# Pseudoscalar pole contributions to the muon g-2 from lattice QCD

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## Muon g-2

- $4.2\sigma$ tension, hadronic contributions dominate $43 \times 10^{-11}$ theory uncertainty.
  - HVP at $O(\alpha^2)$: $40 \times 10^{-11}$ err
  - HLbL at $O(\alpha^3)$: $17 \times 10^{-11}$ err
- Fermilab E989 expects $15 \times 10^{-11}$ err
- Need to reduce theory uncertainties on hadronic contributions

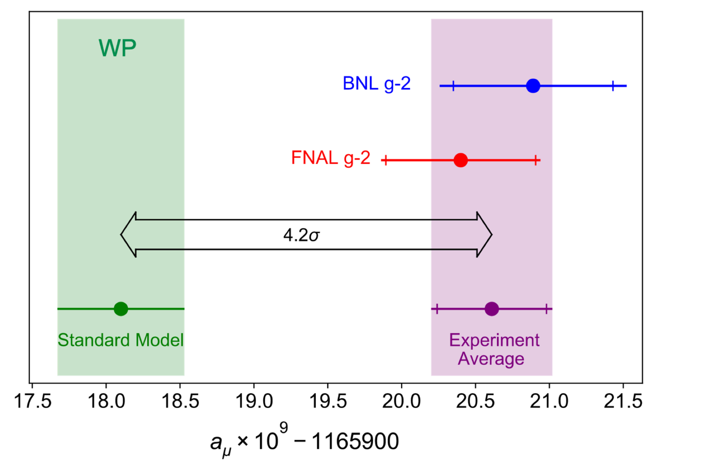
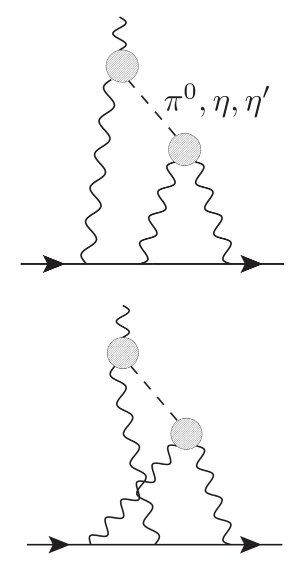



Figure 1: Tension between theory and experimental estimates of the muon g-2 [1].


## Pseudoscalar-pole contribution

- HLbL decomposes into several contributions, $a_\mu^{\mathrm{HLbL}} = a_\mu^{P-\mathrm{pole}} + \dots$



$$a_\mu^{P-\mathrm{pole},(1)} = \int_0^\infty dQ_1 dQ_2 \int_{-1}^{1} d\cos\theta \; w_1(Q_1, Q_2, \cos\theta) \times F_{P\gamma\gamma}(-Q_1^2, -(Q_1+Q_2)^2)\, F_{P\gamma\gamma}(-Q_2^2, 0)$$

$$a_\mu^{P-\mathrm{pole},(2)} = \int_0^\infty dQ_1 dQ_2 \int_{-1}^{1} d\cos\theta \; w_2(Q_1, Q_2, \cos\theta) \times F_{P\gamma\gamma}(-Q_1^2, -Q_2^2)\, F_{P\gamma\gamma}(-(Q_1+Q_2)^2, 0)$$

- Analytically known weight functions.
- Non-perturbative transition form factors $F_{P\gamma\gamma}(q_1^2, q_2^2)$ required.
- Leading contributions from $\pi^0, \eta, \eta'$.

## Pseudoscalar transition form factors

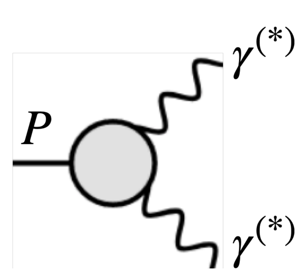


Figure 2: Transition form factor from $P \in \{\pi^0, \eta, \eta'\}$ to two photons.

- Data from CELLO [2], CLEO [3], BaBar [4, 5], Belle [6] for singly-virtual $F_{P\gamma\gamma}(-Q^2, 0)$ at $Q^2 \gtrsim 1.0\mathrm{GeV}^2$.
- Doubly virtual and low-$Q^2$ essentially unconstrained from experiment. However, upcoming BES-III results are promising.
- Complementarity: doubly virtual, low-$Q^2$ easier than singly virtual, large-$Q^2$ on the lattice.

## TFFs from lattice QCD

- Euclidean time current-current vacuum transition amplitude

$$\tilde{A}_{\mu\nu}(\tau) = \left\langle 0 \middle| j_\mu(\tau; \mathbf{q}_1)\, j_\nu(0; \mathbf{0}) \middle| P(\mathbf{p}) \right\rangle$$

- Laplace transform

$$\epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} F_{P\gamma\gamma}(q_1^2, q_2^2) = -i^{n_0} \int_{-\infty}^{\infty} d\tau \; e^{\omega_1 \tau} \tilde{A}_{\mu\nu}(\tau)$$

- Extrapolation from finite-volume "orbits" in $(q_1^2, q_2^2)$ plane by conformal $z$-expansion.

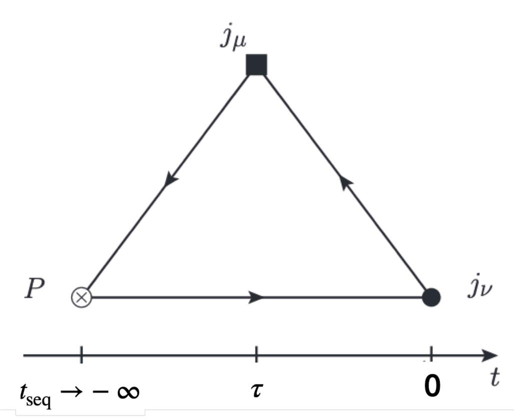



Figure 3: Three-point function for $F_{P\gamma\gamma}$.

## Lattice QCD setup

- $N_f = 2+1+1$ twisted clover, Iwasaki gauge action, physical quark masses.

| ensemble | $L^3 \cdot T/a^4$ | $m_\pi$ [MeV] | $a$ [fm] | $a \cdot L_x$ [fm] | $m_\pi \cdot L_x$ |
|---|---|---|---|---|---|
| cB072.64 | $64^3 \cdot 128$ | 140.2 | 0.080 | 5.09 | 3.62 |
| cC060.80 | $80^3 \cdot 160$ | 136.7 | 0.068 | 5.46 | 3.78 |
| cD054.96 | $96^3 \cdot 192$ | 140.8 | 0.057 | 5.46 | 3.90 |

Figure 4: ETMC ensembles used in this calculation.

- Analysis for $\pi^0$: several lattice spacings, continuum limit in progress.
- Analysis for $\eta$: preliminary results on cB64, finer lattices in progress.
- $\eta'$ currently too noisy to extract reliable data on these ensembles.

## Systematic errors

- Several analysis choices (e.g., fit window, fit model, $z$-expansion order).
- AIC-weighted model averaging, with CDF trick to separate syst./stat. errors, per lattice ensemble used.
- Continuum extrapolation yields additional systematic error.

## Results: $\pi^0$

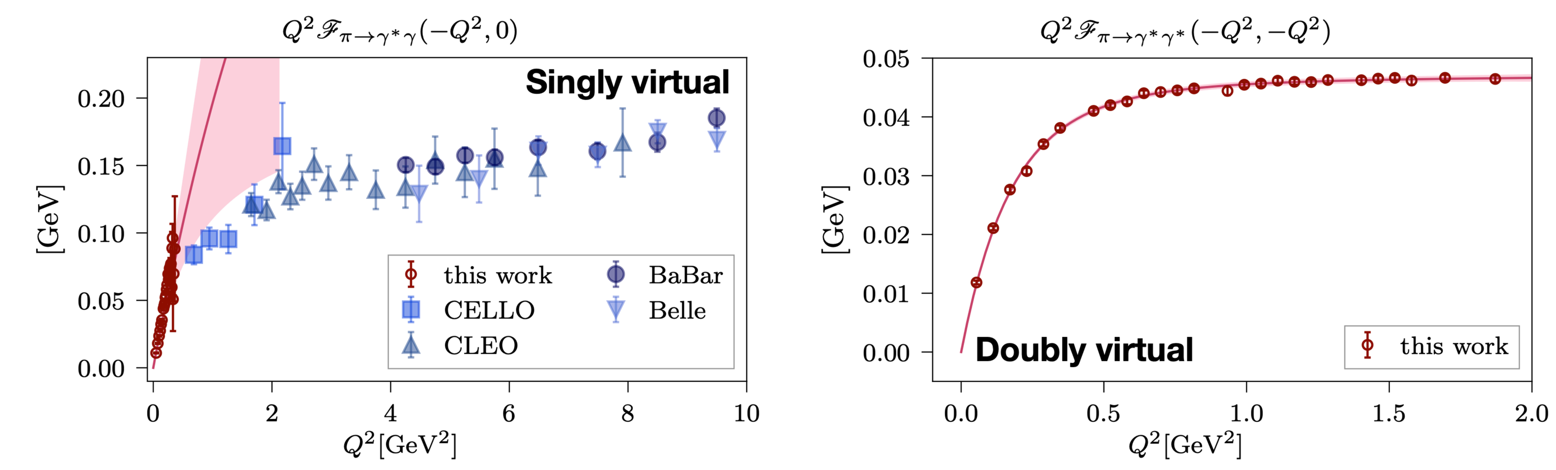



Figure 5: Form factor results from ONE analysis choice.

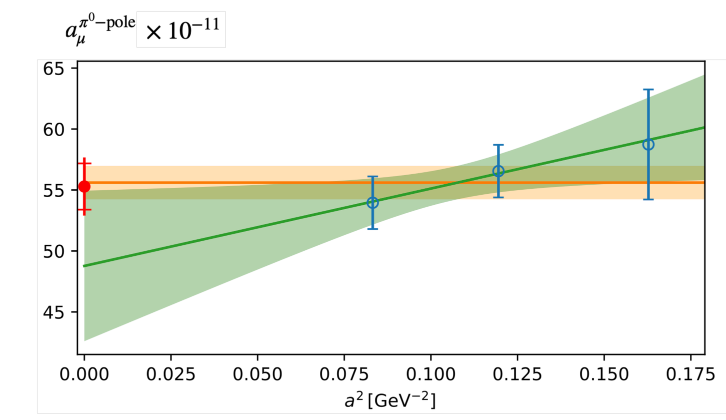


Figure 6: Continuum extrapolation.

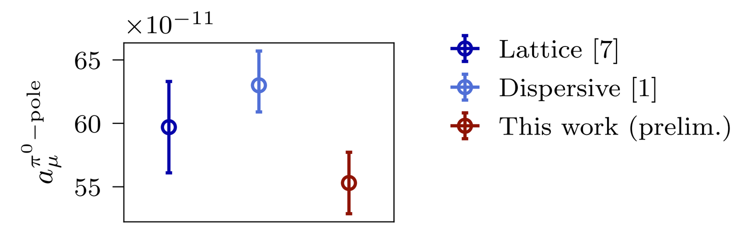



Figure 7: Comparison with known theoretical results.

- Currently analyzing additional statistics on cB64 (rightmost point, Fig. 6).

## Results: $\eta$

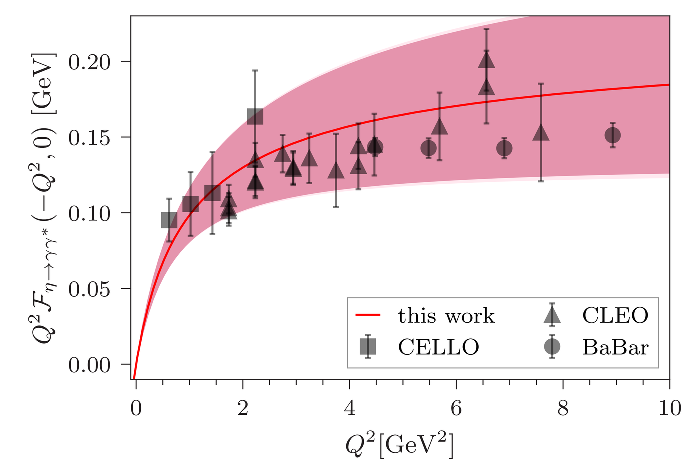



Figure 8: Singly virtual form factor results.

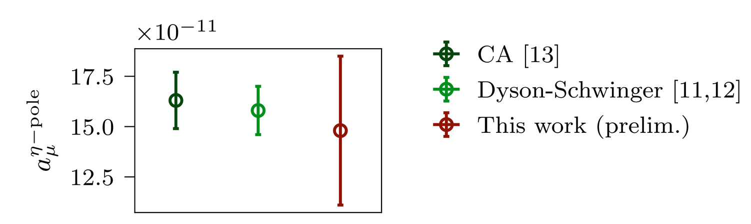



Figure 9: Comparison with known theoretical results.

- Preliminary results do not include lattice discretization uncertainties.

## Conclusions & Outlook

- Our results for the doubly virtual and low-$Q^2$ pseudoscalar TFF are complementary to experimental values.
  - Future combined fits may be of interest.
- This is the first lattice calculation with physical quark masses.
  - We validate $\pi^0$ lattice results extrapolating from unphysical masses [7].
  - Results for the $\eta$ are a first lattice calculation.
- Future directions:
  - Vary lattice setup for better kinematic coverage.
  - Address noise problems with measurements of the $\eta'$?

## References

[1] T. Aoyama et al. *Phys. Rept.*, 887:1–166, 2020.

[2] H. J. Behrend et al. *Z. Phys. C*, 49:401–410, 1991.

[3] J. Gronberg et al. *Phys. Rev. D*, 57:33–54, 1998.

[4] Bernard Aubert et al. *Phys. Rev. D*, 80:052002, 2009.

[5] P. del Amo Sanchez et al. *Phys. Rev. D*, 84:052001, 2011.

[6] S. Uehara et al. *Phys. Rev. D*, 86:092007, 2012.

[7] Antoine Gérardin, Harvey B. Meyer, and Andreas Nyffeler. *Phys. Rev. D*, 100(3):034520, 2019.

[8] Andreas Nyffeler. *Phys. Rev. D*, 94(5):053006, 2016.

[9] G. W. Bennett et al. *Phys. Rev. D*, 73:072003, 2006.

[10] B. Abi et al. *Phys. Rev. Lett.*, 126(14):141801, 2021.

[11] Gernot Eichmann, Christian S. Fischer, Esther Weil, and Richard Williams. *Phys. Lett. B*, 797:134855, 2019.

[12] Khépani Raya, Adnan Bashir, and Pablo Roig. *Phys. Rev. D*, 101(7):074021, 2020.

[13] Pere Masjuan and Pablo Sanchez-Puertas. *Phys. Rev. D*, 95(5):054026, 2017.