RESONANCES OF EXOTIC THREE-BODY SYSTEMS

Jean Servais* and Jérémy Dohet-Eraly
Physique Quantique and Physique Nucléaire Théorique et Physique Mathématique, C.P. 229,
Université libre de Bruxelles (ULB), B-1050 Brussels, Belgium

October 2022

*F.R.S.-FNRS Research fellow (Belgium), jean.servais@ulb.be

INTRODUCTION

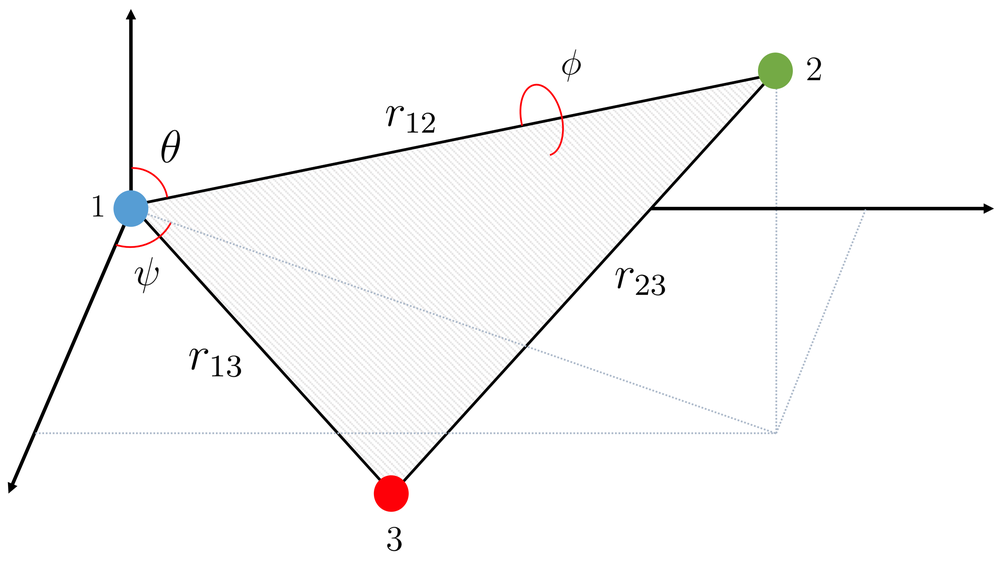
BACKGROUND

- Antimatter facilities at CERN for studying exotic quantum systems
- Efficient tool for determining antimatter properties, e.g.
  - experimental study of the antiprotonic helium atom (made of an α-particle, an electron, and an antiproton), with high-precision laser spectroscopy on narrow resonances of $\bar{p}He^+$ ⟶ best value for the $\bar{p}$ mass [1],
  - same techniques for negatively charged pion [2].

PRESENT WORK

- **Aim:** computing accurately resonant parameters (energies and widths) of
  1. 3-body systems with 2 identical particles (He, H_2^+, Ps^-, etc.),
  2. 3-body exotic quantum systems ($\bar{p}He^+$, $\bar{p}H$, π^-He^+, etc.).
- **Method:** applying the
  1. Lagrange-mesh method [3] in perimetric coordinates [4],
  2. complex scaling method [5] to extract resonance parameters.

THREE-BODY SYSTEM



1. Internal Hamiltonian of the system

$$H = -\frac{1}{2m_1}\Delta_{\boldsymbol{r}_1} - \frac{1}{2m_2}\Delta_{\boldsymbol{r}_2} - \frac{1}{2m_3}\Delta_{\boldsymbol{r}_3} - T_{CM} + \frac{Z_1Z_2}{r_{12}} + \frac{Z_2Z_3}{r_{23}} + \frac{Z_1Z_3}{r_{13}}$$

2. Perimetric coordinates

Euler angles θ, ψ, ϕ

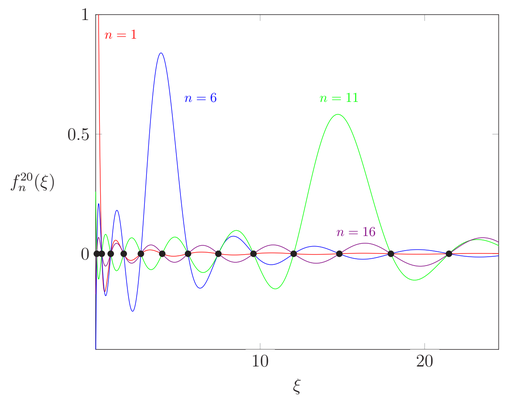
Radial coordinates

$$\begin{cases} x &= r_{12} - r_{23} + r_{13} \\ y &= r_{12} + r_{23} - r_{13} \\ z &= -r_{12} + r_{23} + r_{13} \end{cases}$$

3. Expansion

$$\Psi_{LM} = \sum_{K=0}^{L} \mathcal{D}^{K\pi}_{LM}(\psi,\theta,\phi)\Phi(x,y,z)$$

LAGRANGE-MESH METHOD



$$\Phi(x,y,z) = \sum_{jkn} C_{jkn} f_j^{N_x}\left(\frac{x}{h_x}\right) f_k^{N_y}\left(\frac{y}{h_y}\right) f_n^{N_z}\left(\frac{z}{h_z}\right)$$

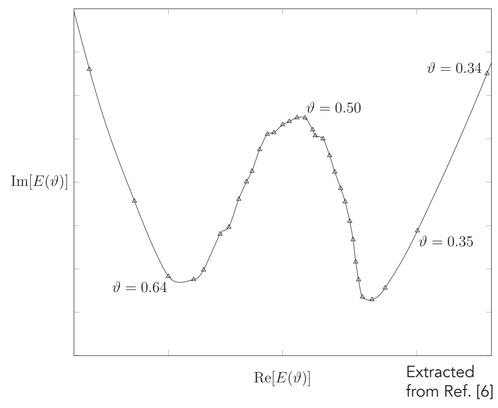
with $f_n^N(\xi) = (-1)^n\sqrt{\xi_n}\frac{L_N(\xi)}{\xi-\xi_n}e^{-\xi/2}$, $n = 1, ..., N$

3D-mesh $(h_x x_j, h_y y_k, h_z z_n)$ and $f_n^N(\xi_k) = \delta_{kn}\lambda_k^{-1/2}$

(Sometimes approximate) Gauss quadrature

$$\langle f_{j'} f_{k'} f_{n'} | V | f_j f_k f_n \rangle \approx V(h_x x_j, h_y y_k, h_z z_n)\delta_{jj'}\delta_{kk'}\delta_{nn'}$$

COMPLEX SCALING METHOD



Extracted from Ref. [6]

Rotation of the coordinates

$$(r_{12}, r_{13}, r_{23}) \to (r_{12}e^{i\vartheta}, r_{13}e^{i\vartheta}, r_{23}e^{i\vartheta})$$

$$H \to e^{-2i\vartheta}T + V(\vartheta)$$

Stable complex eigenvalues

$$E = E_r - \frac{i}{2}\Gamma$$

RESULTS

Helium and Ps^- (L = 0, 1)

- Absolute accuracy between 10^{-11} and 10^{-15}
- For antisymmetric and symmetric states (^1S and ^3S)
- Even and odd, (anti)symmetric states ($^1P^{e,o}$ and $^3P^{e,o}$)

Antiprotonic helium (see Ref. [7])

- Nonrelativistic quasibound / resonant states for L = 0 to 80
- Widths studied for $2 < L < 22$

		Energy (a.u.)		Width (a.u.)		
$^\infty$He	^1S	-7.778 676 355 62	x 10^{-1}	4.541 305 705	x 10^{-3}	[a]
Ref. [8]		-7.778 676 356	x 10^{-1}	4.541 306	x 10^{-3}	
Ps^-	^3S	-6.353 735 370 385	x 10^{-2}	3.141 47	x 10^{-9}	[a]
Ref. [9]		-6.353 735 371	x 10^{-2}	3.134	x 10^{-9}	
Ps^-	$^1P^o$	-6.315 586 163	x 10^{-2}	9.1837	x 10^{-7}	
Ref. [10]		-6.315 59	x 10^{-2}	8.87	x 10^{-7}	
Ps^-	$^3P^e$	-3.1 630 675 990 1	x 10^{-2}	1.791 005 980	x 10^{-4}	
Ref. [11]		-3.1 628 7	x 10^{-2}	1.795	x 10^{-4}	
$\bar{p}He^+$		Energy (a.u.)		Width (a.u.)		
L=3		-1.838 623 03	x 10^2	2	x 10^{-6}	[b]
L=21		-6.595 1		1.1	x 10^{-2}	[b]
L=32		-3.353 757 863 53				[b]
Ref. [12]		-3.353 757 80		2.1	x 10^{-12}	

[a] See Ref. [6].
[b] See Ref. [7].

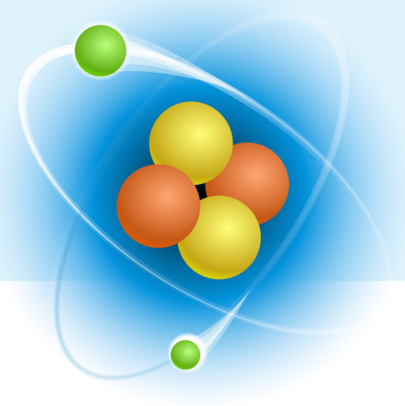
PERSPECTIVES

We plan to develop and extend the current method to study the resonances of

EXOTIC QUANTUM HYBRID SYSTEMS

- Antiprotonic helium atom $\bar{p}He^+$
- Antiprotonic hydrogen ion $\bar{p}H$ [13]
- Pionic helium atom π^-He^+ [14]
- Kaonic helium atom

for any angular momentum L,
by means of the **Kohn variational method**.



References
[1] M. Hori *et al.*, Determination of the Antiproton-to-Electron Mass Ratio by Precision Laser Spectroscopy of pHe^+, Phys. Rev. Lett. 96, 243401 (2006).
[2] M. Hori, A. Sótér, and V. I. Korobov, Proposed method for laser spectroscopy of pionic helium atoms to determine the charged-pion mass, Phys. Rev. A 89, 042515 (2014).
[3] D. Baye and P.-H. Heenen, Generalised meshes for quantum mechanical problems, J. Phys. A 19, 2041 (1986).
[4] C. L. Pekeris, Ground state of two-electron atoms, Phys. Rev. 112, 1649 (1958).
[5] Y. K. Ho, The method of complex coordinate rotation and its applications to atomic collision processes, Phys. Rep. 99, 1 (1983).
[6] J. Dohet-Eraly and J. Servais, Lagrange-mesh calculations of *S*-wave resonances in three-body atomic systems, *submitted to* Jour. Phys. B (2022).
[7] D. Baye, J. Dohet-Eraly, and P. Schoofs, Structure changes along the lowest rotational band of the antiprotonic helium atom, Phys. Rev. A 99, 22508 (2019).
[8] E.Z. Liverts and N. Barnea, Three-body systems with Coulomb interaction. Bound and quasi-bound *S*-states. Comput. Phys. Commun. 184, 2596-2603 (2013).
[9] T. Li and R. Shakeshaft, *S*-wave resonances of the negative positronium ion and stability of a system of two electrons and an arbitrary positive charge, Phys. Rev. A. 71, 052505 (2005).
[10] S. Kar and Y. K. Ho, Doubly excited $^{1,3}P^o$ resonance states of Ps^- in weakly coupled plasmas, J. Phys. A 76 (2006).
[11] S. Kar and Y. K. Ho, Doubly excited $^{1,3}P^e$ resonance states of the positronium negative ion with Coulomb and screened Coulomb potentials. Few-Body Systems 46, 173–181 (2009).
[12] V. I. Korobov and I. Shimamura, Auger transition rates for metastable states of antiprotonic helium He^+p. Phys. Rev. A 56, 4587 (1997).
[13] D. Baye and J. Dohet-Eraly, Quasibound states of an antiproton and a hydrogen atom, Phys. Rev. A 101, 022507 (2020).
[14] D. Baye and J. Dohet-Eraly, Three-body Coulomb description of pionic helium, Phys. Rev. A 103, 022823 (2021).