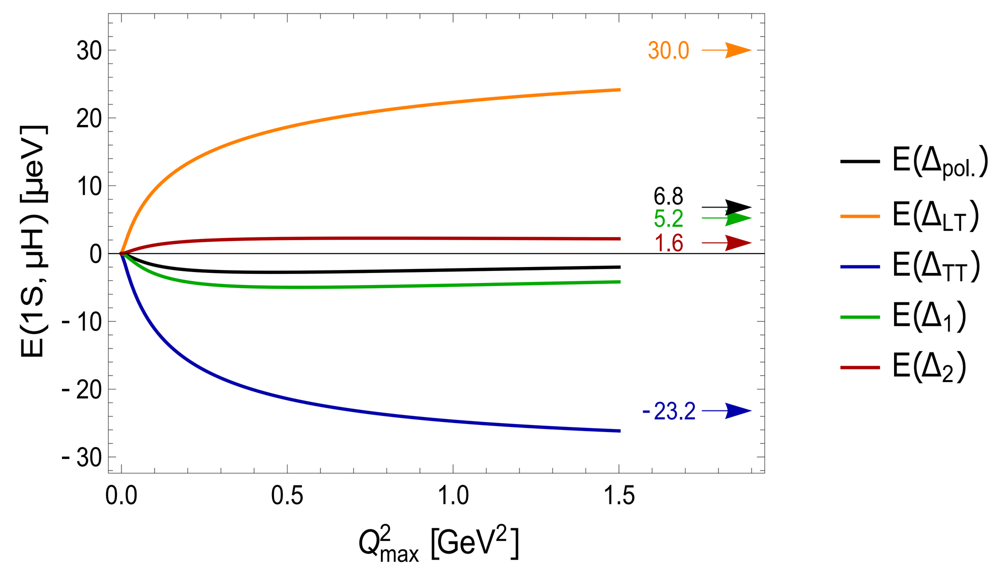
BχPT is a low-energy effective-field theory (EFT). An important requirement for a reliable EFT prediction is that the contributions from beyond the EFT applicability scale (here: $Q_\mathrm{max} > m_\rho = 775$ MeV) have to be within the expected uncertainty. At LO in BχPT, Δ_1 and Δ_2 are numerically small and one has to consider instead the contributions from the longitudinal-transverse and helicity-difference cross sections σ_{LT} and σ_{TT}:

$$\Delta_\mathrm{pol.} = \Delta_{LT} + \Delta_{TT} + \Delta_{F_2} = \frac{\alpha m}{2\pi(1+\kappa)M}(\delta_{LT}+\delta_{TT}+\delta_{F_2}), \tag{4a}$$

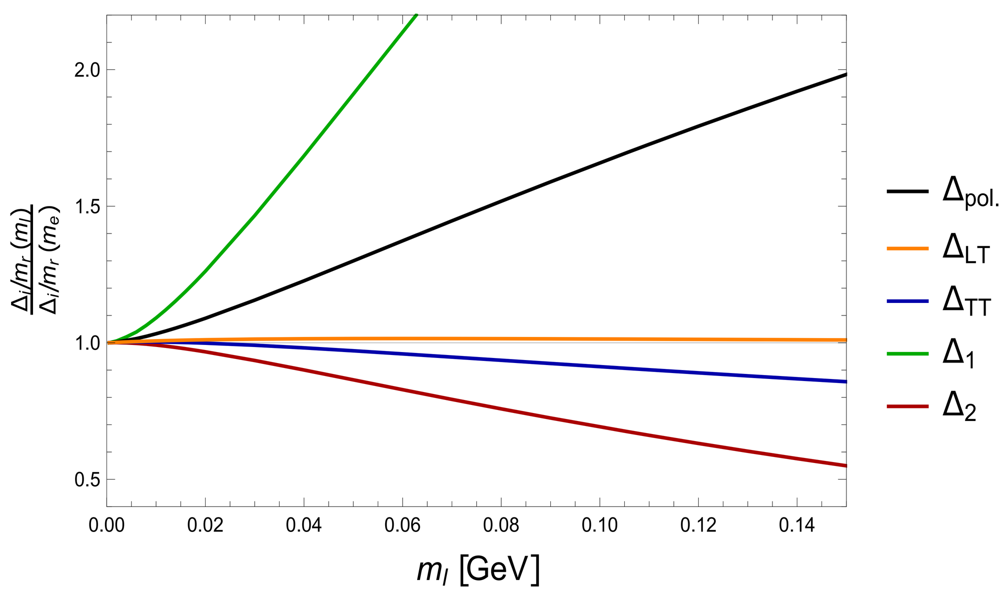
$$\delta_{LT} = \int_0^\infty \mathrm{d}Q \int_0^{x_0} \mathrm{d}x\, K_{LT}(x,Q^2)\, \sigma_{LT}(x,Q^2), \tag{4b}$$

$$\delta_{TT} = \int_0^\infty \mathrm{d}Q \int_0^{x_0} \mathrm{d}x\, K_{TT}(x,Q^2)\, \sigma_{TT}(x,Q^2), \tag{4c}$$

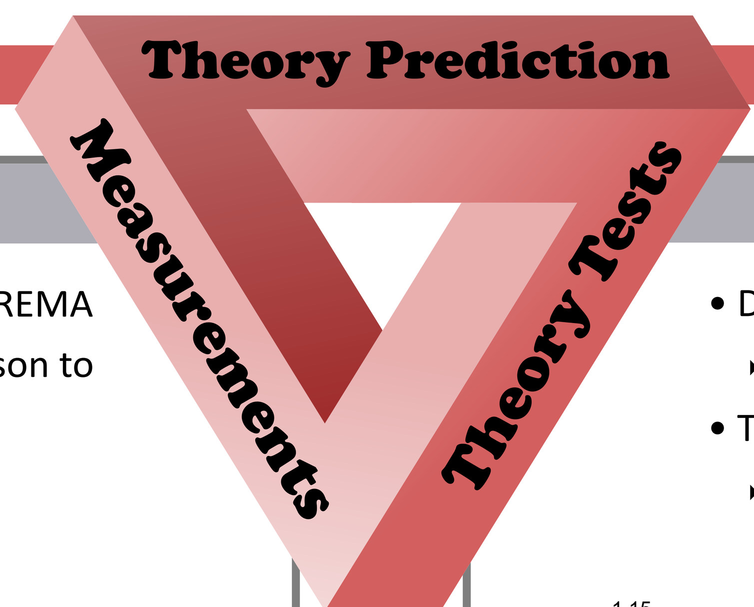
$$\delta_{F_2} = \int_0^\infty \mathrm{d}Q\, K_{F_2}(Q^2)\, F_2^2(Q^2). \tag{4d}$$



One way to refine the theory predictions is to use a scaling procedure based on the empirical $1S$ HFS in H. The Zemach radius and polarizability effects scale essentially with the reduced mass of the bound state m_r. At LO in BχPT, the numerically large contributions Δ_{LT} and Δ_{TT} satisfy the expected scaling behaviour at the level of 1 and 10 %, respectively.

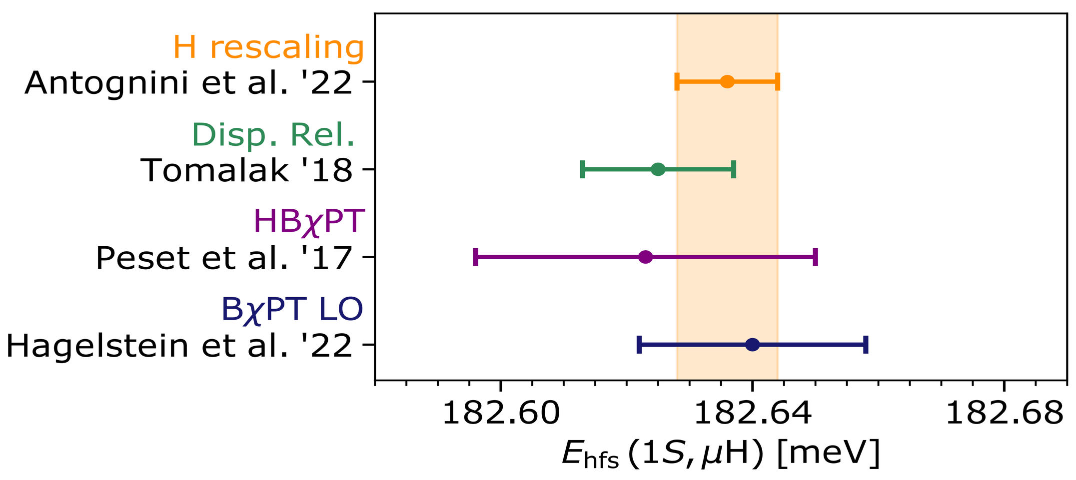


for the figures see Ref. [5]



4. Guiding the Experiment

A precise theory prediction is needed to guide the experiment. Presently, the CREMA collaboration will need to cover a frequency search range of 40 GHz in comparison to the narrow linewidth of 200 MHz. It will require up to 8 weeks to search for the transition and further 3 weeks to acquire the necessary statistics.



0.16 meV (40 GHz) search range

As it can be seen from the figure, the best prediction is obtained by rescaling the empirical value for the Zemach and polarizability effects from the H $1S$ HFS. For the general case:

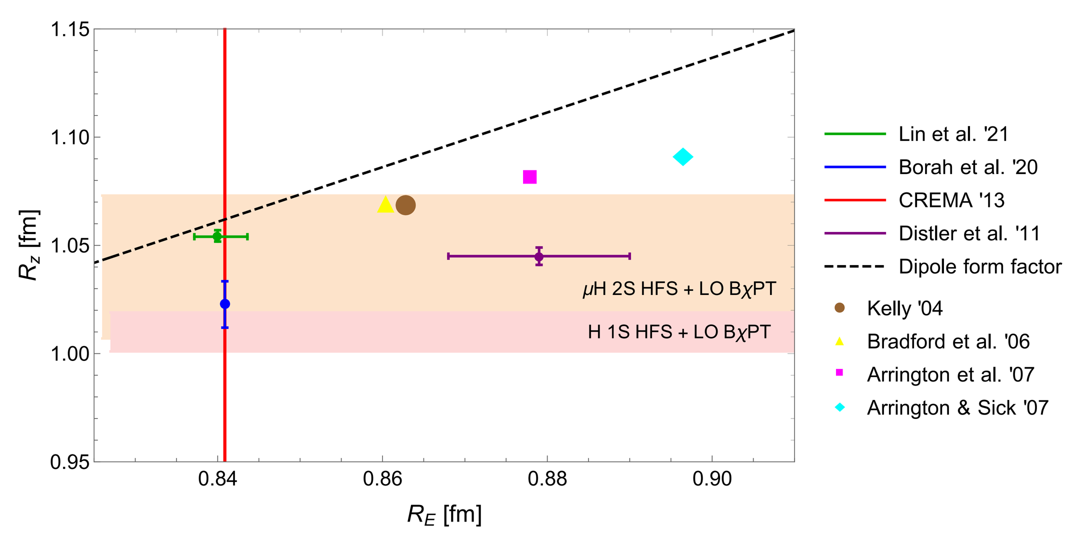
$$E^{Z+\mathrm{pol.}}_{nS\text{-HFS}}(\mathrm{X}) = \frac{E_\mathrm{F}(\mathrm{X})}{n^3}[b_{nS}(\mathrm{X})\Delta_\mathrm{Z}(\mathrm{X}) + c_{nS}(\mathrm{X})\Delta_\mathrm{pol.}(\mathrm{X})], \quad \mathrm{X} = \mathrm{H}, \mu\mathrm{H} \tag{5}$$

with b_{nS} and c_{nS} being the radiative correction factors that differ for H and μH, as well as for the different nS-levels.

for the figure see Ref. [5]

5. Checking the Theory

- Discriminate between theory predictions for polarizability effect
  - disentangle Zemach & polarizability effect by combining $1S$ HFS in H & μH
- Test HFS theory
  - combining $1S$ HFS in H & μH with theory prediction for polarizability effect



A small polarizability effect implies a smaller Zemach radius R_Z.

TABLE I. Determinations of the proton Zemach radius R_Z, in units of fm.

ep scattering		μH 2S hfs		H 1S hfs	
Lin et al. '21	Borah et al. '20	Antognini et al. '13	LO BχPT	Volotka et al. '04	LO BχPT
$1.054^{+0.003}_{-0.002}$	1.0227(107)	1.082(37)	1.040(33)	1.045(16)	1.010(9)

for the figure and the table see Ref. [5]

This work is supported by the Swiss National Science Foundation through the Ambizione Grant PZ00P2_19338.