Light cone tensor network and time evolution

Mari-Carmen Bañuls

work with Miguel Frías-Pérez

PRB 106, 115117 (2022)

Max Planck Institut
of Quantum Optics
(Garching)

MPQ

MCQST

EPFL 29.11.2022

TNS are very useful in the quantum many-body context…

works for GS, low energy, thermal equilibrium…

Verstraete, Cirac, PRB 2006 Hastings PRB 2006
Hastings J. Stat. Phys 2007 Molnar *et al.* PRB 2015

area laws

applicability for QFT problems

LGT: systematically probed in 1D, progress in 2D

review: MCB, K. Cichy arXiv:1910.00257

suitable for other QFT problems arXiv:1908.04536,1912.08836

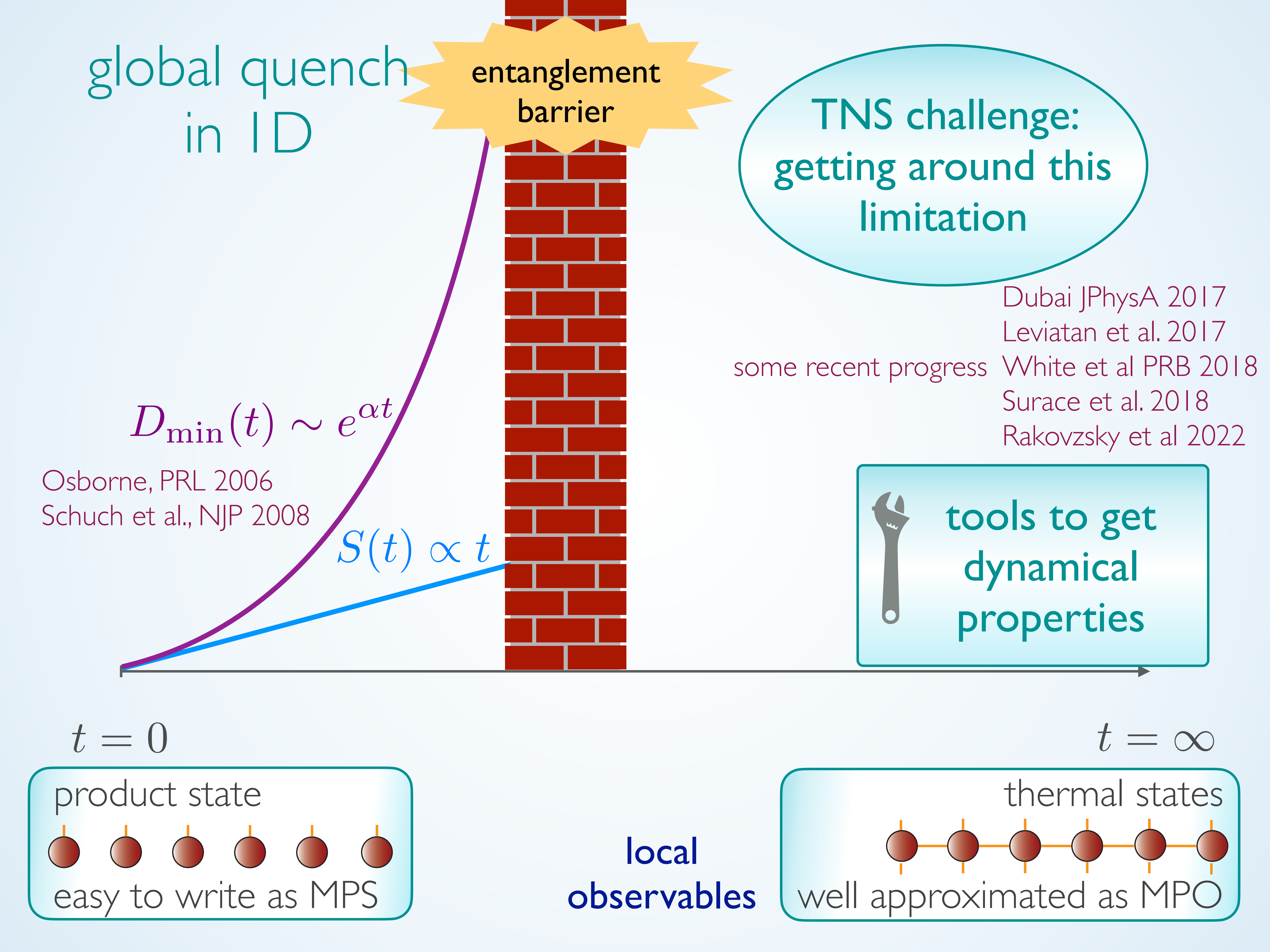
high energy eigenstates, quenches…

Osborne, PRL 2006 Vidmar *et al.*, PRL 2017
Schuch *et al.*, NJP 2008

volume law

entanglement growth in non-equilibrium scenarios limits the applicability of MPS

fundamental questions: thermalization, ETH…



global quench in 1D
entanglement barrier
TNS challenge: getting around this limitation
some recent progress
Dubai JPhysA 2017
Leviatan et al. 2017
White et al PRB 2018
Surace et al. 2018
Rakovzsky et al 2022
$D_{\min}(t) \sim e^{\alpha t}$
Osborne, PRL 2006
Schuch et al., NJP 2008
$S(t) \propto t$
tools to get dynamical properties
$t = 0$
$t = \infty$
product state
easy to write as MPS
local observables
thermal states
well approximated as MPO

alternative: give up description of the full state

light-cone TN for non-equilibrium evolution of local observables

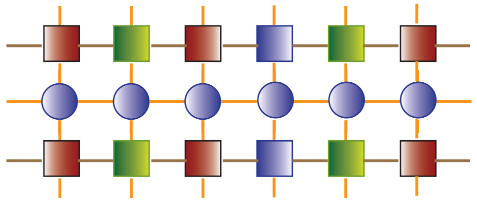
M. Frías-Pérez, MCB, PRB 106, 115117 (2022)

light-cone TN for non-equilibrium evolution of local observables

M. Frías-Pérez, MCB, PRB 106, 115117 (2022)

evolving operators: Heisenberg picture

Hartmann et al, PRL 2009

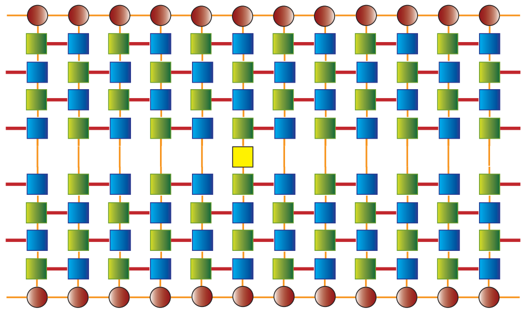


also for mixed states

operator space entanglement

Prosen Pizorn, PRL 2008

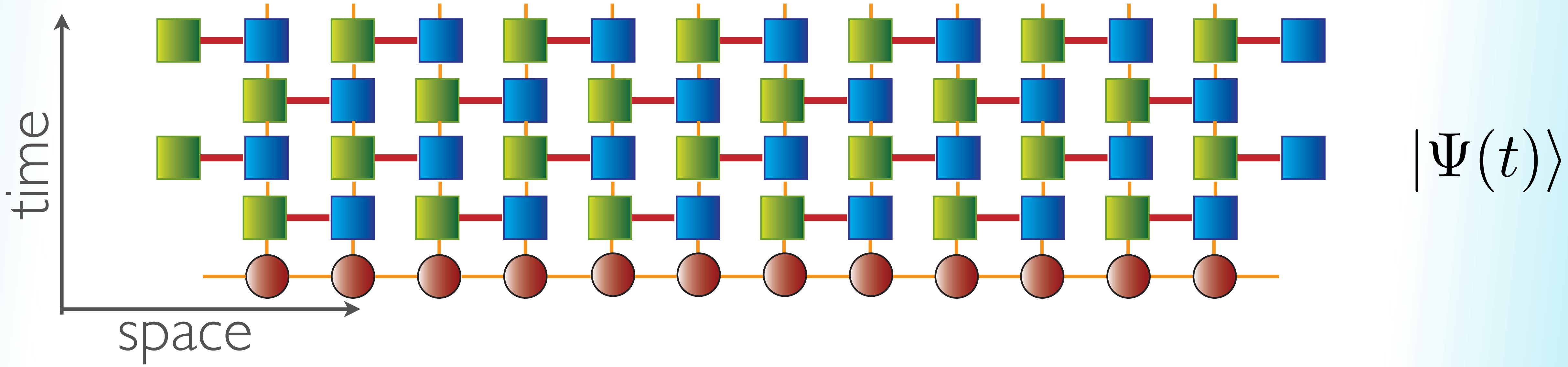
observables as TN to contract



different *entanglement* quantities

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes et al., NJP 2012
Hastings, Mahajan 2014
Frías-Pérez, MCB PRB 2022

time-dependent observable as a TN

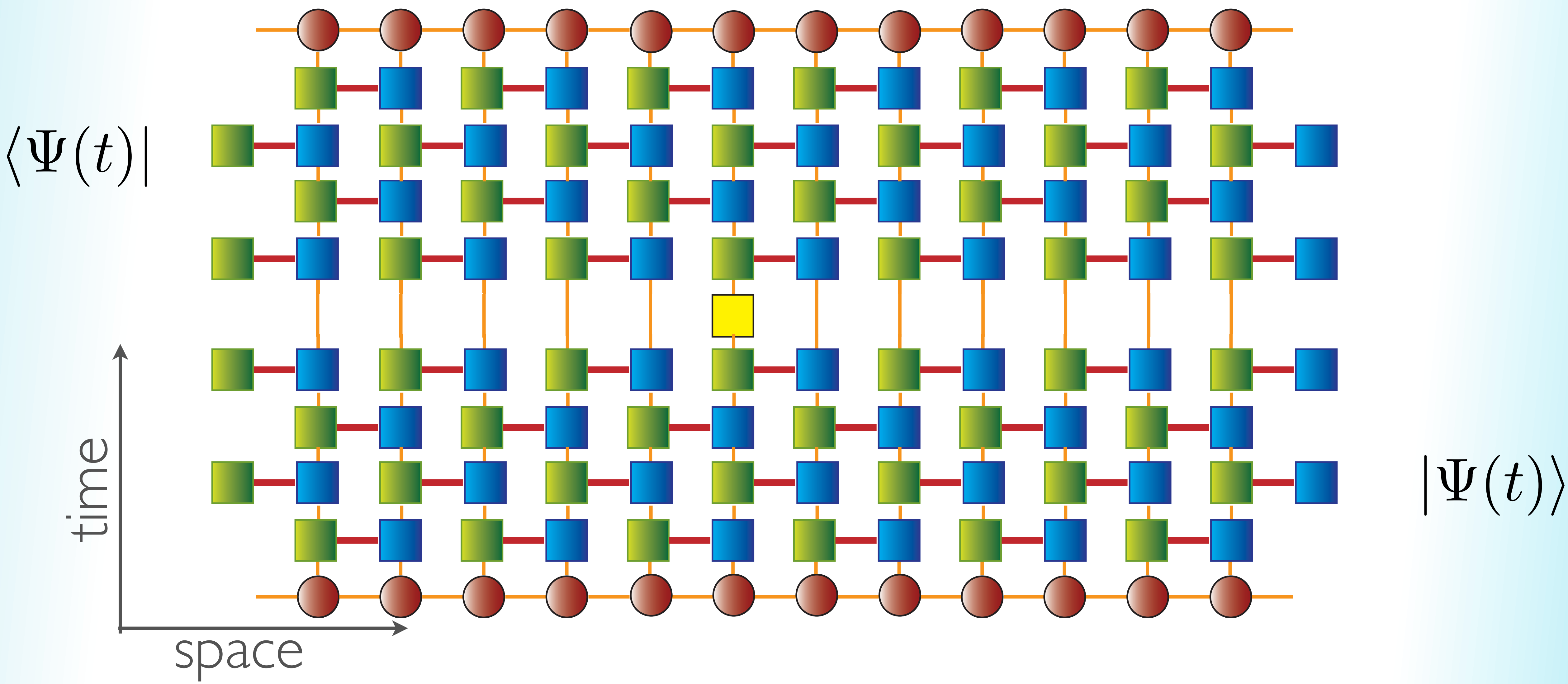


time-dependent observable as a TN

TN describe observables, not states

exact contraction not possible
#P complete

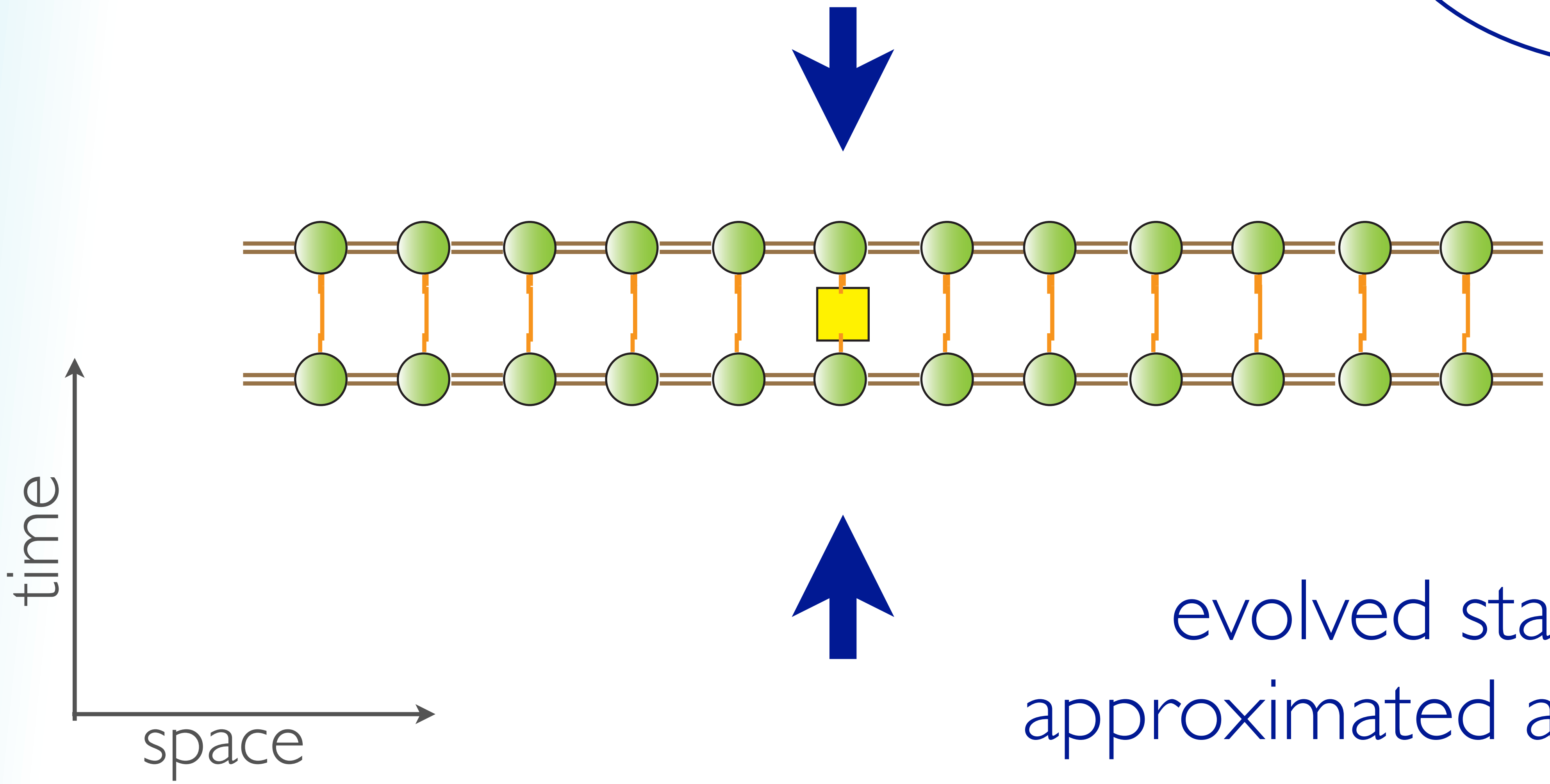
$$\langle \Psi(t) | O | \Psi(t) \rangle$$



time-dependent observable as a TN

different approximate contraction strategies

standard (TEBD, tDMRG)



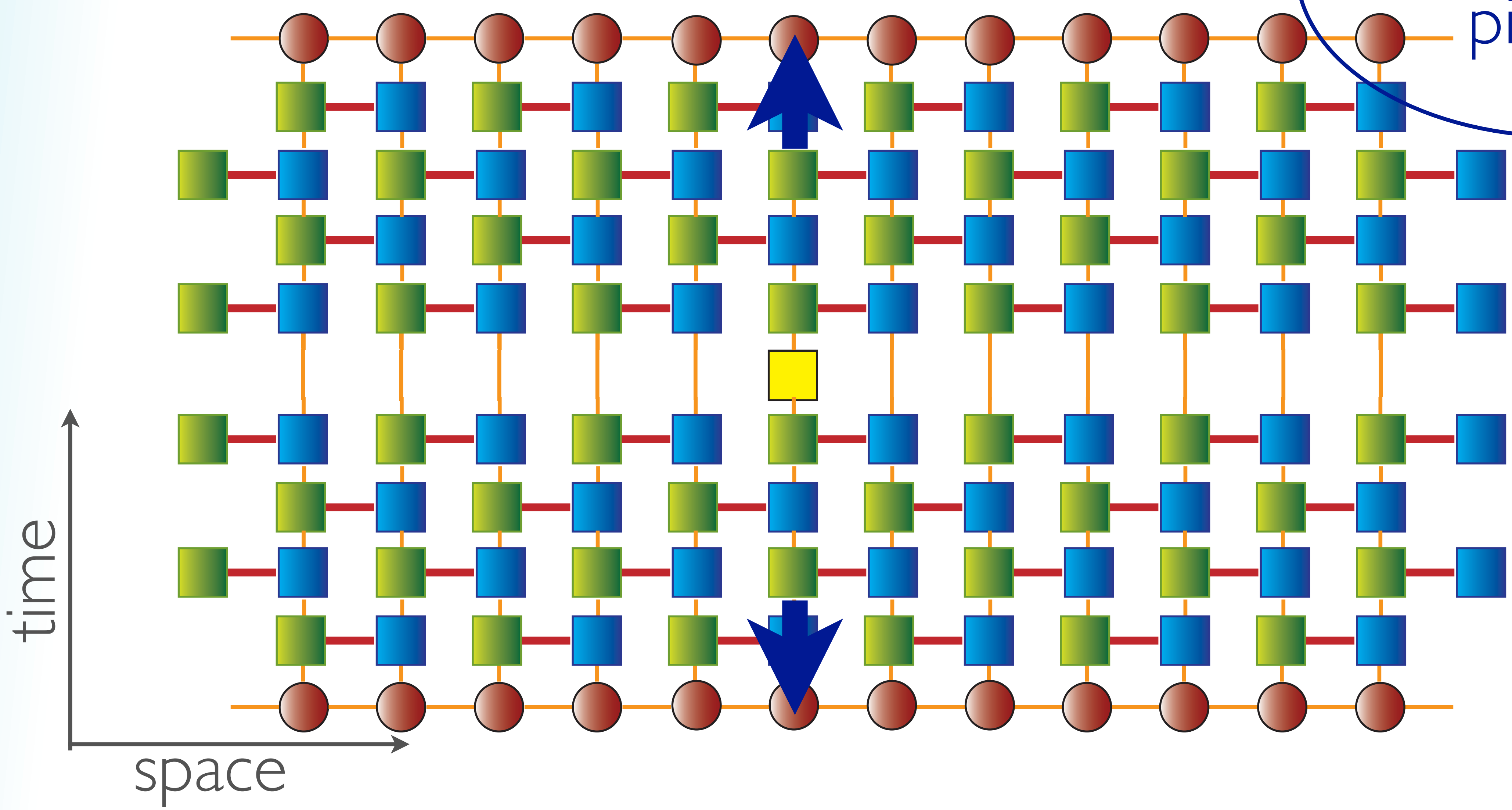
evolved state approximated as MPS

time-dependent observable as a TN

different approximate contraction strategies

evolved operator as MPO

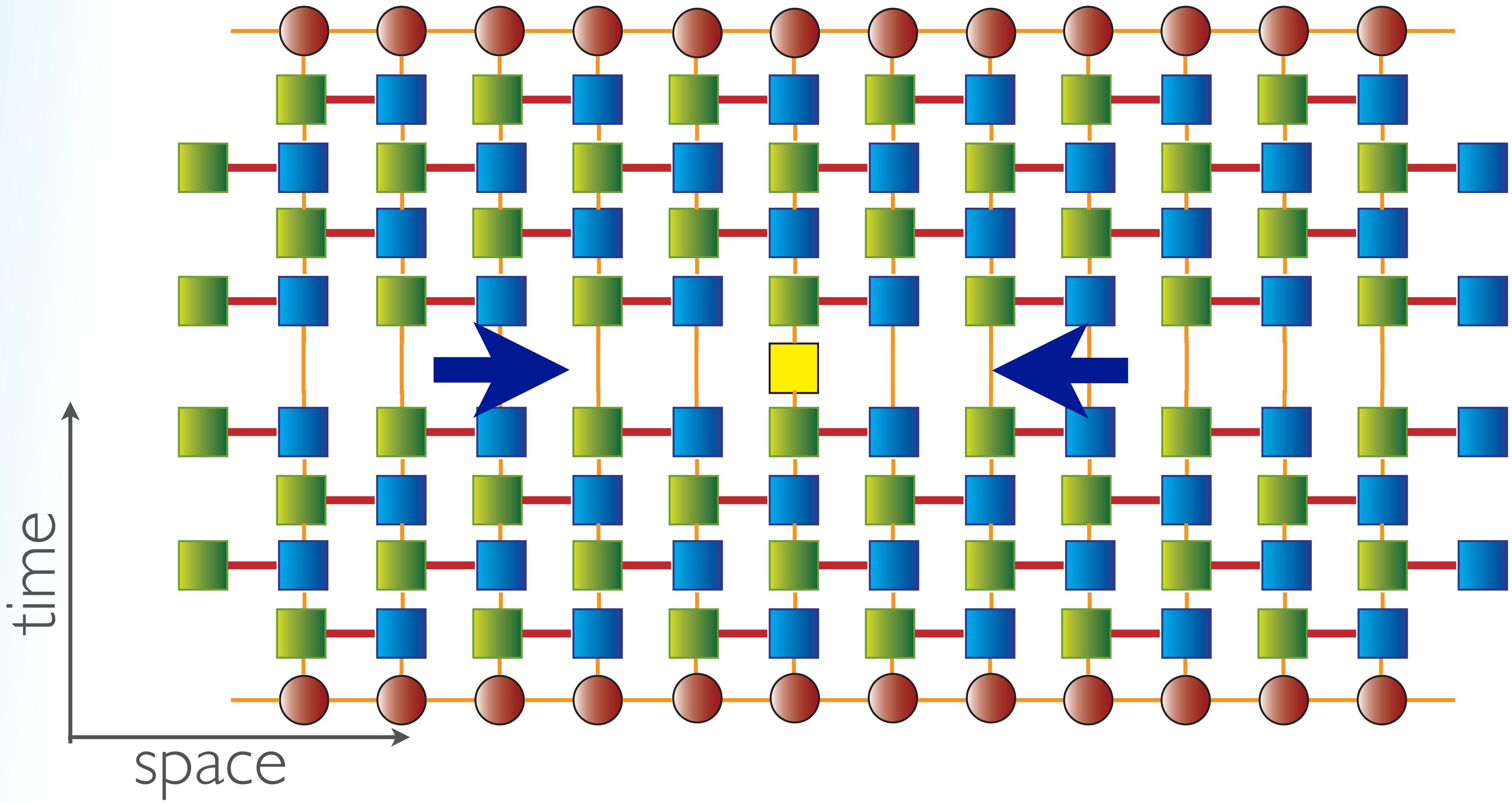
Heisenberg picture



time-dependent observable as a TN

for infinite systems, transverse folding approach

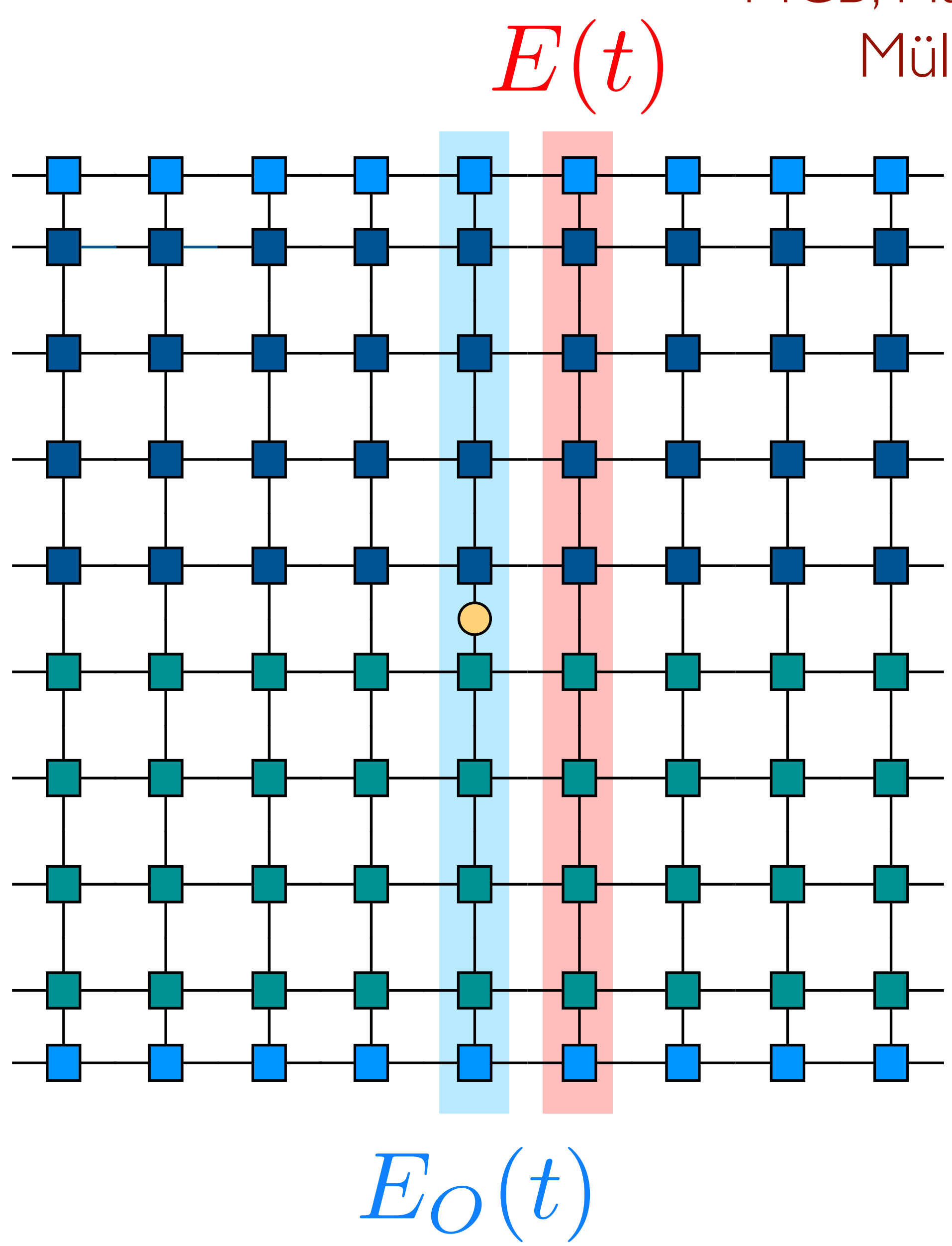
MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012
Hastings, Mahajan 2014



transverse folding approach

for infinite systems, transverse folding approach

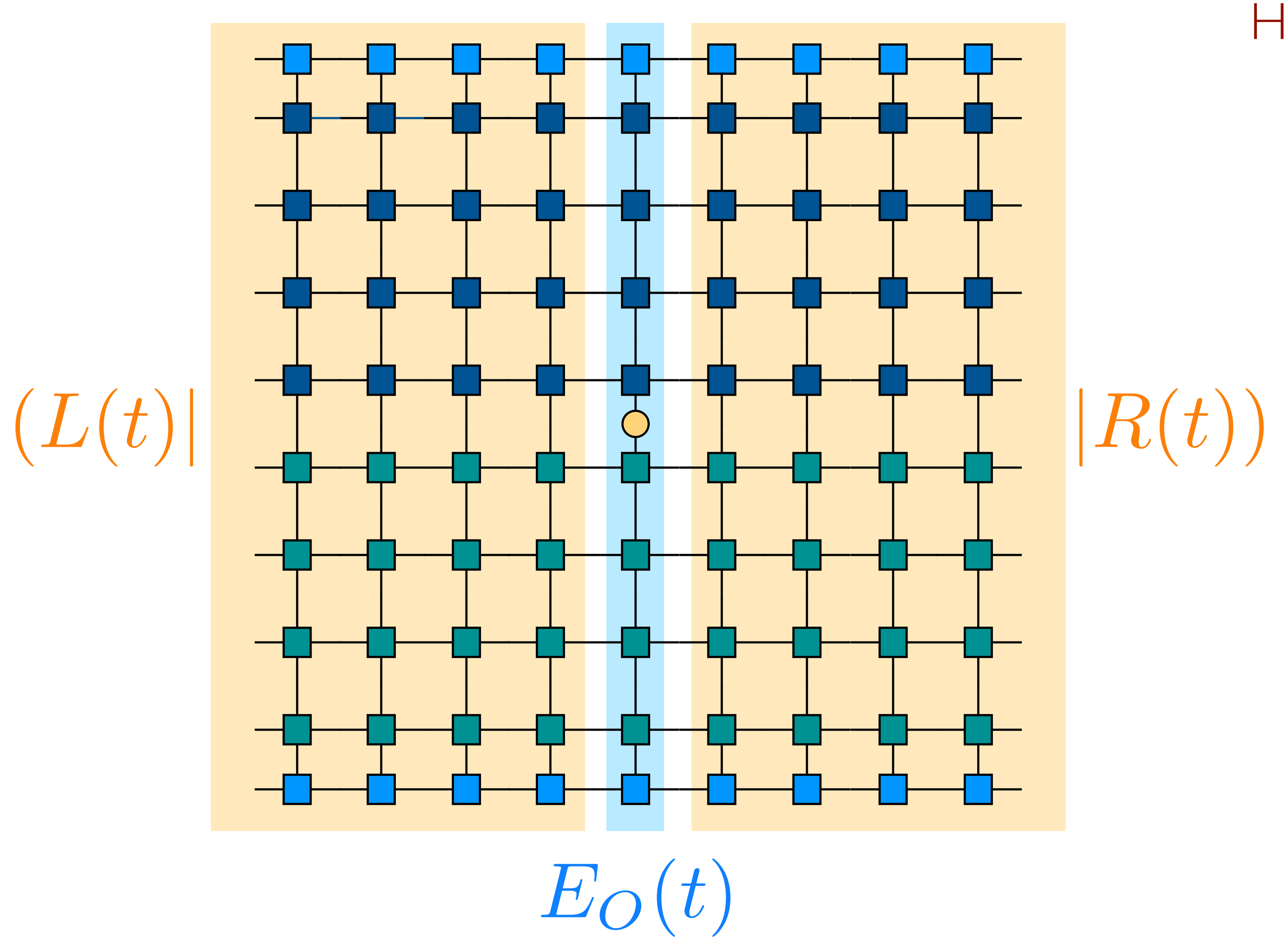
MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012
Hastings, Mahajan 2014



transverse folding approach

for infinite systems, transverse folding approach

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012
Hastings, Mahajan 2014

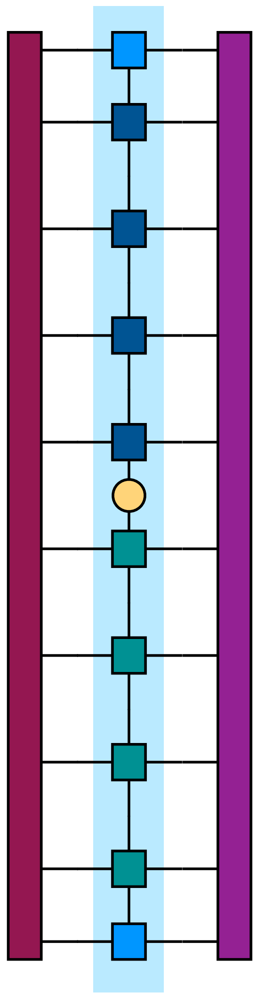


transverse folding approach

for infinite systems, transverse folding approach

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012
Hastings, Mahajan 2014

$(L(t)|$



$|R(t))$

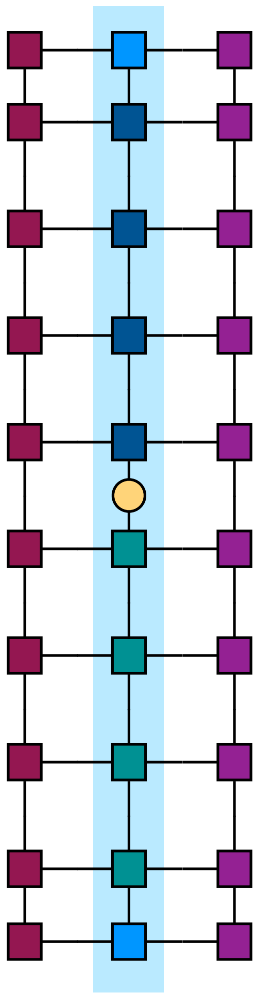
$E_O(t)$

transverse folding approach

for infinite systems, transverse folding approach

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012
Hastings, Mahajan 2014

$(L(t)|$

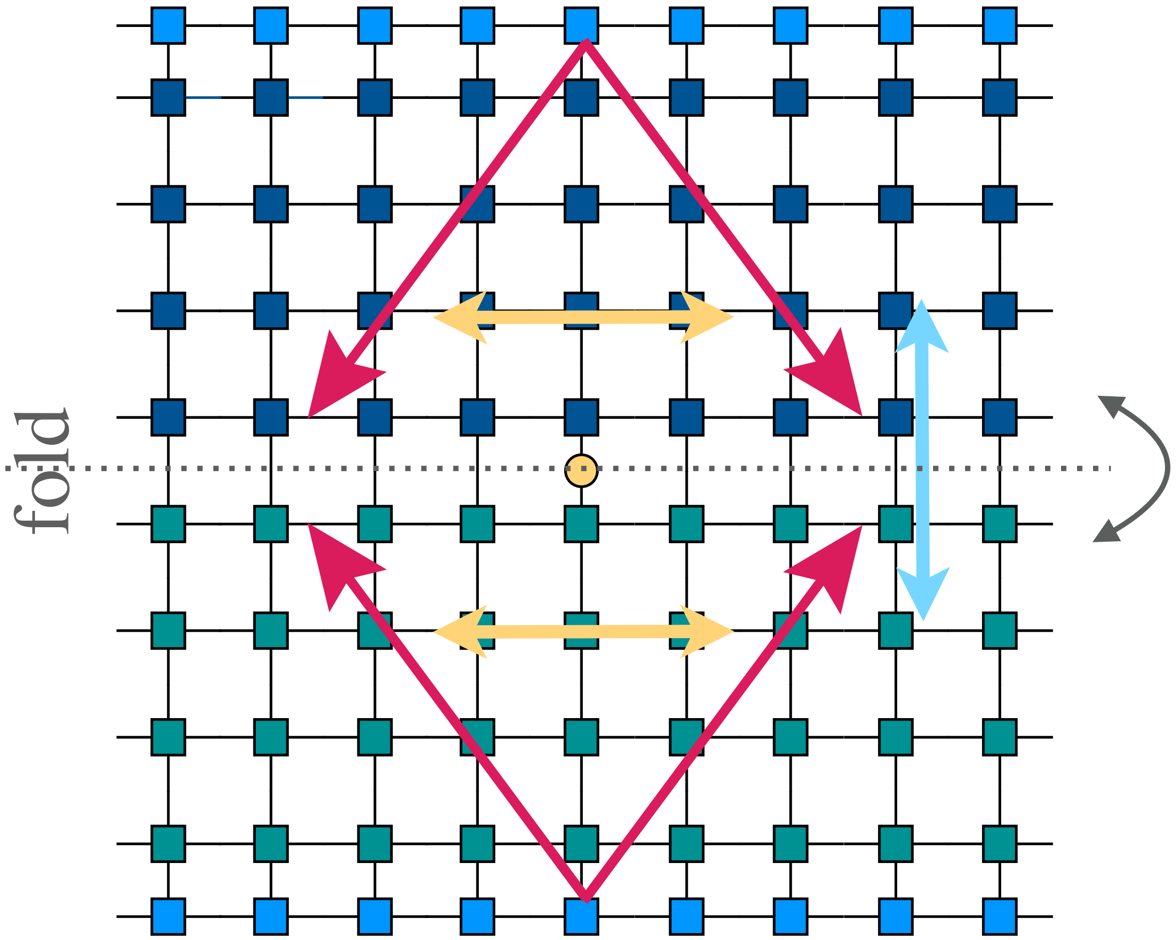


$|R(t))$

$E_O(t)$

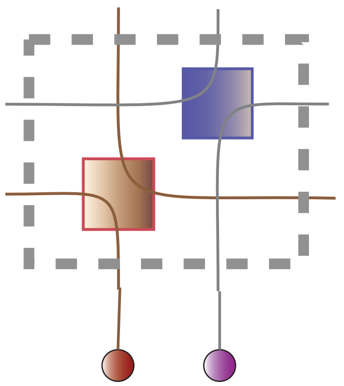
transverse folding approach

intuition: model free propagating excitations



transverse folding approach

intuition: toy model



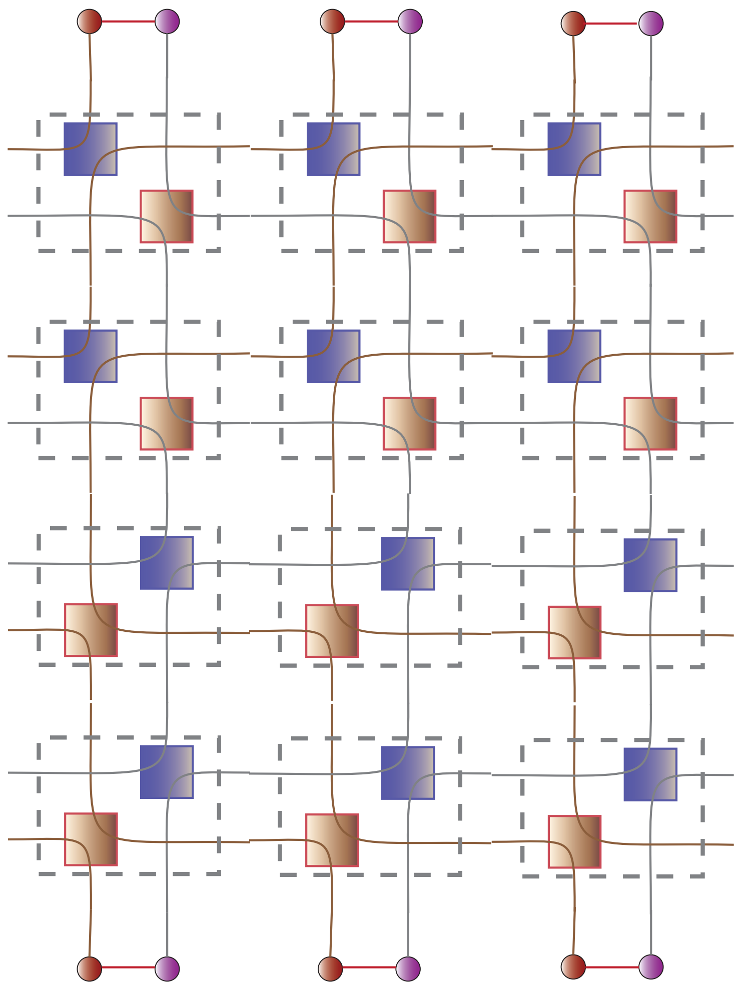
a particular case of
dual unitary circuit

Bertini, Kos, Prosen, PRL 2019

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

transverse folding approach

intuition: toy model



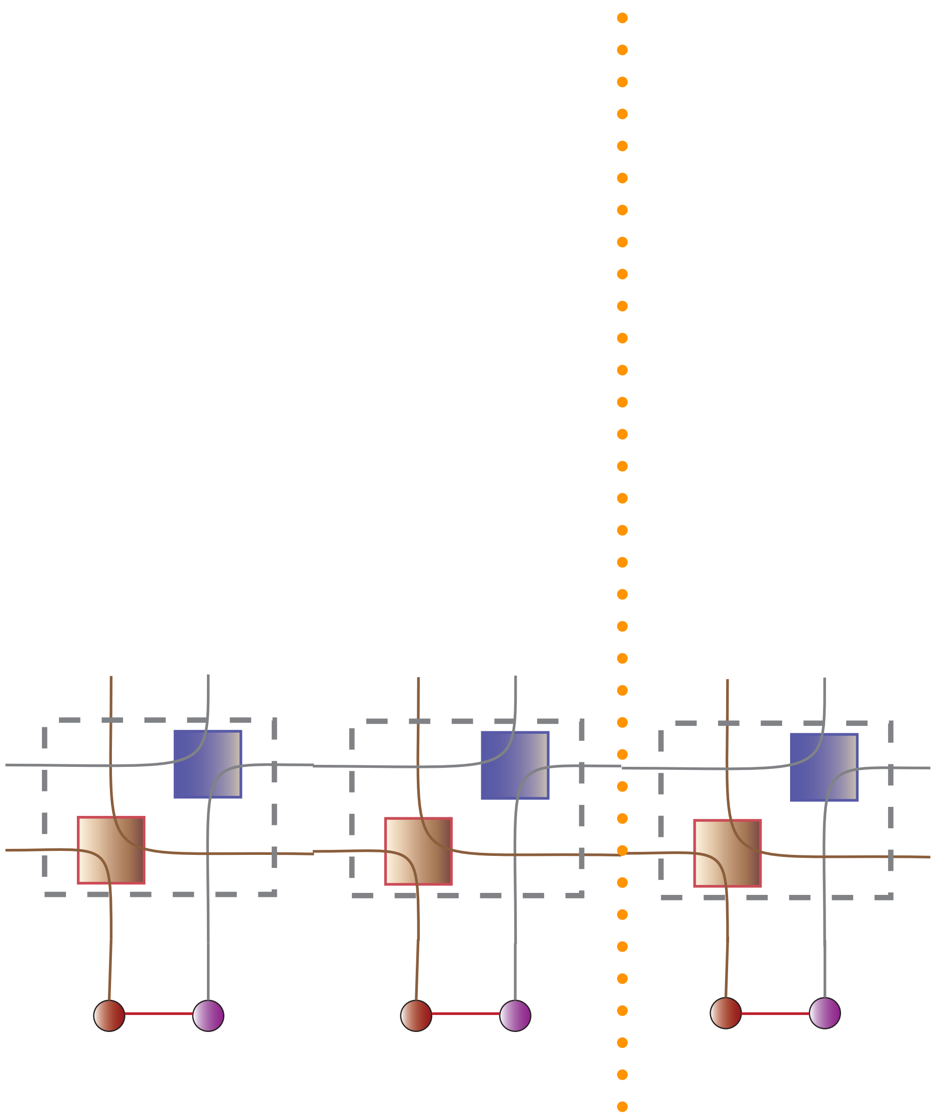
a particular case of
dual unitary circuit

Bertini, Kos, Prosen, PRL 2019

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

transverse folding approach

intuition: toy model

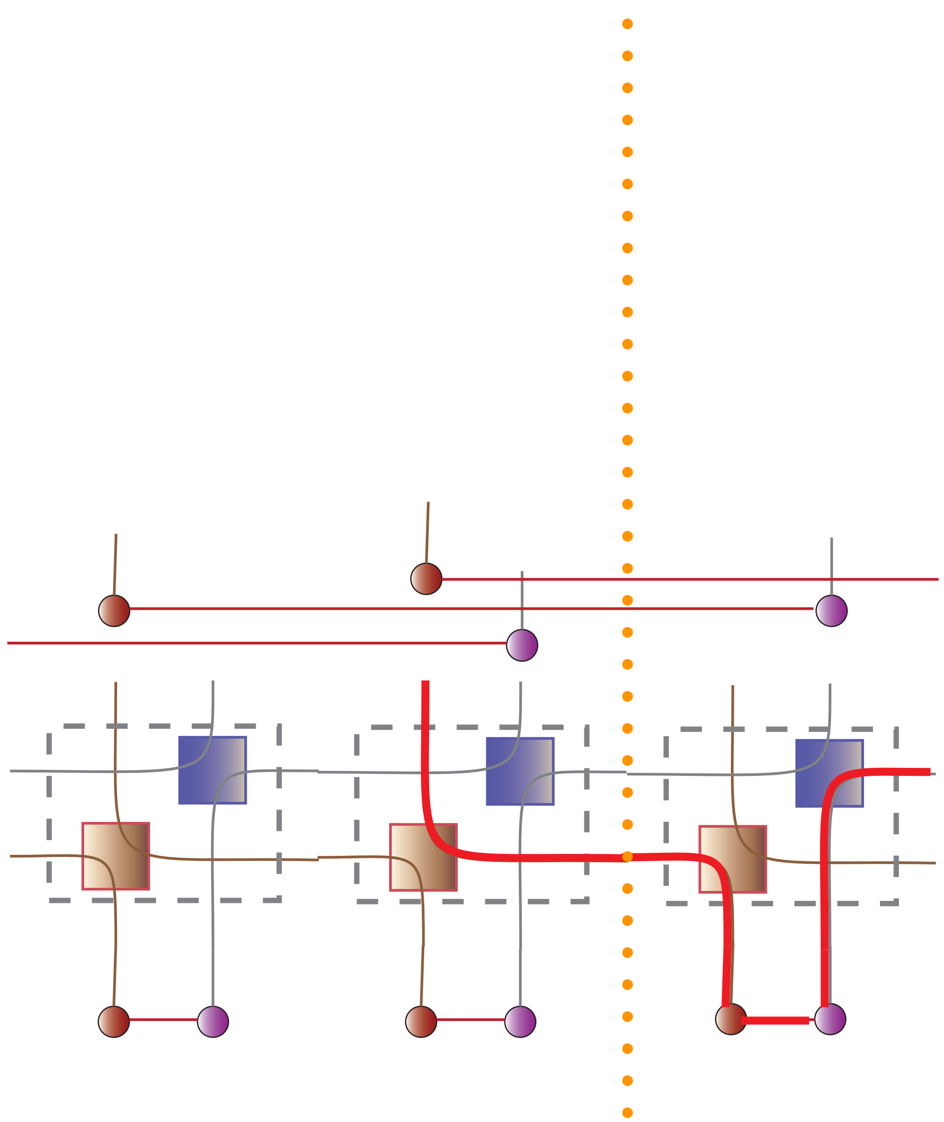


$|\ell\rangle \otimes |0\rangle + |0\rangle \otimes |r\rangle$

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

transverse folding approach

intuition: toy model

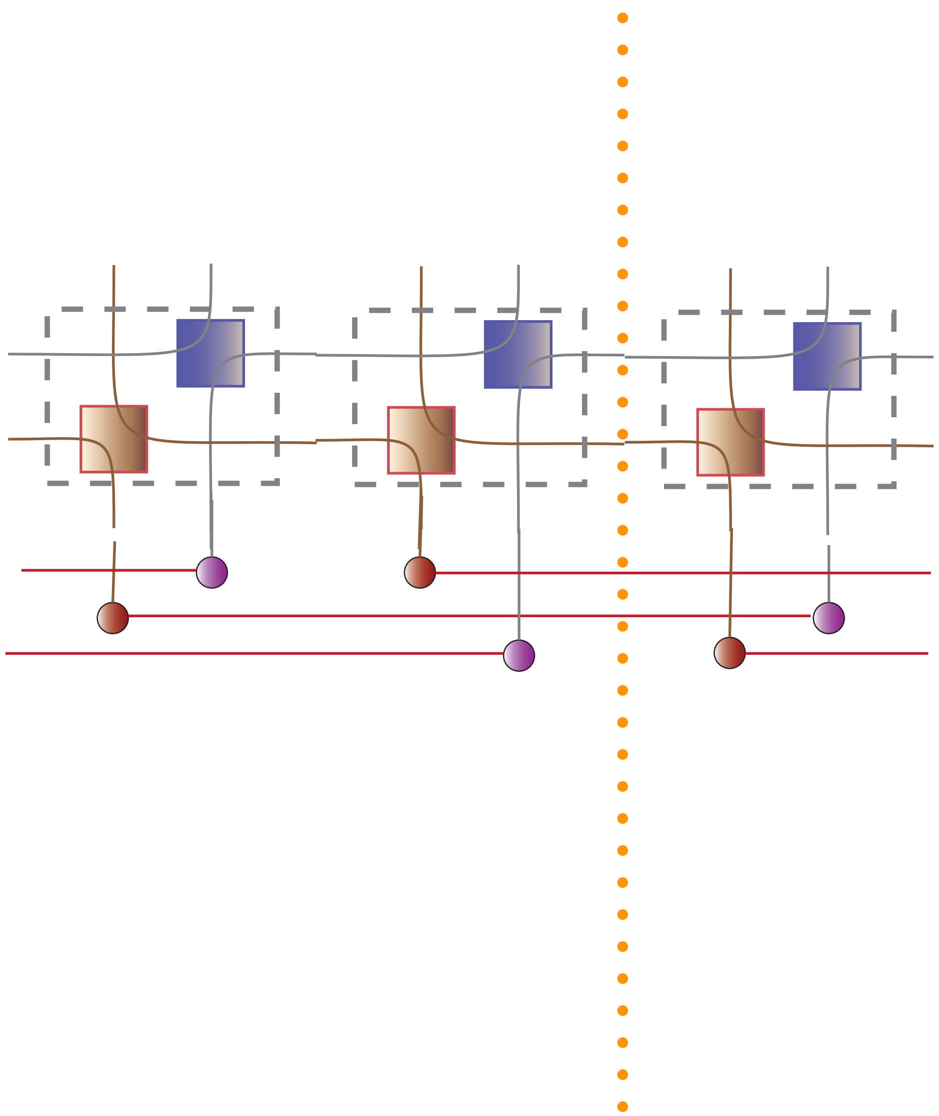


$|\ell\rangle \otimes |0\rangle + |0\rangle \otimes |r\rangle$

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

transverse folding approach

intuition: toy model

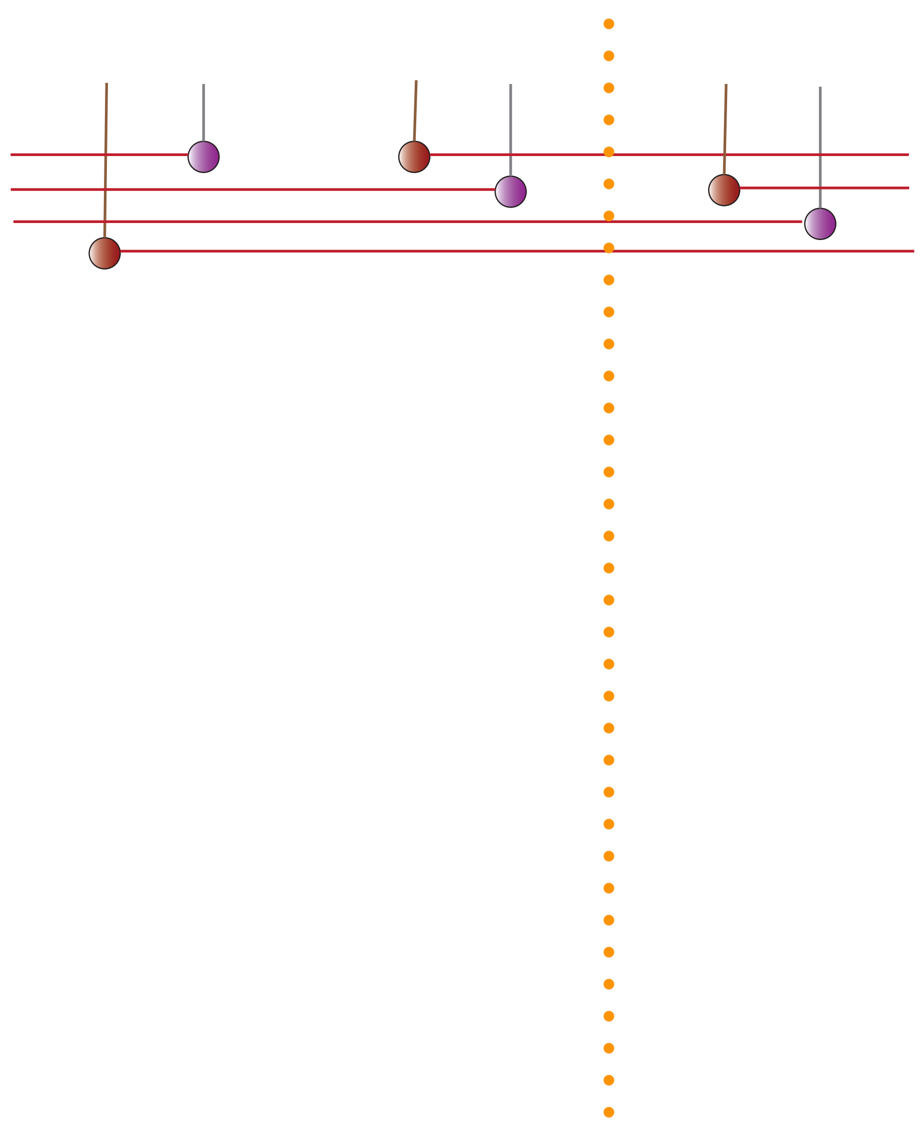


$|\ell\rangle \otimes |0\rangle + |0\rangle \otimes |r\rangle$

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

transverse folding approach

intuition: toy model



$$|\ell\rangle \otimes |0\rangle + |0\rangle \otimes |r\rangle$$

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

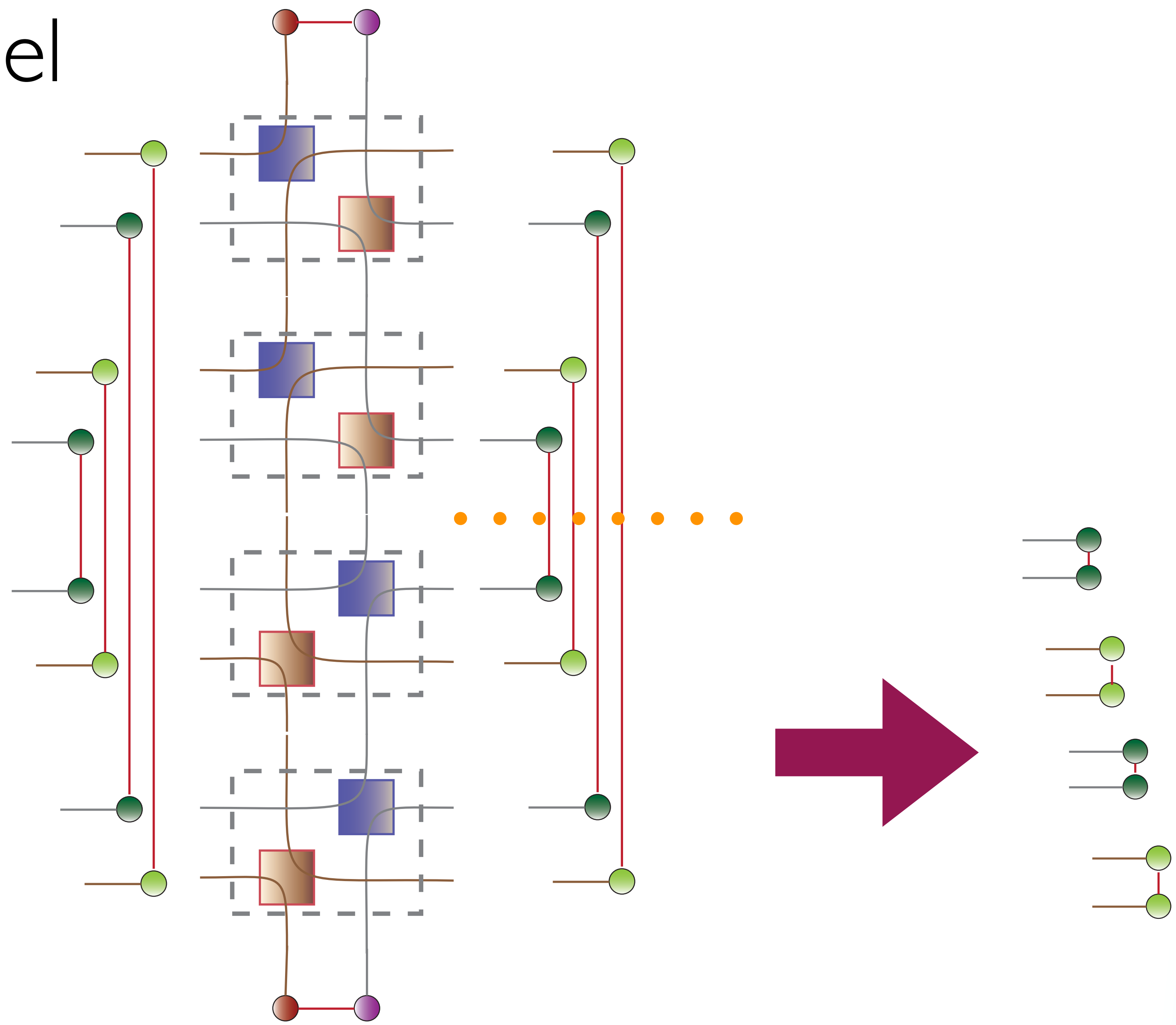
transverse folding approach

intuition: toy model

time direction

entanglement also in the transverse eigenvector

folding can reduce the entanglement in this case

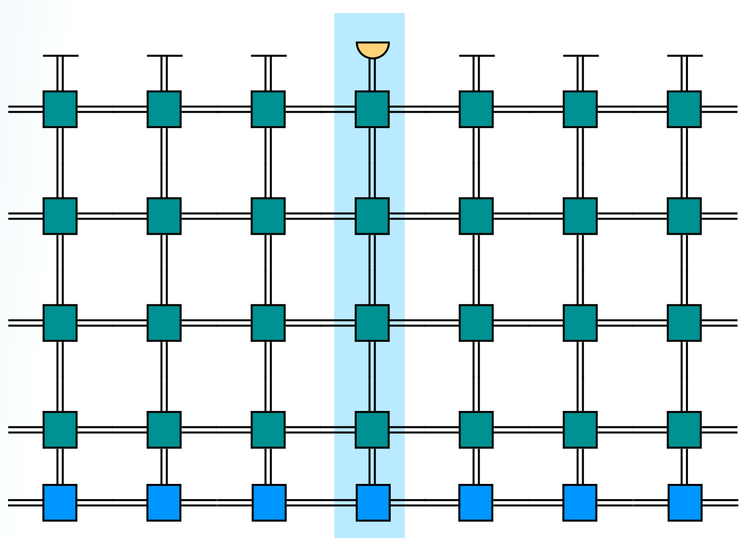


MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

transverse folding approach

free propagating excitations

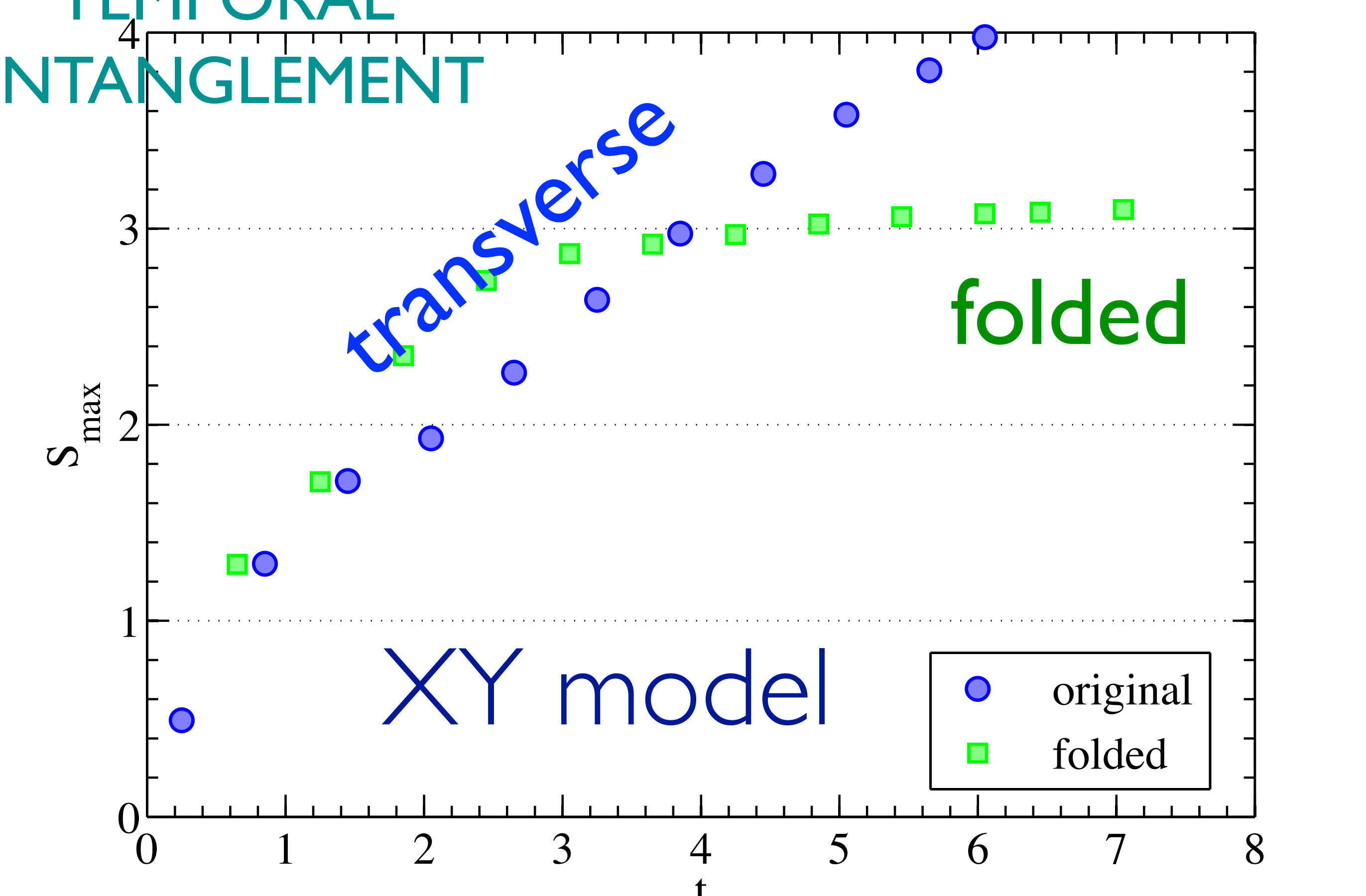
recent interest: influence functional
Sonner et al, Ann. Phys 2021
Lerose et al. PRX 2021
Ye, Chan, J. Chem. Phys. 2021



$E_O(t)$

closest real case: global quench
in free fermionic models

TEMPORAL
ENTANGLEMENT

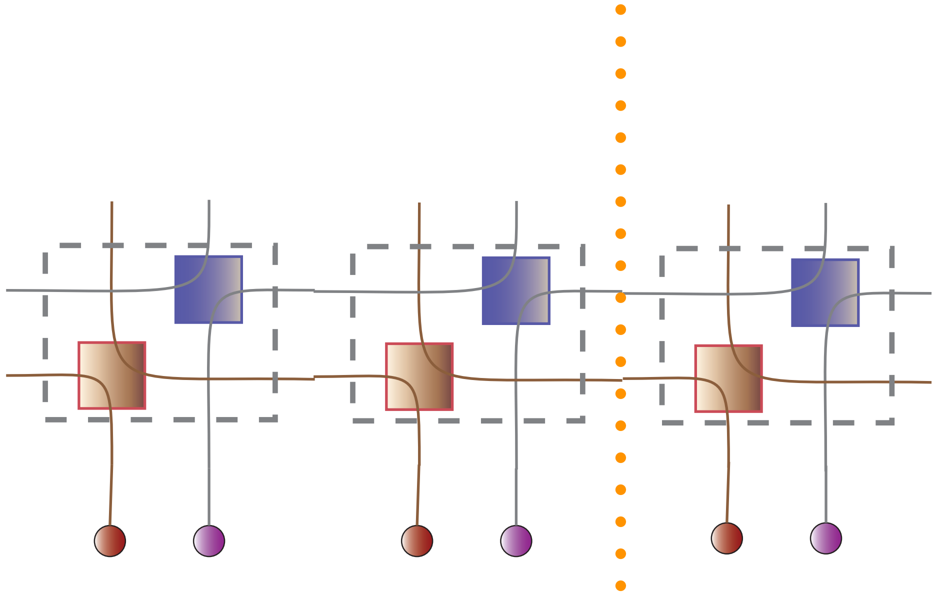


Müller-Hermes, Cirac, MCB, NJP 2012
see also Giudice et al., PRL128, 220401 (2022)

transverse folding approach

intuition: toy model

a second case $|\ell\rangle \otimes |r\rangle$



MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

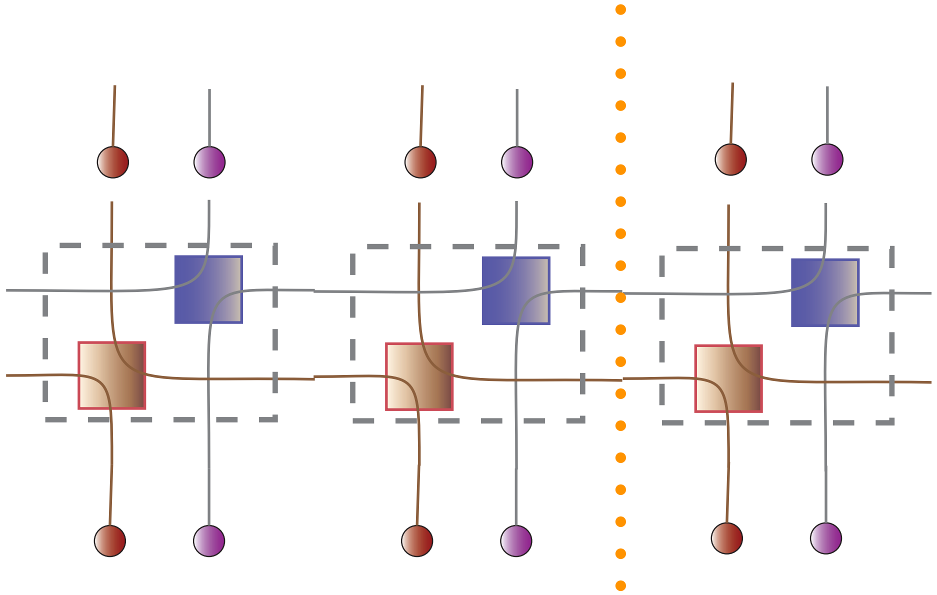
transverse folding approach

intuition: toy model

a second case $|\ell\rangle \otimes |r\rangle$

eigenstate of the evolution

no entanglement created in space



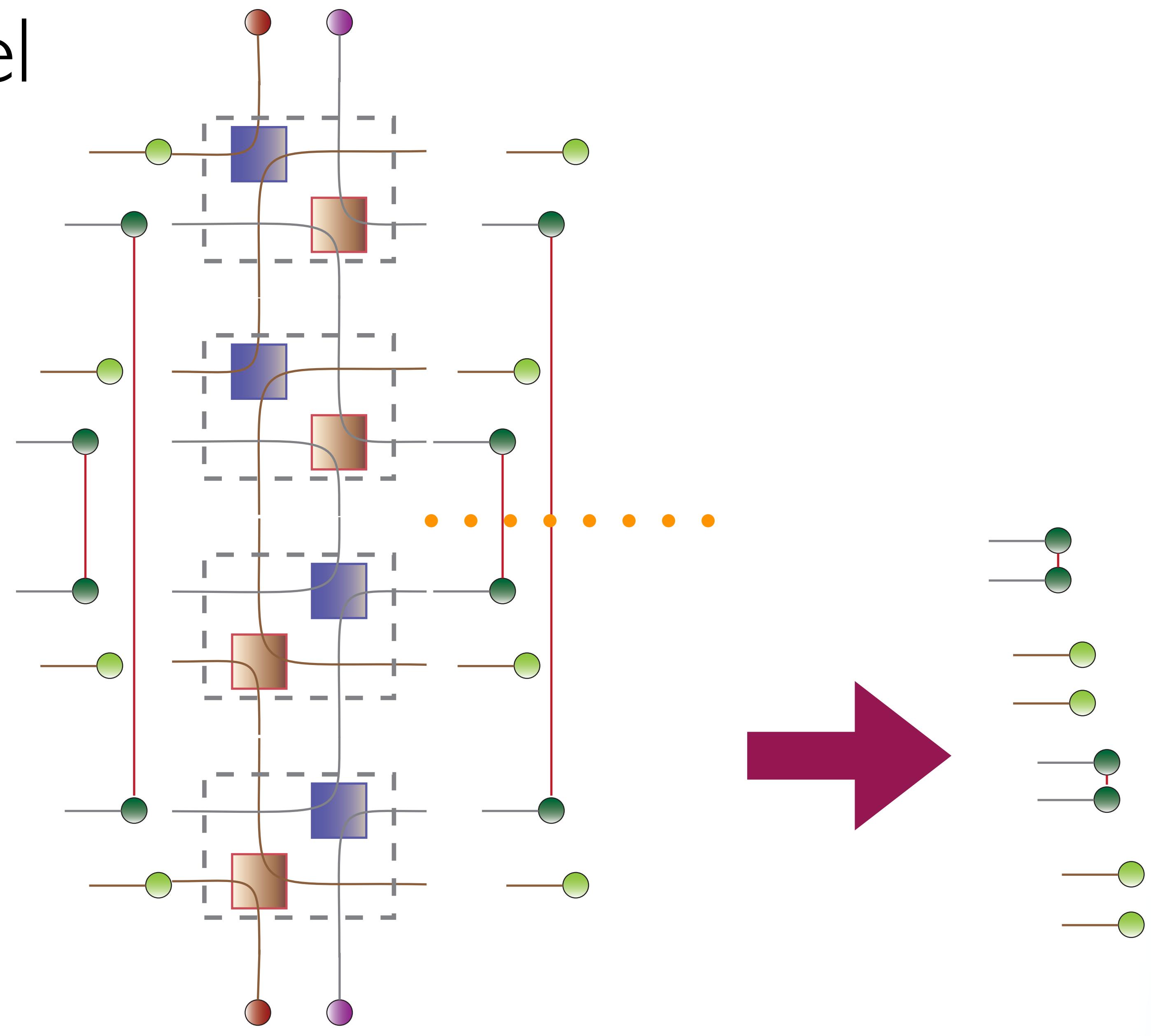
MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

transverse folding approach

intuition: toy model

fast growing entanglement in transverse direction

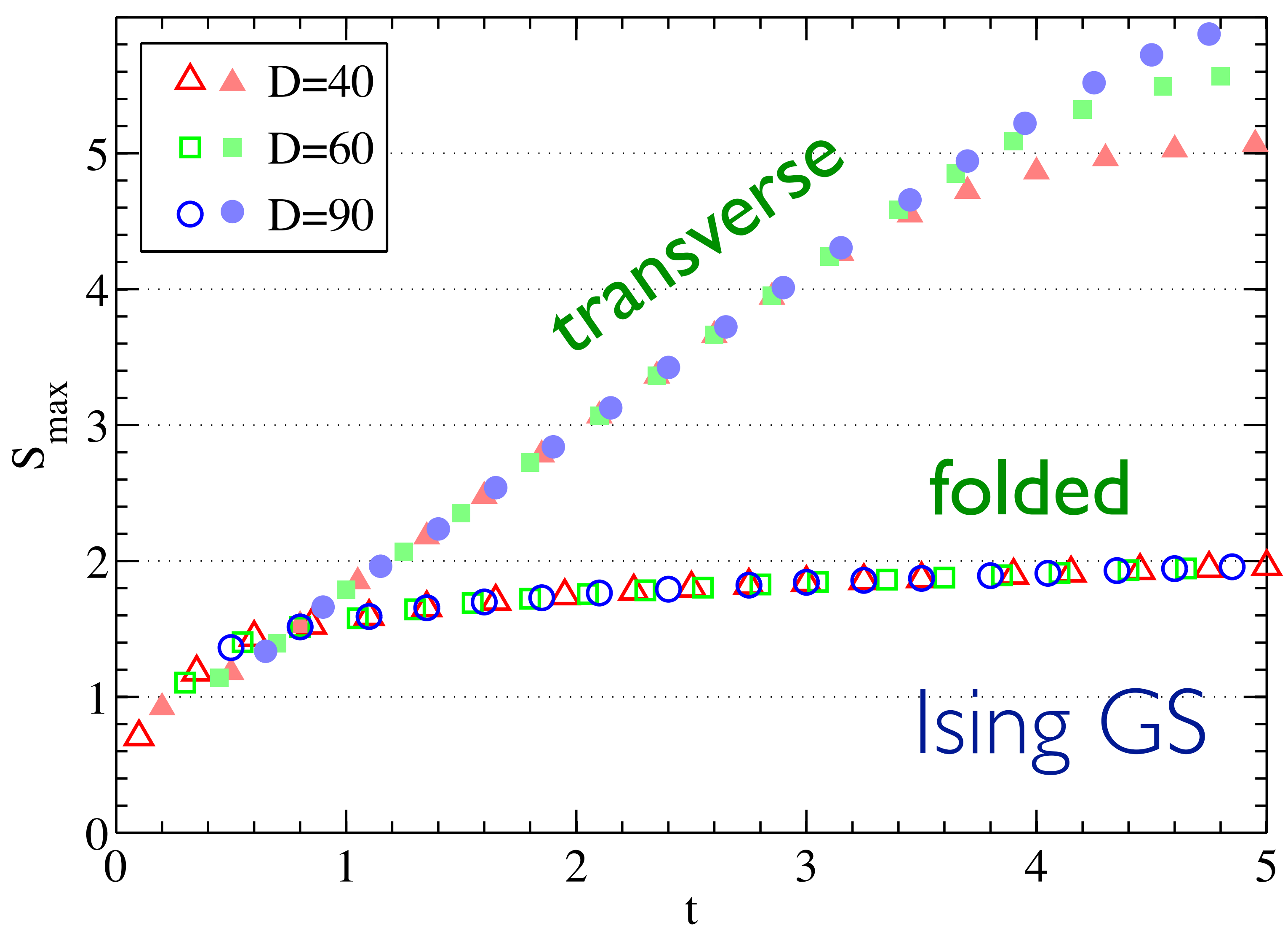
folding works



MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

transverse folding approach

intuition: toy mo



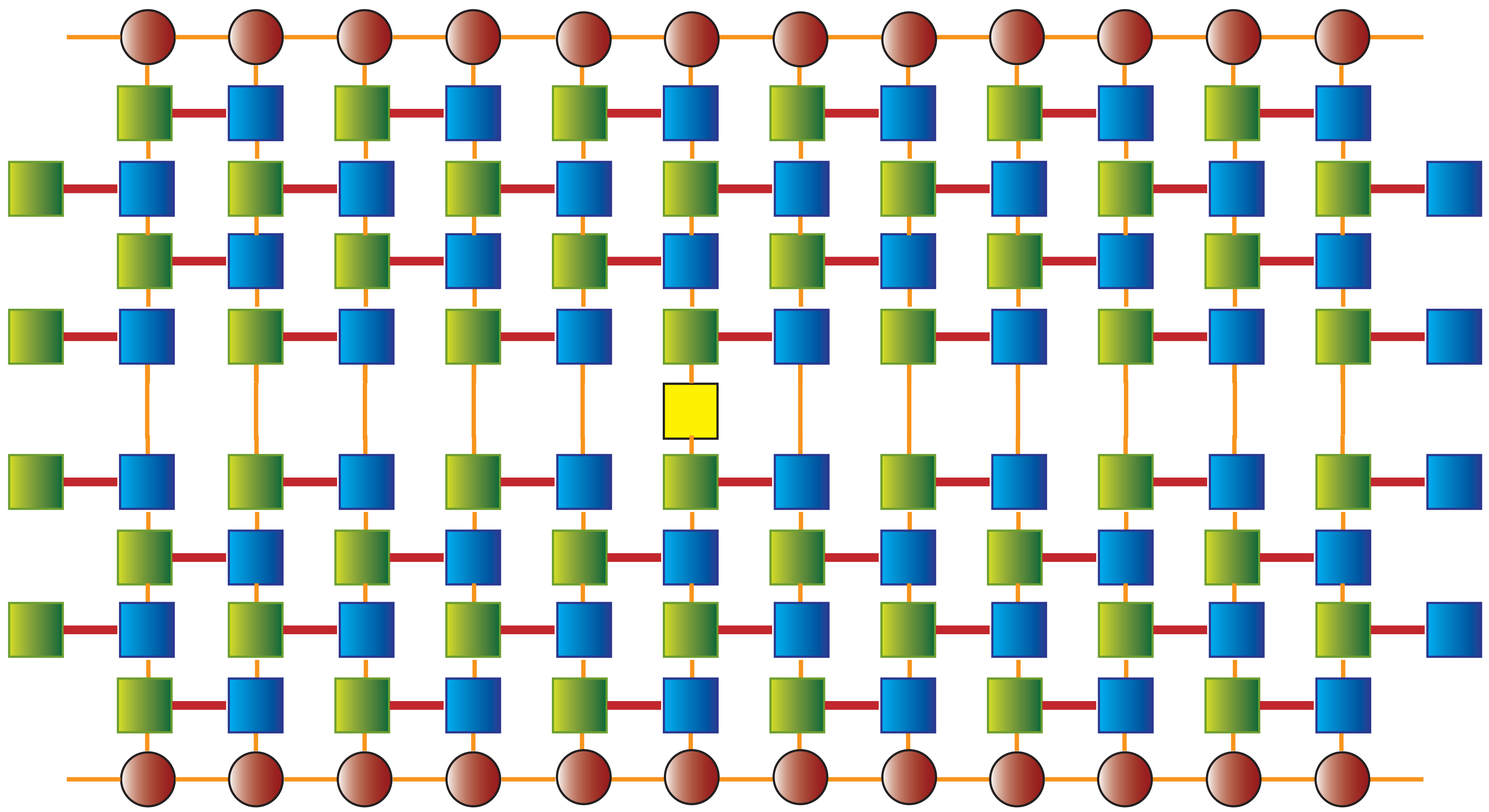
fast growing entanglement in transverse direction

folding works

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012

transverse folding + light cone

cancelling local unitaries

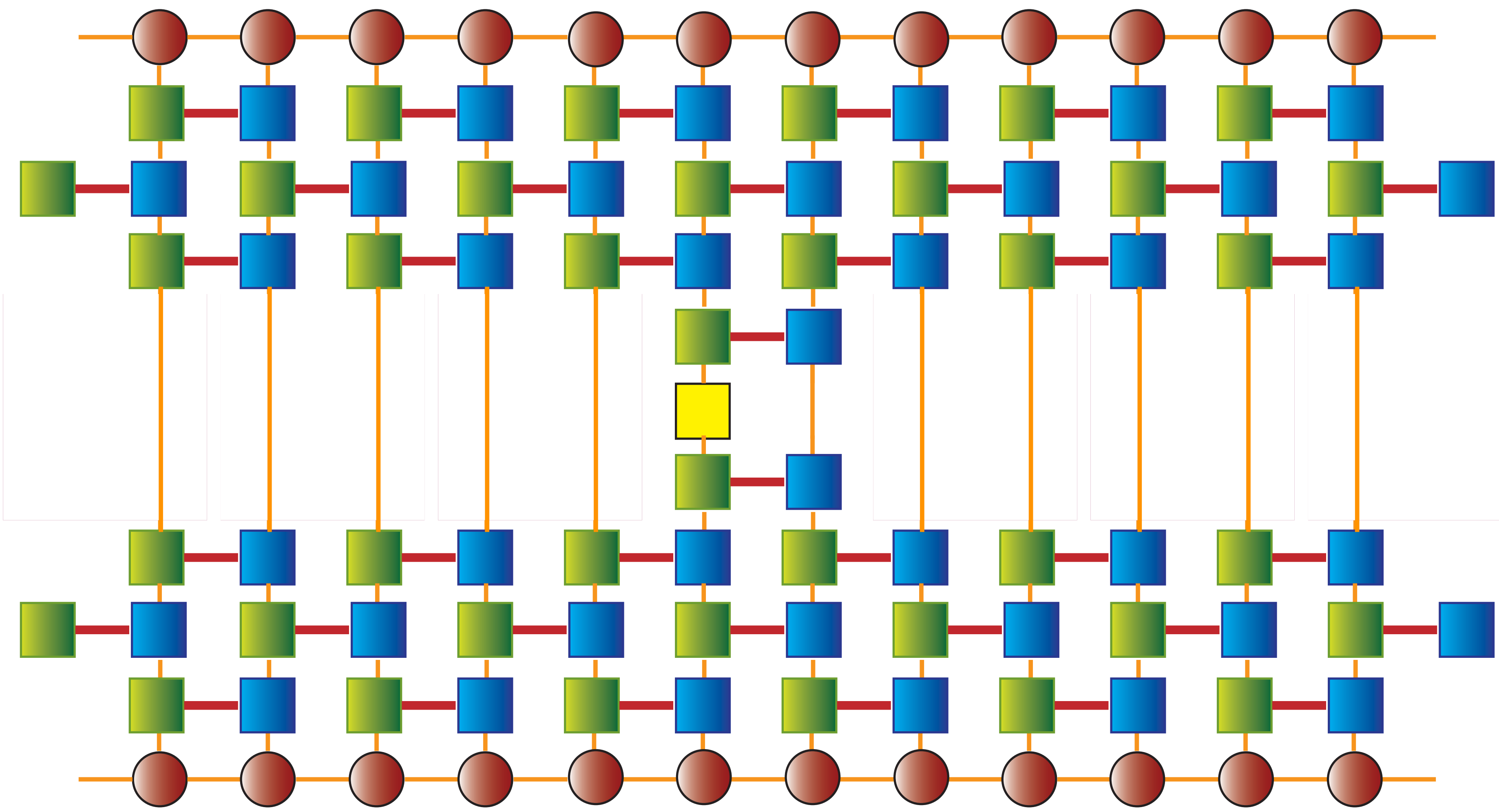


similar idea: Lerose, Sonner, Abanin, 2201.04150

Frías-Pérez, MCB, PRB 106, 115117 (2022)

transverse folding + light cone

cancelling local unitaries

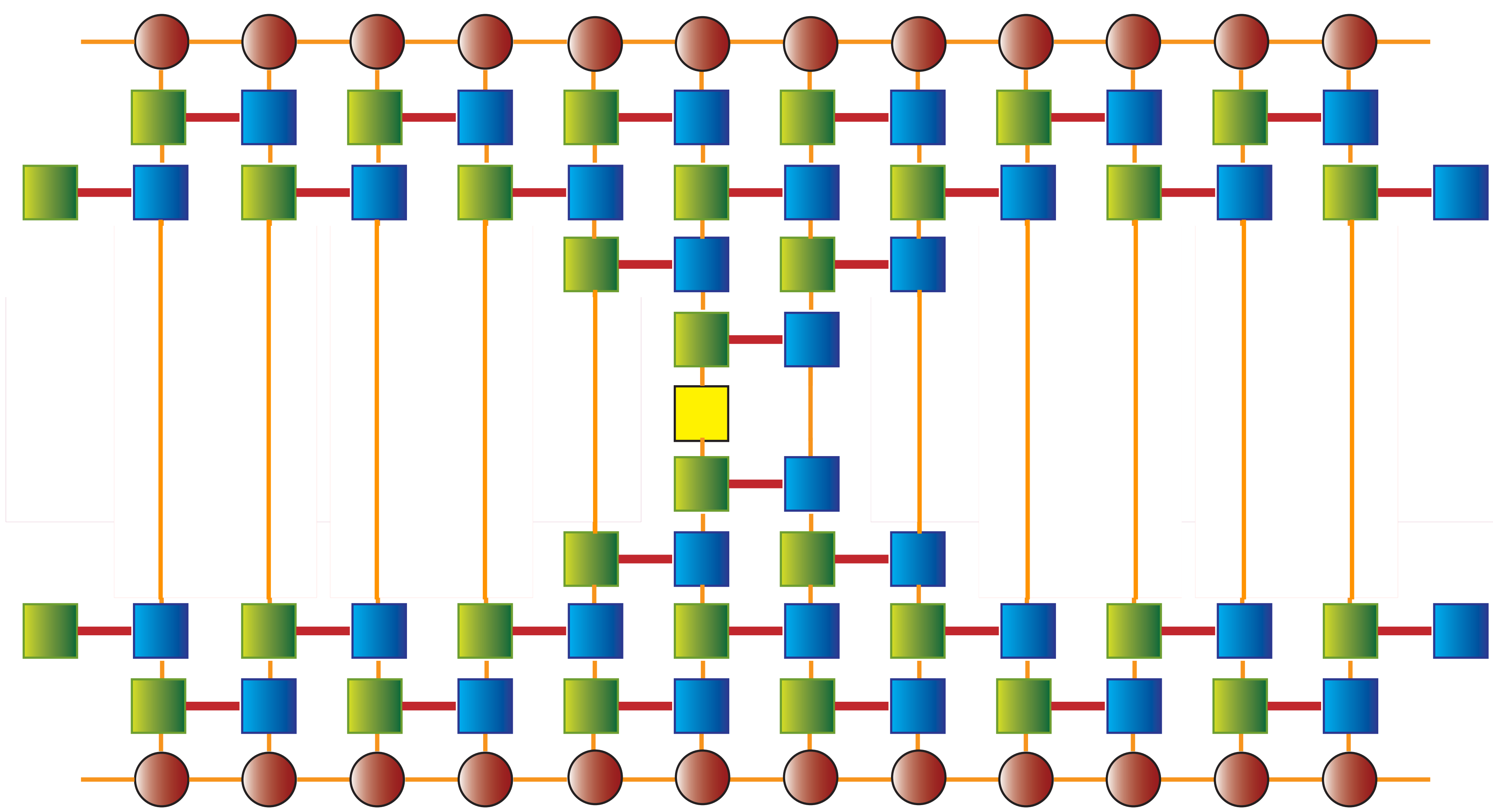


similar idea: Lerose, Sonner, Abanin, 2201.04150

Frías-Pérez, MCB, PRB 106, 115117 (2022)

transverse folding + light cone

cancelling local unitaries

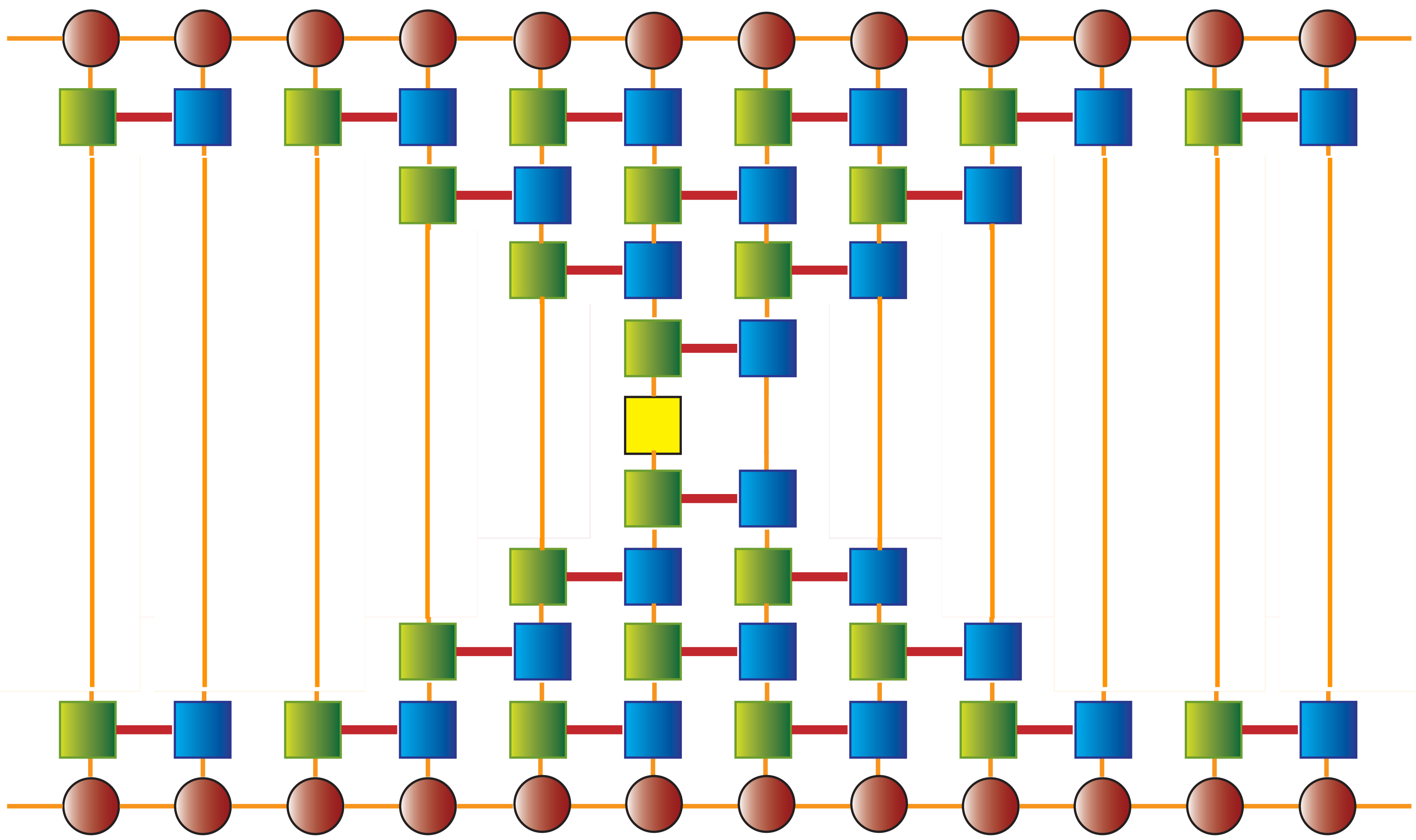


similar idea: Lerose, Sonner, Abanin, 2201.04150

Frías-Pérez, MCB, PRB 106, 115117 (2022)

transverse folding + light cone

cancelling local unitaries

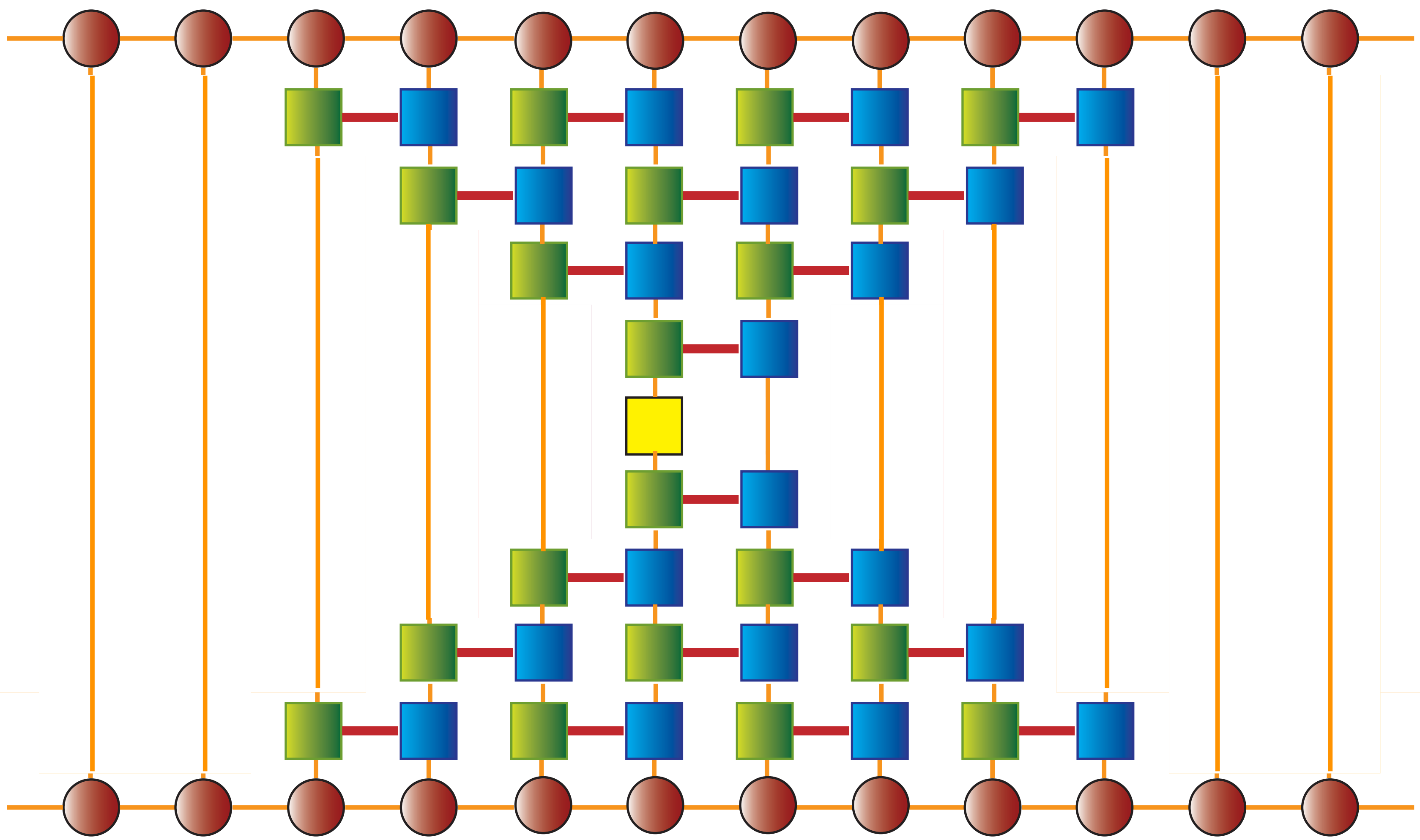


similar idea: Lerose, Sonner, Abanin, 2201.04150

Frías-Pérez, MCB, PRB 106, 115117 (2022)

transverse folding + light cone

cancelling local unitaries



similar idea: Lerose, Sonner, Abanin, 2201.04150

Frías-Pérez, MCB, PRB 106, 115117 (2022)

transverse folding + light cone =TLCC

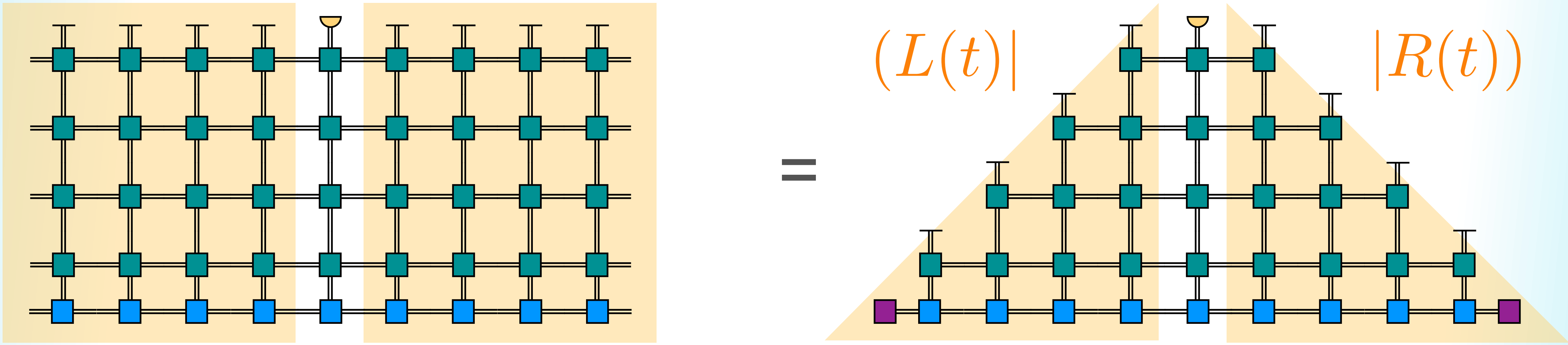
cancelling local unitaries

gain in efficiency

systematic increment of t

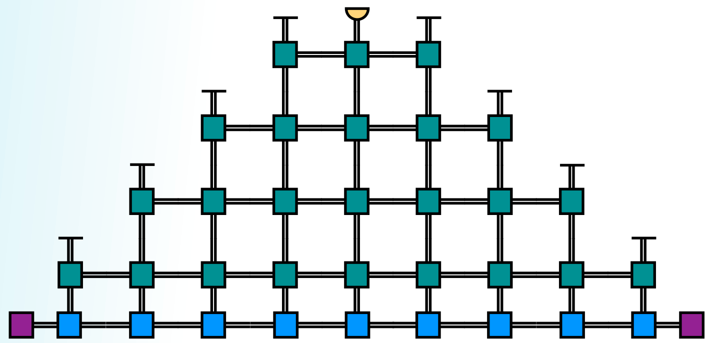
improved convergence with Hastings' truncation

Hastings, Mahajan 2014



Frías-Pérez, MCB, PRB 106, 115117 (2022)

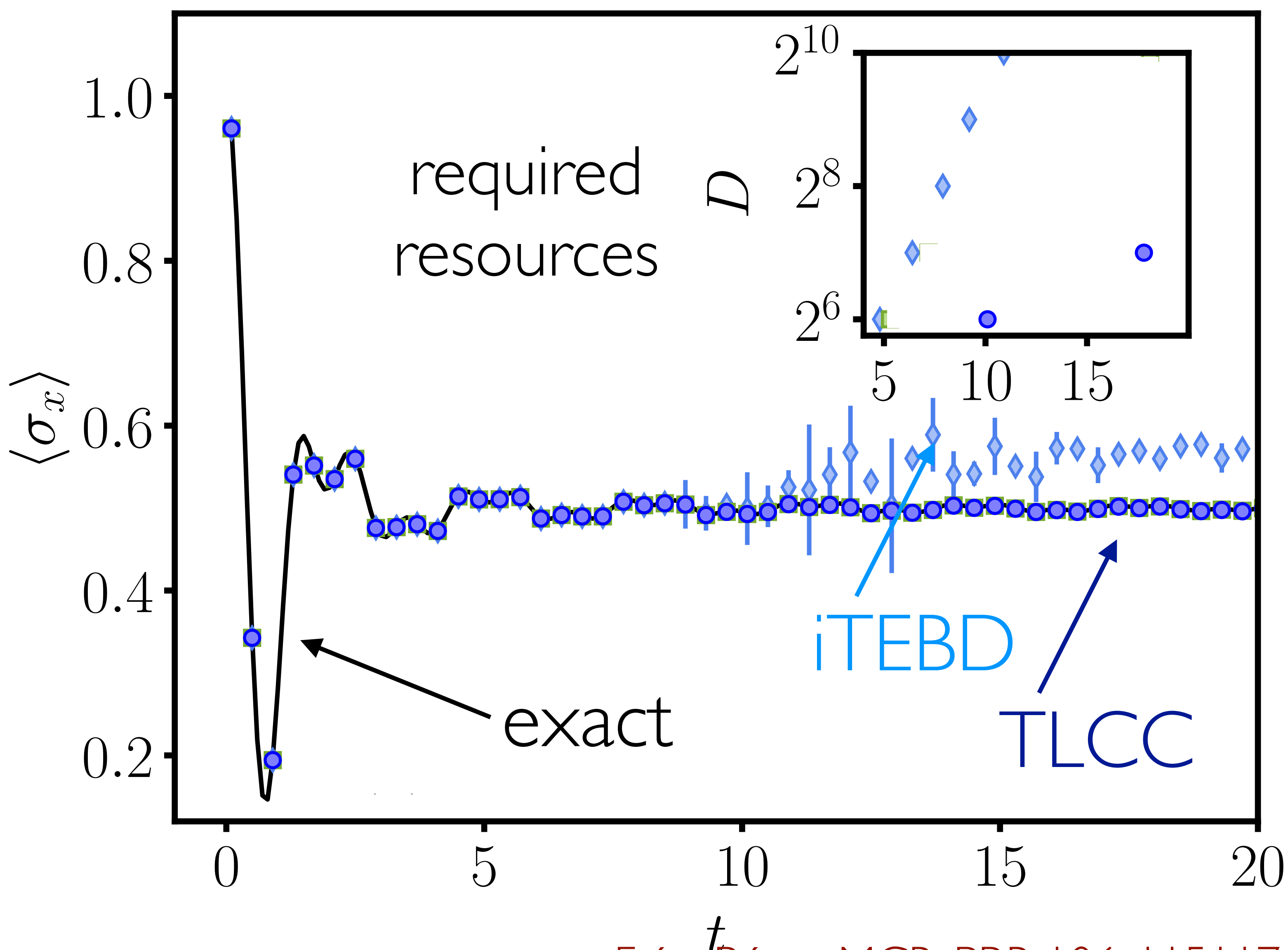
transverse folding + light cone =TLCC



$$H_{\text{Ising}} = J \sum_{i=1}^{N-1} \sigma_z^{[i]} \sigma_z^{[i+1]} + g \sum_i^N \sigma_x^{[i]} + h \sum_i^N \sigma_z^{[i]}$$

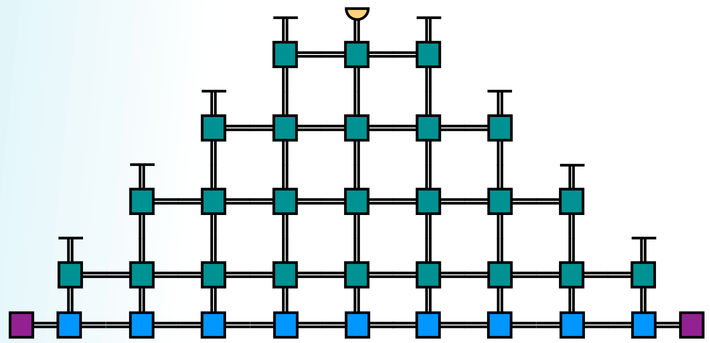
$$(J, g, h) = (1, 0.5, 0)$$

quench from $|X+\rangle$



Frías-Pérez, MCB, PRB 106, 115117 (2022)

transverse folding + light cone =TLCC

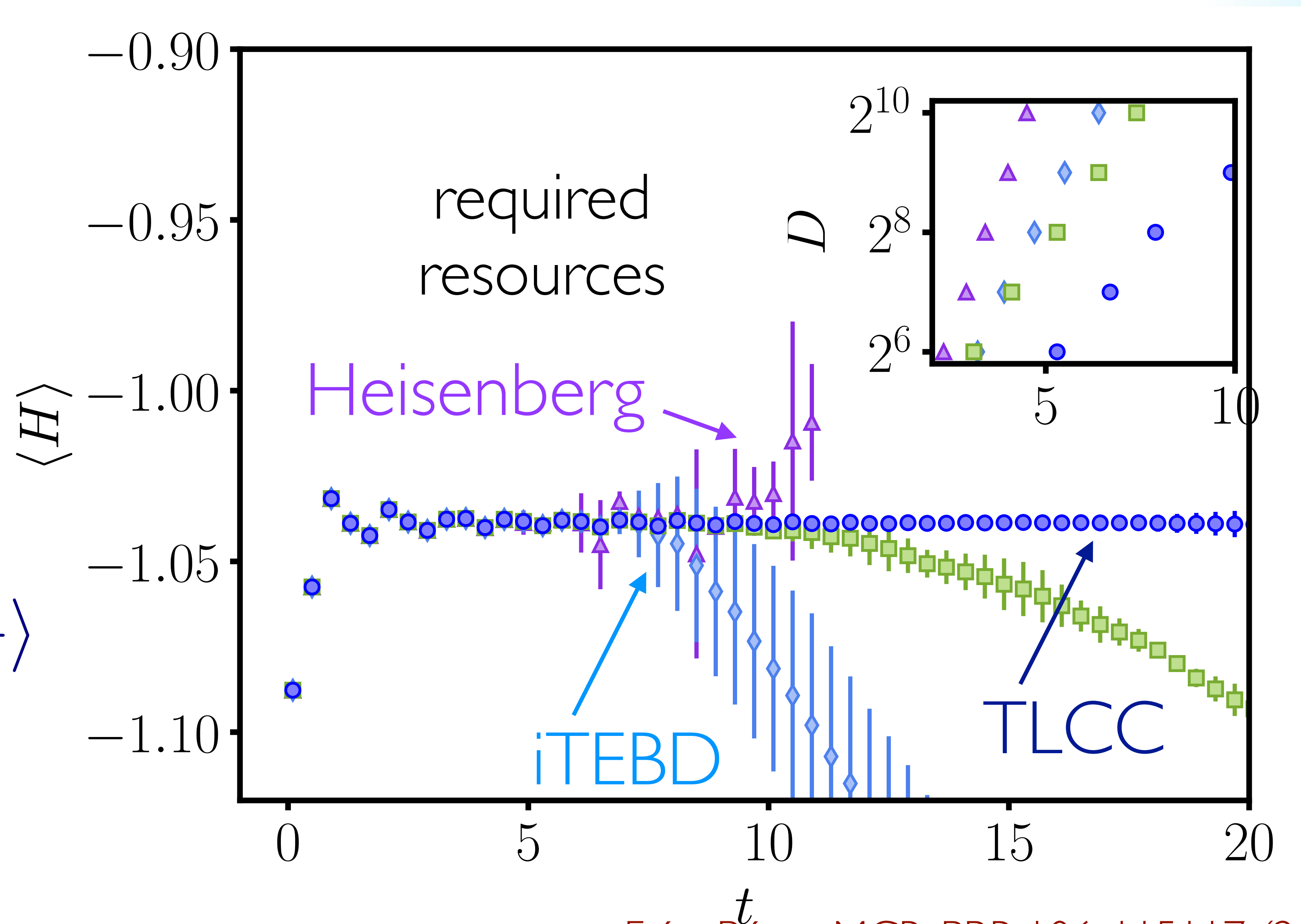


$$H_{\text{Ising}} = J \sum_{i=1}^{N-1} \sigma_z^{[i]} \sigma_z^{[i+1]} + g \sum_i^N \sigma_x^{[i]} + h \sum_i^N \sigma_z^{[i]}$$

$(J, g, h) =$

$(1, -1.05, 0.5)$

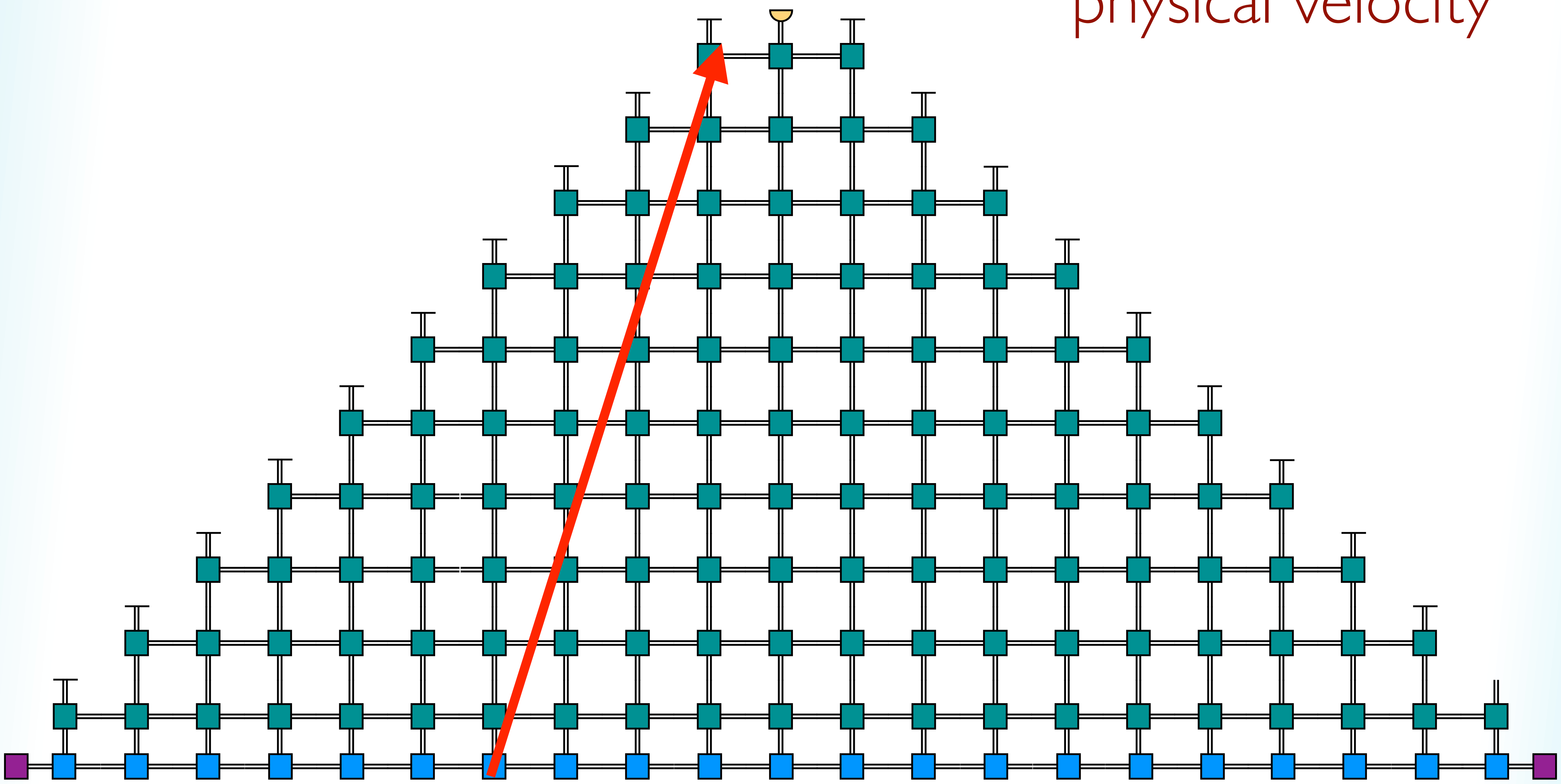
quench from $|X+\rangle$



Frías-Pérez, MCB, PRB 106, 115117 (2022)

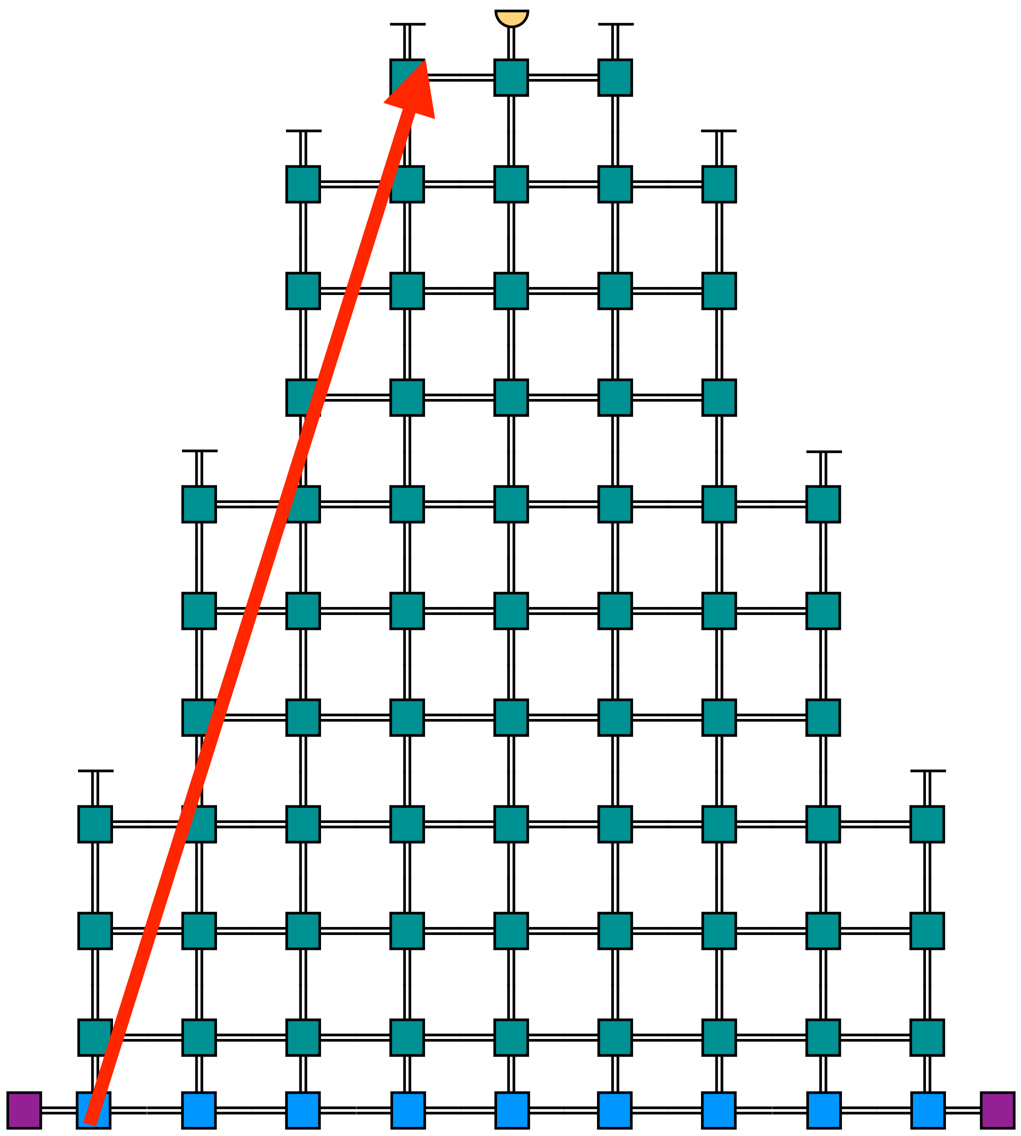
physical light cone

Lieb-Robinson: maximal physical velocity



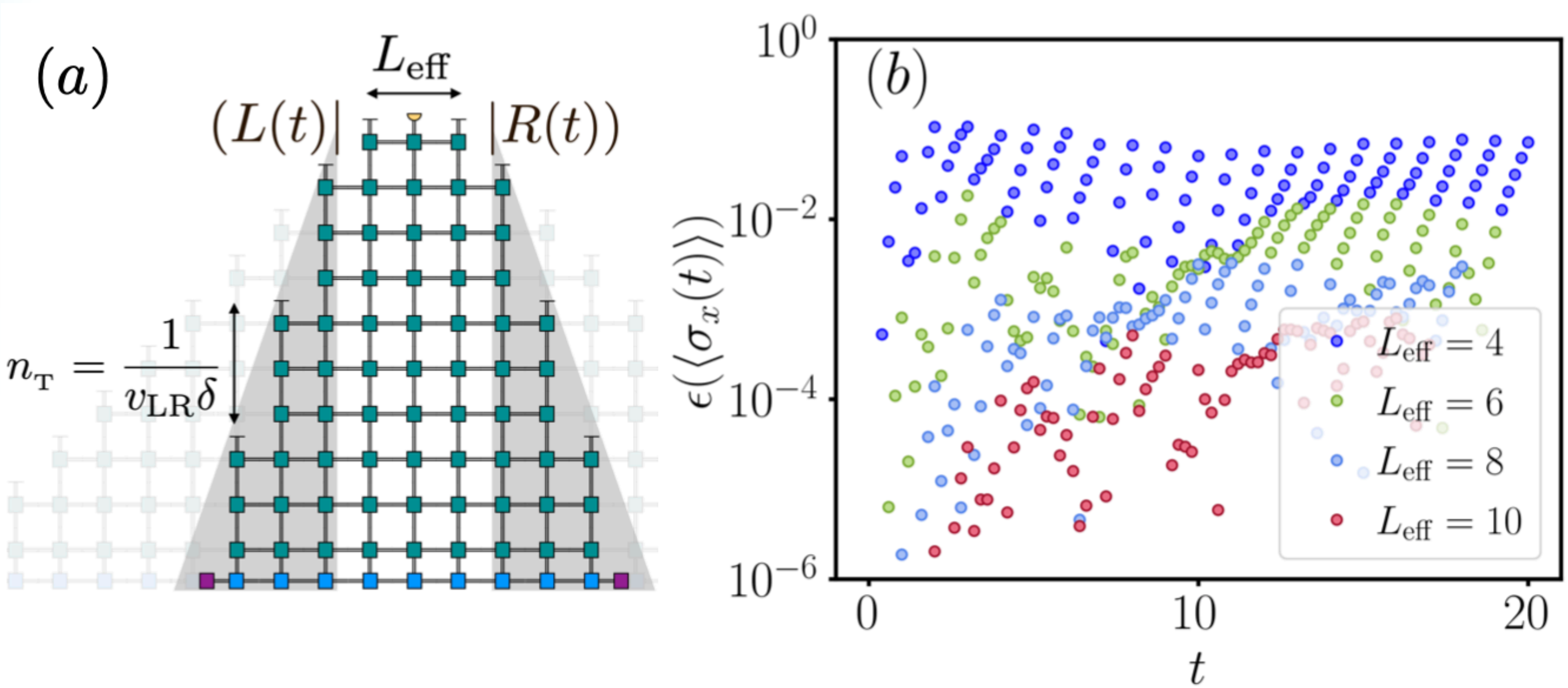
Frías-Pérez, MCB, PRB 106, 115117 (2022)

physical light cone



Frías-Pérez, MCB, PRB 106, 115117 (2022)

physical light cone



LC with exponential corrections

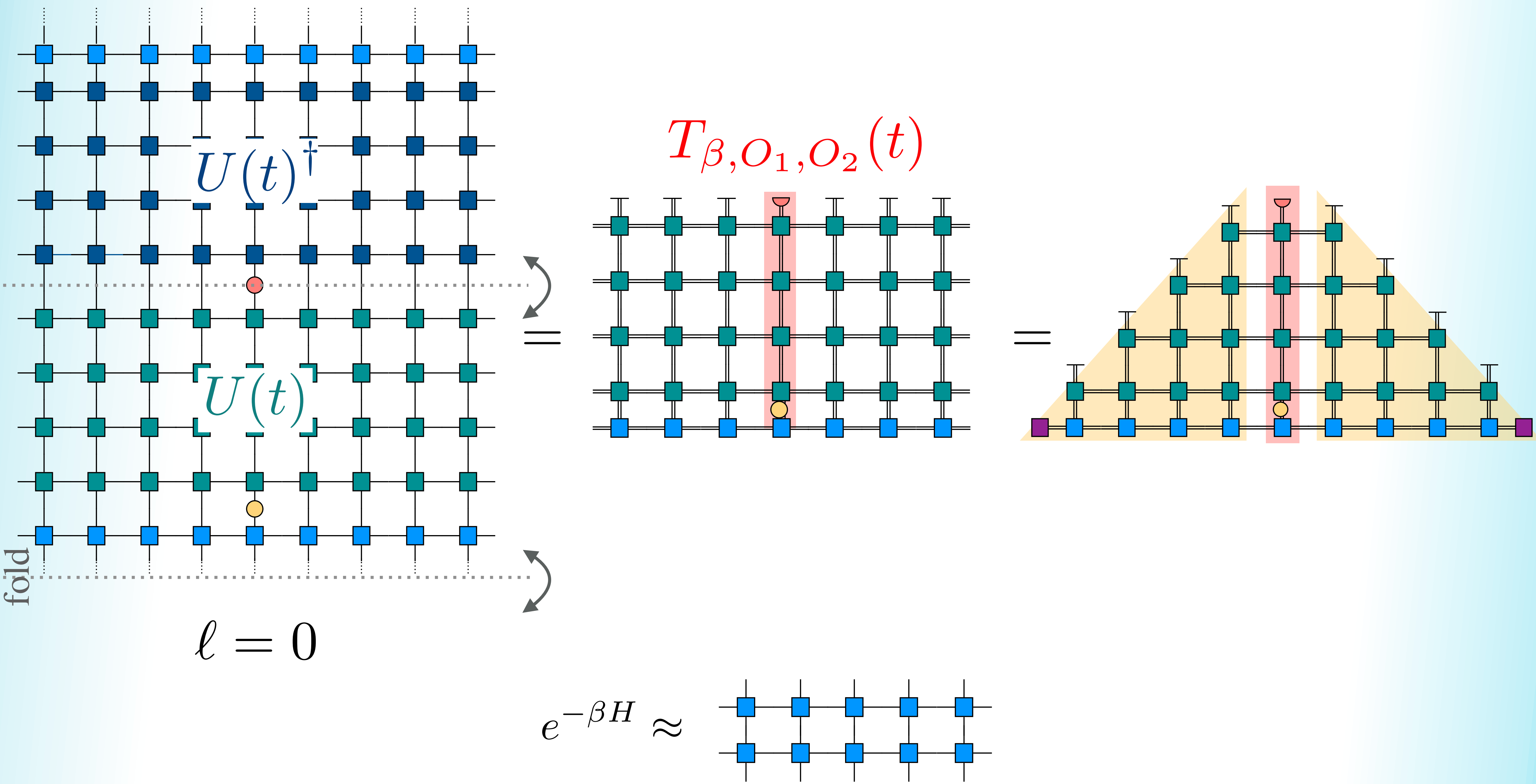
finite size window to decrease error

Frías-Pérez, MCB, PRB 106, 115117 (2022)

light cone can be exploited for other quantities

computing response functions

$$C_{1,2}(t,\ell,\beta) = \mathrm{tr}(\rho_\beta O_2^{[\ell]}(t) O_1^{[0]}(0))$$

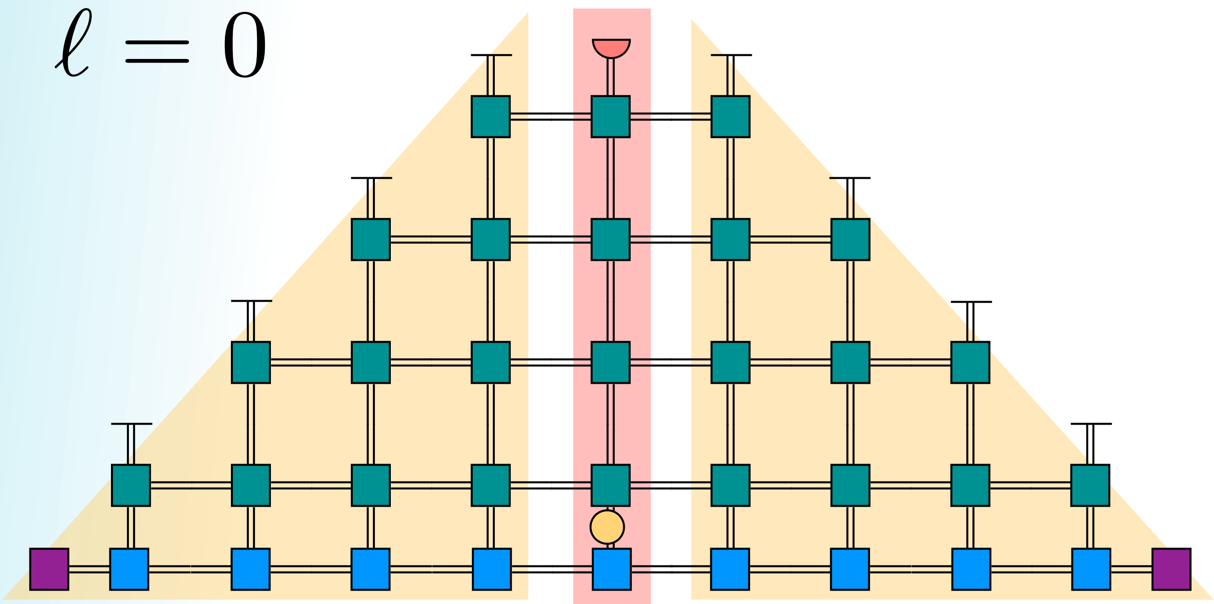


Frías-Pérez, MCB, PRB 106, 115117 (2022)

computing response functions

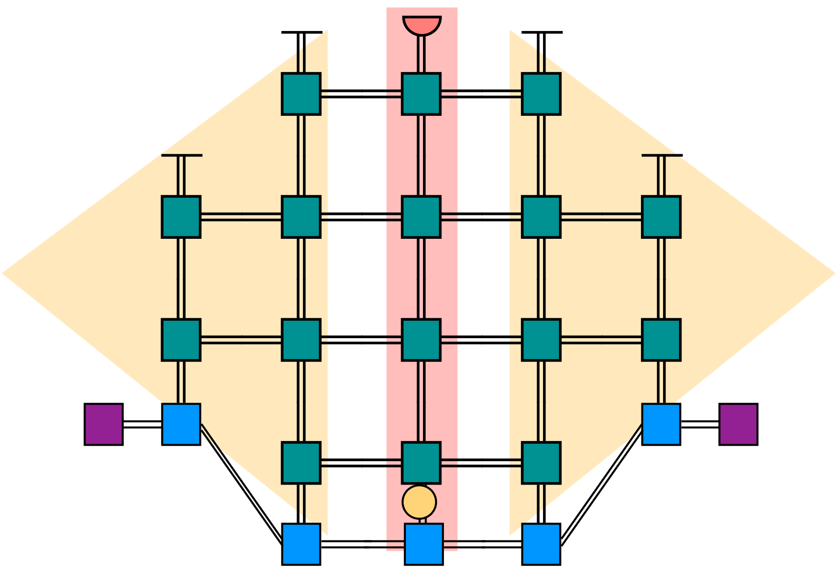
$$C_{1,2}(t,\ell,\beta) = \mathrm{tr}(\rho_\beta O_2^{[\ell]}(t) O_1^{[0]}(0))$$

$\ell = 0$



$T_{\beta,O_1,O_2}(t)$

$\approx$



see poster by Miguel Frías

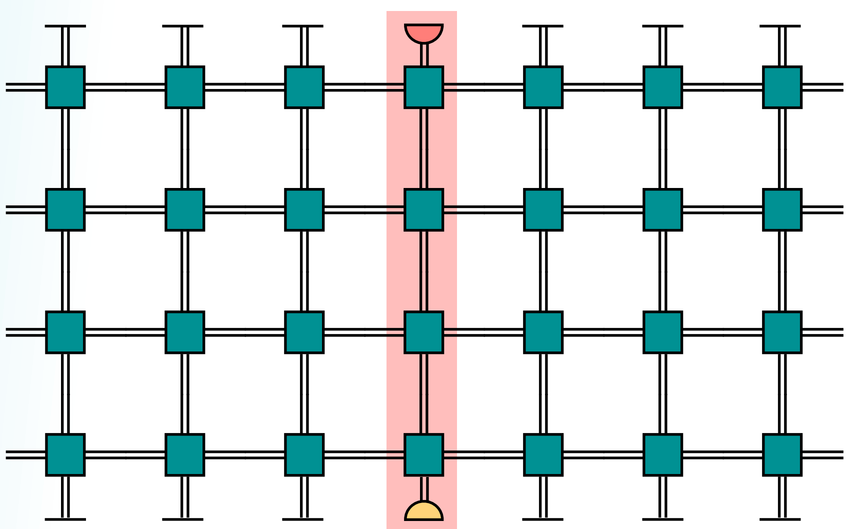
double lightcone

Frías-Pérez, MCB, PRB 106, 115117 (2022)

computing response functions

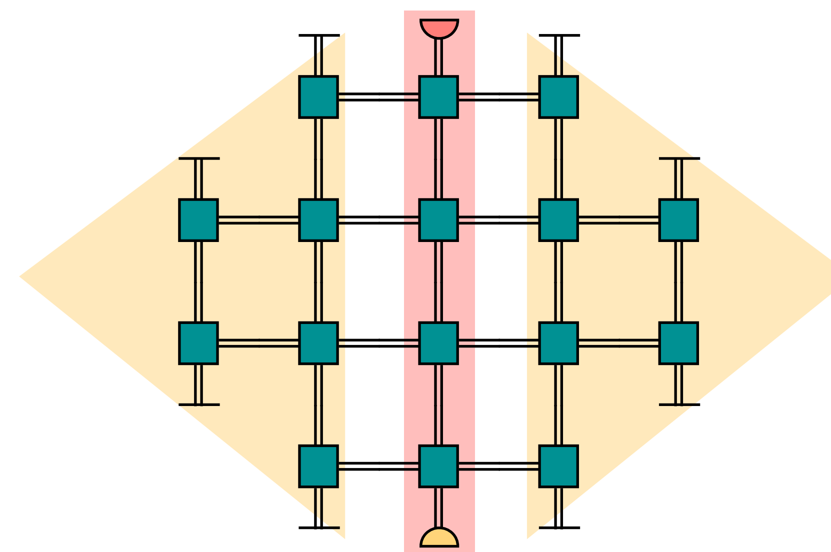
$$C_{1,2}(t,\ell,\beta) = \mathrm{tr}(\rho_\beta O_2^{[\ell]}(t) O_1^{[0]}(0))$$

$\ell = 0$



$T_{\beta=0,O_1,O_2}(t)$

$=$



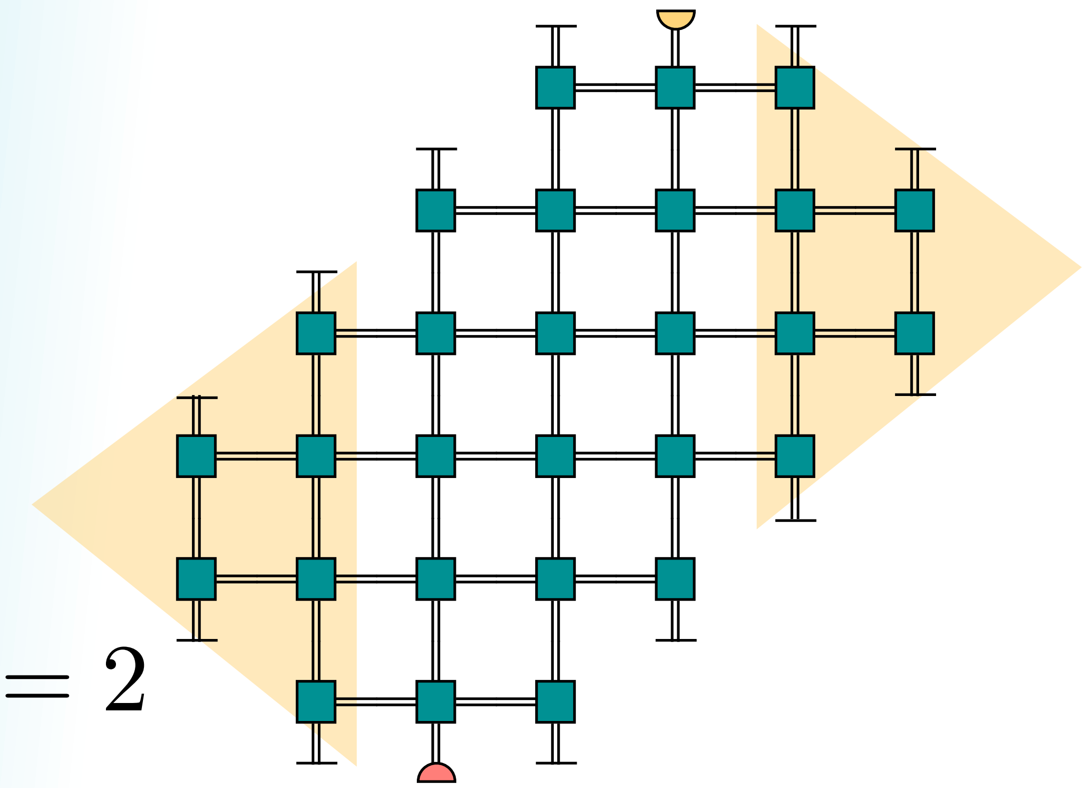
infinite temperature

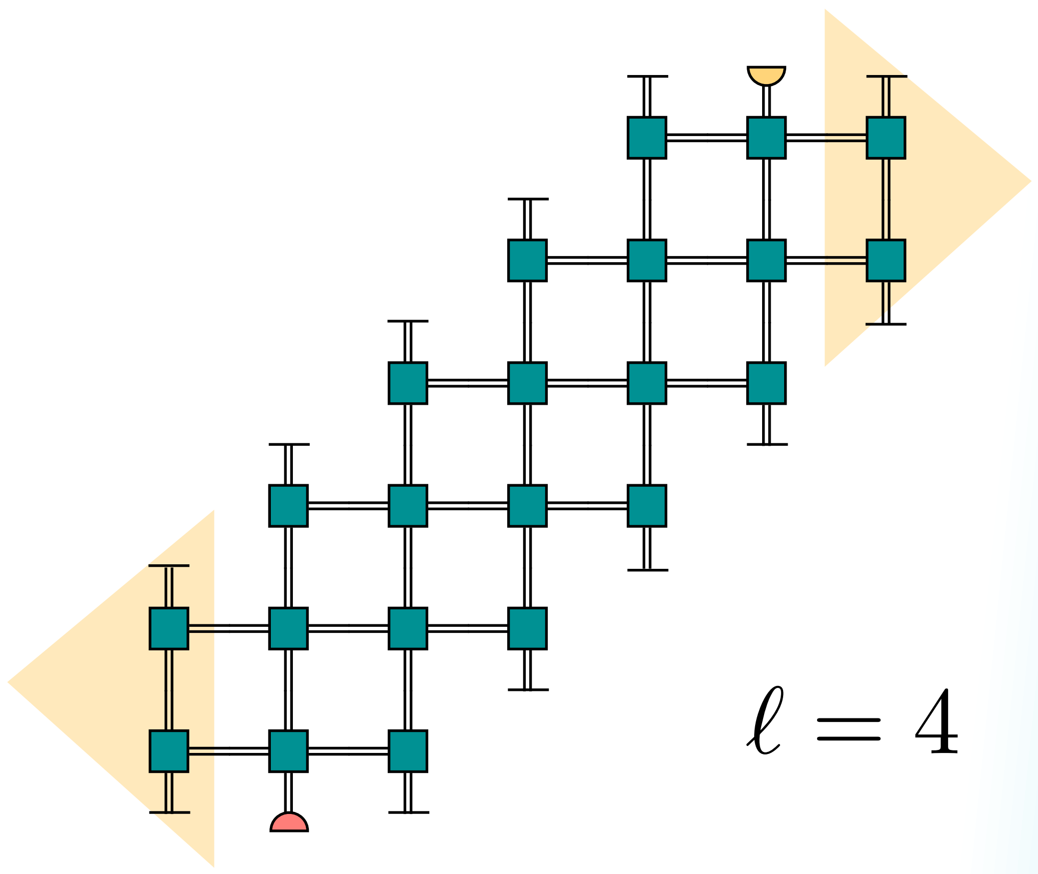
$\beta = 0$

Frías-Pérez, MCB, PRB 106, 115117 (2022)

computing response functions

$$C_{1,2}(t,\ell,\beta) = \mathrm{tr}(\rho_\beta O_2^{[\ell]}(t) O_1^{[0]}(0))$$





arbitrary distance

Frías-Pérez, MCB, PRB 106, 115117 (2022)

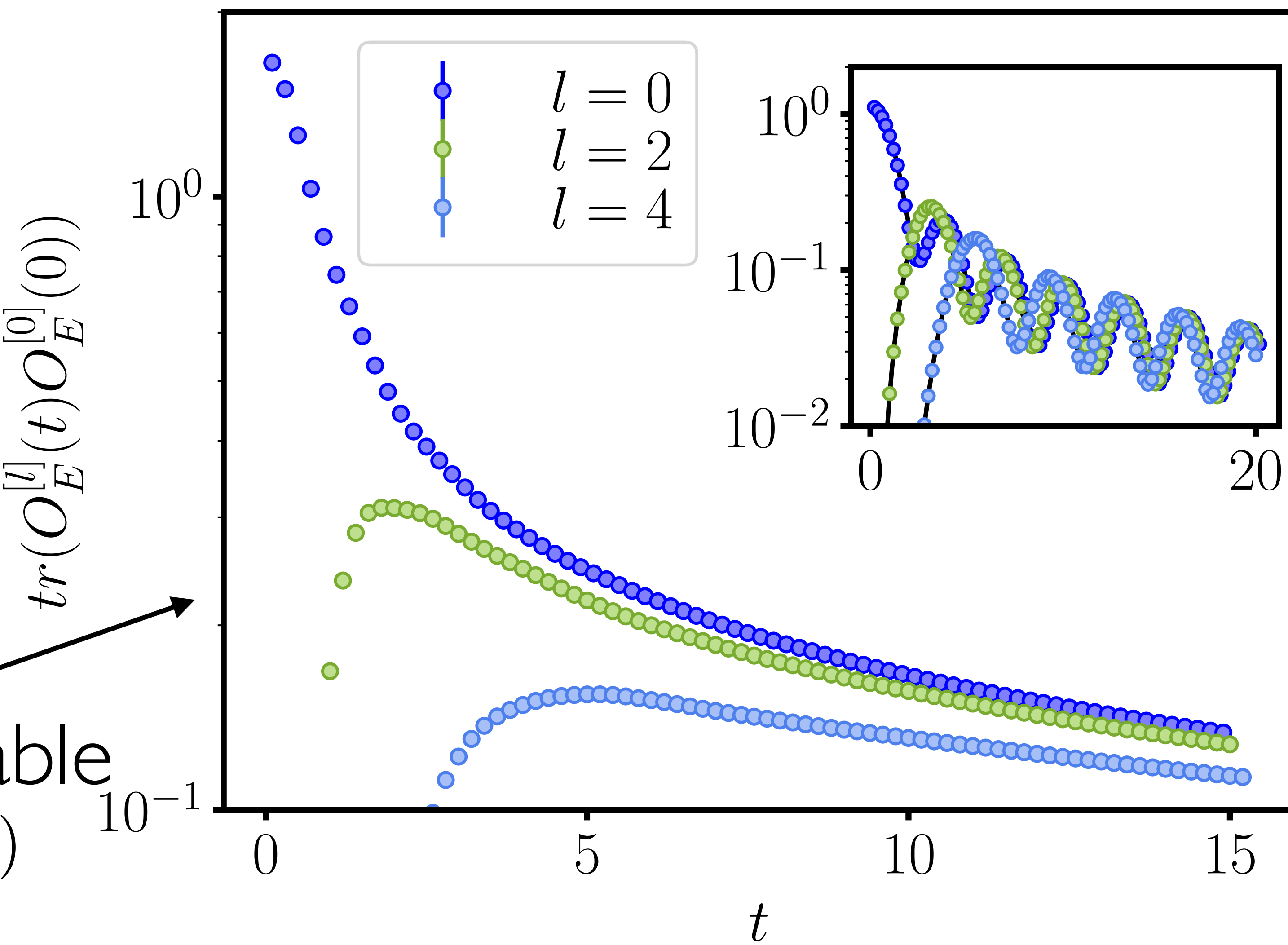
computing response functions

tilted Ising

energy density

infinite temperature

$$O_E^{[i]} := J\sigma_i^z\sigma_{i+1}^z + \frac{g}{2}(\sigma_i^x + \sigma_{i+1}^x) + \frac{h}{2}(\sigma_i^z + \sigma_{i+1}^z)$$



Frías-Pérez, MCB, PRB 106, 115117 (2022)

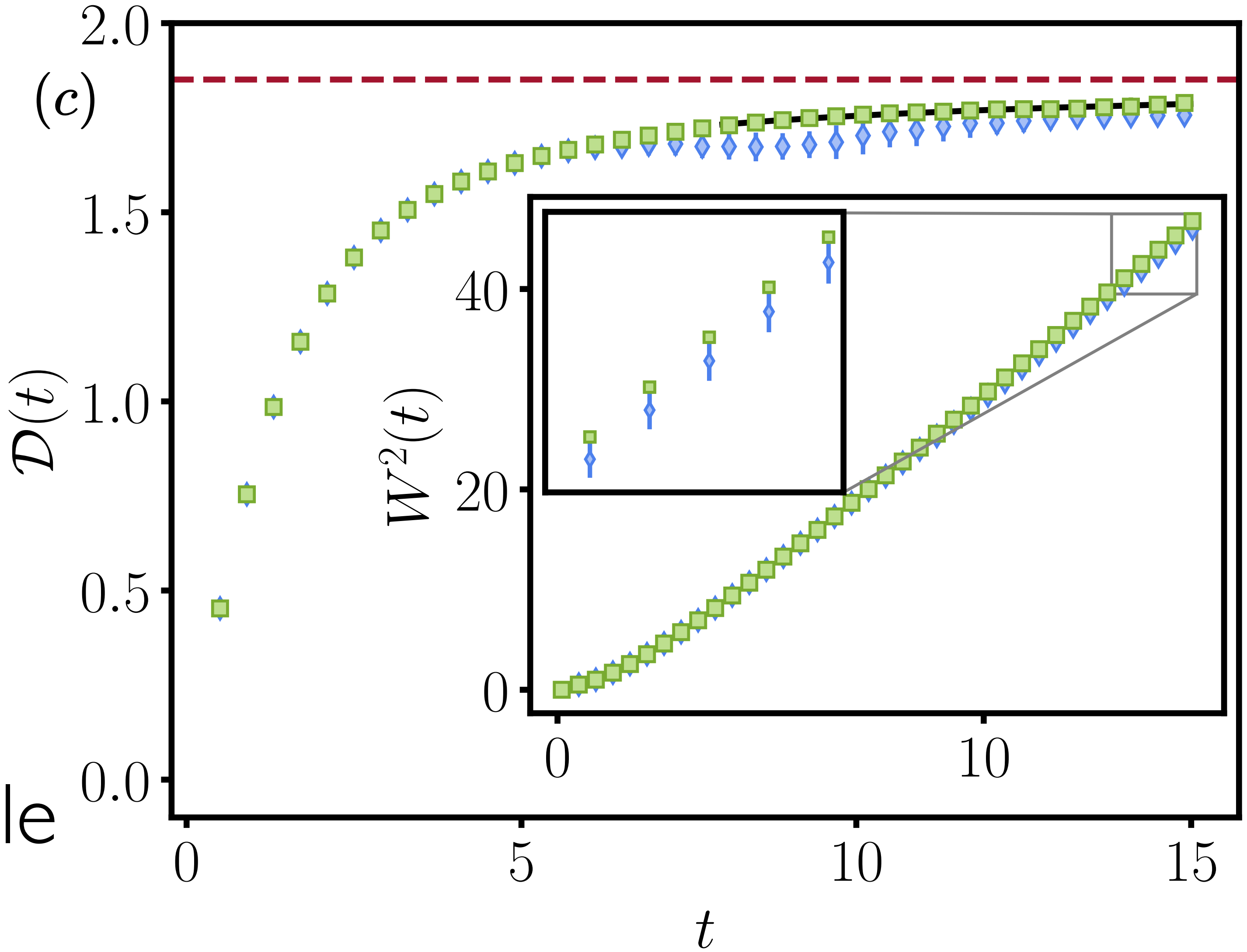
computing response functions

tilted Ising

energy density

infinite temperature

$$O_E^{[i]} := J\sigma_i^z\sigma_{i+1}^z + \frac{g}{2}(\sigma_i^x + \sigma_{i+1}^x) + \frac{h}{2}(\sigma_i^z + \sigma_{i+1}^z)$$



diffusion constant

non-integrable

$(1, -1.05, 0.5)$

Frías-Pérez, MCB, PRB 106, 115117 (2022)

alternative: give up description of the full state

spectral properties of the QMB Hamiltonian

Yang, Iblisdir, Cirac, MCB, PRL124, 100602 (2020)
Papaefstathiou, Robaina, Cirac, MCB, PRD104, 014514 (2021)
Çakan, Cirac, MCB, PRB 103, 115113 (2021)
Lu, MCB, Cirac, PRX Quantum 2, 02032 (2021)
Yang, Cirac, MCB, PRB 106, 024307 (2022)

light-cone TN for non-equilibrium evolution of local observables

M. Frías-Pérez, MCB, PRB 106, 115117 (2022)

spectral properties of the QMB Hamiltonian

Yang, Iblisdir, Cirac, MCB, PRL124, 100602 (2020)
Papaefstathiou, Robaina, Cirac, MCB, PRD104, 014514 (2021)
Çakan, Cirac, MCB, PRB 103, 115113 (2021)
Lu, MCB, Cirac, PRX Quantum 2, 02032 (2021)
Yang, Cirac, MCB, PRB 106, 024307 (2022)

spectral properties of
the QMB Hamiltonian

Yang, Iblisdir, Cirac, MCB, PRL124, 100602 (2020)
Papaefstathiou, Robaina, Cirac, MCB, PRD104, 014514 (2021)
Çakan, Cirac, MCB, PRB 103, 115113 (2021)
Lu, MCB, Cirac, PRX Quantum 2, 02032 (2021)
Yang, Cirac, MCB, PRB 106, 024307 (2022)

exploring properties of quantum many-body systems at finite energy density

generalized density of states

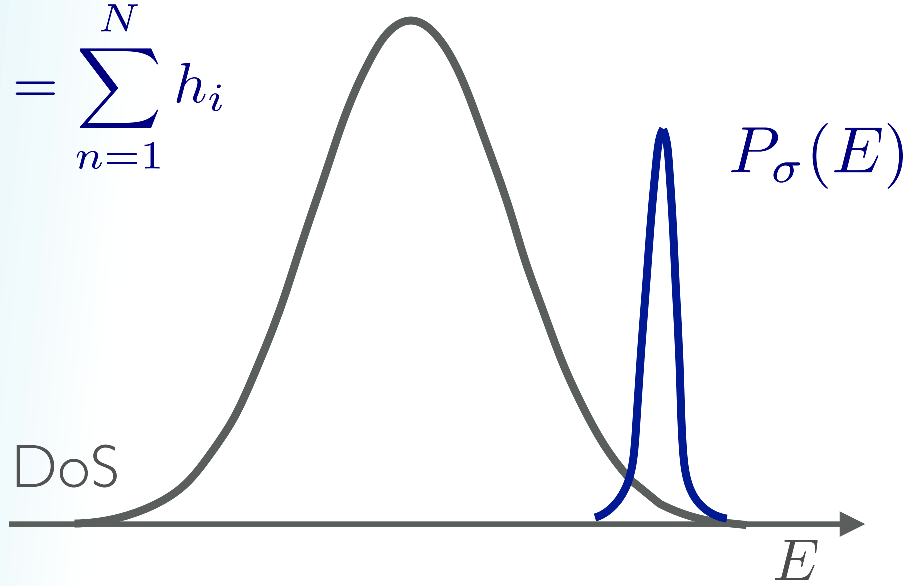
can be connected to equilibrium and non-equilibrium properties

generalized density of states

$$\sum_n \delta(E - E_n)\langle E_n|O|E_n\rangle = \mathrm{tr}\left(O\ \hat{\delta}(H - E)\right)$$

$$\hat{\delta}(H - E) \rightarrow \hat{P}_\sigma(E)$$

$$H = \sum_{n=1}^{N} h_i$$



$$\hat{P}_\sigma(E) \propto e^{-\frac{(H-E)^2}{2\sigma^2}}$$

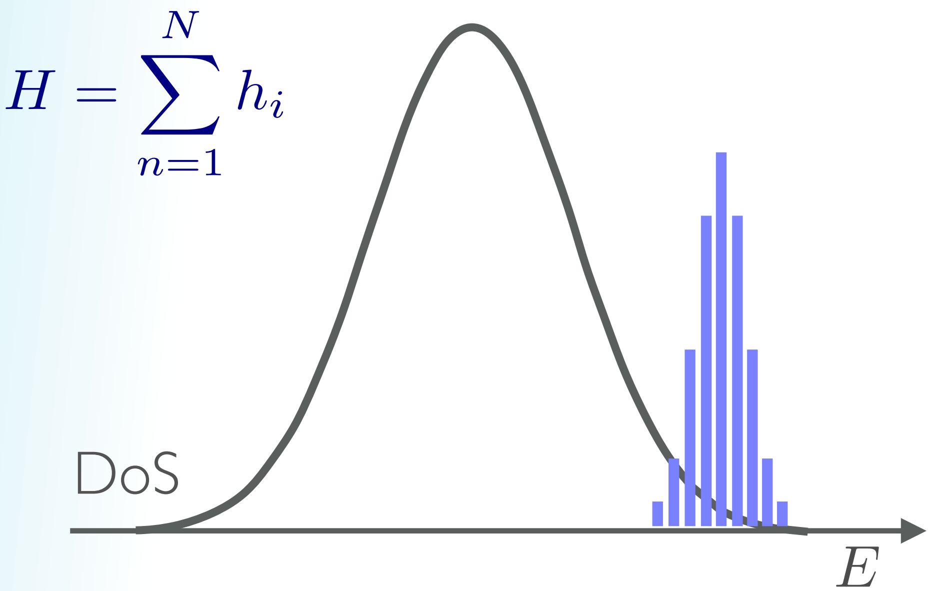
energy filter

1 filter as ensemble

diagonal in energy eigenbasis ⇒ microcanonical

$$\frac{\mathrm{tr}\left(OP_\delta(E)\right)}{\mathrm{tr}P_\delta(E)} \Rightarrow O(E)$$

$$\mathrm{tr}P_\delta(E) \Rightarrow \text{DOS}$$



equivalent to diagonal ensemble of a certain pure state

reached only after long time evolution

Lu et al. PRX Q2, 02032 (2021); Yang et al PRB 106, 024307 (2022)

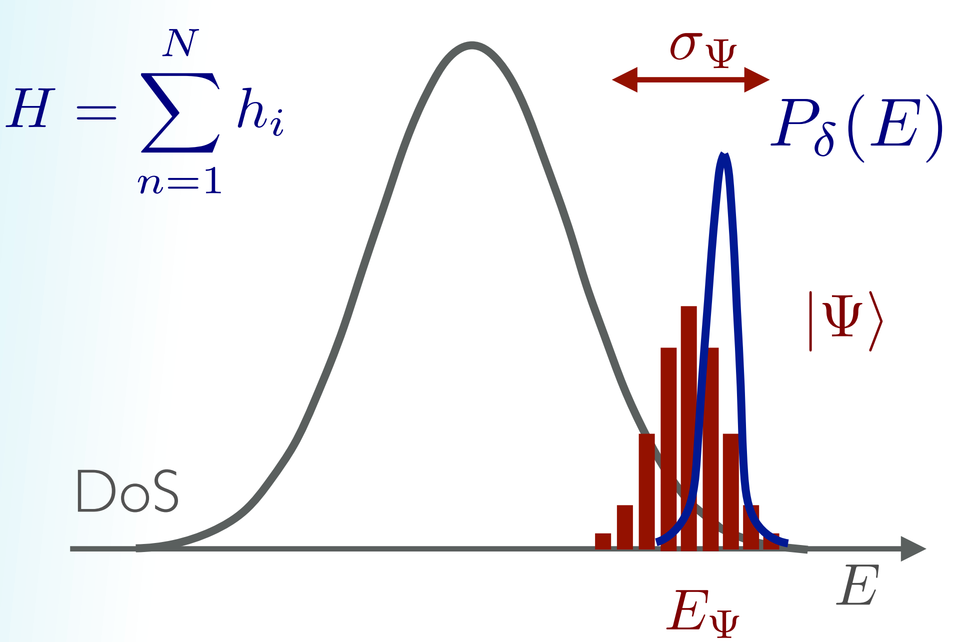
energy filter

2 filtering a state

decrease energy variance ⇒ microcanonical

$$\langle P_\delta(E)\Psi|O|P_\delta(E)\Psi\rangle \Rightarrow O(E)$$

$$\langle \Psi|P_\delta(E)|\Psi\rangle \Rightarrow \text{LDOS}$$



BUT in general, entanglement of filtered state grows

$$S \leq \frac{k_1}{\delta} + \log\sqrt{N} + k_2$$

MCB, Huse, Cirac, PRB 101, 144305 (2020)

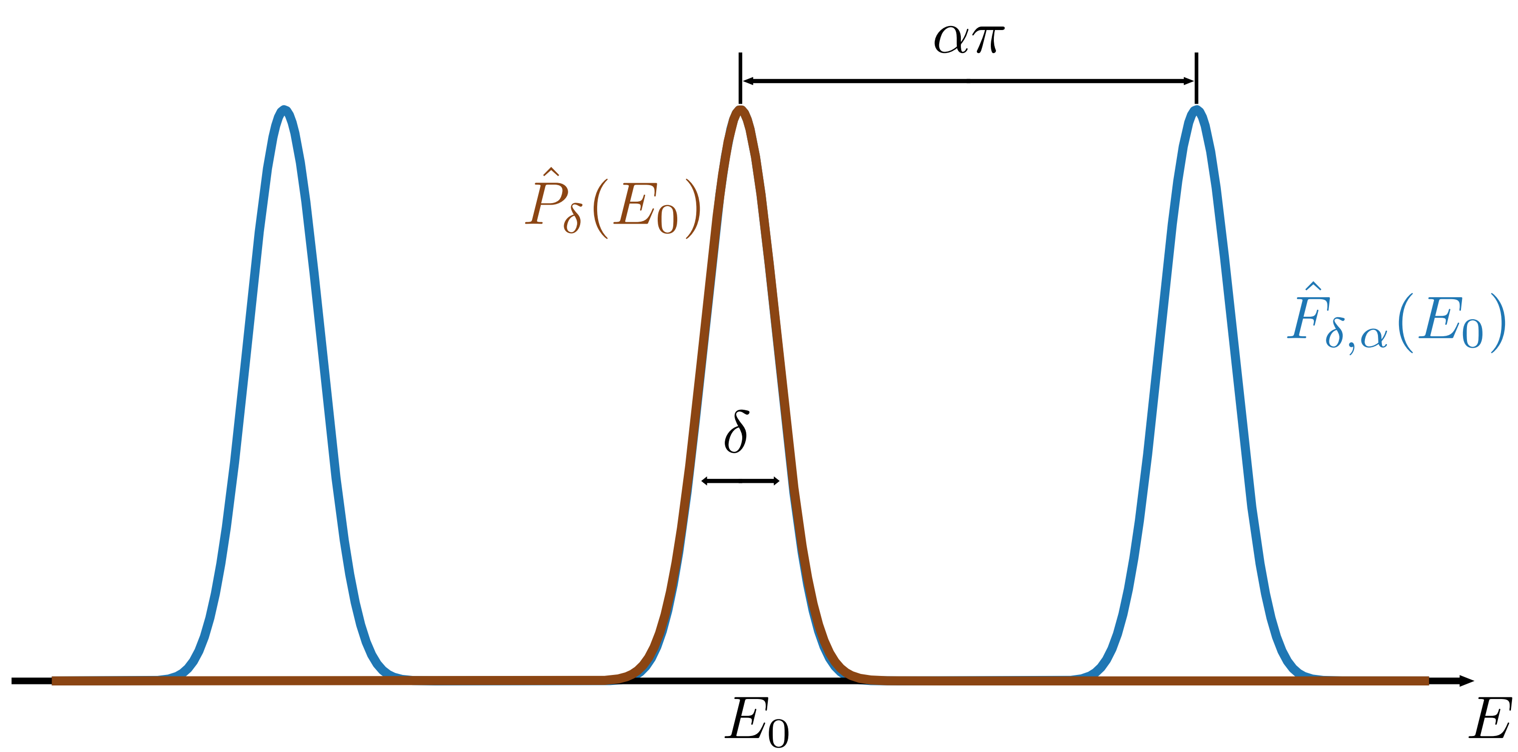
Lu et al. PRX Q2, 02032 (2021); Yang et al PRB 106, 024307 (2022)

implementing the filter

Gaussian operator ⇒ not local

⇒ cosine approximation

$$\cos^M(x) \approx e^{-Mx^2/2} \qquad x < \pi/2$$



Lu et al. PRX Q2, 02032 (2021); Yang et al PRB 106, 024307 (2022)

implementing the filter

Gaussian filter ⇒ approximated by series of evolutions

$$\exp\left[-\frac{(H-E)^2}{2\delta^2}\right] \approx \sum_{m=-R}^{R} c_m e^{-i2mE/\alpha} e^{i2mH/\alpha}$$

nr of terms $R = \frac{x\alpha}{\delta}$

largest time $t_{\max} = \frac{2x}{\delta}$

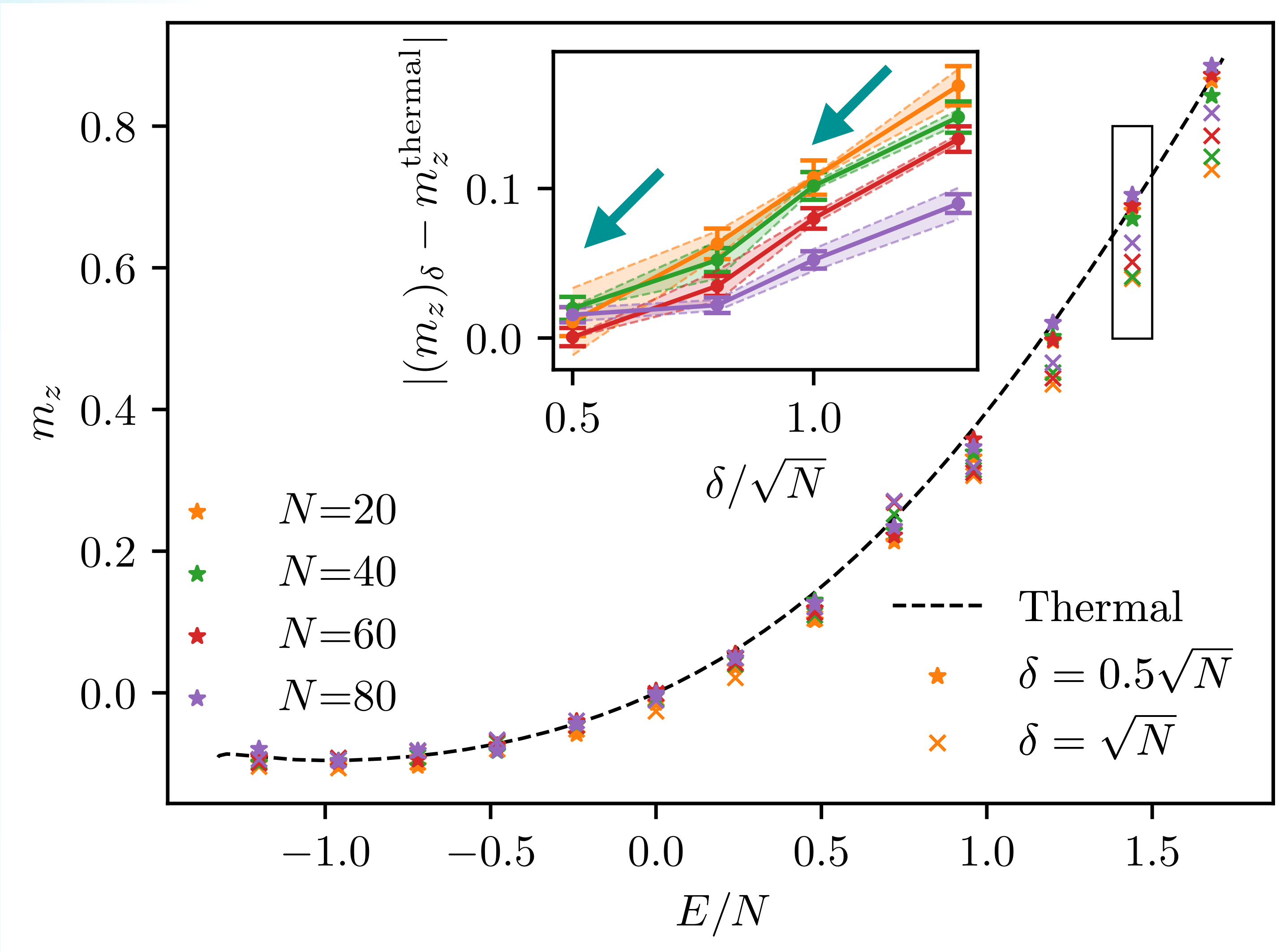
scaling factor $\alpha \sim N, \sqrt{N}$

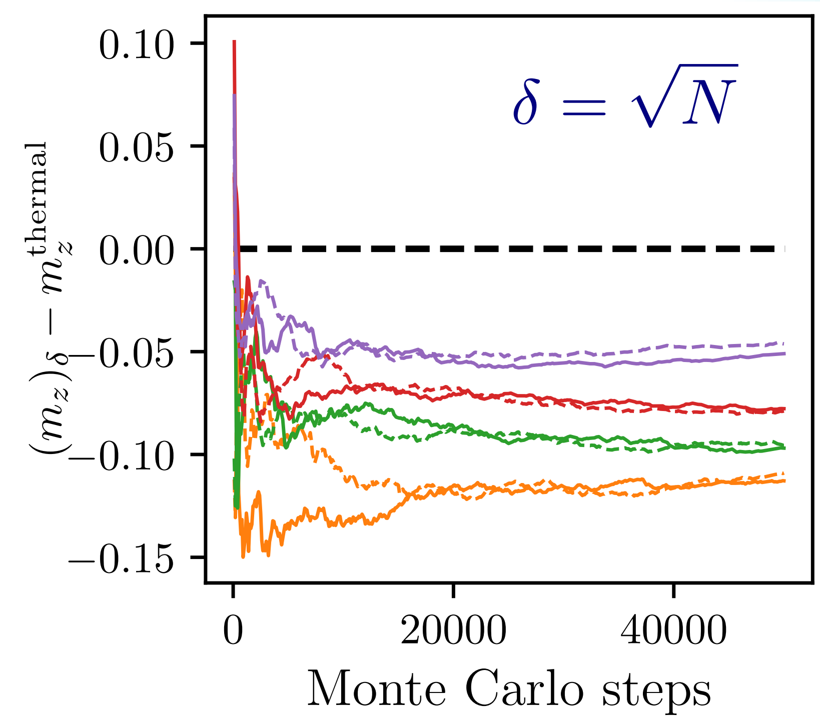
can be run in a quantum simulator or simulated with TNS

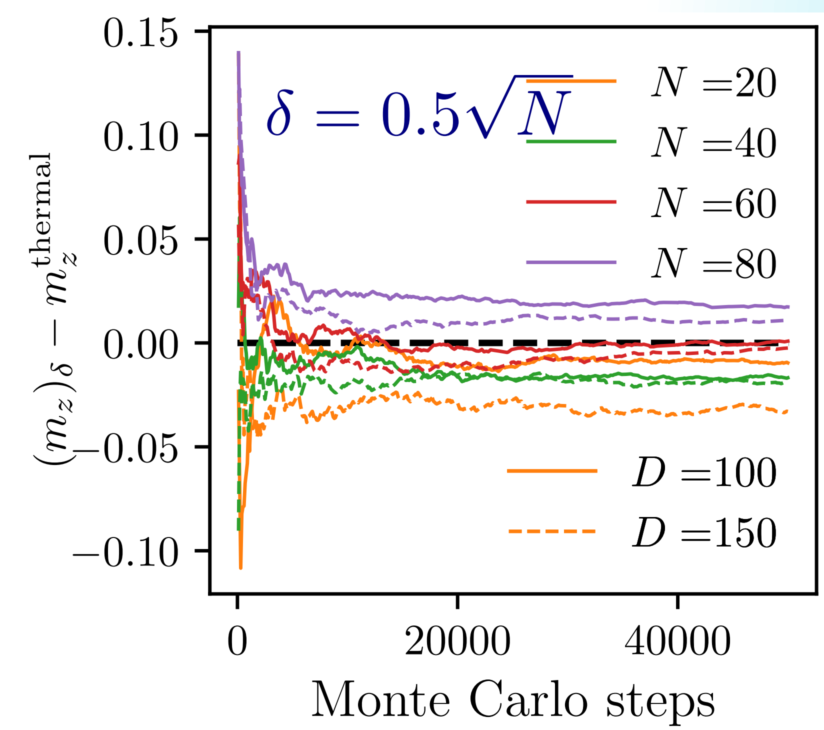
Lu et al. PRX Q2, 02032 (2021); Yang et al PRB 106, 024307 (2022)

classical (TNS) simulation

non-integrable Ising model







microcanonical properties

average magnetization

MPO + sampling over product states

Yang et al PRB 106, 024307 (2022)

Thanks for your attention!

MPQ MCQST

alternative use of TN to get dynamical properties

Frías-Pérez, MCB, PRB 106, 115117 (2022)

see poster by Miguel Frías

key: entanglement in space vs time

light cone TN contraction improved efficiency

global quenches and thermal correlators

physical upper-bound for velocity can be used

also in this spirit:
spectral properties of a QMB Hamiltonian

Yang, Iblisdir, Cirac, MCB, PRL124, 100602 (2020)
Papaefstathiou, Robaina, Cirac, MCB, PRD104, 014514 (2021)
Çakan, Cirac, MCB, PRB 103, 115113 (2021)
Yang, Cirac, MCB, PRB 106, 024307 (2022)