

# SUSY-QCD Corrections to Pseudoscalar Higgs Production via Gluon Fusion

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# Introduction

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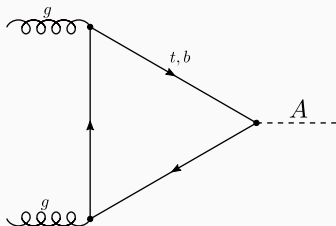
# Motivation

MSSM has extended Higgs sector

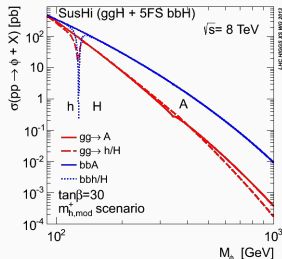
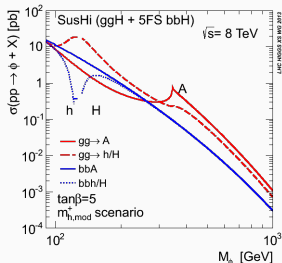
$$h, H, \mathbf{A}, H^\pm$$

→ Find SUSY-QCD corrections to the production cross section numerically

Production at the LHC via



# Motivation



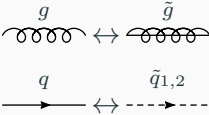
- gluon fusion important production channel at the LHC, together with  $gg \rightarrow b\bar{b}A$
- (pure) QCD corrections are large (10 - 30%)

[Spira '93] [Harlander et al. '09] [Anastasiou, Melnikov '02]

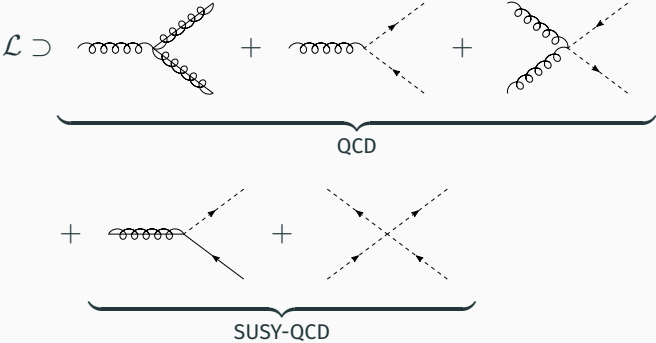
- Can be reused for relevant decay channels  $A \rightarrow \gamma\gamma, gg$

# Susy QCD

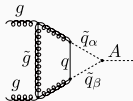
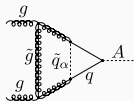
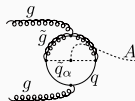
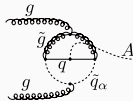
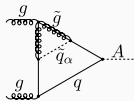
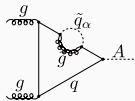
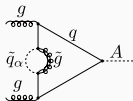
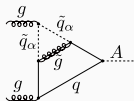
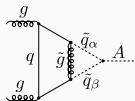
Supersymmetry  $\rightarrow$  every particle gets a (heavy) super-partner



New Interactions:



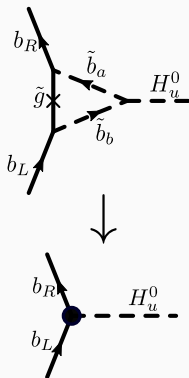
# Diagrams



## Existing Results

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- Non decoupling for  $M_{\text{SUSY}} \rightarrow \infty$
- Absorb into effective Yukawa coupling
- Resums large  $\tan\beta$  contributions
- includes all leading powers of  $\alpha_s \mu \tan\beta$  [Carena, Garcia, Nierste, Wagner '00][Guasch, Häflinger, Spira '03]



$$\Delta_q = \frac{C_F}{2} \frac{\alpha_s}{\pi} m_{\tilde{g}} \mu r_q I(m_{\tilde{q}_1}, m_{\tilde{q}_2}, m_{\tilde{g}}) + \Delta_q^{\text{elw.}}$$

$$r_b = \tan\beta \quad r_t = \cot\beta$$



## Existing Two-Loop Results

Analytic results exist as asymptotic expansion valid upto

$$\mathcal{O}\left(\frac{M_A^2}{M^2}\right) \quad \& \quad \mathcal{O}\left(\frac{m_t^2}{M^2}\right) \quad \mathcal{O}\left(\frac{m_b^2}{M_A^2}\right) \quad \& \quad \mathcal{O}\left(\frac{m_b}{M}\right)$$

[Harlander, Hofmann '06]

[Degrassi, Vita, Slavich '11]

# Calculation

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# $\gamma_5$ in Dimensional Regularization

$\gamma_5$  inconsistent in  $D$ -dim:

$$2(D - 4) \operatorname{tr}(\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 0$$

- Larin scheme

- $\bar{\psi} \gamma_5 \psi = \frac{i}{4!} \epsilon_{\mu\nu\rho\sigma} \bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \psi$

- $\epsilon^{\mu_1 \nu_1 \rho_1 \sigma_1} \epsilon_{\mu_2 \nu_2 \rho_2 \sigma_2} = -\det \begin{pmatrix} g_{\mu_2}^{\mu_1} & g_{\nu_2}^{\mu_1} & g_{\rho_2}^{\mu_1} & g_{\sigma_2}^{\mu_1} \\ g_{\mu_2}^{\nu_1} & g_{\nu_2}^{\nu_1} & g_{\rho_2}^{\nu_1} & g_{\sigma_2}^{\nu_1} \\ g_{\mu_2}^{\rho_1} & g_{\nu_2}^{\rho_1} & g_{\rho_2}^{\rho_1} & g_{\sigma_2}^{\rho_1} \\ g_{\mu_2}^{\sigma_1} & g_{\nu_2}^{\sigma_1} & g_{\rho_2}^{\sigma_1} & g_{\sigma_2}^{\sigma_1} \\ g_{\mu_2}^{\sigma_1} & g_{\nu_2}^{\sigma_1} & g_{\rho_2}^{\sigma_1} & g_{\sigma_2}^{\sigma_1} \end{pmatrix}$

- Only  $D$ -dimensional objects

[Larin '93]

- Breitenlohner Maison scheme:  $\{\gamma_5, \gamma^\mu\} \neq 0$

[Breitenlohner, Maison '77]

- Kreimer scheme: Noncyclic trace [Kreimer '94]

# Adler Bardeen theorem

Adler Bardeen Theorem: no anomalous corrections at NLO

Peccei Quinn symmetry:  $\partial_\mu j_{PQ}^\mu = -\delta\mathcal{L} + \frac{\alpha_s}{2\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$

$$m_A^2 \xrightarrow{p=0} \text{[Diagram: a grey circle with two wavy lines] } = \mathcal{M}_{p \rightarrow 0}$$

$$= \sum_{Q=t,b} \frac{m_Q(1+r_Q^2)}{v} \mu \underbrace{\text{[Diagram: a grey circle with two wavy lines and two dashed lines labeled } \tilde{q}_1, \tilde{q}_2 \text{]}_{\rightarrow \Delta_Q} + \frac{r_t + r_b}{v} \frac{\alpha_s}{4\pi} \text{[Diagram: a grey circle with two wavy lines and a loop of gluons]}$$

$\Rightarrow$  No corrections beyond  $\Delta_b$  and  $\Delta_t$  for  $m_A^2 \ll m_Q^2 \ll M_{\text{SUSY}}^2$

# Numerical Integration

After Feynman Parametrization:

$$I = \int_0^1 d^d x \frac{f(x)}{N^{n+2\varepsilon}(x)}$$

upto 5 parameters

parametrized, such that  $N(x)$  is quadratic polynomial in Feynman parameters

Example:

$$N(x) = M_A^2 (x_1 - 1) x_2 (1 - x_3) (x_3 + x_1 (x_3 + x_4 - 1) (x_5 - 1)) x_5 \\ + M_{\tilde{g}}^2 x_1 x_5 + M_Q^2 (x_5 - x_1 x_5) + M_{\tilde{q}_\alpha}^2 (x_1 - 1) x_1 (x_5 - 1)$$

## UV singularities

To get finite and divergent part expand in  $\varepsilon$

Divergent integrals can arise from factors  $x^{-1+\varepsilon} \Rightarrow$  Endpoint subtraction

$$\int_0^1 dx x^{-1+\varepsilon} f(x) = \int_0^1 dx \underbrace{x^{-1+\varepsilon} (f(x) - f(0))}_{\text{regular in } x} + \underbrace{\int_0^1 dx x^{-1+\varepsilon} f(0)}_{=\frac{f(0)}{\varepsilon}}$$

```
Iteration 8: 80000000 integrand evaluations so far
[1] 107.257 +- 87.4996          chisq 2.60227 (7 df)
[2] 52.3746 +- 81.1029          chisq 4.40932 (7 df)
[3] 59011.6 +- 53836.4          chisq 7.80152 (7 df)
[4] 81290.5 +- 140535          chisq 5.79475 (7 df)
```

```
Iteration 9: 90000000 integrand evaluations so far
[1] 106.833 +- 82.401          chisq 2.60247 (8 df)
[2] 16.4968 +- 75.2997          chisq 5.82753 (8 df)
[3] 43936.9 +- 53032.7          chisq 10.4471 (8 df)
[4] 57839.8 +- 139436          chisq 7.58229 (8 df)
```

```
Iteration 10: 100000000 integrand evaluations so far
[1] 72.2666 +- 79.8743          chisq 5.51663 (9 df)
[2] 24.5327 +- 69.712          chisq 5.90723 (9 df)
[3] 37029.3 +- 52329.3          chisq 11.0909 (9 df)
[4] 34889.4 +- 118027          chisq 7.67784 (9 df)
```

# Thresholds

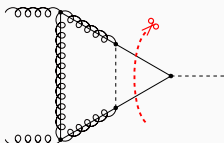
after expansion in  $\varepsilon$

$$I = \int_0^1 d^d x \frac{g(x) + h(x) \ln(N(x))}{N^n(x)}$$

What happens for  $N(x) = 0$ ?

Microcausality: Masses from propagators are given small imaginary part

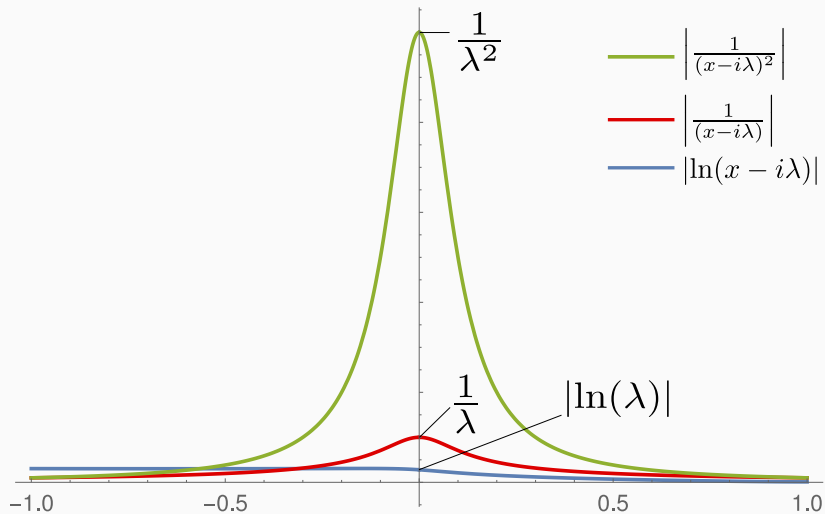
$$m^2 \rightarrow m^2(1 - i\lambda)$$



Usually  $\lambda \rightarrow 0$ , but for numerical integration we set  $\lambda$  sufficiently small but finite.



# Thresholds



# Integration by parts

$$1 = \frac{1}{\underbrace{R}_{\text{better zeros}}} \left( \underbrace{p_0 N}_{\text{Cancels one power of the denominator}} + \underbrace{\vec{p} \cdot \vec{\nabla} N}_{\text{Integration by parts}} \right)$$

$$\frac{\vec{\nabla} N}{N^n} = \frac{1}{n-1} \vec{\nabla} \frac{1}{N^{n-1}}$$

Example in one dimension:

$$N(x) = ax^2 + bx + c$$

$$\Rightarrow 1 = \frac{1}{\underbrace{4ac - b^2}_{\text{constant}}} \left( \underbrace{4a}_{p_0} \cdot N(x) - \underbrace{(2ax + b)}_{p_1} \cdot \partial_x N(x) \right)$$

```
Iteration 8: 80000000 integrand evaluations so far
[1] -0.419694 +- 0.000978278      chisq 4.17753 (7 df)
[2] 0.0699153 +- 0.000950702     chisq 4.28112 (7 df)
[3] 1.972 +- 0.050616      chisq 5.16887 (7 df)
[4] -1.3137 +- 0.0504296      chisq 6.73128 (7 df)
```

```
Iteration 9: 90000000 integrand evaluations so far
[1] -0.420145 +- 0.00071454      chisq 4.63387 (8 df)
[2] 0.0696467 +- 0.000687736     chisq 4.44859 (8 df)
[3] 1.98024 +- 0.0339793      chisq 5.21711 (8 df)
[4] -1.3158 +- 0.0342964      chisq 6.73449 (8 df)
```

```
Iteration 10: 100000000 integrand evaluations so far
[1] -0.420004 +- 0.00057225      chisq 4.74287 (9 df)
[2] 0.0694324 +- 0.00054715     chisq 4.71313 (9 df)
[3] 1.97658 +- 0.02691      chisq 5.24823 (9 df)
[4] -1.33638 +- 0.0284265      chisq 7.88515 (9 df)
```

# Richardson Extrapolation

- Stability for larger values of regulator  $\lambda$
- Extrapolate from stable integration regions
- Assumes polynomial behavior for small  $\lambda$

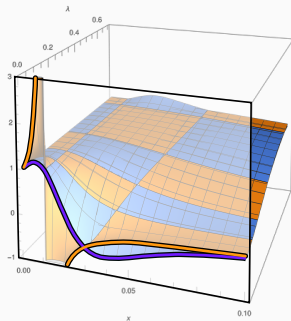
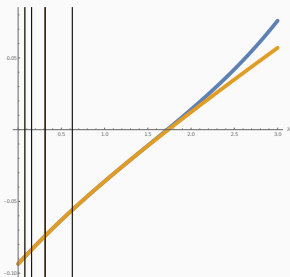
$$R_n(G)(\lambda) = \frac{t^n R_{n-1}(G)(\lambda) - R_{n-1}(G)(t^n \lambda)}{t^n - 1} = G(0) + \mathcal{O}(\lambda^n)$$

$$R_0(G)(\lambda) = G(\lambda)$$

$$\text{e.g. } R_3(G)(\lambda) = \frac{64G(\lambda)}{21} - \frac{8}{3}G(2\lambda) + \frac{2}{3}G(4\lambda) - \frac{1}{21}G(8\lambda) \approx G(0)$$

# Richardson Extrapolation

$$\underbrace{G(\lambda)}_{\text{well approximated}} = \int dx \underbrace{g(x, \lambda)}_{\text{badly approximated}}$$



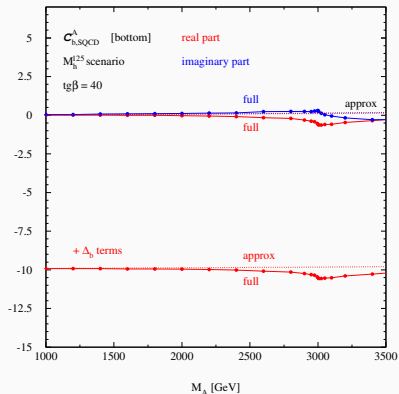
→ can gain stability

Does not work near threshold, because  $G(\lambda) \sim \sqrt{\lambda}$

## Results

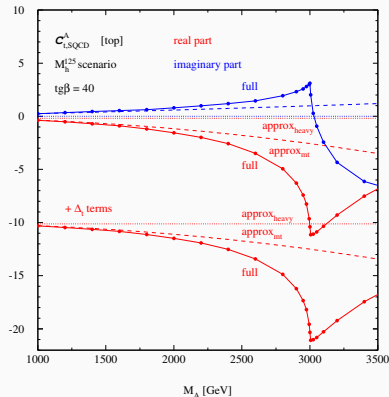
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$$\mathcal{C}_{Q,SQCD}, \tan\beta = 40$$



$$m_{\tilde{b}_1} = 1459 \text{ GeV}$$

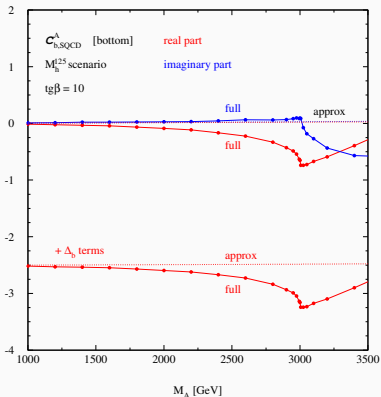
$$m_{\tilde{b}_2} = 1531 \text{ GeV}$$



$$m_{\tilde{t}_1} = 1353 \text{ GeV}$$

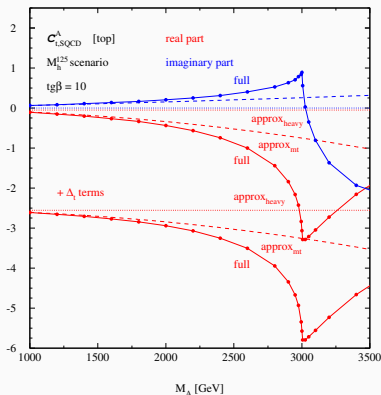
$$m_{\tilde{t}_2} = 1650 \text{ GeV}$$

# $\mathcal{C}_{Q,SQCD}, \tan\beta = 10$



$$m_{\tilde{b}_1} = 1466 \text{ GeV}$$

$$m_{\tilde{b}_2} = 1504 \text{ GeV}$$

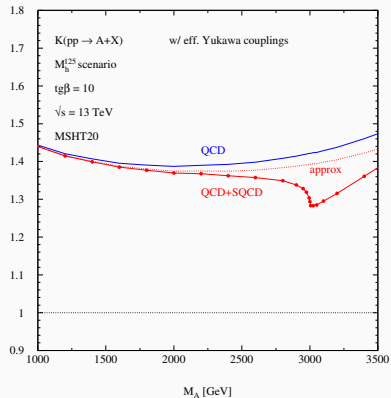
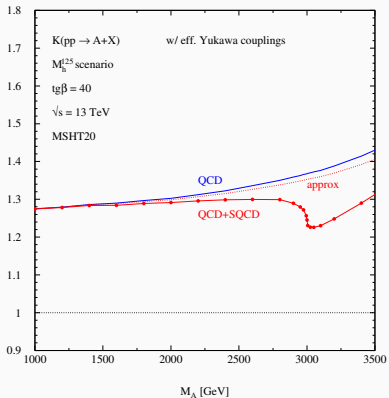


$$m_{\tilde{t}_1} = 1353 \text{ GeV}$$

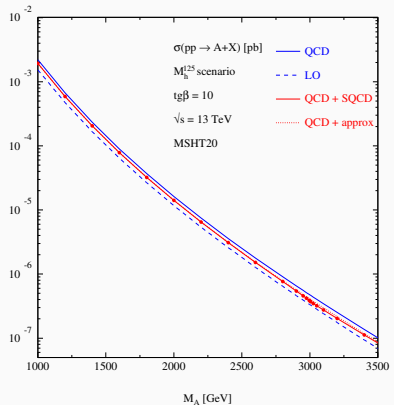
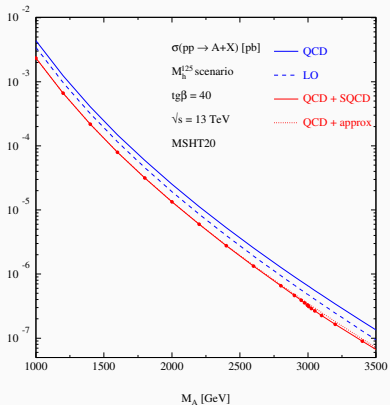
$$m_{\tilde{t}_2} = 1650 \text{ GeV}$$



# K-factors



# Crosssections



## Conclusion

- Existing approximations valid for  $M_A$  well below the virtual squark thresholds
- Dominant contributions can be absorbed in effective coupling ( $\rightsquigarrow$  electroweak and NNLO corrections)
- SUSY-QCD remainders significant for large  $M_A$

◇ THANK YOU FOR YOUR ATTENTION ◇