Measurement of the charge radius of ^{185}Re and ^{187}Re from their muonic hyperfine spectrum

Master Thesis
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1. **Relating Rhenium muonic hyperfine spectrum to the charge radius**

2. **Modeling detector response using Lead spectrum: identification of lineshape and linewidth broadening**
    - *2.1 Experimental dataset*
    - *2.2 Lead muonic spectrum: energy regions and transitions*
    - *2.3 Histograms and time cut selection: coincidental prompt events*
    - *2.4 Fitting model and method*
    - *2.5 Transitions fits results*
    - *2.6 Detector lineshape as a function of energy*

3. **Rhenium fitting and outlook**

1. Relating Rhenium muonic hyperfine spectrum to the charge radius

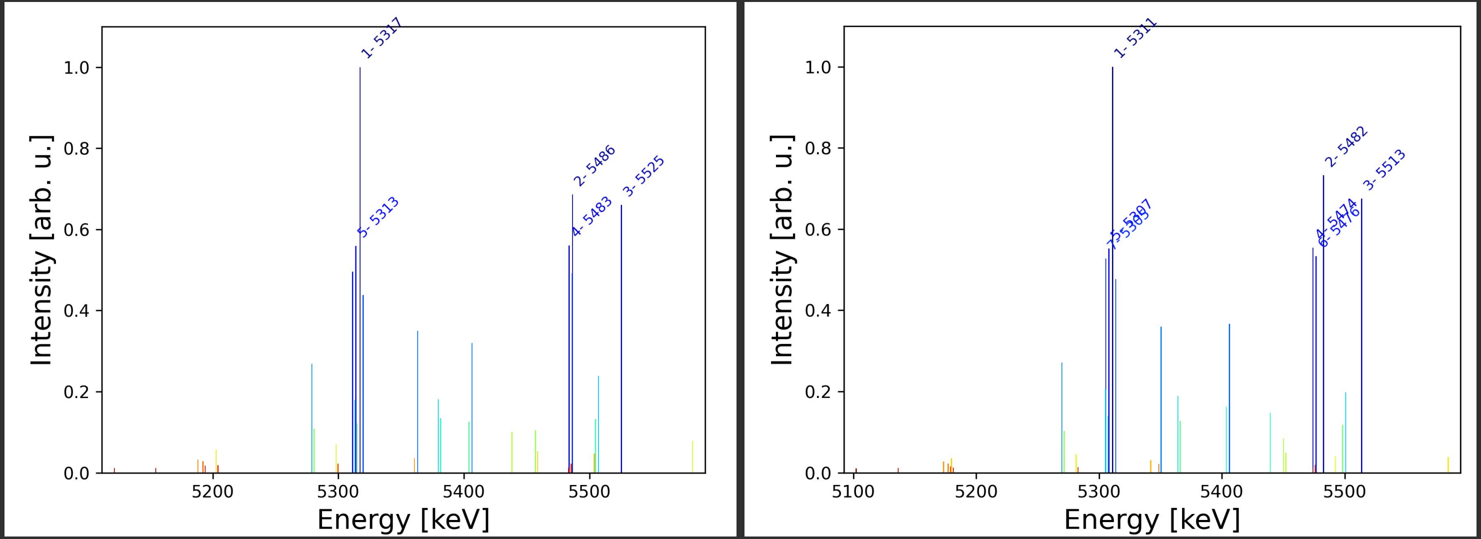
$$E^{if}(R) = E_0^{if} + E_1^{if}\Delta R + E_2^{if}\Delta R^2$$
$$I^{if}(R) = I_0^{if} + I_1^{if}\Delta R + I_2^{if}\Delta R^2$$

$$\Delta R := \frac{R - R_0}{R_0}$$

$$R_0 = \begin{cases} 5.3596 & \text{for } {}^{185}\text{Re} \\ 5.3698 & \text{for } {}^{187}\text{Re} \end{cases}$$

Calculations and theoretical predictions by Natalia S. Oreshkina.
Nuclear polarization not included in the calculations.

I. Angeli, K.P. Marinova,
Table of experimental nuclear ground state charge radii: An update,
Atomic Data and Nuclear Data Tables, Volume 99, Issue 1,
2013,Pages 69-95, ISSN 0092-640X


^{185}Re

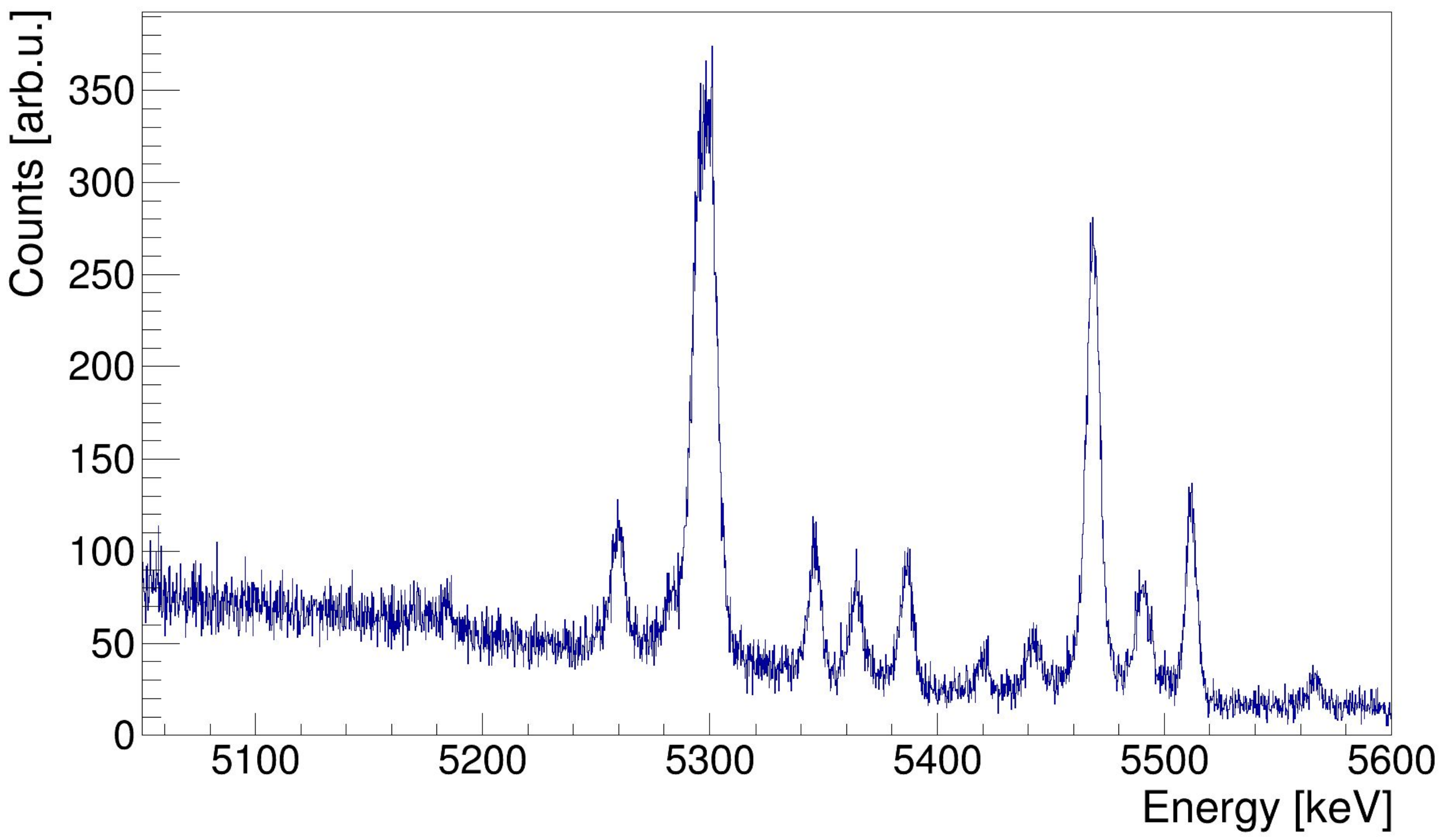
^{187}Re

Predicted 2p-1s muonic X-ray spectra for R = R₀

(color map from lesser to higher intensity peaks)

$2p \rightarrow 1s$

^{185}Re



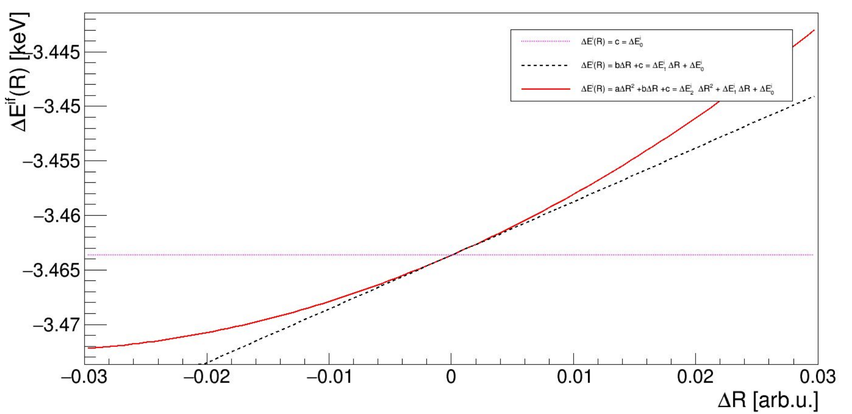
Energy and intensity of transitions dependence on charge radius

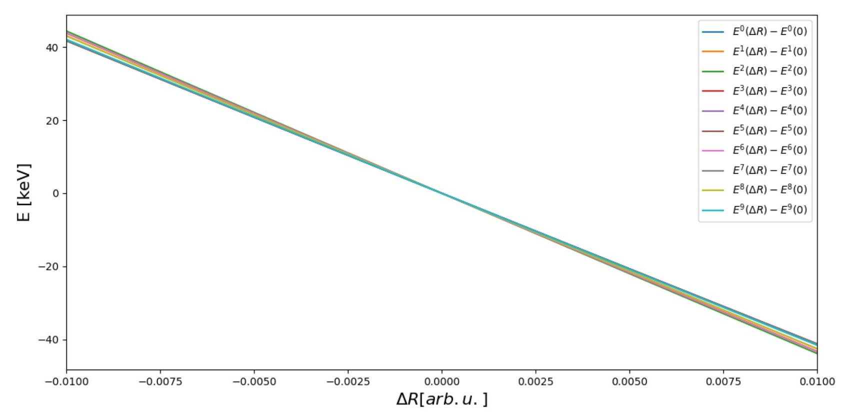
Deviation from the highest intensity transition as a function of charge radius:

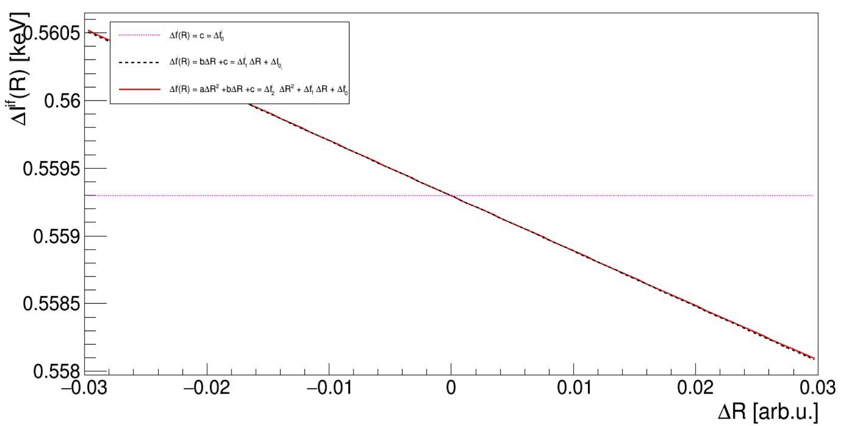
$$\Delta E^i(R) = \Delta E^i_0 + \Delta E^i_1 \Delta R + \Delta E^i_2 \Delta R^2$$

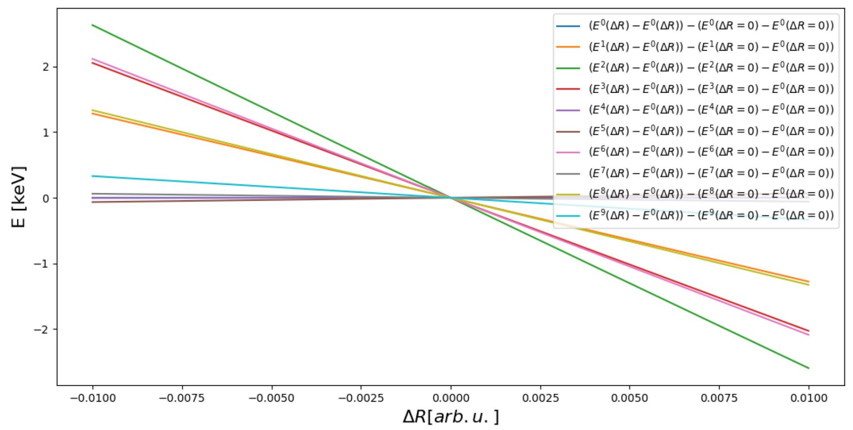
$$\Delta I^i(R) = \Delta I^i_0 + \Delta I^i_1 \Delta R + \Delta I^i_2 \Delta R^2$$

Rhenium 2p-1s









2. Modeling detector response using ^{208}Pb: determination of lineshape

2.1 Experimental dataset

Dataset from two germanium detectors from the 2016 muX experiment

- Germanium left detector (GeL) : larger, 75% efficiency
- Germanium right detector (GeR) : smaller, 20% efficiency

Set of points, i.e recorded counts (number of events) associated to *coincidental time difference* Delta t and energy E of incoming photon:

- $\Delta t = t_\gamma - t_\mu$: difference between detection time of gamma-rays associated to detection time of a muon
- Energy
- Counts

2.2 Lead muonic spectrum: energy regions and transitions

Goal: understand and model the detectors’ response (to be able to fit Rhenium). We need to select transitions well suited for analysis: high intensity (high probability of occurrence) and proper energy range.
Higher energy (lower levels) transitions to explore overall behaviour in region of interest (2p-1s region).
Lower energy (higher levels) transitions to focus on sigma energy dependence over all energy range.

- $4f_{5/2} \rightarrow 3d_{3/2}$
- $3d_{3/2} \rightarrow 2p_{1/2}$
- $2p_{1/2} \rightarrow 1s_{1/2}$

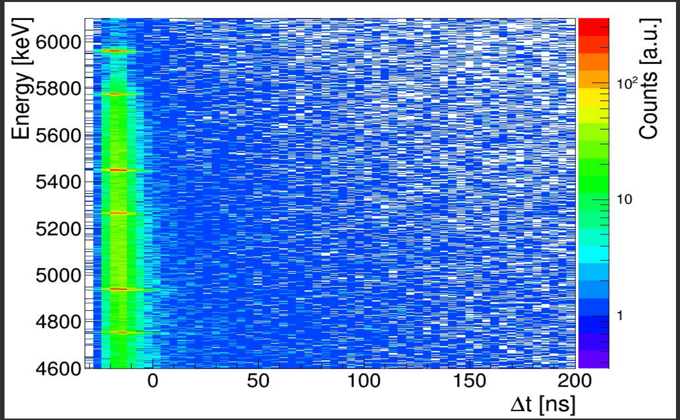
- $4f_{7/2} \rightarrow 3d_{5/2}$
- $3d_{5/2} \rightarrow 2p_{3/2}$
- $2p_{3/2} \rightarrow 1s_{1/2}$

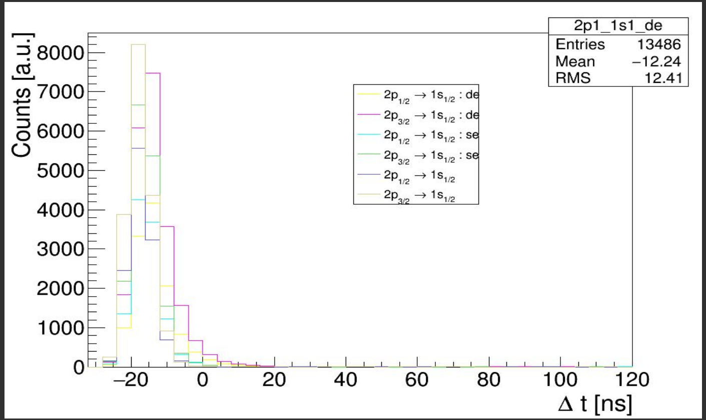
2.3 Histograms and time cuts selection: coincidental prompt events

We need to select suitable time cuts for our histograms to have increased signal to background ratio of the transitions, leading to more accurate fits.
We therefore make a comparison of several time cuts of the coincidental time of prompt detection events. Trade-off between amount of data point associated with transitions and unwanted background noise.

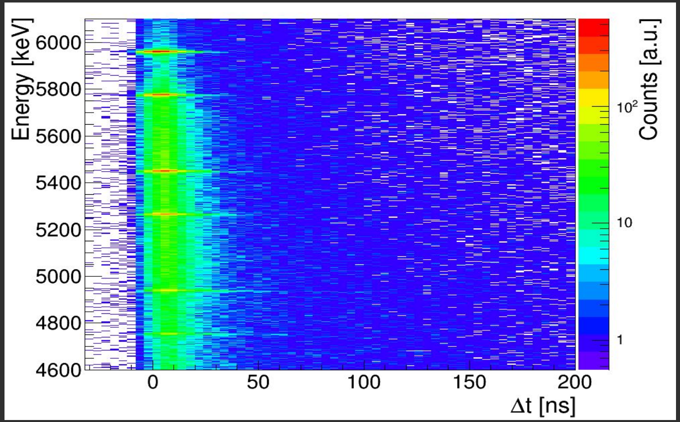
$2p \to 1s$

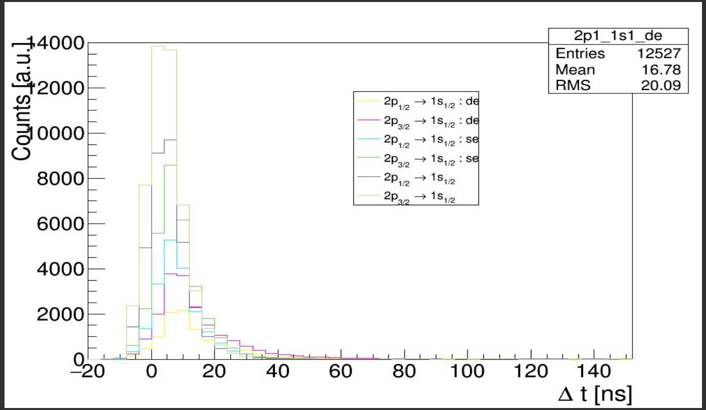
Right germanium detector





Left germanium detector





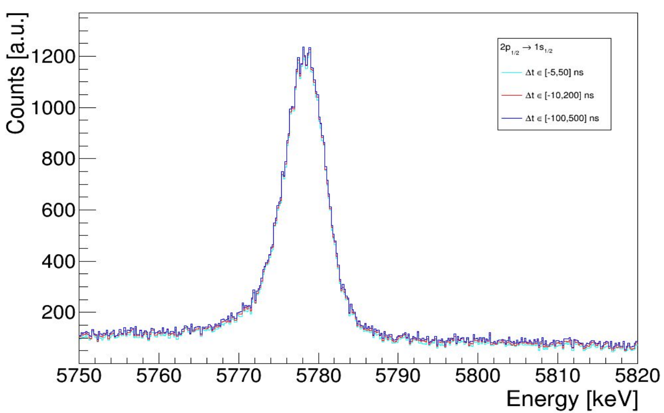
Important:

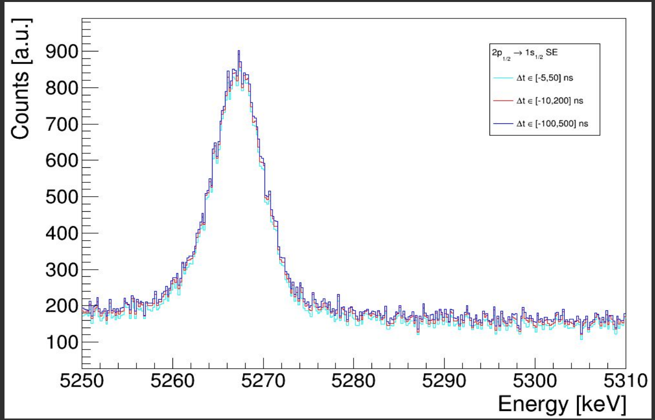
- Right germanium detector data is shifted to earlier detection times compared to left detector data
- Due to time walk effect in the detector, the responsiveness of the detectors decreases with energy, i.e the detection time of particles is slower for lower energy gamma rays

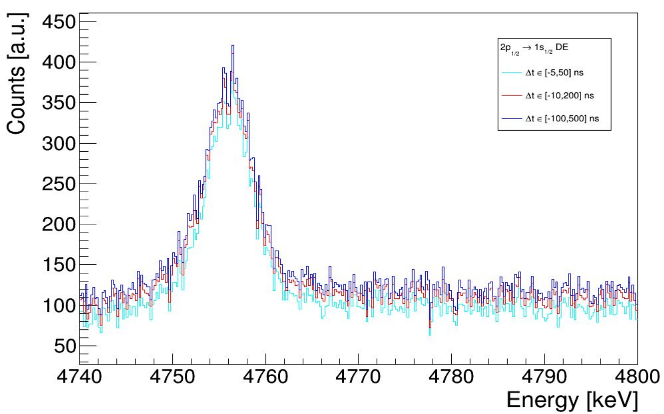
$2p_{1/2} \to 1s_{1/2}$

Left detector

Influence of timecut on number of counts



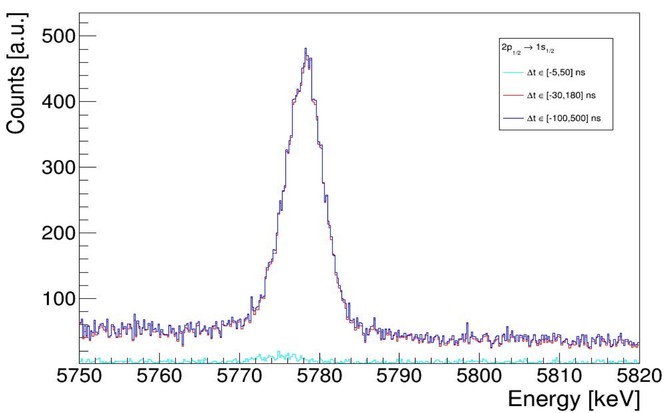


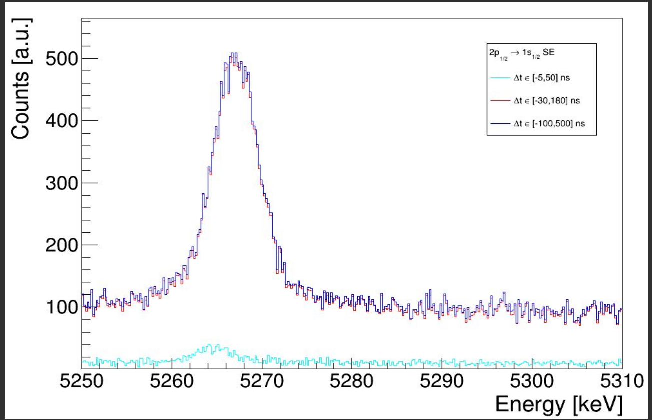


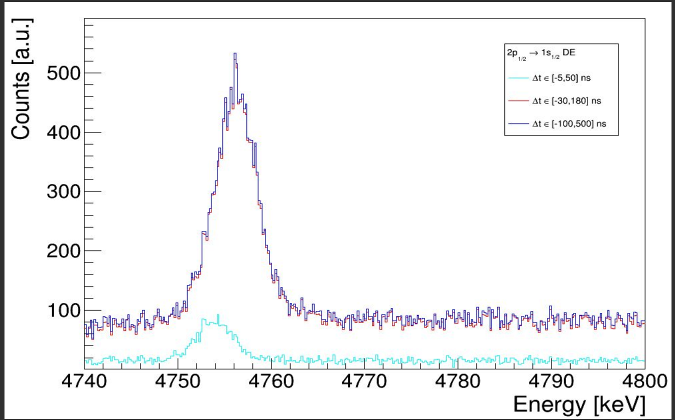
Right detector

$2p_{1/2} \rightarrow 1s_{1/2}$

Influence of timecut on number of counts







2.4 Fitting model

HYPERMET model with Voigt profile:

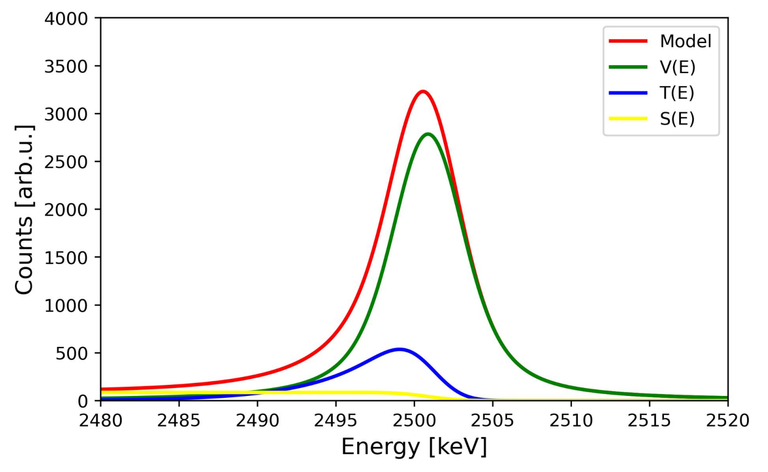
$$\boxed{\begin{aligned} f(E) &= f_V V(E;\mu,\sigma,L_w) + (1-f_V)T(E;\mu,\sigma,b) + S(E;\mu,\sigma) \\ &= f_V G(E;\mu,\sigma) * L(E;\mu,L_w) + (1-f_V)\frac{1}{2b}e^{\frac{E-\mu}{b}+\frac{\sigma^2}{2b^2}}\, erf\left(\frac{E-\mu}{\sqrt{2}\sigma}+\frac{\sigma}{\sqrt{2}b}\right) + \frac{A}{2}erf\left(\frac{E-\mu}{\sqrt{2}\sigma}\right) \end{aligned}}$$

5 free parameters:

- μ : mean value, energy of the transition
- σ : standard deviation
- fV : weight of Voigt function
- A : step function height
- b : “slope” of the tail function

Known parameter:

L_w : transition natural Lorentzian linewidth



Background: linear (to be investigated further) :

- B(E), two free parameters

Final model:

- Model(E) = nSig f(E) + nB B(E) ; 8 fitting parameters
- Method: MIGRAD minimization algorithm from MINUIT package

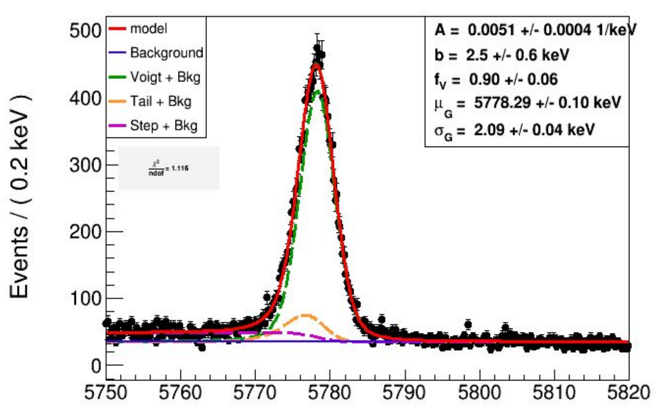
2.5 Transitions fits results

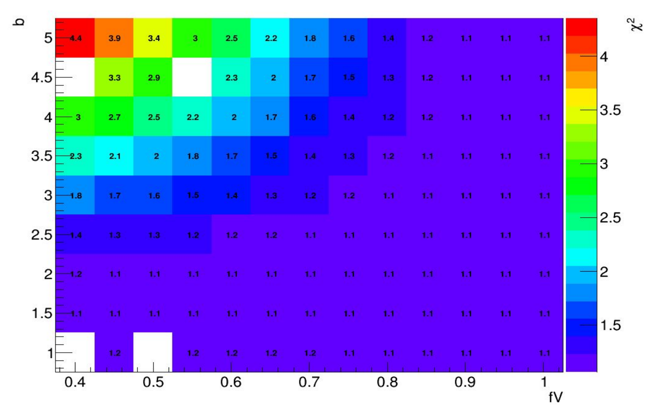
For each transition we performed:

- A free fit of all parameters
- A scan over 2 fixed parameters (fV, b) to observe minimum chi² trends in parameter space
- Fits of 2p-1s transitions with common fixed fitted parameters (simultaneous fits)

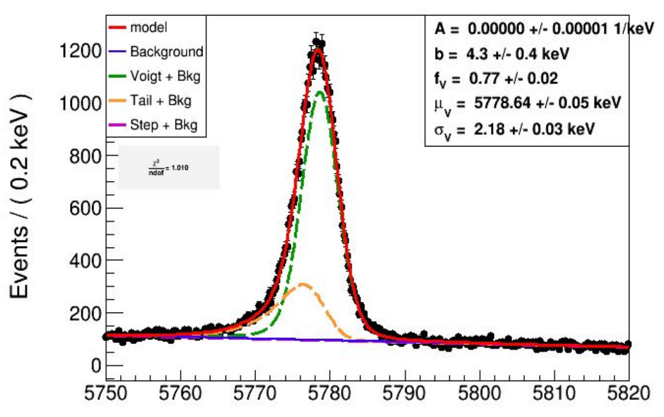
$2p_{1/2} \to 1s_{1/2}$

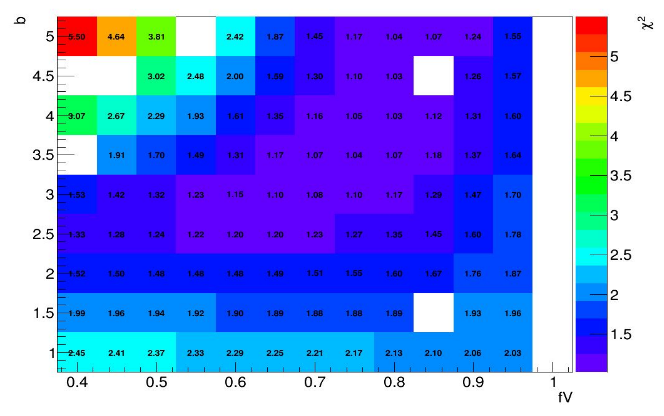
Right germanium detector



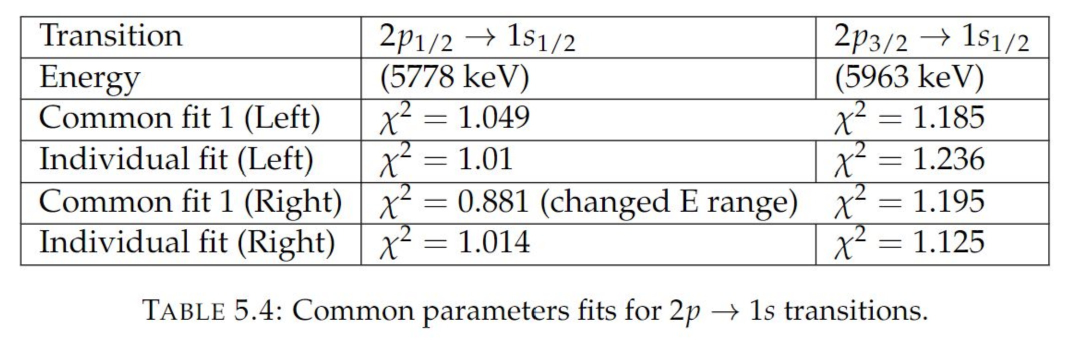


Left germanium detector



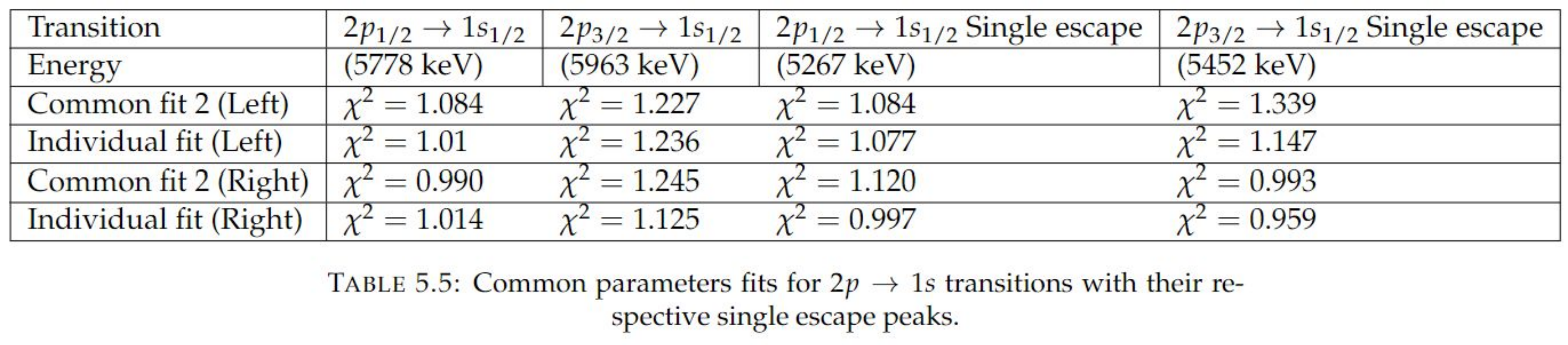


Simultaneous fits common fitted parameters {fV, b , A}



Transition	$2p_{1/2} \rightarrow 1s_{1/2}$	$2p_{3/2} \rightarrow 1s_{1/2}$
Energy	(5778 keV)	(5963 keV)
Common fit 1 (Left)	$\chi^2 = 1.049$	$\chi^2 = 1.185$
Individual fit (Left)	$\chi^2 = 1.01$	$\chi^2 = 1.236$
Common fit 1 (Right)	$\chi^2 = 0.881$ (changed E range)	$\chi^2 = 1.195$
Individual fit (Right)	$\chi^2 = 1.014$	$\chi^2 = 1.125$

TABLE 5.4: Common parameters fits for $2p \rightarrow 1s$ transitions.



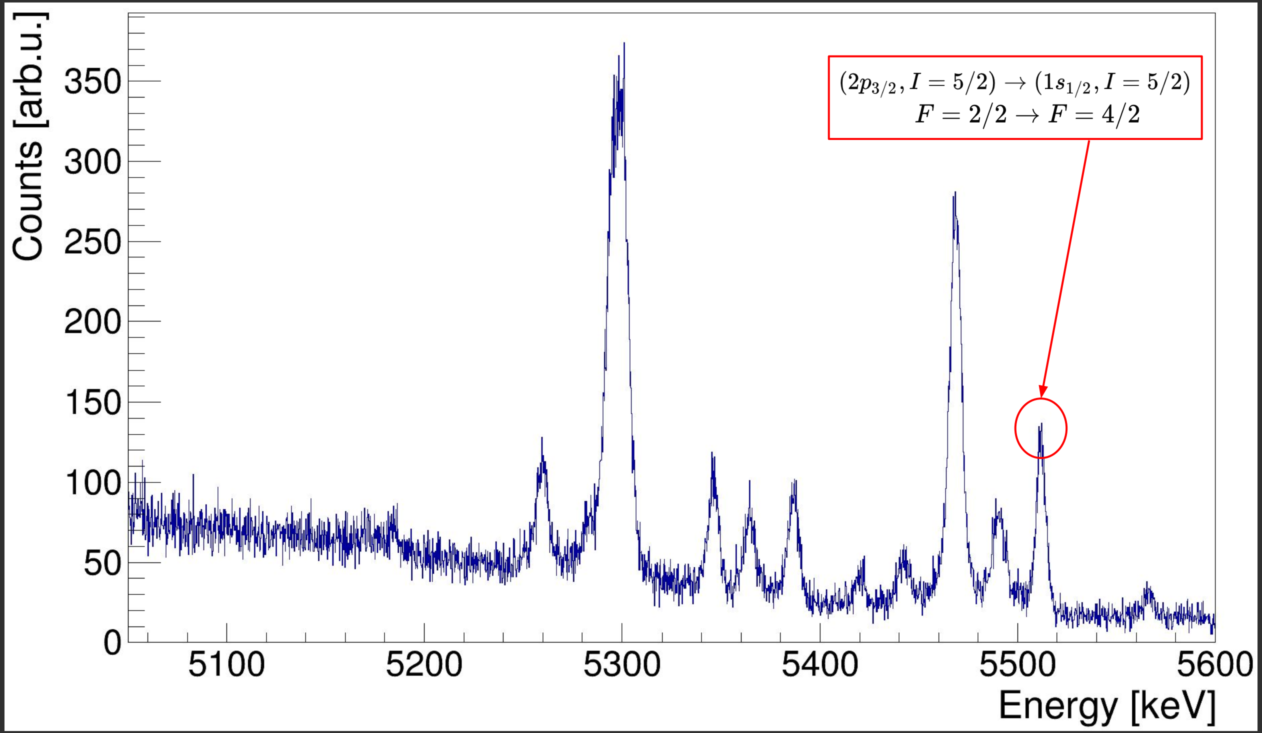
Transition	$2p_{1/2} \rightarrow 1s_{1/2}$	$2p_{3/2} \rightarrow 1s_{1/2}$	$2p_{1/2} \rightarrow 1s_{1/2}$ Single escape	$2p_{3/2} \rightarrow 1s_{1/2}$ Single escape
Energy	(5778 keV)	(5963 keV)	(5267 keV)	(5452 keV)
Common fit 2 (Left)	$\chi^2 = 1.084$	$\chi^2 = 1.227$	$\chi^2 = 1.084$	$\chi^2 = 1.339$
Individual fit (Left)	$\chi^2 = 1.01$	$\chi^2 = 1.236$	$\chi^2 = 1.077$	$\chi^2 = 1.147$
Common fit 2 (Right)	$\chi^2 = 0.990$	$\chi^2 = 1.245$	$\chi^2 = 1.120$	$\chi^2 = 0.993$
Individual fit (Right)	$\chi^2 = 1.014$	$\chi^2 = 1.125$	$\chi^2 = 0.997$	$\chi^2 = 0.959$

TABLE 5.5: Common parameters fits for $2p \rightarrow 1s$ transitions with their respective single escape peaks.

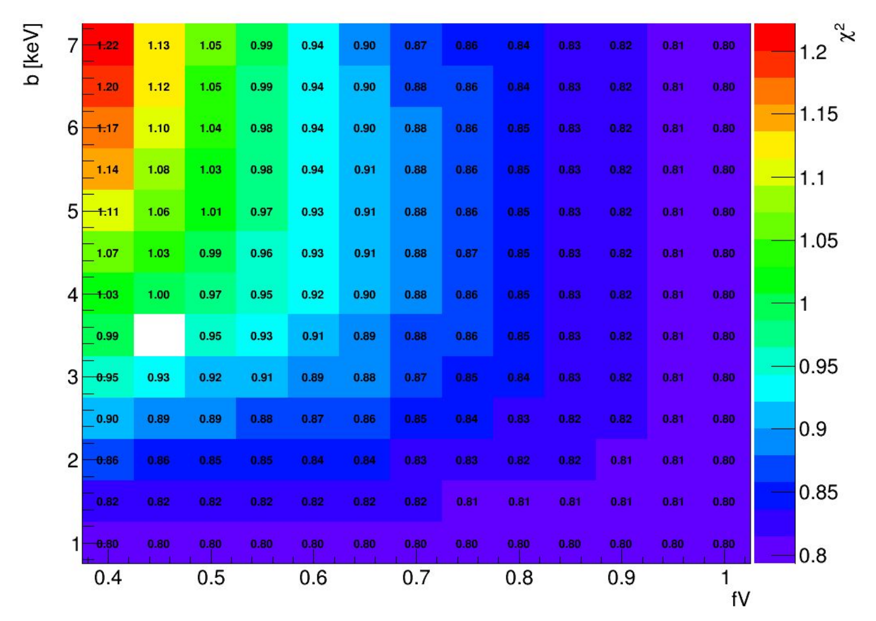
Test: rhenium isolated peak

$$F = 2/2 \rightarrow F = 4/2$$

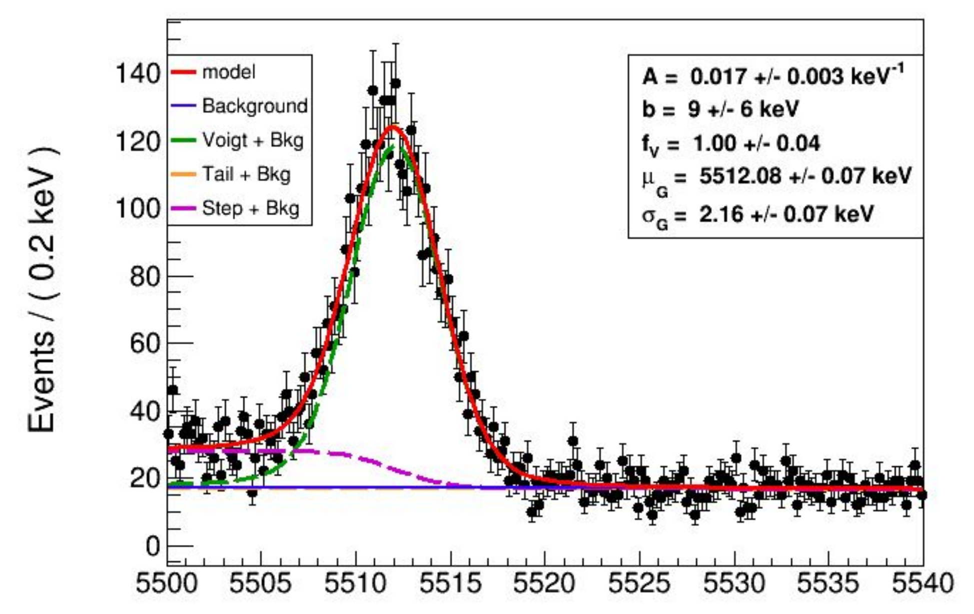
$$(2p_{3/2}, I = 5/2) \rightarrow (1s_{1/2}, I = 5/2)$$



$(2p_{3/2}, I = 5/2) \rightarrow (1s_{1/2}, I = 5/2)$
$F = 2/2 \rightarrow F = 4/2$



b [keV]	0.4		0.5		0.6		0.7		0.8		0.9		1
7	1.22	1.13	1.05	0.99	0.94	0.90	0.87	0.86	0.84	0.83	0.82	0.81	0.80
	1.20	1.12	1.05	0.99	0.94	0.90	0.88	0.86	0.84	0.83	0.82	0.81	0.80
6	1.17	1.10	1.04	0.98	0.94	0.90	0.88	0.86	0.85	0.83	0.82	0.81	0.80
	1.14	1.08	1.03	0.98	0.94	0.91	0.88	0.86	0.85	0.83	0.82	0.81	0.80
5	1.11	1.06	1.01	0.97	0.93	0.91	0.88	0.86	0.85	0.83	0.82	0.81	0.80
	1.07	1.03	0.99	0.96	0.93	0.91	0.88	0.87	0.85	0.83	0.82	0.81	0.80
4	1.03	1.00	0.97	0.95	0.92	0.90	0.88	0.86	0.85	0.83	0.82	0.81	0.80
	0.99		0.95	0.93	0.91	0.89	0.88	0.86	0.85	0.83	0.82	0.81	0.80
3	0.95	0.93	0.92	0.91	0.89	0.88	0.87	0.85	0.84	0.83	0.82	0.81	0.80
	0.90	0.89	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80
2	0.86	0.86	0.85	0.85	0.84	0.84	0.83	0.83	0.82	0.82	0.81	0.81	0.80
	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.81	0.81	0.81	0.81	0.81	0.80
1	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80



2.6 Detector lineshape as a function of energy

$$\boxed{\sigma(E) = \frac{FWHM(E)}{2.35} = \frac{\sqrt{\Delta E_{noise}^2 + \Delta E_{ion}^2 + \Delta E_{incomplete}^2}}{2.35}}$$

$$\Delta E_{ion} = 2.35\sqrt{\epsilon F E}$$

$$\Delta E_{noise} = \epsilon c_{noise}$$

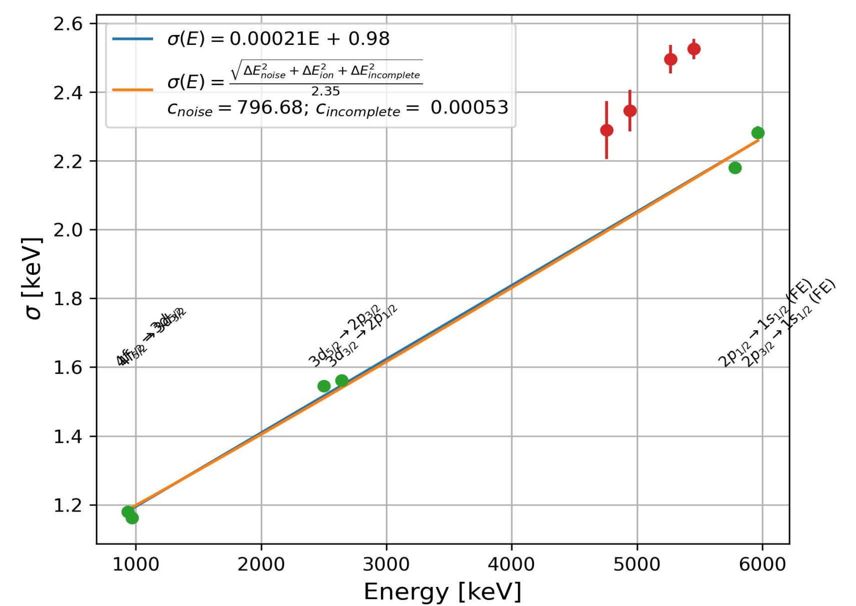
$$\Delta E_{incomplete} = E c_{incomplete}$$

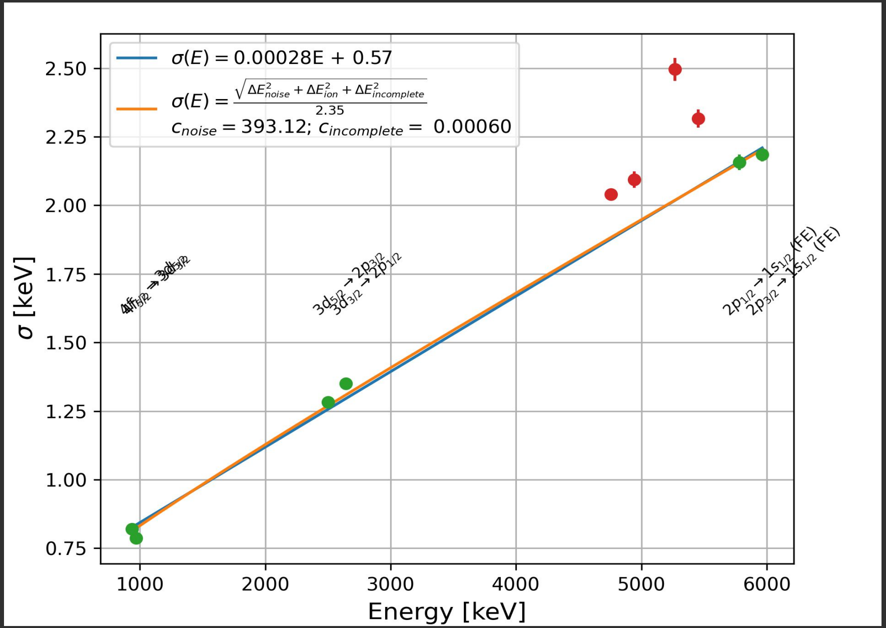
$$F = 0.13$$

$$\epsilon = 2.95 \cdot 10^{-3}$$

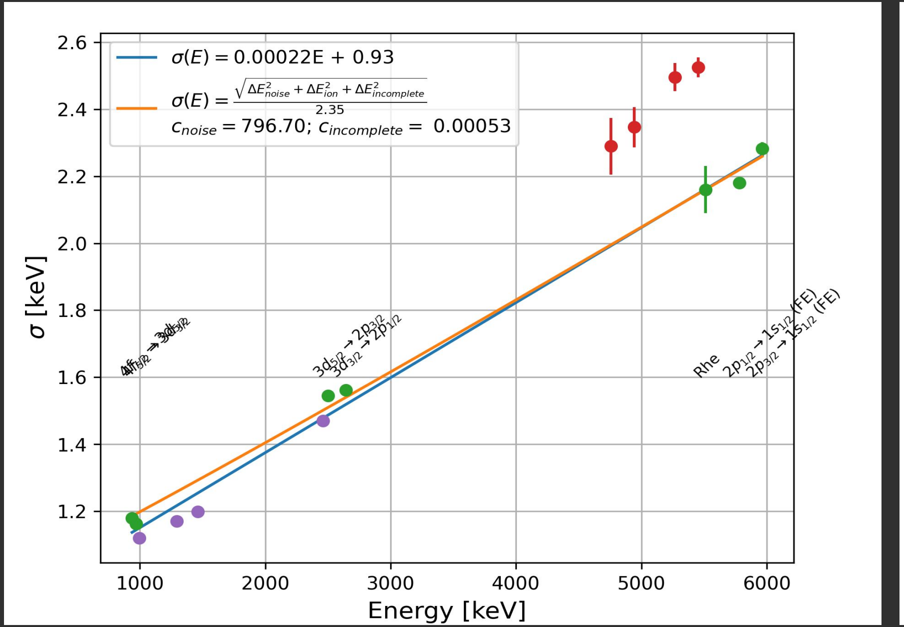
Sigma dependence on energy

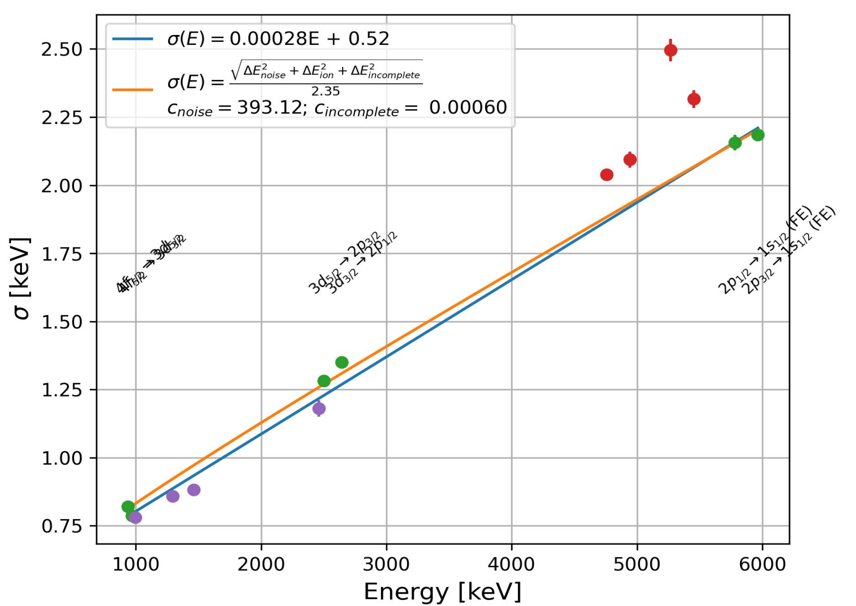
Single escape peaks are wider due to doppler broadening (from the motion of electrons in the crystals that recombine with the positrons, giving the produced photons a larger energy distribution).
Double escape peaks are broadened due to the geometry of the detector.

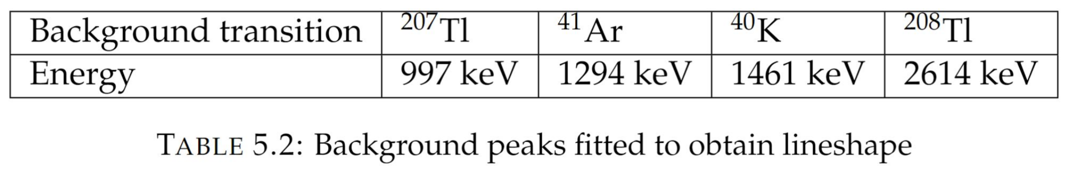




Fits with anticoincidence and background lines



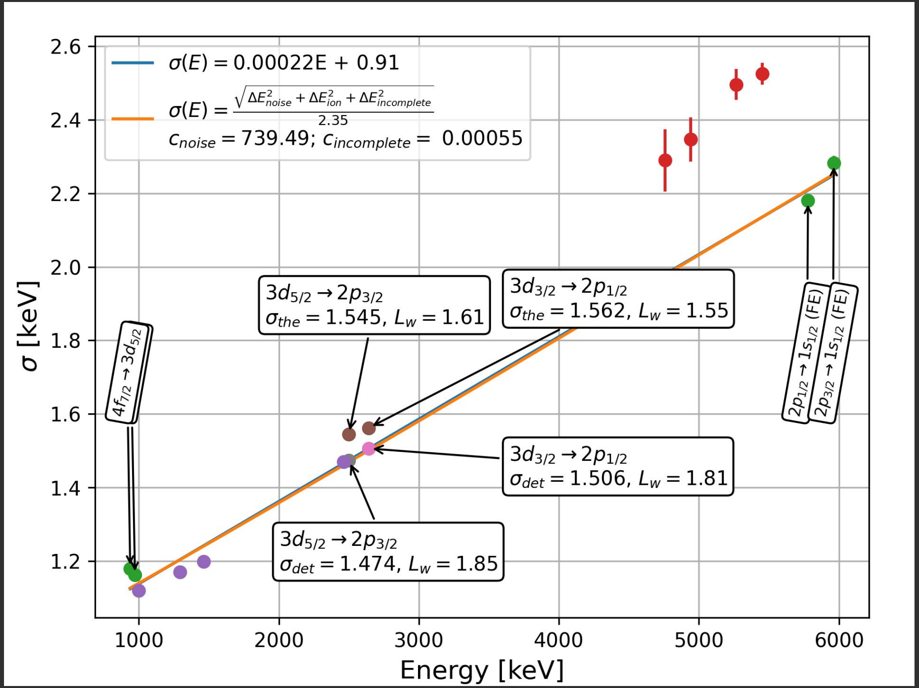


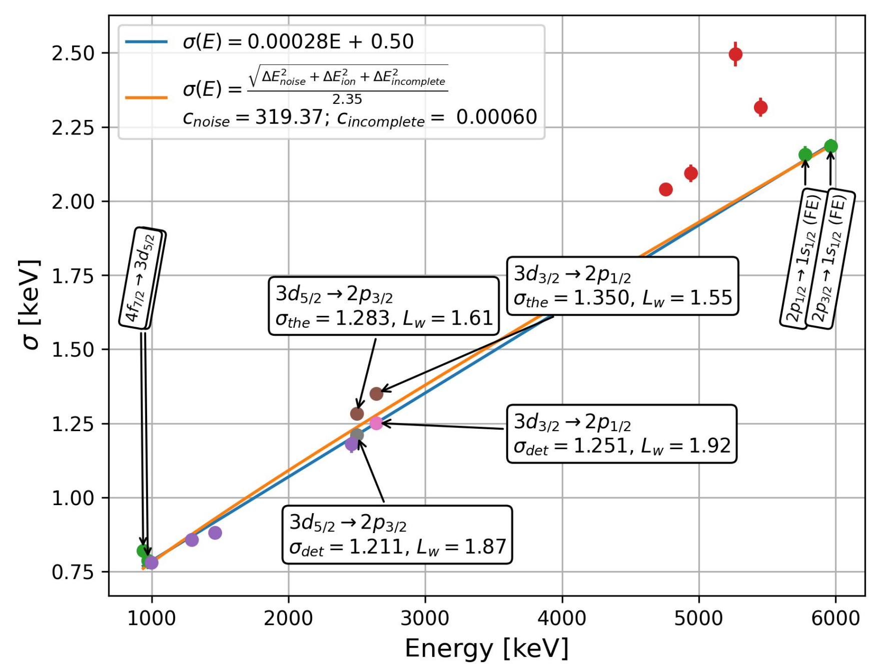


Background transition	^{207}Tl	^{41}Ar	^{40}K	^{208}Tl
Energy	997 keV	1294 keV	1461 keV	2614 keV

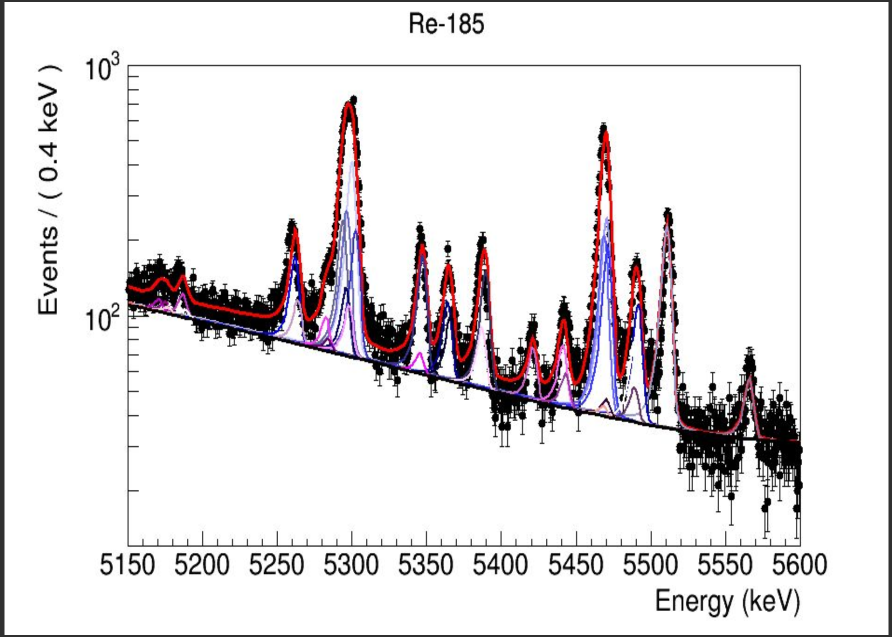
TABLE 5.2: Background peaks fitted to obtain lineshape

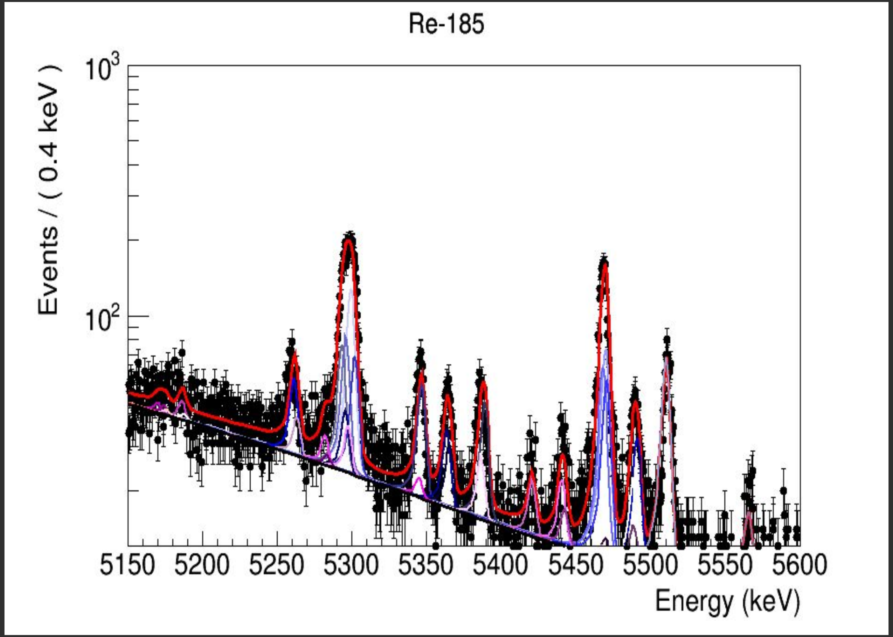
Natural linewidth





3. Fitting Rhenium 2p-1s hyperfine spectrum





Outlook

Systematic studies and errors

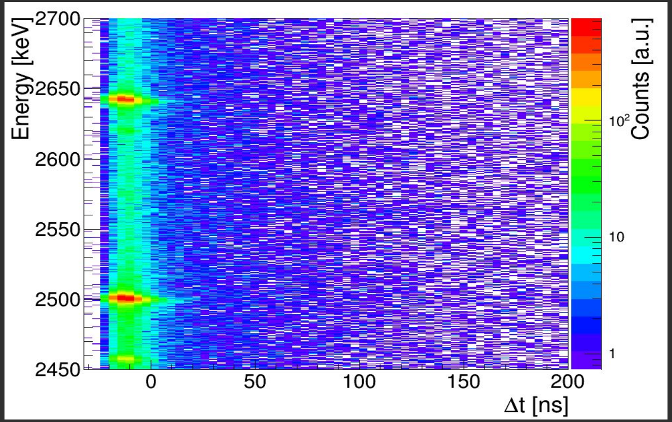
- Detector response (lineshape) and model (correlation analysis to identify systematic effects)
- Background model
- Energy range chosen for fits

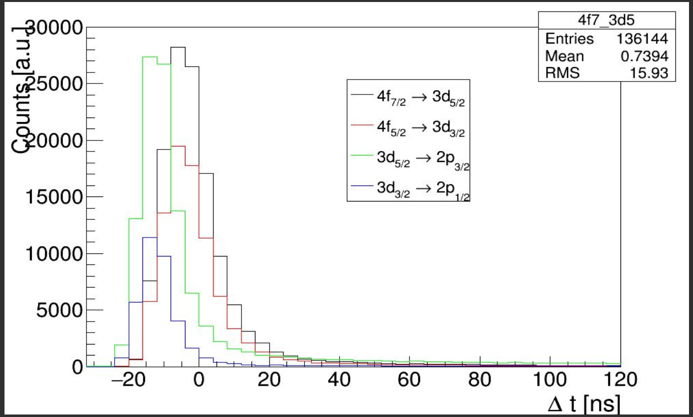
Further improvement (beyond this thesis)

- Include nuclear polarization in the transitions energy calculations
- Fitting method (e.g SATLAS package for fitting and correlation analysis)

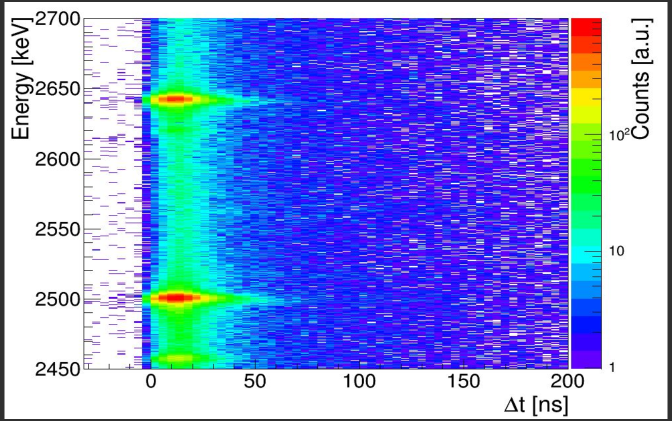
$3d \rightarrow 2p$

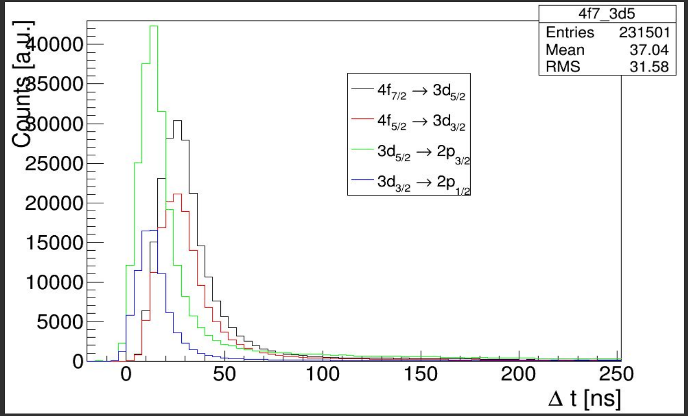
Right germanium detector





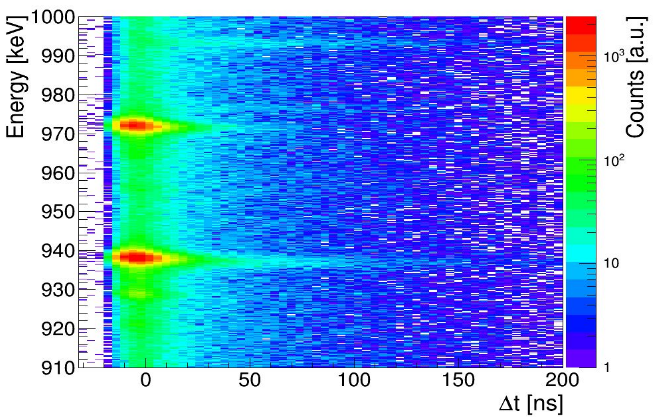
Left germanium detector

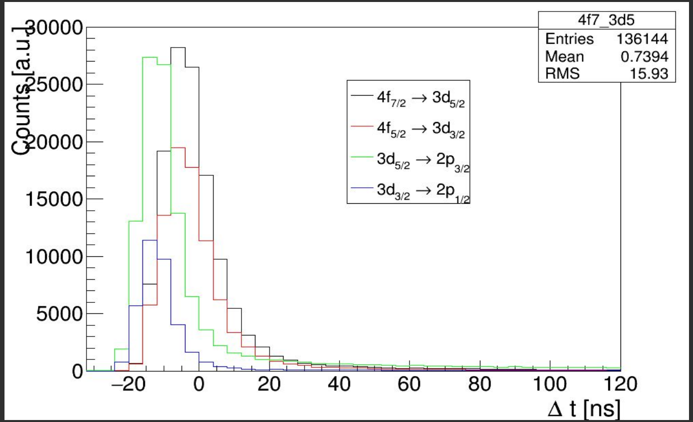




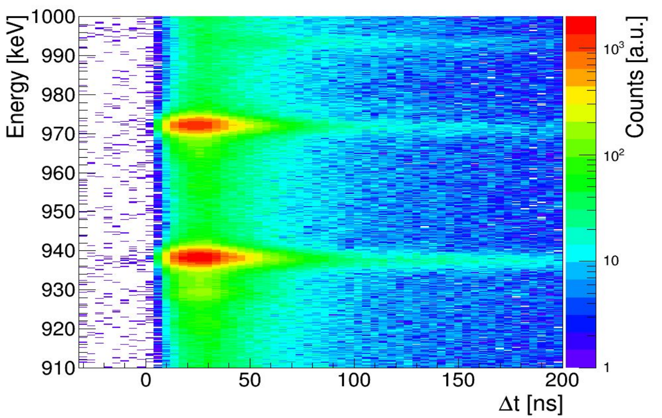
$4f \rightarrow 3d$

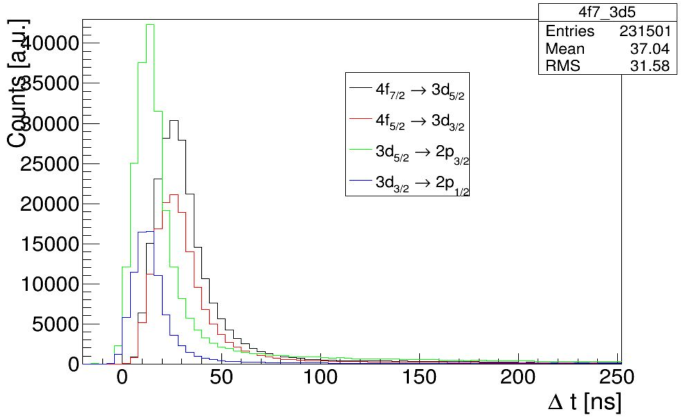
Right germanium detector





Left germanium detector

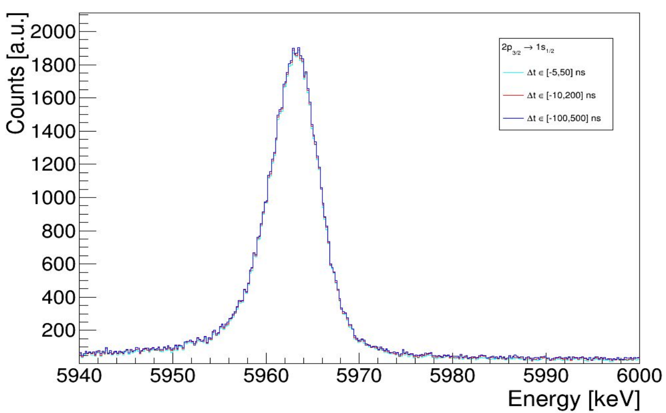


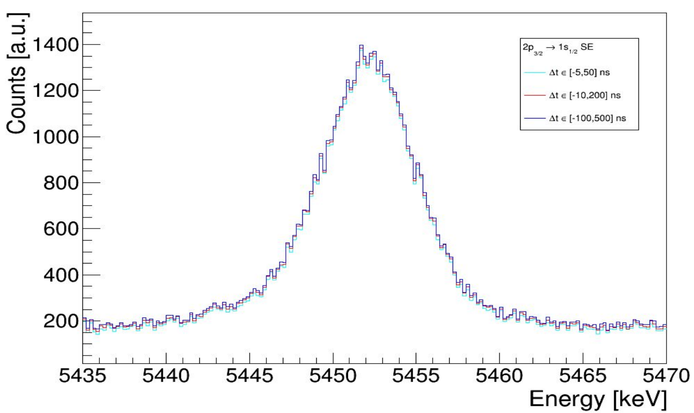


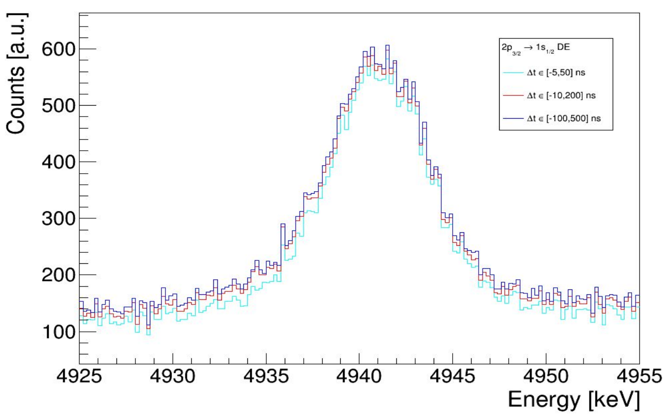
Left detector

$2p_{3/2} \rightarrow 1s_{1/2}$

Influence of timecut on number of counts



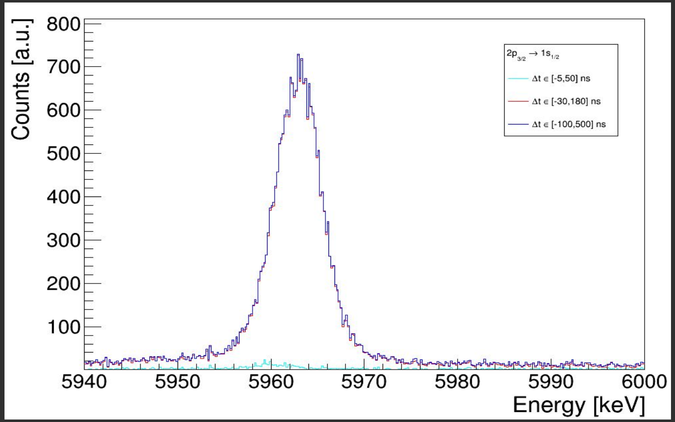


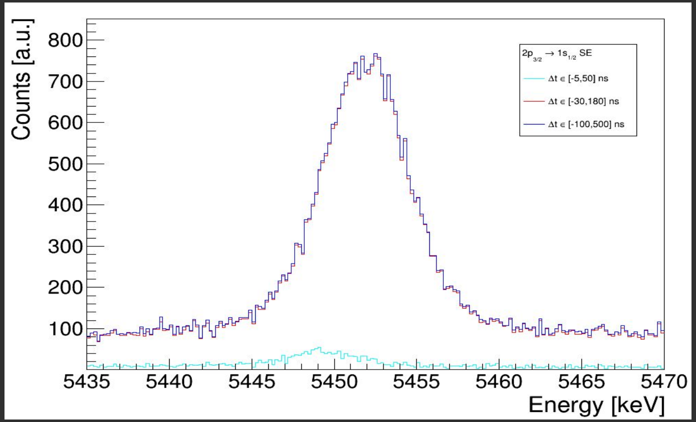


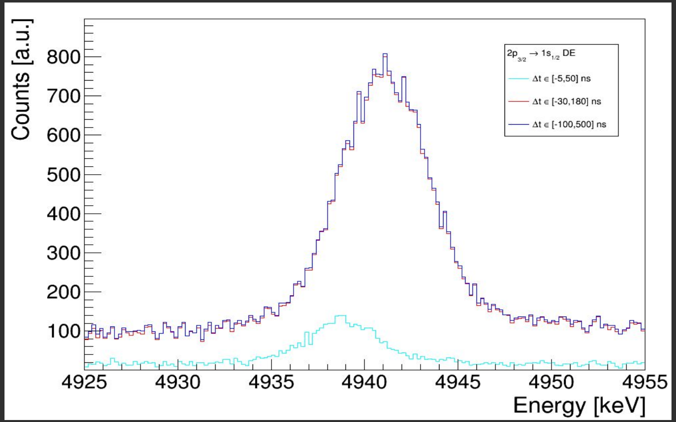
Right detector

$2p_{3/2} \rightarrow 1s_{1/2}$

Influence of timecut on number of counts





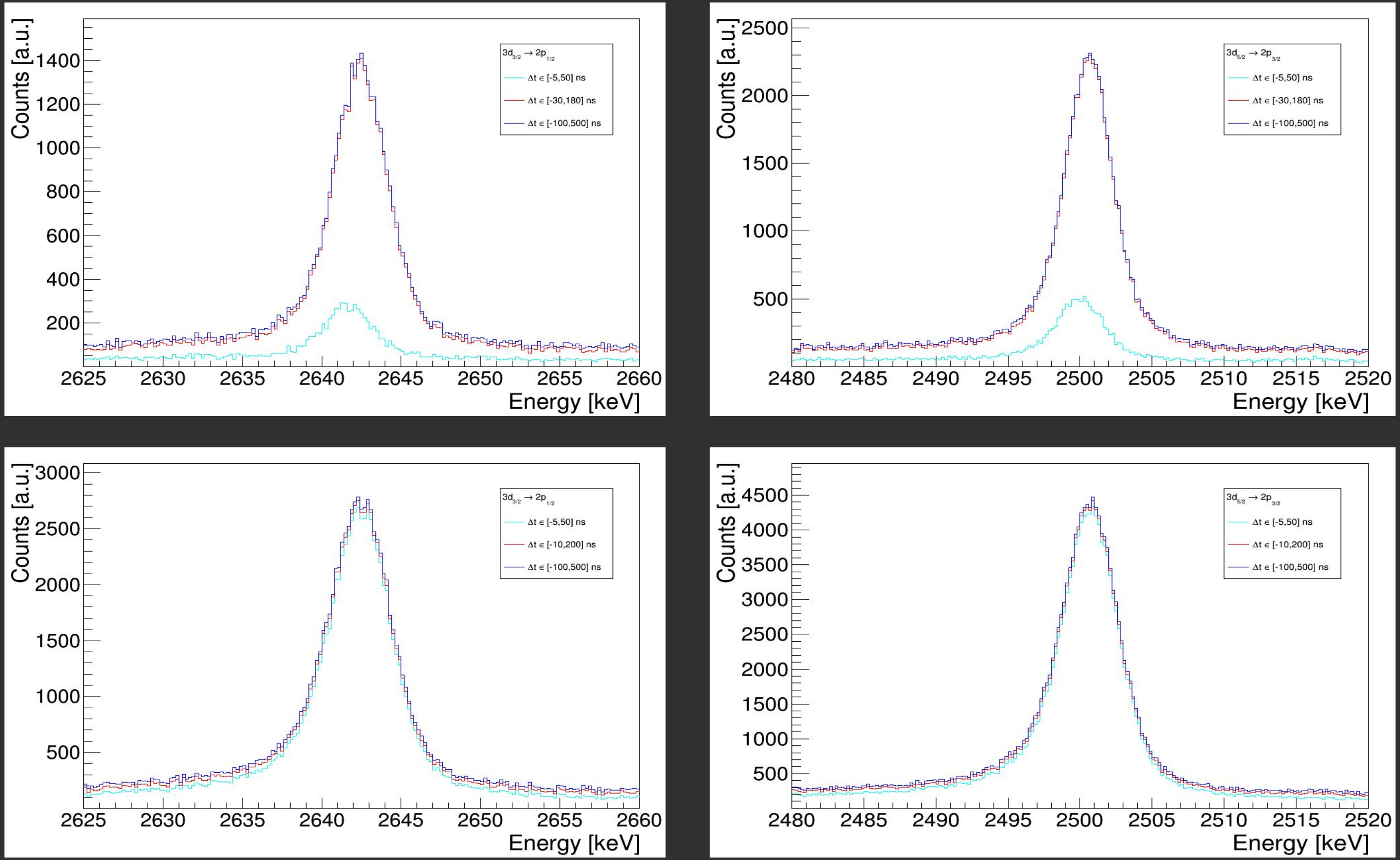


$3d \rightarrow 2p$

Influence of timecut on number of counts

Right germanium detector

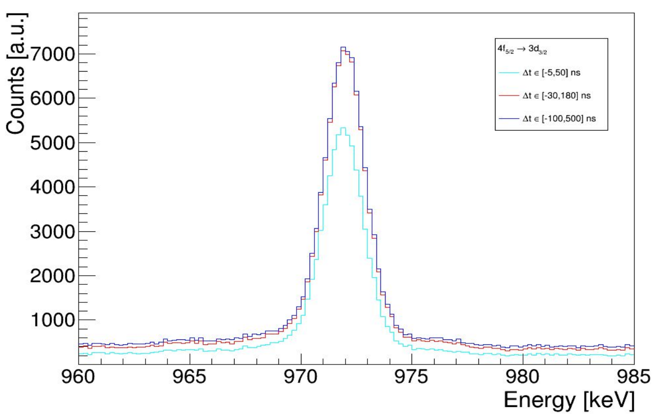
Left germanium detector

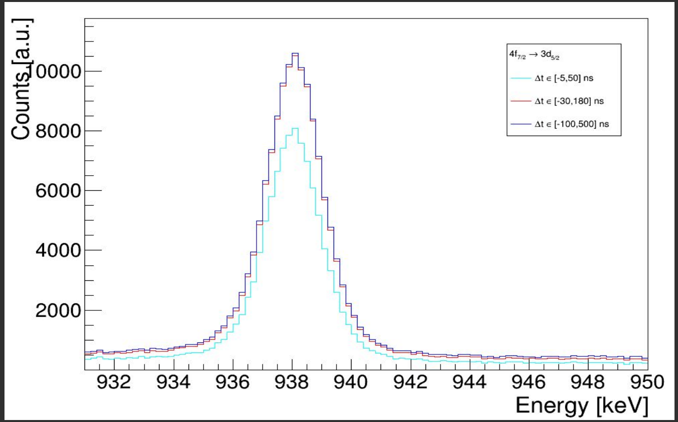


$4f \to 3d$

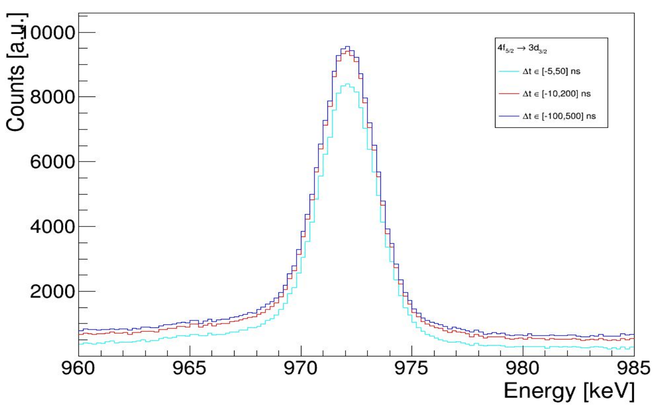
Influence of timecut on number of counts

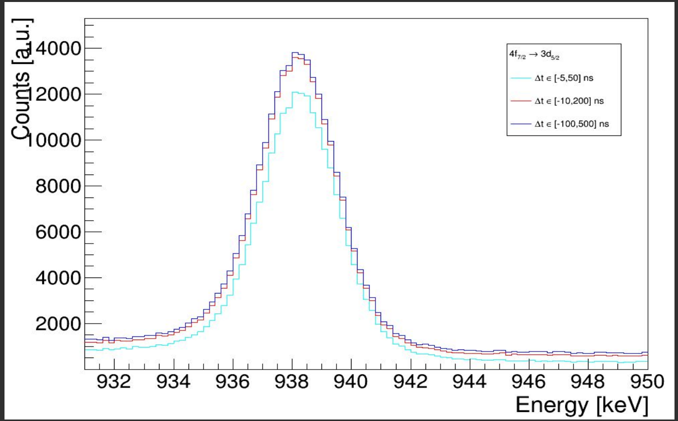
Right germanium detector





Left germanium detector

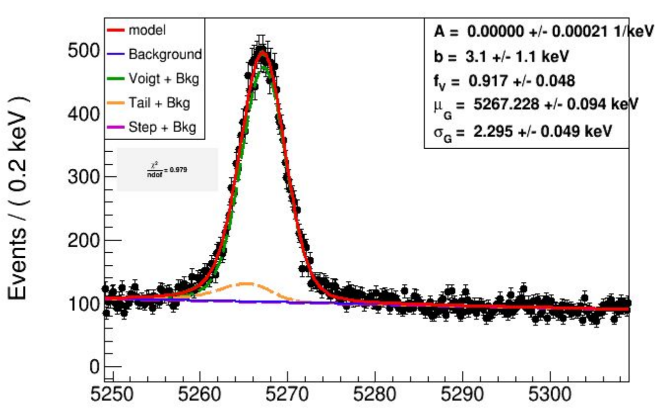


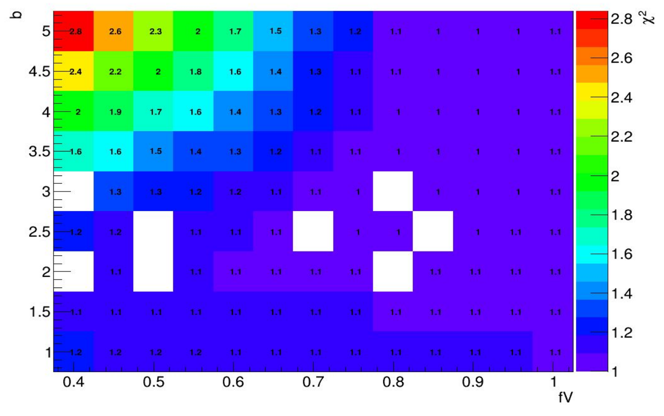


Single escape

$$2p_{1/2} \rightarrow 1s_{1/2}$$

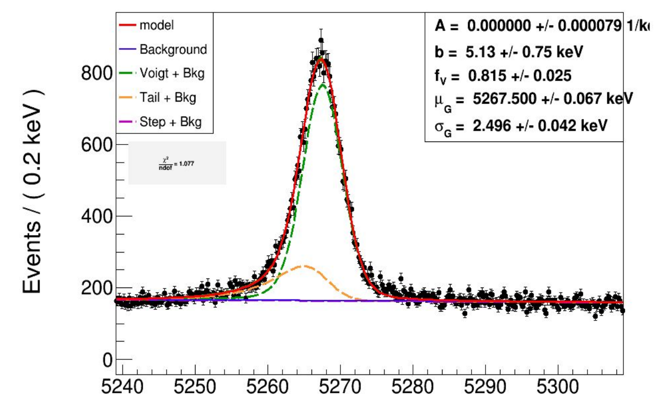
Right germanium detector

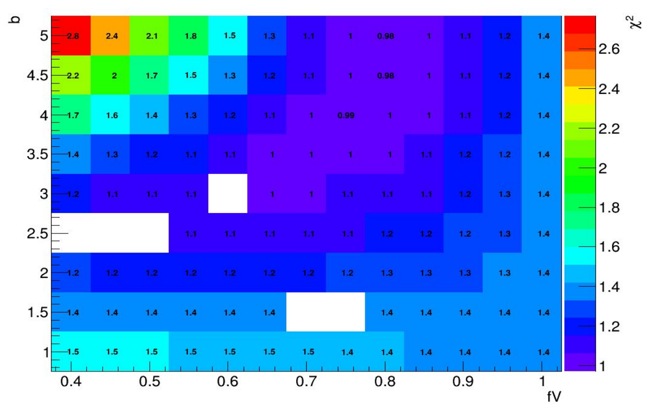




b \ fV	0.4		0.5		0.6		0.7		0.8		0.9		1
5	2.8	2.6	2.3	2	1.7	1.5	1.3	1.2	1.1	1	1	1	1.1
4.5	2.4	2.2	2	1.8	1.6	1.4	1.3	1.1	1.1	1	1	1	1.1
4	2	1.9	1.7	1.6	1.4	1.3	1.2	1.1	1	1	1	1	1.1
3.5	1.6	1.6	1.5	1.4	1.3	1.2	1.1	1.1	1	1	1	1	1.1
3		1.3	1.3	1.2	1.2	1.1	1.1	1		1	1	1	1.1
2.5	1.2	1.2		1.1	1.1	1.1		1	1		1	1.1	1.1
2		1.1		1.1	1.1	1.1	1.1	1.1		1.1	1.1	1.1	1.1
1.5	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
1	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1

Left germanium detector

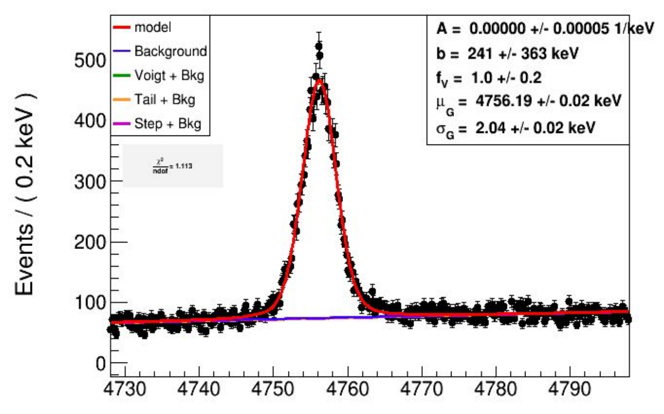


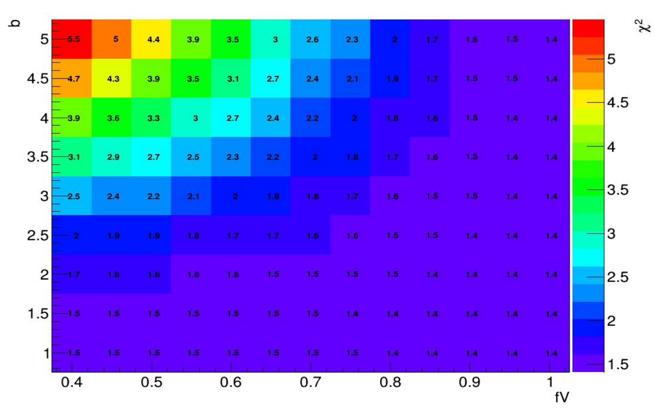


b \ fV	0.4		0.5		0.6		0.7		0.8		0.9		1
5	2.8	2.4	2.1	1.8	1.5	1.3	1.1	1	0.98	1	1.1	1.2	1.4
4.5	2.2	2	1.7	1.5	1.3	1.2	1.1	1	0.98	1	1.1	1.2	1.4
4	1.7	1.6	1.4	1.3	1.2	1.1	1	0.99	1	1	1.1	1.2	1.4
3.5	1.4	1.3	1.2	1.1	1.1	1	1	1	1	1.1	1.2	1.2	1.4
3	1.2	1.1	1.1	1.1		1	1	1.1	1.1	1.1	1.2	1.3	1.4
2.5				1.1	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.3	1.4
2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.3	1.3	1.3	1.3	1.4
1.5	1.4	1.4	1.4	1.4	1.4	1.4			1.4	1.4	1.4	1.4	1.4
1	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4

Double escape $2p_{1/2} \rightarrow 1s_{1/2}$

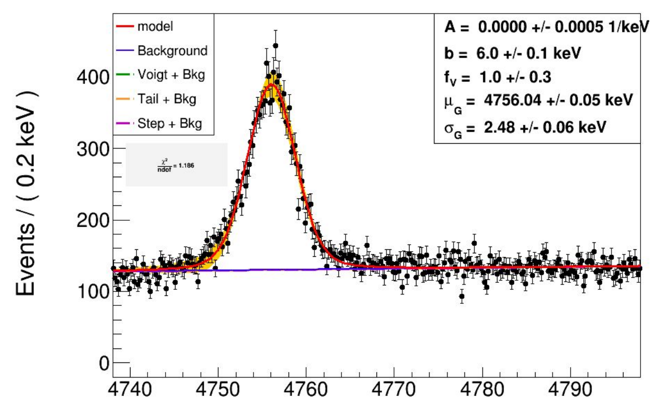
Right germanium detector

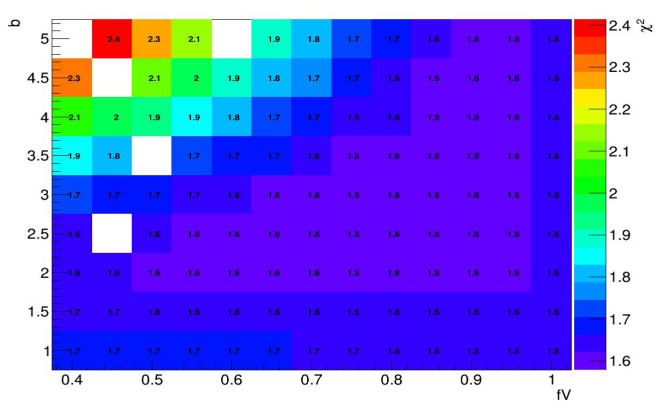




b	0.4		0.5		0.6		0.7		0.8		0.9		1
5	5.5	5	4.4	3.9	3.5	3	2.6	2.3	2	1.7	1.6	1.5	1.4
4.5	4.7	4.3	3.9	3.5	3.1	2.7	2.4	2.1	1.9	1.7	1.5	1.5	1.4
4	3.9	3.6	3.3	3	2.7	2.4	2.2	2	1.8	1.6	1.5	1.4	1.4
3.5	3.1	2.9	2.7	2.5	2.3	2.2	2	1.8	1.7	1.6	1.5	1.4	1.4
3	2.5	2.4	2.2	2.1	2	1.9	1.8	1.7	1.6	1.5	1.5	1.4	1.4
2.5	2	1.9	1.9	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4	1.4	1.4
2	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4
1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4
1	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4

Left germanium detector

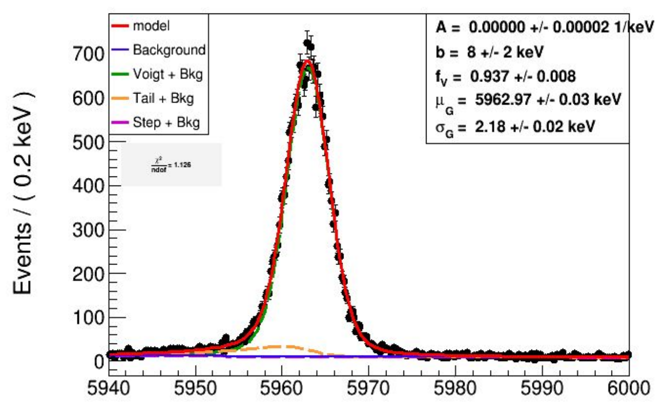




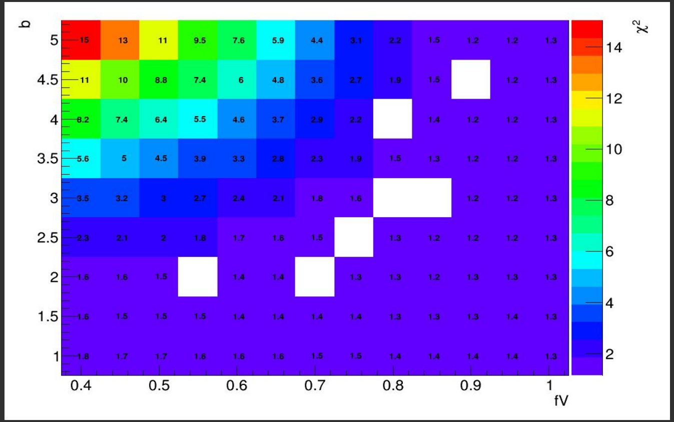
b	0.4		0.5		0.6		0.7		0.8		0.9		1
5		2.4	2.3	2.1		1.9	1.8	1.7	1.7	1.6	1.6	1.6	1.6
4.5	2.3		2.1	2	1.9	1.8	1.7	1.7	1.6	1.6	1.6	1.6	1.6
4	2.1	2	1.9	1.9	1.8	1.7	1.7	1.6	1.6	1.6	1.6	1.6	1.6
3.5	1.9	1.8		1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.6	1.6	1.6
3	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
2.5	1.6		1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
2	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
1.5	1.7	1.7	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
1	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.6

$2p_{3/2} \to 1s_{1/2}$

Right germanium detector

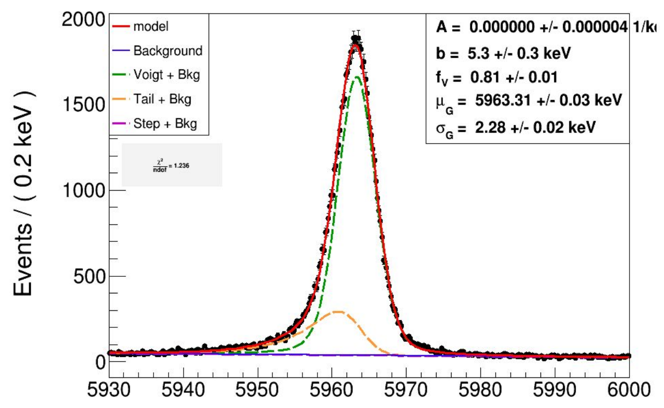


A = 0.00000 +/- 0.00002 1/keV
b = 8 +/- 2 keV
f_V = 0.937 +/- 0.008
μ_G = 5962.97 +/- 0.03 keV
σ_G = 2.18 +/- 0.02 keV

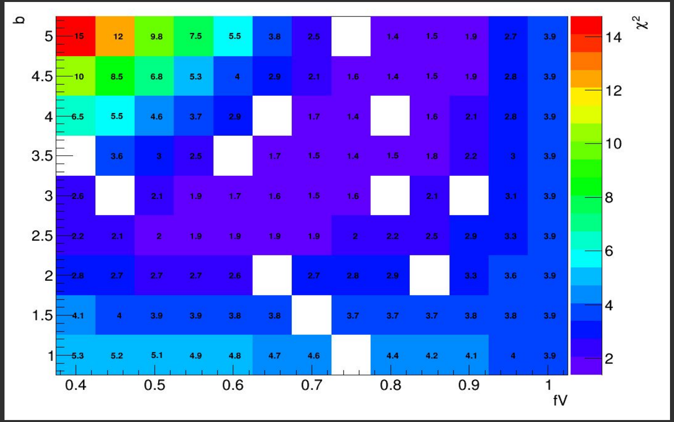


b	0.4		0.5		0.6		0.7		0.8		0.9		1
5	15	13	11	9.5	7.6	5.9	4.4	3.1	2.2	1.5	1.2	1.2	1.3
4.5	11	10	8.8	7.4	6	4.8	3.6	2.7	1.9	1.5		1.2	1.3
4	8.2	7.4	6.4	5.5	4.6	3.7	2.9	2.2		1.4	1.2	1.2	1.3
3.5	5.6	5	4.5	3.9	3.3	2.8	2.3	1.9	1.5	1.3	1.2	1.2	1.3
3	3.5	3.2	3	2.7	2.4	2.1	1.8	1.6			1.2	1.2	1.3
2.5	2.3	2.1	2	1.8	1.7	1.6	1.5		1.3	1.2	1.2	1.2	1.3
2	1.6	1.6	1.5		1.4	1.4		1.3	1.3	1.2	1.3	1.3	1.3
1.5	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.4	1.3
1	1.8	1.7	1.7	1.6	1.6	1.6	1.5	1.5	1.4	1.4	1.4	1.4	1.3

Left germanium detector



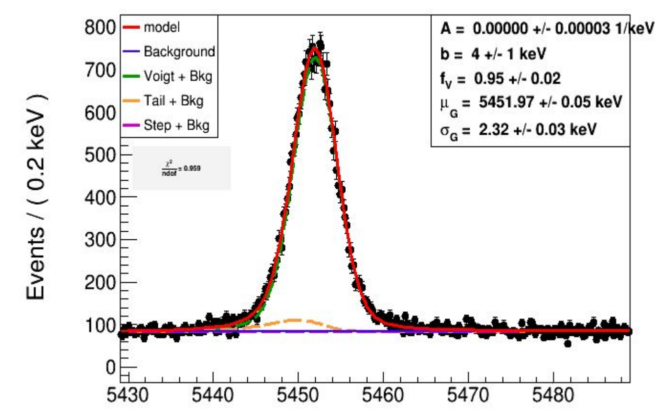
A = 0.000000 +/- 0.000004 1/k
b = 5.3 +/- 0.3 keV
f_V = 0.81 +/- 0.01
μ_G = 5963.31 +/- 0.03 keV
σ_G = 2.28 +/- 0.02 keV



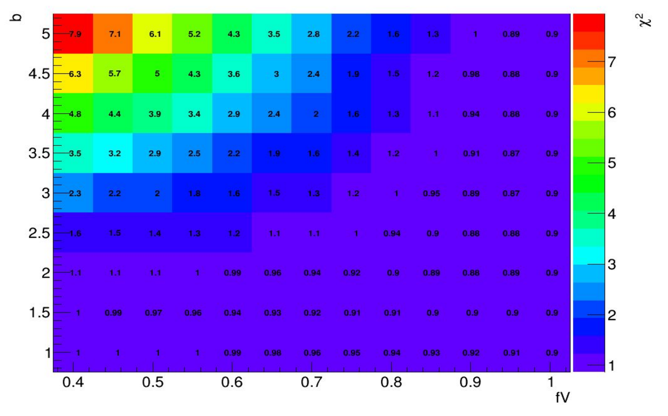
b	0.4		0.5		0.6		0.7		0.8		0.9		1
5	15	12	9.8	7.5	5.5	3.8	2.5		1.4	1.5	1.9	2.7	3.9
4.5	10	8.5	6.8	5.3	4	2.9	2.1	1.6	1.4	1.5	1.9	2.8	3.9
4	6.5	5.5	4.6	3.7	2.9		1.7	1.4		1.6	2.1	2.8	3.9
3.5		3.6	3	2.5		1.7	1.5	1.4	1.5	1.8	2.2	3	3.9
3	2.6		2.1	1.9	1.7	1.6	1.5	1.6		2.1		3.1	3.9
2.5	2.2	2.1	2	1.9	1.9	1.9	1.9	2	2.2	2.5	2.9	3.3	3.9
2	2.8	2.7	2.7	2.7	2.6		2.7	2.8	2.9		3.3	3.6	3.9
1.5	4.1	4	3.9	3.9	3.8	3.8		3.7	3.7	3.7	3.8	3.8	3.9
1	5.3	5.2	5.1	4.9	4.8	4.7	4.6		4.4	4.2	4.1	4	3.9

Single escape $2p_{3/2} \to 1s_{1/2}$

Right germanium detector

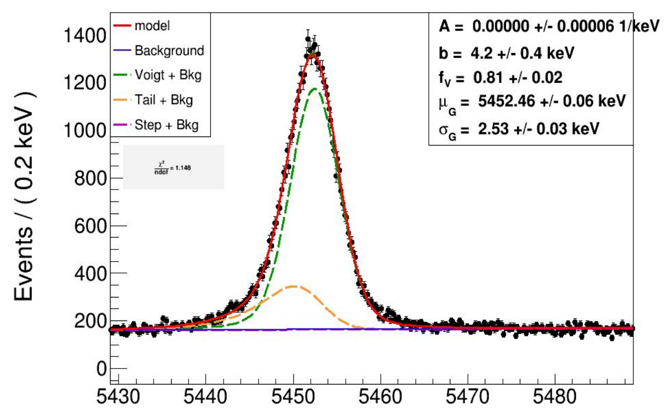


$A =$ 0.00000 +/- 0.00003 1/keV
$b =$ 4 +/- 1 keV
$f_V =$ 0.95 +/- 0.02
$\mu_G =$ 5451.97 +/- 0.05 keV
$\sigma_G =$ 2.32 +/- 0.03 keV

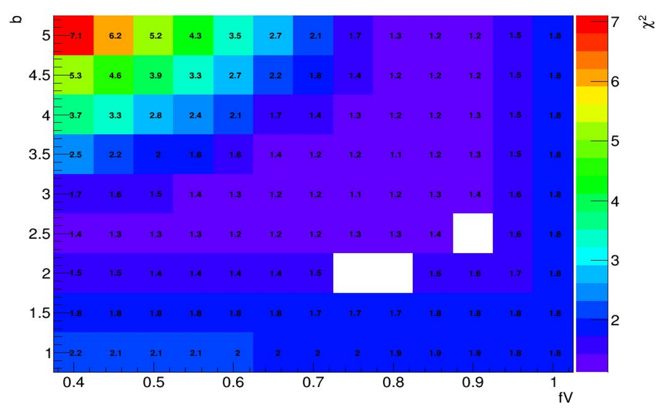


5	7.9	7.1	6.1	5.2	4.3	3.5	2.8	2.2	1.6	1.3	1	0.89	0.9
4.5	6.3	5.7	5	4.3	3.6	3	2.4	1.9	1.5	1.2	0.98	0.88	0.9
4	4.8	4.4	3.9	3.4	2.9	2.4	2	1.6	1.3	1.1	0.94	0.88	0.9
3.5	3.5	3.2	2.9	2.5	2.2	1.9	1.6	1.4	1.2	1	0.91	0.87	0.9
3	2.3	2.2	2	1.8	1.6	1.5	1.3	1.2	1	0.95	0.89	0.87	0.9
2.5	1.6	1.5	1.4	1.3	1.2	1.1	1.1	1	0.94	0.9	0.88	0.88	0.9
2	1.1	1.1	1.1	1	0.99	0.96	0.94	0.92	0.9	0.89	0.88	0.89	0.9
1.5	1	0.99	0.97	0.96	0.94	0.93	0.92	0.91	0.91	0.9	0.9	0.9	0.9
1	1	1	1	1	0.99	0.98	0.96	0.95	0.94	0.93	0.92	0.91	0.9

Left germanium detector



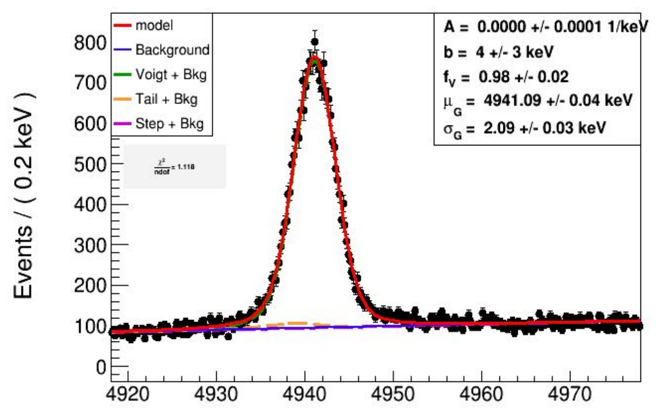
$A =$ 0.00000 +/- 0.00006 1/keV
$b =$ 4.2 +/- 0.4 keV
$f_V =$ 0.81 +/- 0.02
$\mu_G =$ 5452.46 +/- 0.06 keV
$\sigma_G =$ 2.53 +/- 0.03 keV

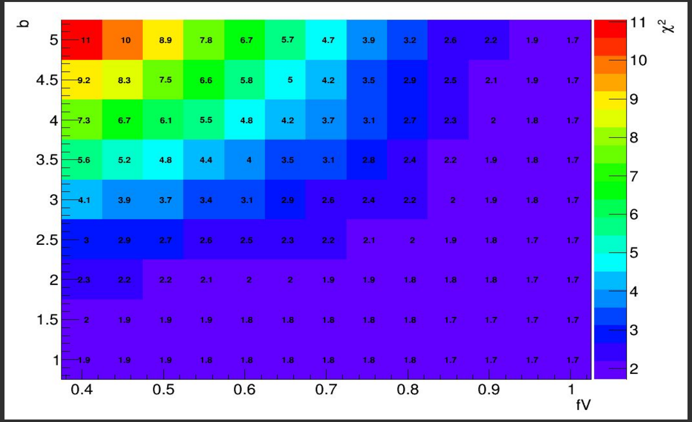


5	7.1	6.2	5.2	4.3	3.5	2.7	2.1	1.7	1.3	1.2	1.2	1.5	1.8
4.5	5.3	4.6	3.9	3.3	2.7	2.2	1.8	1.4	1.2	1.2	1.2	1.5	1.8
4	3.7	3.3	2.8	2.4	2.1	1.7	1.4	1.3	1.2	1.2	1.3	1.5	1.8
3.5	2.5	2.2	2	1.8	1.6	1.4	1.2	1.2	1.1	1.2	1.3	1.5	1.8
3	1.7	1.6	1.5	1.4	1.3	1.2	1.2	1.1	1.2	1.3	1.4	1.6	1.8
2.5	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.3	1.3	1.4		1.6	1.8
2	1.5	1.5	1.4	1.4	1.4	1.4	1.5			1.6	1.6	1.7	1.8
1.5	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.7	1.7	1.8	1.8	1.8	1.8
1	2.2	2.1	2.1	2.1	2	2	2	2	1.9	1.9	1.9	1.8	1.8

Double escape $2p_{3/2} \rightarrow 1s_{1/2}$

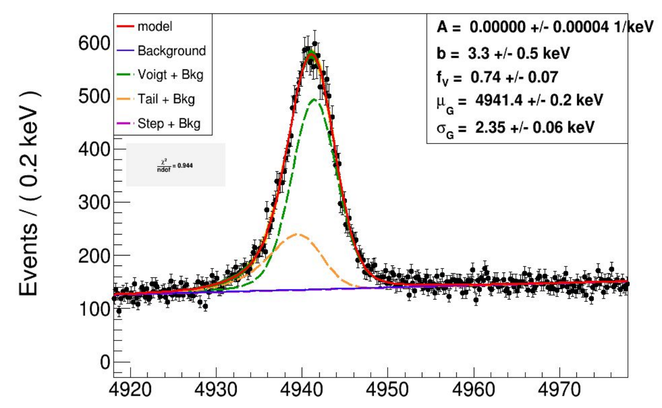
Right germanium detector

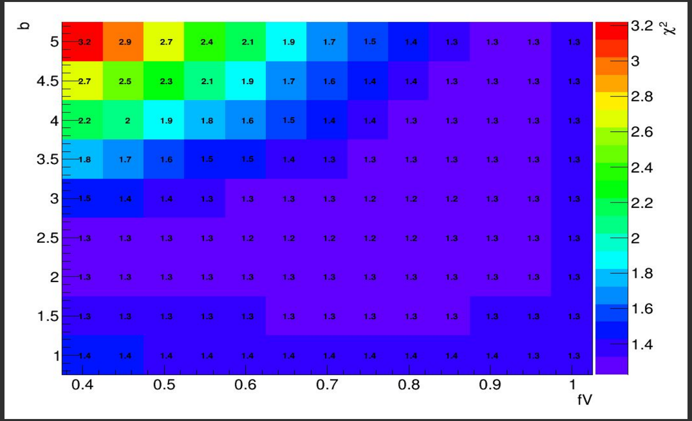




b \ fV	0.4		0.5		0.6		0.7		0.8		0.9		1
5	11	10	8.9	7.8	6.7	5.7	4.7	3.9	3.2	2.6	2.2	1.9	1.7
4.5	9.2	8.3	7.5	6.6	5.8	5	4.2	3.5	2.9	2.5	2.1	1.9	1.7
4	7.3	6.7	6.1	5.5	4.8	4.2	3.7	3.1	2.7	2.3	2	1.8	1.7
3.5	5.6	5.2	4.8	4.4	4	3.5	3.1	2.8	2.4	2.2	1.9	1.8	1.7
3	4.1	3.9	3.7	3.4	3.1	2.9	2.6	2.4	2.2	2	1.9	1.8	1.7
2.5	3	2.9	2.7	2.6	2.5	2.3	2.2	2.1	2	1.9	1.8	1.7	1.7
2	2.3	2.2	2.2	2.1	2	2	1.9	1.9	1.8	1.8	1.8	1.7	1.7
1.5	2	1.9	1.9	1.9	1.8	1.8	1.8	1.8	1.8	1.7	1.7	1.7	1.7
1	1.9	1.9	1.9	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.7	1.7	1.7

Left germanium detector

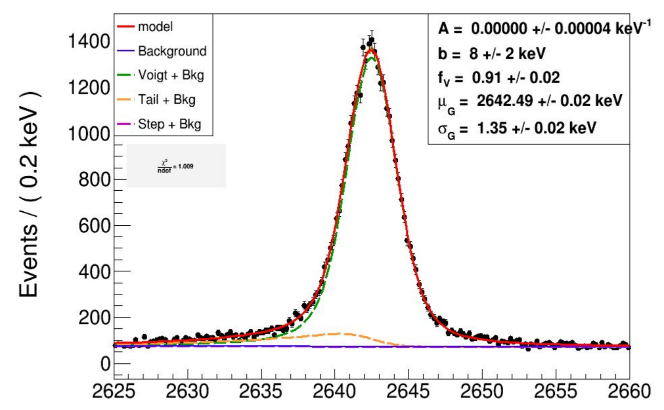


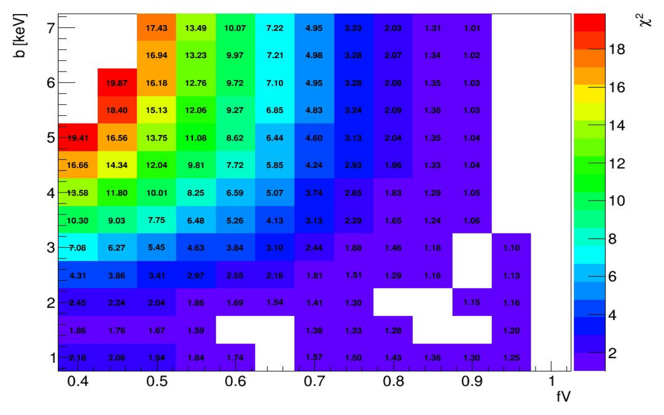


b \ fV	0.4		0.5		0.6		0.7		0.8		0.9		1
5	3.2	2.9	2.7	2.4	2.1	1.9	1.7	1.5	1.4	1.3	1.3	1.3	1.3
4.5	2.7	2.5	2.3	2.1	1.9	1.7	1.6	1.4	1.4	1.3	1.3	1.3	1.3
4	2.2	2	1.9	1.8	1.6	1.5	1.4	1.4	1.3	1.3	1.3	1.3	1.3
3.5	1.8	1.7	1.6	1.5	1.5	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3
3	1.5	1.4	1.4	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.3	1.3	1.3
2.5	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.3	1.3	1.3	1.3
2	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
1.5	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
1	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.3	1.3

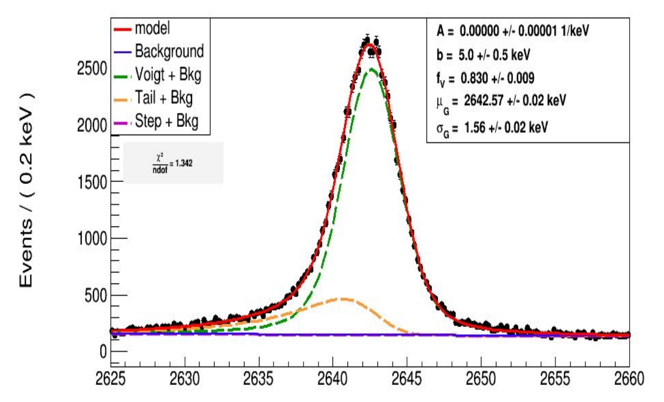
$3d_{3/2} \to 2p_{1/2}$

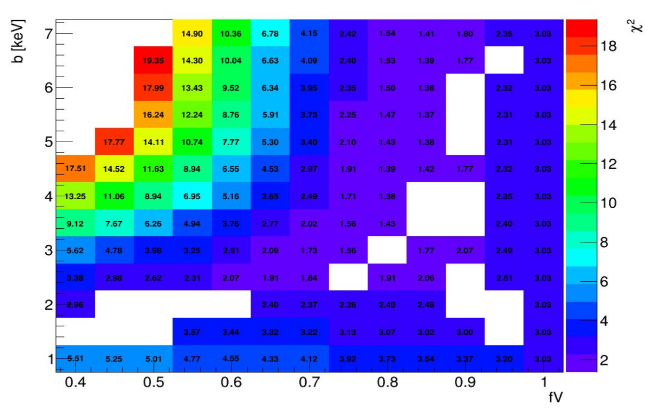
Right germanium detector





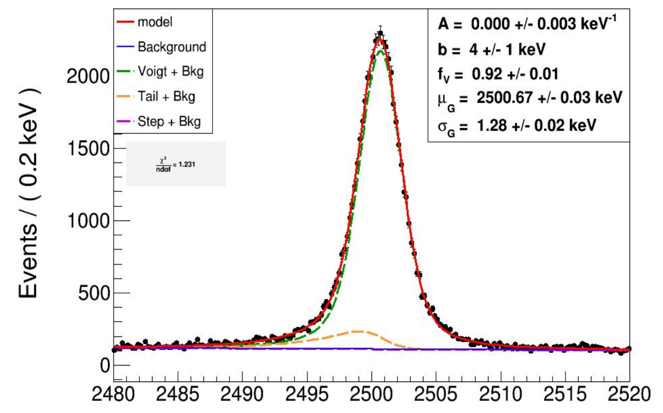
Left germanium detector

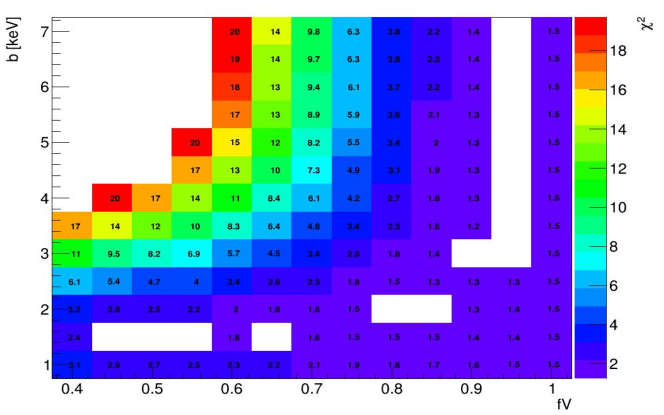




$3d_{5/2} \rightarrow 2p_{3/2}$

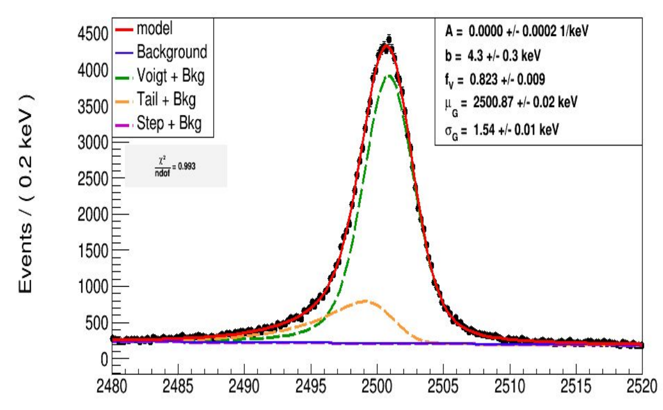
Right germanium detector

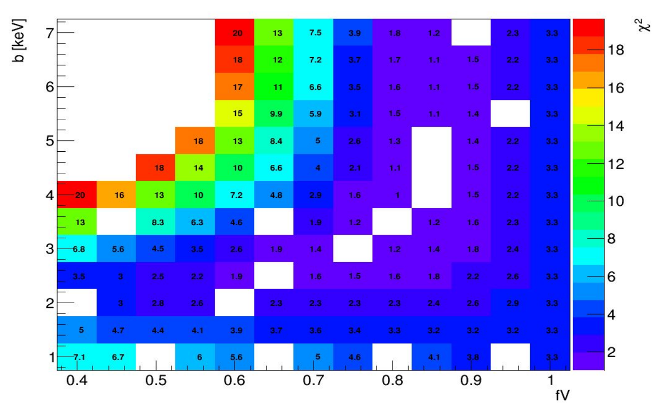




b [keV]	0.4		0.5		0.6		0.7		0.8		0.9		1
7					20	14	9.8	6.3	3.8	2.2	1.4		1.5
					19	14	9.7	6.3	3.8	2.2	1.4		1.5
6					18	13	9.4	6.1	3.7	2.2	1.4		1.5
					17	13	8.9	5.9	3.6	2.1	1.3		1.5
5				20	15	12	8.2	5.5	3.4	2	1.3		1.5
				17	13	10	7.3	4.9	3.1	1.9	1.3		1.5
4		20	17	14	11	8.4	6.1	4.2	2.7	1.8	1.3		1.5
	17	14	12	10	8.3	6.4	4.8	3.4	2.3	1.6	1.2		1.5
3	11	9.5	8.2	6.9	5.7	4.5	3.4	2.5	1.8	1.4			1.5
	6.1	5.4	4.7	4	3.4	2.8	2.3	1.8	1.5	1.3	1.3	1.3	1.5
2	3.2	2.8	2.5	2.2	2	1.8	1.6	1.5			1.3	1.4	1.5
	2.4				1.8		1.6	1.5	1.5	1.5	1.4	1.4	1.5
1	3.1	2.9	2.7	2.5	2.3	2.2	2.1	1.9	1.8	1.7	1.6	1.5	1.5

Left germanium detector

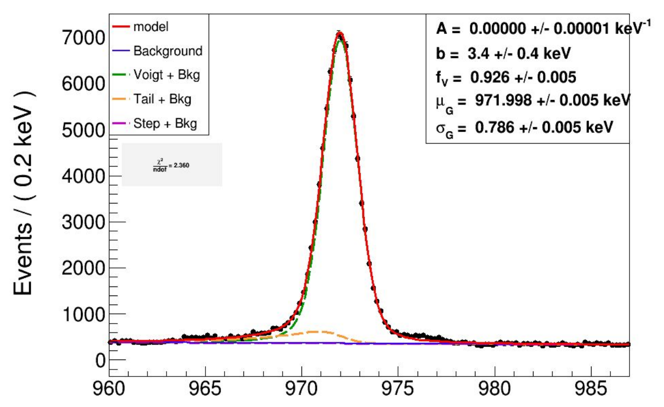


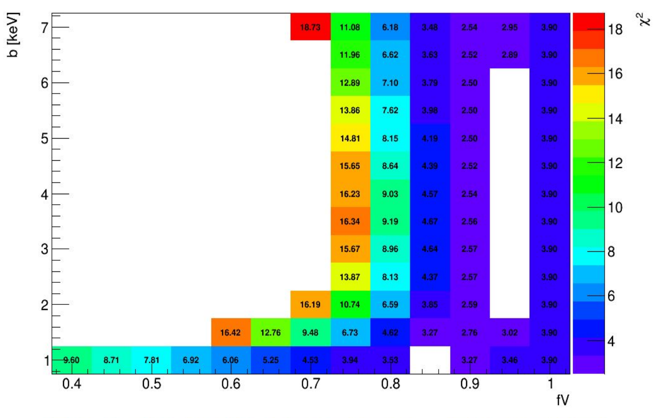


b [keV]	0.4		0.5		0.6		0.7		0.8		0.9		1
7					20	13	7.5	3.9	1.8	1.2		2.3	3.3
					18	12	7.2	3.7	1.7	1.1	1.5	2.2	3.3
6					17	11	6.6	3.5	1.6	1.1	1.5	2.2	3.3
					15	9.9	5.9	3.1	1.5	1.1	1.4		3.3
5				18	13	8.4	5	2.6	1.3		1.4	2.2	3.3
			18	14	10	6.6	4	2.1	1.1		1.5	2.2	3.3
4	20	16	13	10	7.2	4.8	2.9	1.6	1		1.5	2.2	3.3
	13		8.3	6.3	4.6		1.9	1.2		1.2	1.6	2.3	3.3
3	6.8	5.6	4.5	3.5	2.6	1.9	1.4		1.2	1.4	1.8	2.4	3.3
	3.5	3	2.5	2.2	1.9		1.6	1.5	1.6	1.8	2.2	2.6	3.3
2		3	2.8	2.6		2.3	2.3	2.3	2.3	2.4	2.6	2.9	3.3
	5	4.7	4.4	4.1	3.9	3.7	3.6	3.4	3.3	3.2	3.2	3.2	3.3
1	7.1	6.7		6	5.6		5	4.6		4.1	3.8		3.3

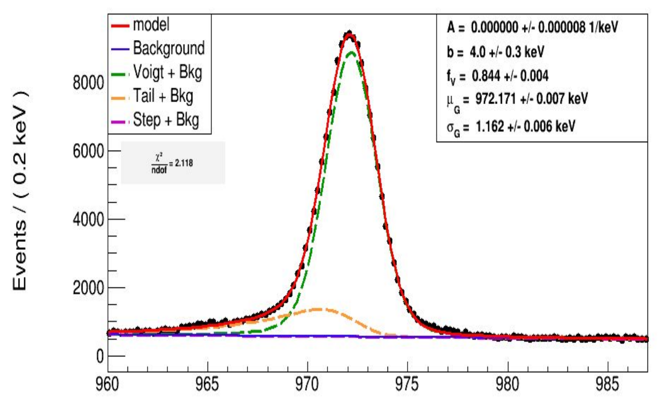
$4f_{5/2} \to 3d_{3/2}$

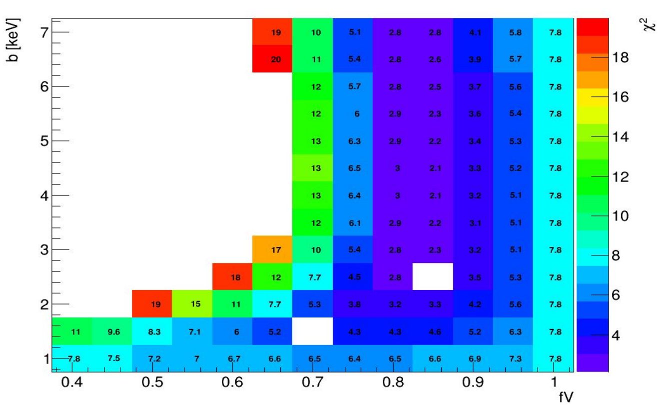
Right germanium detector





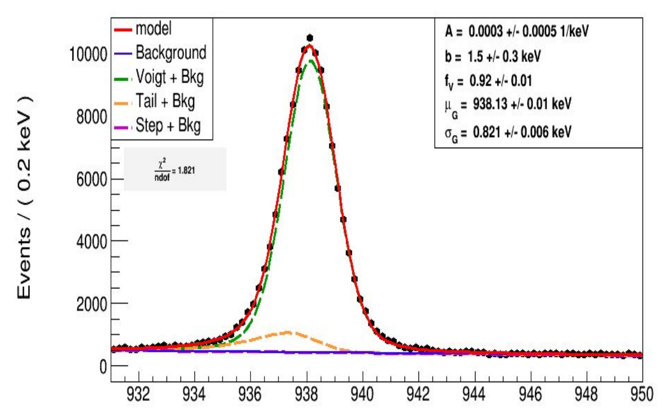
Left germanium detector

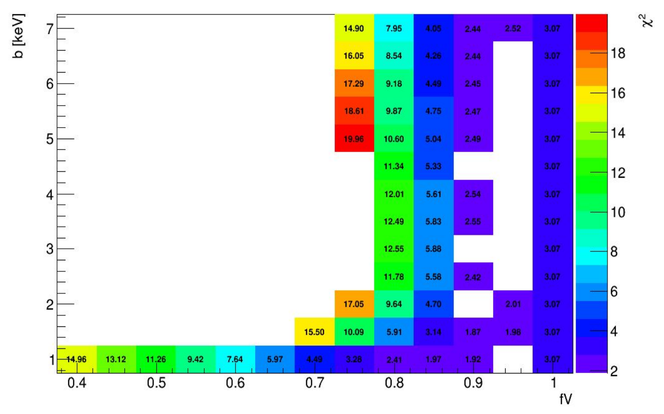




$4f_{7/2} \rightarrow 3d_{5/2}$

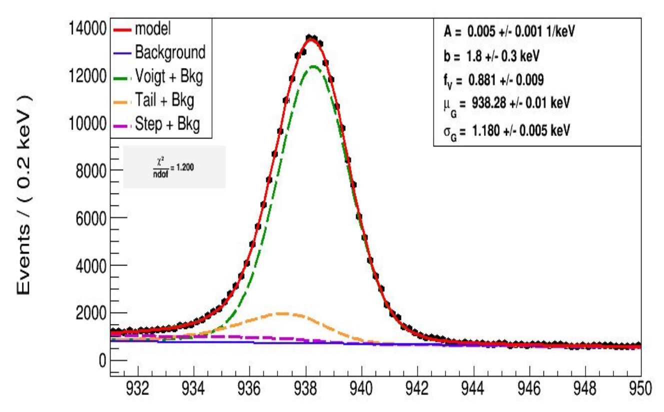
Right germanium detector

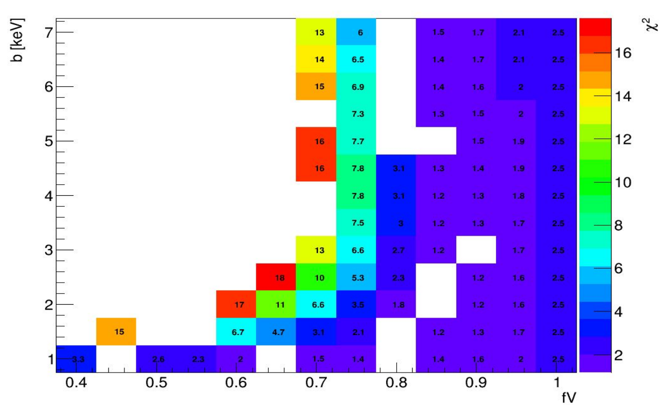




b [keV]	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
7								14.90	7.95	4.05	2.44	2.52	3.07
6.5								16.05	8.54	4.26	2.44		3.07
6								17.29	9.18	4.49	2.45		3.07
5.5								18.61	9.87	4.75	2.47		3.07
5								19.96	10.60	5.04	2.49		3.07
4.5									11.34	5.33			3.07
4									12.01	5.61	2.54		3.07
3.5									12.49	5.83	2.55		3.07
3									12.55	5.88			3.07
2.5									11.78	5.58	2.42		3.07
2								17.05	9.64	4.70		2.01	3.07
1.5							15.50	10.09	5.91	3.14	1.87	1.98	3.07
1	14.96	13.12	11.26	9.42	7.64	5.97	4.49	3.28	2.41	1.97	1.92		3.07

Left germanium detector





b [keV]	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
7							13	6		1.5	1.7	2.1	2.5
6.5							14	6.5		1.4	1.7	2.1	2.5
6							15	6.9		1.4	1.6	2	2.5
5.5								7.3		1.3	1.5	2	2.5
5							16	7.7			1.5	1.9	2.5
4.5							16	7.8	3.1	1.3	1.4	1.9	2.5
4								7.8	3.1	1.2	1.3	1.8	2.5
3.5								7.5	3	1.2	1.3	1.7	2.5
3							13	6.6	2.7	1.2		1.7	2.5
2.5						18	10	5.3	2.3		1.2	1.6	2.5
2					17	11	6.6	3.5	1.8		1.2	1.6	2.5
1.5		15			6.7	4.7	3.1	2.1		1.2	1.3	1.7	2.5
1	3.3		2.6	2.3	2		1.5	1.4		1.4	1.6	2	2.5