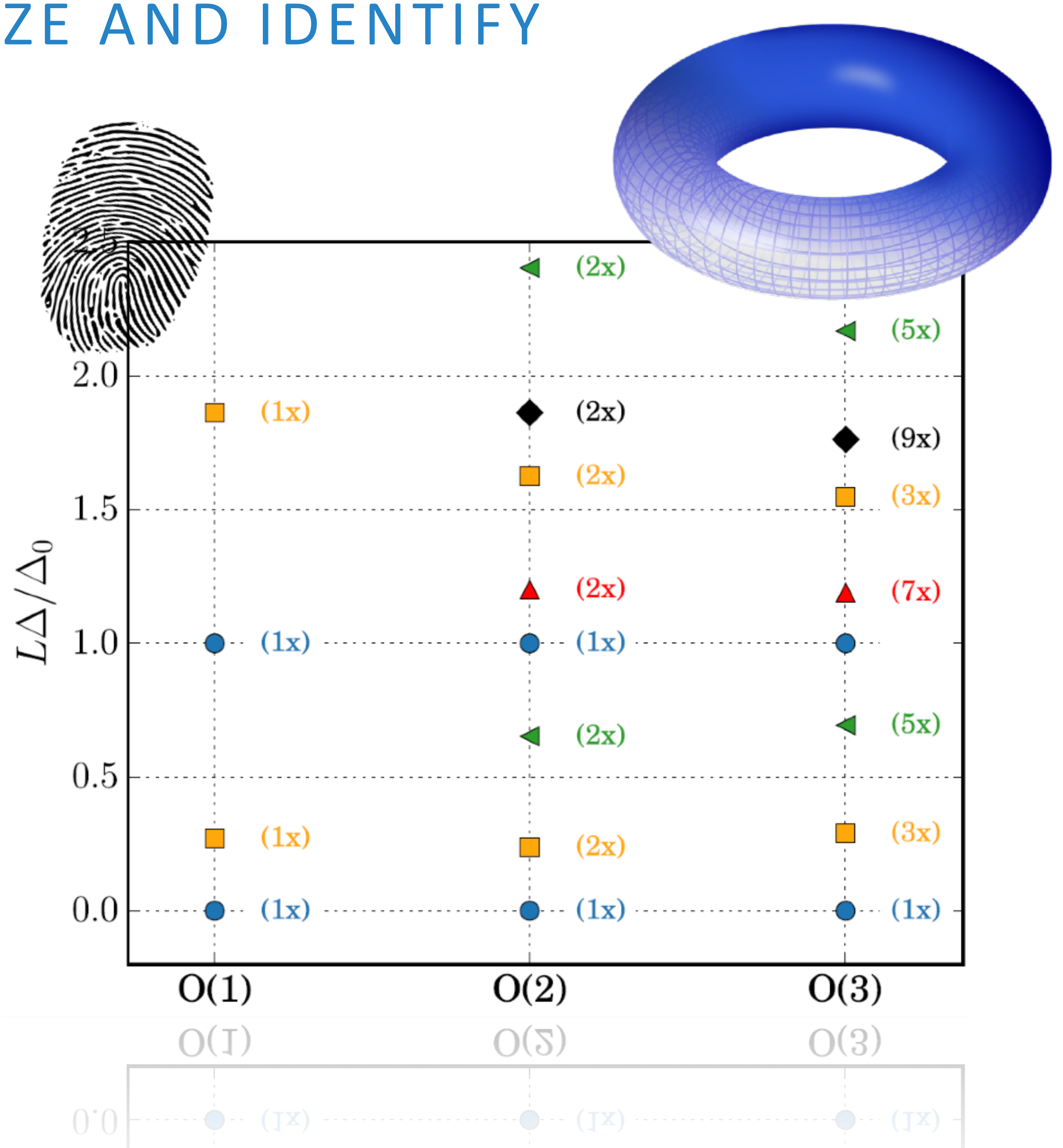
Torus Spectroscopy of 2+1D Quantum Critical Points

A NOVEL APPROACH TO CHARACTERIZE AND IDENTIFY UNIVERSALITY CLASSES

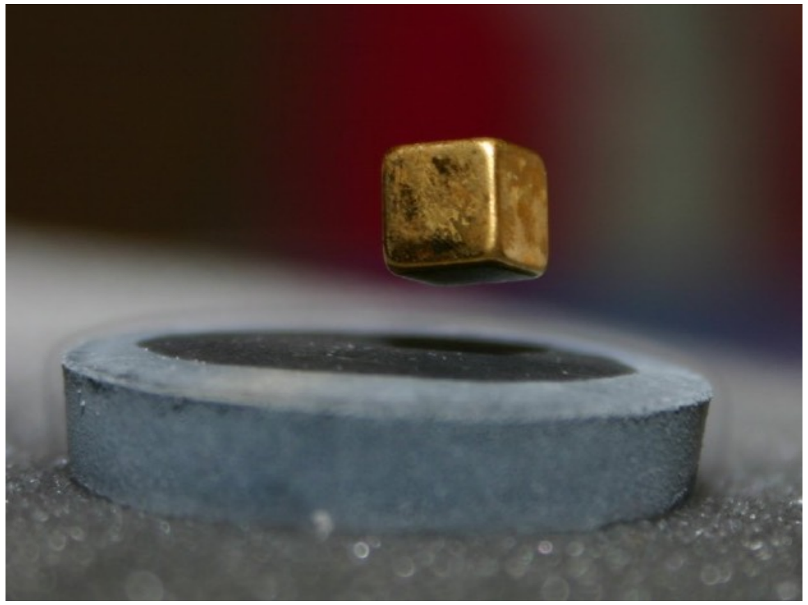


Michael Schuler
Institut für Theoretische Physik
Universität Innsbruck

Workshop "Entanglement Scaling and **Criticality** with Tensor Networks”
Lausanne, 29.11.2022

Motivation – Phase Transitions

Superconductivity



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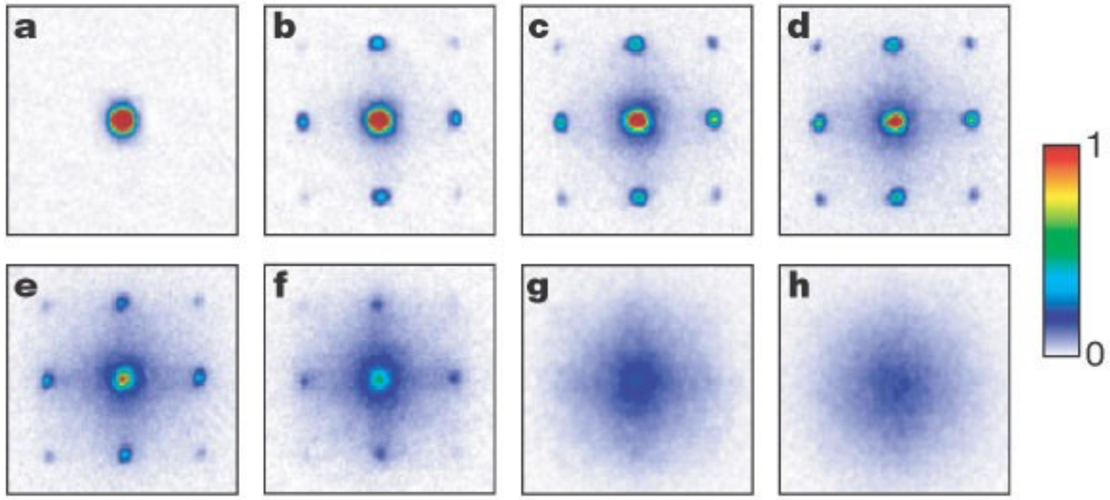
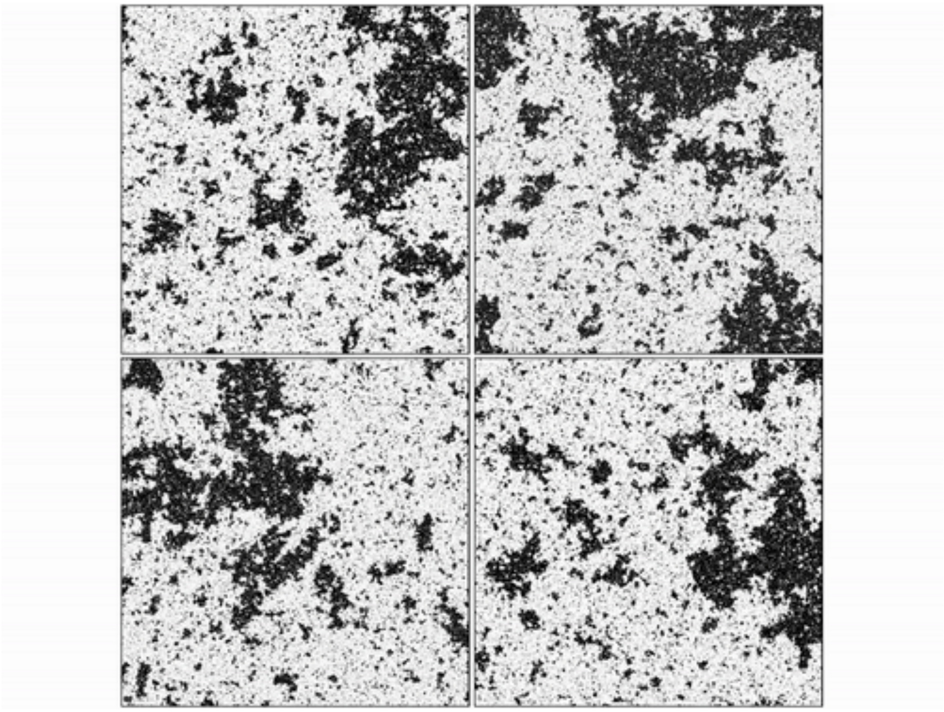
Mott - Superfluid



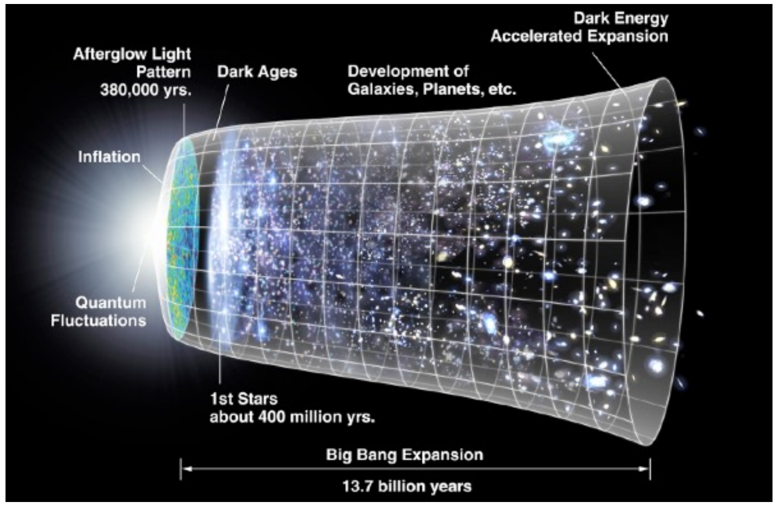
Greiner et.al.: Nature **415**. 39 (2002)

Scale invariance in the critical Ising model



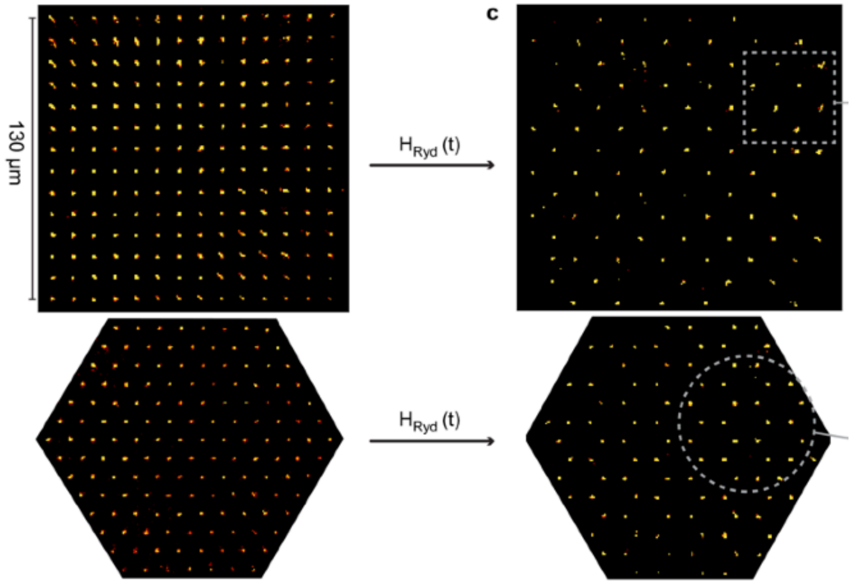
© gfycat/Douglas Ashton (YouTube)

Electroweak phase transition



https://www.jpl.nasa.gov

Quantum Simulators

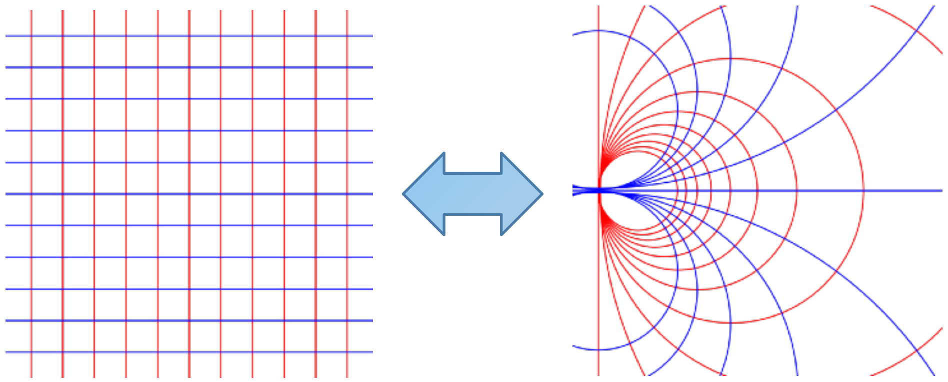


P. Scholl, MS, et.al.; Nature **595**, 233 (2021)

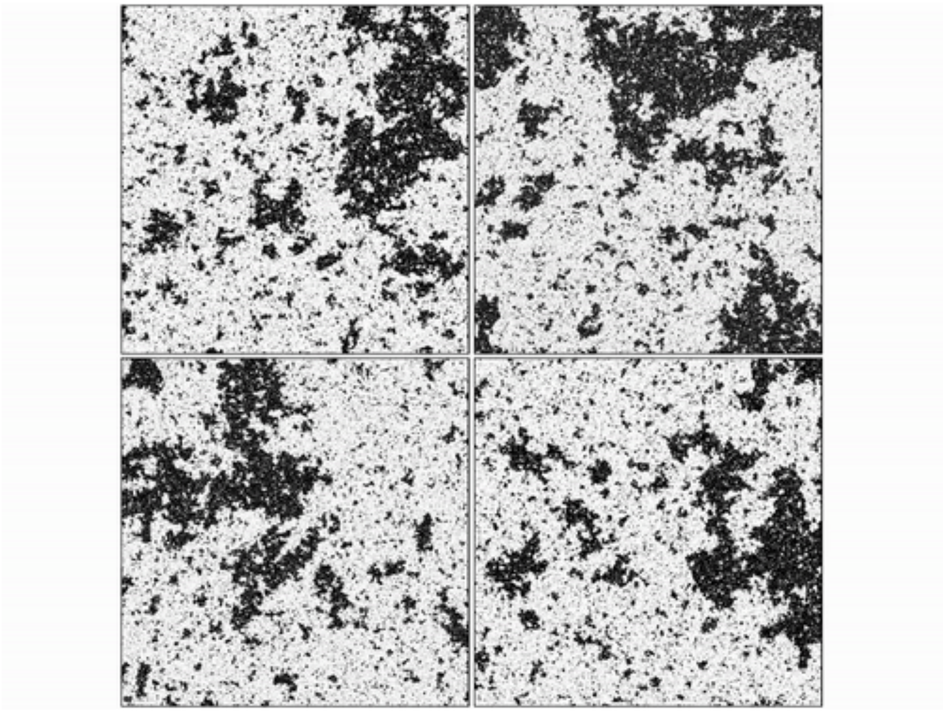
Critical Points — Conformal symmetry

- Continuous (quantum) phase transitions → Critical points
- Diverging correlation length → Scale invariance
- Expectation: Scale invariant systems even obey conformal invariance
- Conformal symmetries: Locally angle preserving maps

➡ Description: Field theory with conformal symmetries = Conformal field theory (CFT)



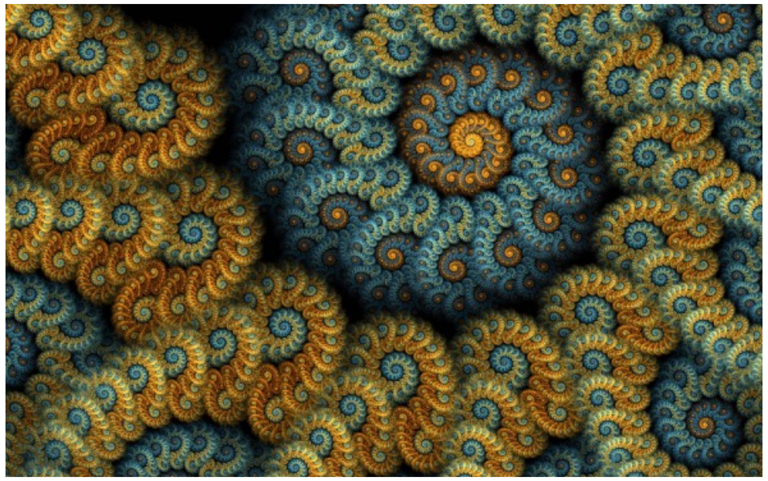
Scale invariance in the critical Ising model



Critical behaviour in bird flocks

Cavagna et.al., PNAS **107**, 11865 (2010)

Scale invariance in fractals



Critical Points — Universality

- Universality — At criticality many microscopically different systems show identical behaviour, independent of microscopic details

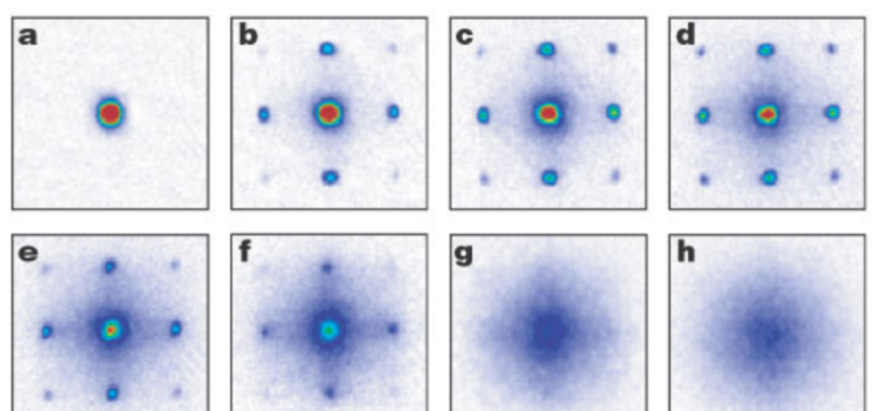
➡ Universality classes — Defined by a specific (conformal) field theory

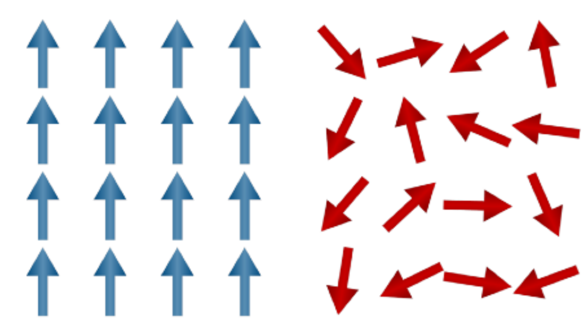
- Usually characterised by critical exponents = Set of universal numbers = Some properties of the CFT

$$m \propto (-t)^{\beta}$$
$$\xi \propto |t|^{-\nu}$$
$$C \propto t^{-\alpha}$$
$$\langle m(0)m(r)\rangle \propto r^{-(d-2+\eta)}$$

	ν	η
$O(1) \equiv \mathbb{Z}_2$	$0.629988(65)$	$0.03631(5)$
$O(2)$	$0.6719(7)$	$0.0381(2)$
$O(3)$	$0.7113(16)$	$0.0375(5)$

Universal @ $h = h_c$





XY

Critical Points — Universality

- Universality — At criticality many microscopically different systems show identical behaviour, independent of microscopic details

➡ Universality classes — Defined by a specific (conformal) field theory

- Usually characterised by critical exponents = S… universal numbers = Some proper…

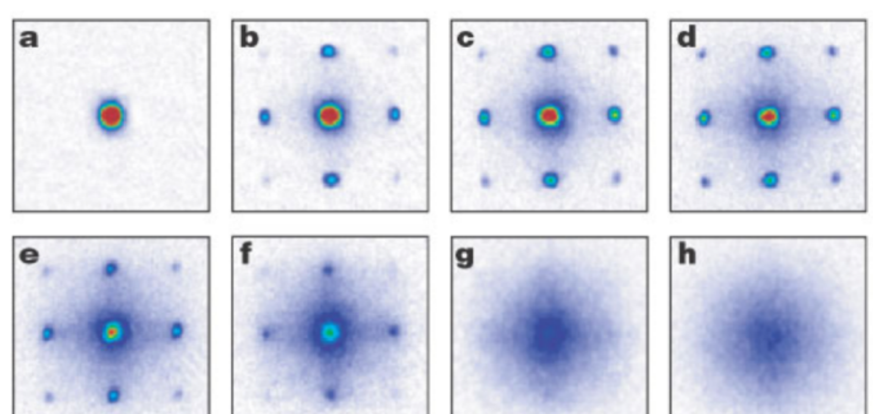
$$m \propto (-t)^{\beta}$$

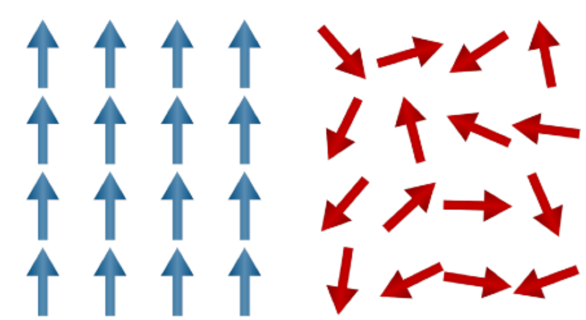
$$\xi \propto |t|^{-\nu}$$

		η
	…29988(65)	0.03631(5)
	0.6719(7)	0.0381(2)
$O(3)$	0.7113(16)	0.0375(5)

We have developed a new tool to characterize & identify universality classes of QCPs in (2+1)D, based on spectroscopy!

Universal @ $h = h_c$





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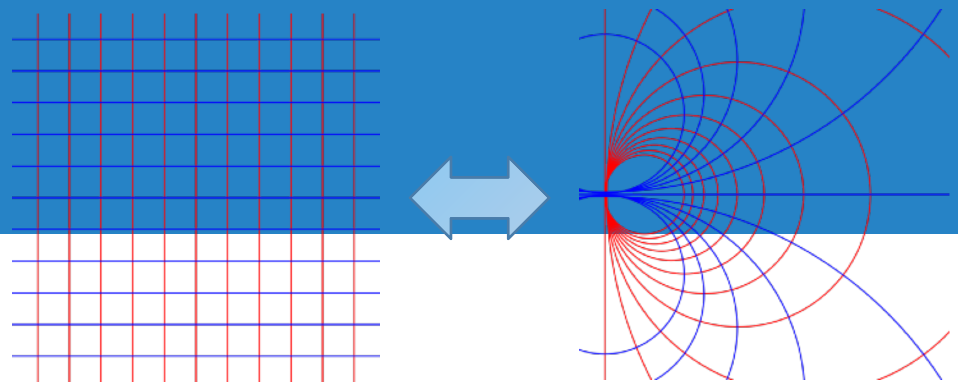
CFT in a tiny nutshell

CFT in a tiny nutshell



- Field theories with conformal symmetry
- Conformal symmetries = Locally angle preserving transformations
- In 2 (space-time) dimensions the conformal group is infinitely large
  - Generated by holomorphic functions
  - ➡ Allows to solve many CFTs analytically

$$z \to f(z), \quad \frac{\partial f}{\partial \bar{z}} = 0$$

- In 3 (space-time) dimensions the conformal group is finite
  - ➡ Much less restrictions
  - ➡ Typically not solvable

This talk

- Defined by central charge (especially 2D), operator content, operator product expansion, …

$$S_L = \frac{c}{3} \log \frac{L}{a}$$

- Operator content = Set of fields (local operators), their scaling dimensions, and further quantum numbers (spin, parity, …)

$$\langle O_i(x) O_i(0) \rangle \propto \frac{1}{|x|^{2\Delta_i}}$$

- Scaling dimensions related to critical exponents, subleading scaling corrections, …

Operator	Spin l	$\mathbb{Z}_2$	Δ	Exponent
σ	0	$-$	0.5182(3)	$\Delta = 1/2 + \eta/2$
σ'	0	$-$	$\gtrsim 4.5$	$\Delta = 3 + \omega_A$
ε	0	$+$	1.413(1)	$\Delta = 3 - 1/\nu$
ε'	0	$+$	3.84(4)	$\Delta = 3 + \omega$
ε''	0	$+$	4.67(11)	$\Delta = 3 + \omega_2$
$T_{\mu\nu}$	2	$+$	3	n/a
$C_{\mu\nu\kappa\lambda}$	4	$+$	5.0208(12)	$\Delta = 3 + \omega_{\mathrm{NR}}$

El-Showk et.al., PRD **86**, 025022 (2012)

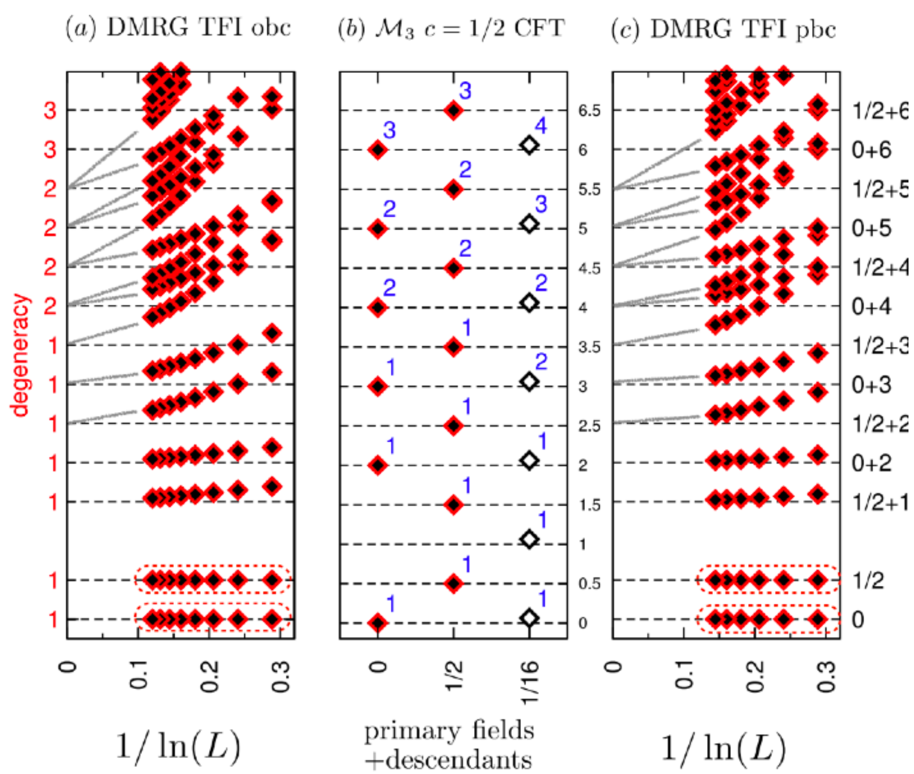
CFT — Entanglement spectrum

- Entanglement Hamiltonian of a region $\tilde{H}_A$, from reduced density matrix

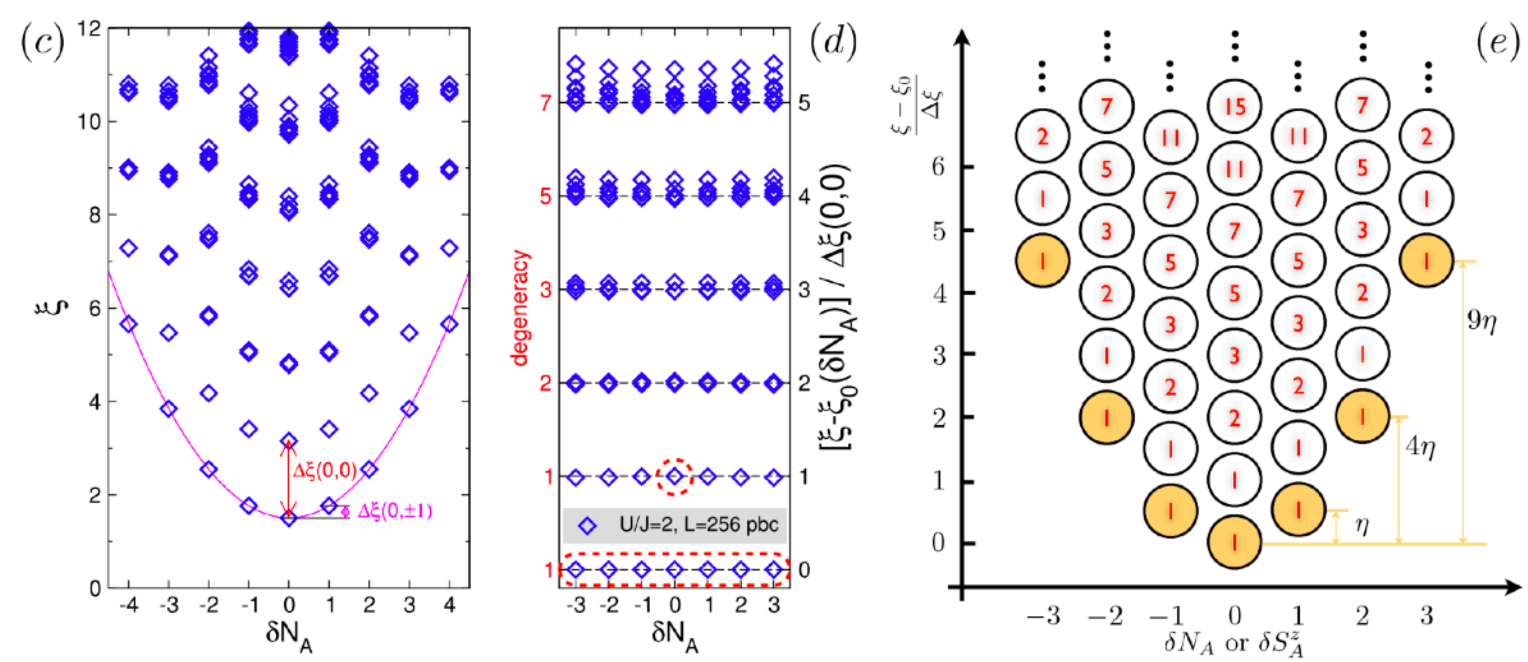
$$\rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi| = e^{-\tilde{H}_A}$$

- Entanglement spectrum = Eigenvalues of $\tilde{H}_A$
- Directly accessible with MPS methods
- Can show characteristic signatures of corresponding phases (topological states of matter, SPT, continuous symmetry breaking - tower of states, …)
- Conjecture for QCPs (mostly 1+1D): (Extremal) entanglement spectrum similar to spectrum of a corresponding boundary CFT

1+1D Ising



Bose-Hubbard chain — $c = 1$ CFT



A. M. Läuchli, arXiv:1303.0741

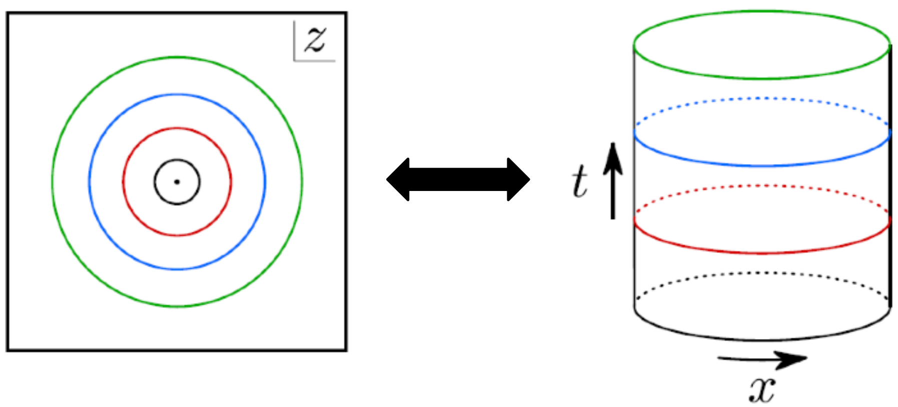
CFT — Energy spectrum — Radial quantization

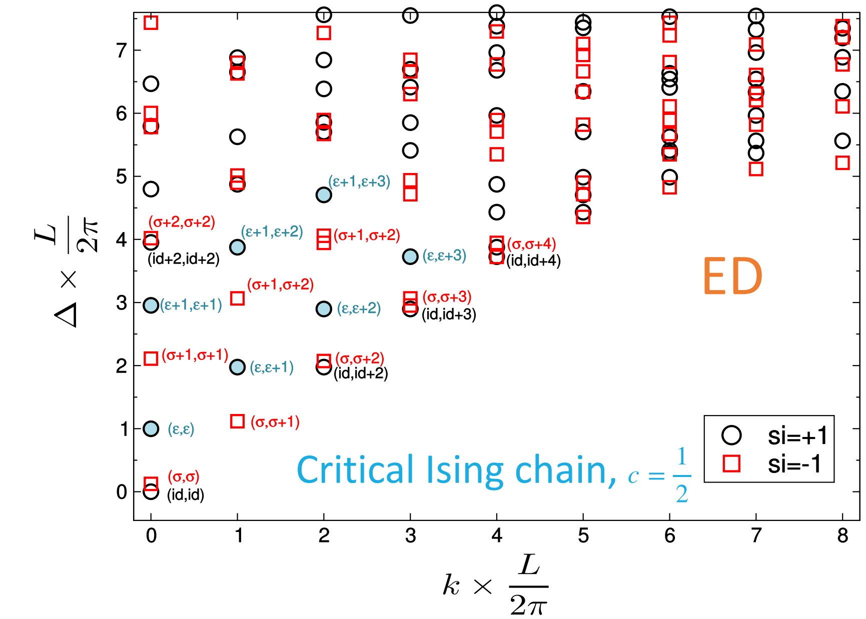
- Radial quantization
  - Conformal mapping

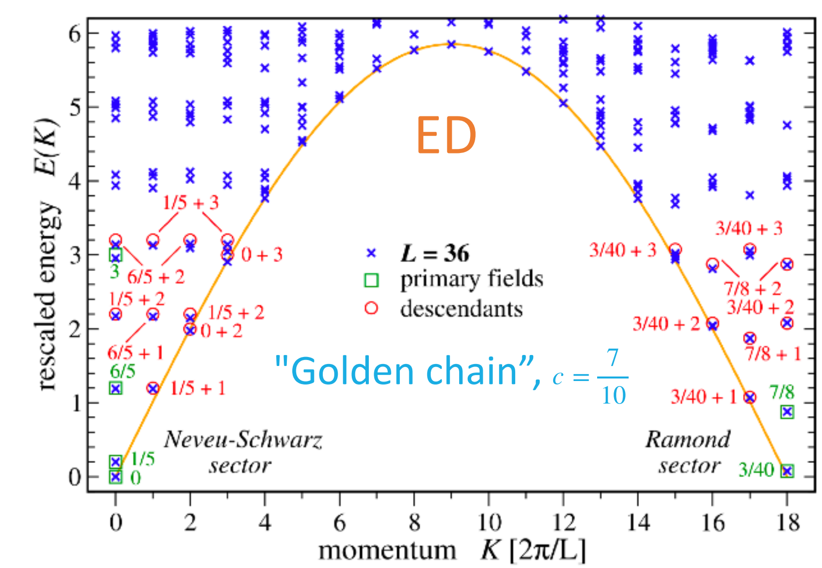
$$\mathbb{R}^D \leftrightarrow S^{D-1} \times \mathbb{R}$$

Cardy, J. Phys. A **18**, L757 (1985)

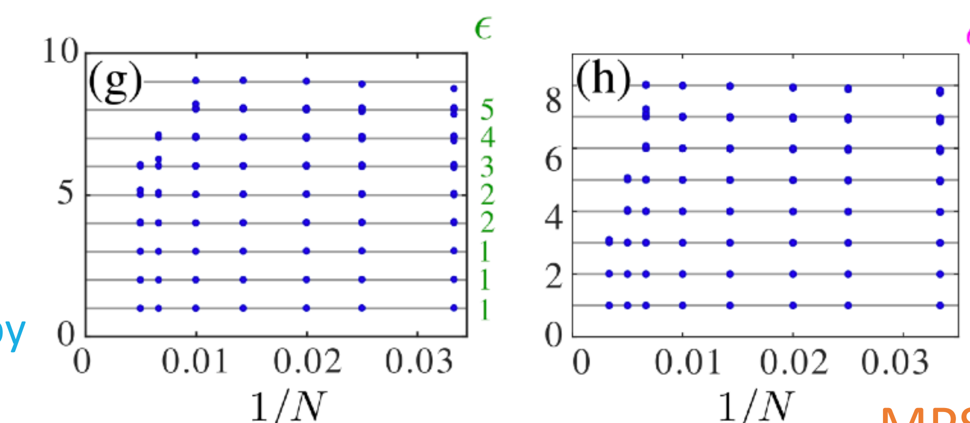
$D = 2$







Feiguin et.al.; PRL **98**, 160409 (2007)



Chepiga, Mila, PRB **96**, 054425 (2017)

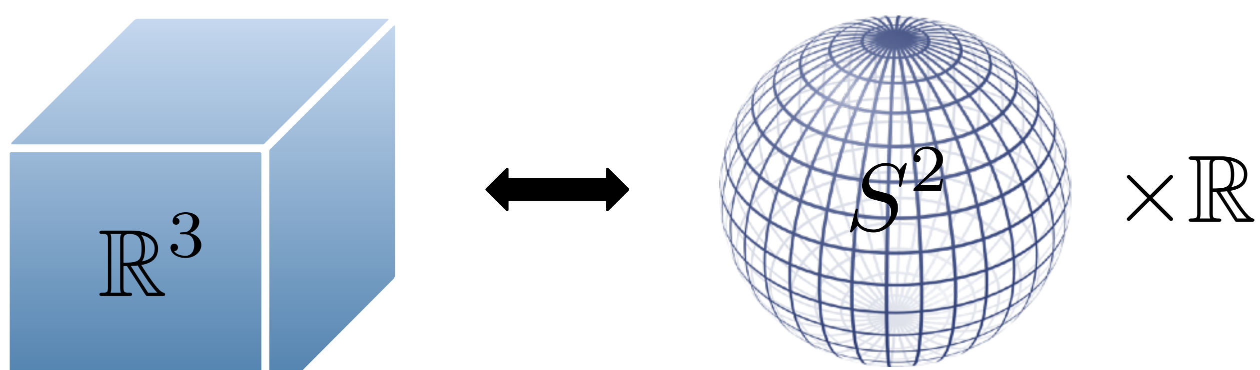
- Scaling dimensions in radial quantized geometry correspond to eigenvalues of time translation operator, i.e. Hamiltonian energy spectrum
- Accessible with
  - ED — full spectrum, small systems
  - QMC — if no sign problem & possible to compute gaps reasonably well
  - MPS, DMRG ? — effective local Hamiltonian, in certain circumstances, boundary CFT spectrum [Chepiga, Mila, PRB **96**, 054425 (2017), …]
  - iMPS ? — energy spectrum of local Hamiltonian, transfer matrix spectra (time and/or space direction) [talk by L. Tagliacozzo, poster by A. Eberharter]

CFT — Energy spectrum — Radial quantization

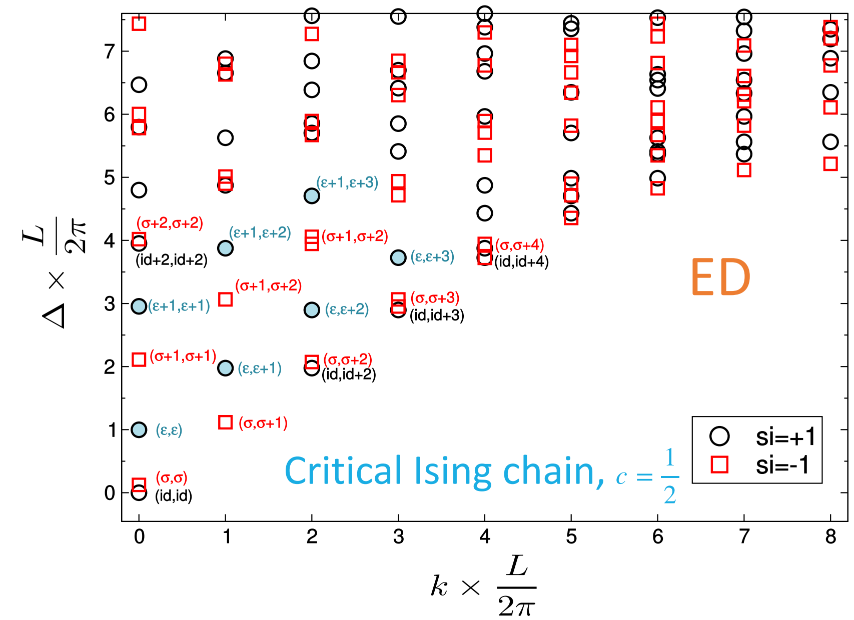
- Radial quantization
  - Conformal mapping
  - For $D = 3$:

$$\boxed{\mathbb{R}^D \leftrightarrow S^{D-1} \times \mathbb{R}}$$

Cardy, J. Phys. A **18**, L757 (1985)

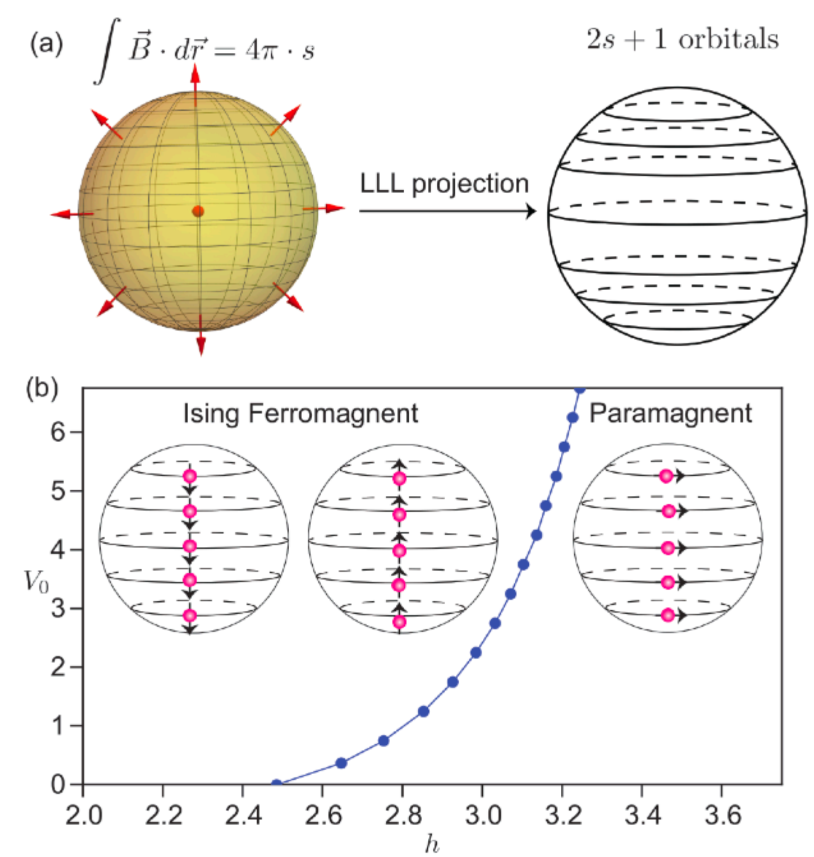


spatial time



- Hamiltonian simulations on a 2d sphere – Difficult

Brower et.al., Proc. Sci. (2016)



Zhu et.al., arXiv:2210.13482

- Very recently: 3D Ising CFT via spinful electrons in lowest (spherical) Landau levels [Zhu et.al, arXiv:2210.13482]

CFT — Energy spectrum — Radial quantization

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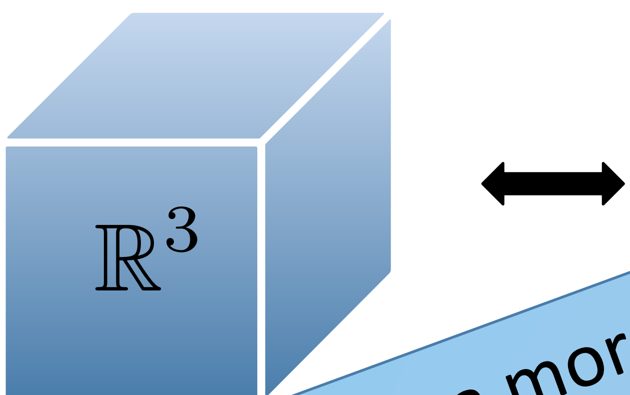
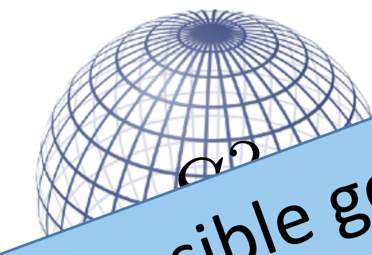
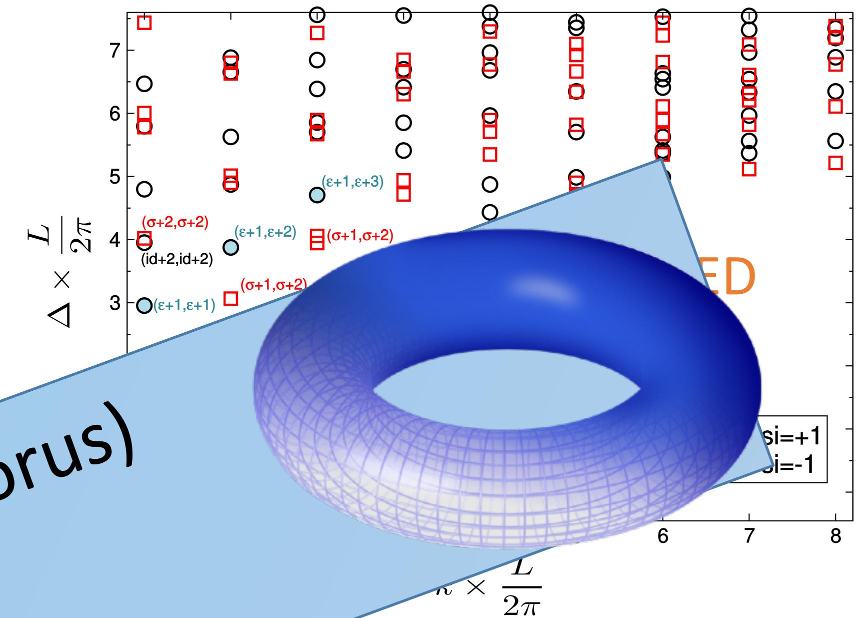






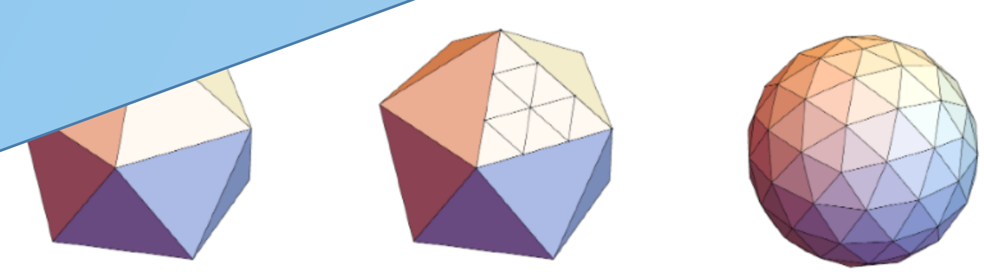
Use a more accessible geometry (2d torus)

No conformal mapping

Can we utilize the energy spectrum?

...ime

...ere – Difficult



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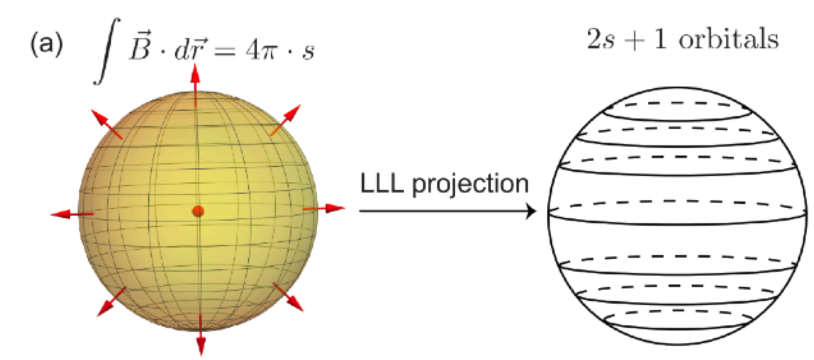

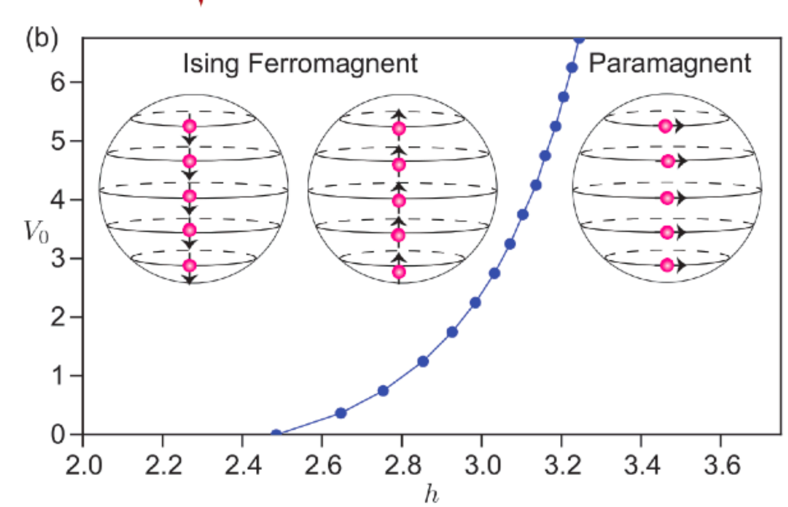

Zhu et.al., arXiv:2210.13482

Critical torus energy spectroscopy

Critical torus energy spectrosocopy

PRL 117, 210401 (2016) PHYSICAL REVIEW LETTERS week ending 18 NOVEMBER 2016

Universal Signatures of Quantum Critical Points from Finite-Size Torus Spectra: A Window into the Operator Content of Higher-Dimensional Conformal Field Theories

Michael Schuler,[1] Seth Whitsitt,[2] Louis-Paul Henry,[1] Subir Sachdev,[2,3] and Andreas M. Läuchli[1]
[1]*Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*
[2]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[3]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
(Received 24 March 2016; revised manuscript received 3 October 2016; published 16 November 2016)

The low-energy spectra of many body systems on a torus, of finite size L, are well understood in magnetically ordered and gapped topological phases. However, the spectra at quantum critical points separating such phases are largely unexplored for $(2+1)$D systems. Using a combination of analytical and numerical techniques, we accurately calculate and analyze the low-energy torus spectrum at an Ising critical

PHYSICAL REVIEW B 103, 125128 (2021)

Torus spectroscopy of the Gross-Neveu-Yukawa quantum field theory: Free Dirac versus chiral Ising fixed point

Michael Schuler,[1,2] Stephan Hesselmann,[3] Seth Whitsitt,[4] Thomas C. Lang,[1] Stefan Wessel,[3] and Andreas M. Läuchli[1]
[1]*Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*
[2]*Vienna Center for Quantum Science and Technology, Atominstitut, TU Wien, 1040 Wien, Austria*
[3]*Institut für Theoretische Festkörperphysik, JARA-FIT and JARA-HPC, RWTH Aachen University, 52056 Aachen, Germany*
[4]*Joint Qua*

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SciPost SciPost Phys. ?, ??? (20??)

Emergent XY* transition driven by symmetry fractionalization and anyon condensation

Michael Schuler[1*], Louis-Paul Henry[2], Yuan-Ming Lu[3] and Andreas M. Läuchli[4,5]

1 Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria
2 Pasqal, 2 avenue Augustin Fresnel, 91120 Palaiseau, France
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Abstract

Anyons in a topologically ordered phase can carry fractional quantum numbers with respect to the symmetry group of the considered system, one example being the fractional charge of the quasiparticles and quasiholes in the fractional quantum Hall effect. When such symmetry-fractionalized anyons condense, the resulting phase must spontaneously break the symmetry and display a local order parameter. In this paper, we study the

PHYSICAL REVIEW B 96, 035142 (2017)

Spectrum of the Wilson-Fisher conformal field theory on the torus

Seth Whitsitt,[1] Michael Schuler,[2] Louis-Paul Henry,[2] Andreas M. Läuchli,[2] and Subir Sachdev[1,3]
[1]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2]*Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*
[3]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada N2L 2Y5*
(Received 26 March 2017; published 24 July 2017)

We study the finite-size spectrum of the $O(N)$-symmetric Wilson-Fisher conformal field theory (CFT) on the $(d=2)$-spatial-dimension torus using the expansion in $\epsilon = 3-d$. This is done by deriving a set of universal effective Hamiltonians describing fluctuations of the zero-momentum modes. The effective Hamiltonians take the form of N-dimensional quantum anharmonic oscillators, which are shown to be strongly coupled at the critical point for small ϵ. The low-energy spectrum is solved numerically for $N = 1,2,3,4$. Using exact diagonalization, we also numerically study explicit lattice models known to be in the O(2) and O(3) universality class, obtaining estimates of the low-lying critical spectrum. The analytic and numerical results show excellent agreement and the critical low-energy torus spectra are qualitatively different among the studied CFTs, identifying them as a useful fi

Science TECHNICAL COMMENTS

Cite as: S. Hesselmann *et al.*, *Science* 10.1126/science.aav6869 (2019).

Comment on "The role of electron-electron interactions in two-dimensional Dirac fermions"

S. Hesselmann[1], T. C. Lang[2], M. Schuler[2], S. Wessel[1], A. M. Läuchli[2*]
[1]Institute for Theoretical Solid State Physics, JARA-FIT and JARA-HPC, RWTH Aachen University, 52056 Aachen, Germany. [2]Institute for Theoretical Physics, University of Innsbruck, 6020 Innsbruck, Austria.
*Corresponding author. Email: andreas.laeuchli@uibk.ac.at

Tang *et al.* (Research Articles, 10 August 2018, p. 570) report on the properties of Dirac fermions with
velocity of
s-Neveu

PHYSICAL REVIEW B 94, 085134 (2016)

Transition from the $\mathbb{Z}_2$ spin liquid to antiferromagnetic order: Spectrum on the torus

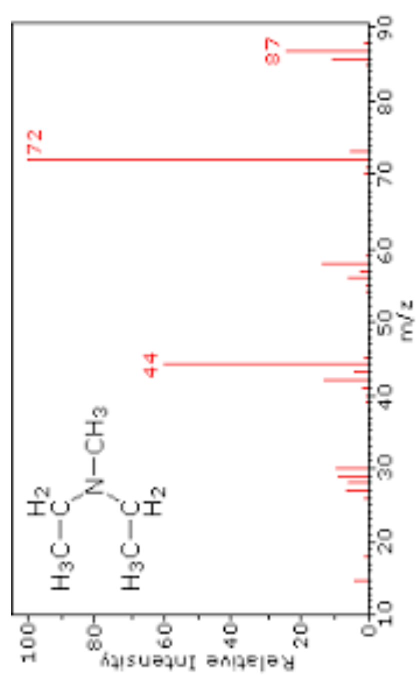
Seth Whitsitt[1] and Subir Sachdev[1,2]
[1]*Department of Physics, Harvard University, Cambridge, Massachusetts, 02138, USA*
[2]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
(Received 7 April 2016; published 19 August 2016)

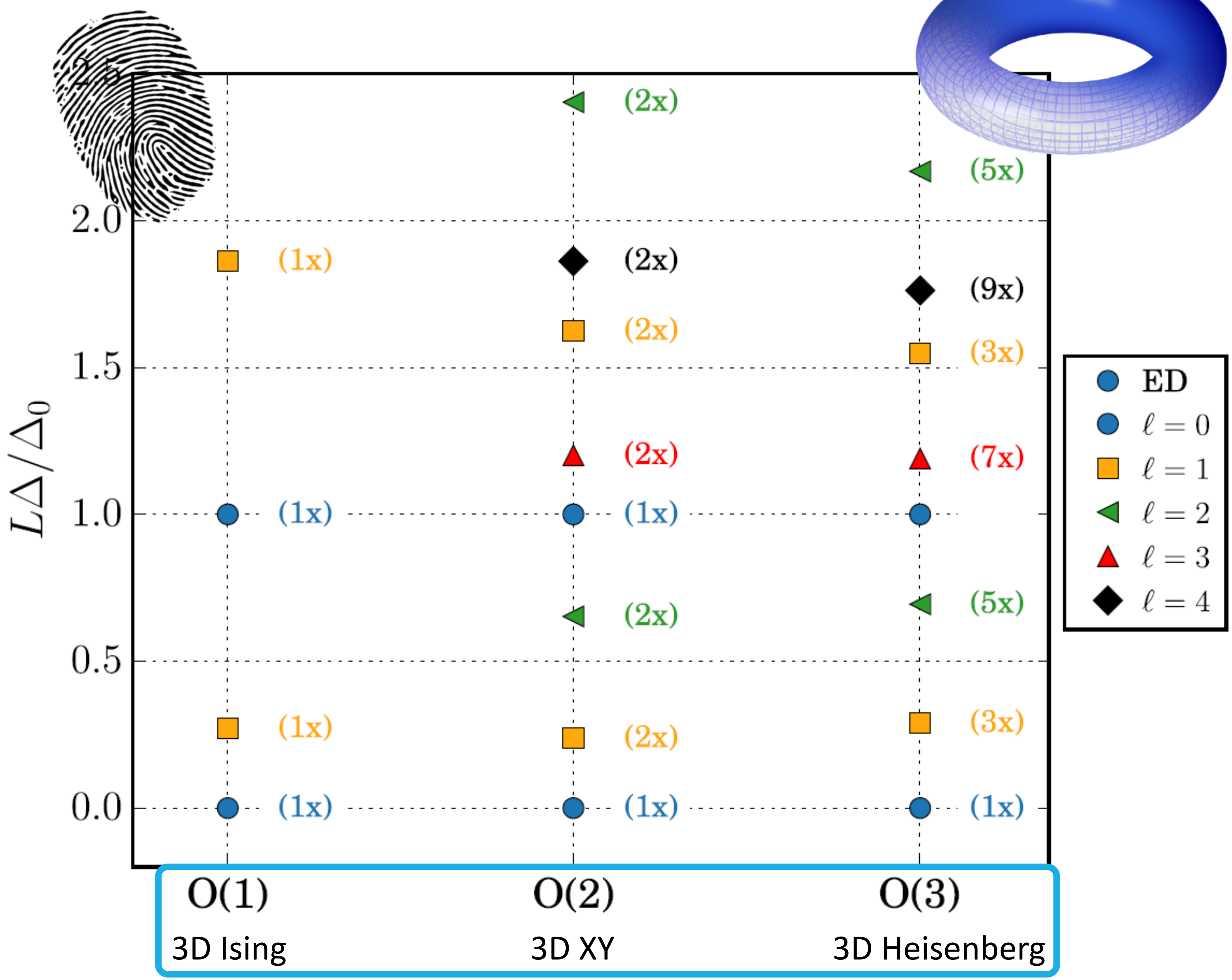
We describe the finite-size spectrum in the vicinity of the quantum critical point between a $\mathbb{Z}_2$ spin liquid and a coplanar antiferromagnet on the torus. We obtain the universal evolution of all low-lying states in an antiferromagnet with global SU(2) spin rotation symmetry, as it moves from the fourfold topological degeneracy in a gapped $\mathbb{Z}_2$ spin liquid to the Anderson "tower-of-states" in the ordered antiferromagnet. Due to the existence of nontrivial order on either side of this transition, this critical point cannot be described in a conventional Landau-Ginzburg-Wilson framework. Instead, it is described by a theory involving fractionalized degrees of freedom known as the $O(4)^*$ model, whose spectrum is altered in a significant way by its proximity to a topologically ordered phase. We compute the spectrum by relating it to the spectrum of the O(4) Wilson-Fisher fixed point on the torus, modified with a selection rule on the states, and with nontrivial boundary conditions corresponding to topological sectors in the spin liquid. The spectrum of the critical $O(2N)$ model is calculated directly at $N = \infty$, which then allows a reconstruction of the full spectrum of the $O(2N)^*$ model at leading order in $1/N$. This spectrum is a unique characteristic of the vicinity of a fractionalized quantum critical point, as well as a universal signature of the existence of proximate $\mathbb{Z}_2$ topological and antiferromagnetically ordered phases, and can be compared with numerical computations on quantum antiferromagnets on two-dimensional lattices.

Critical torus energy spectra for different universality classes

- Qualitatively different torus spectra among universality classes
- Rich multiplicity / quantum number structure
- Fingerprint — Unique identification

Analogy -
Mass spectroscopy:





Different Wilson-Fisher critical points / Universality classes

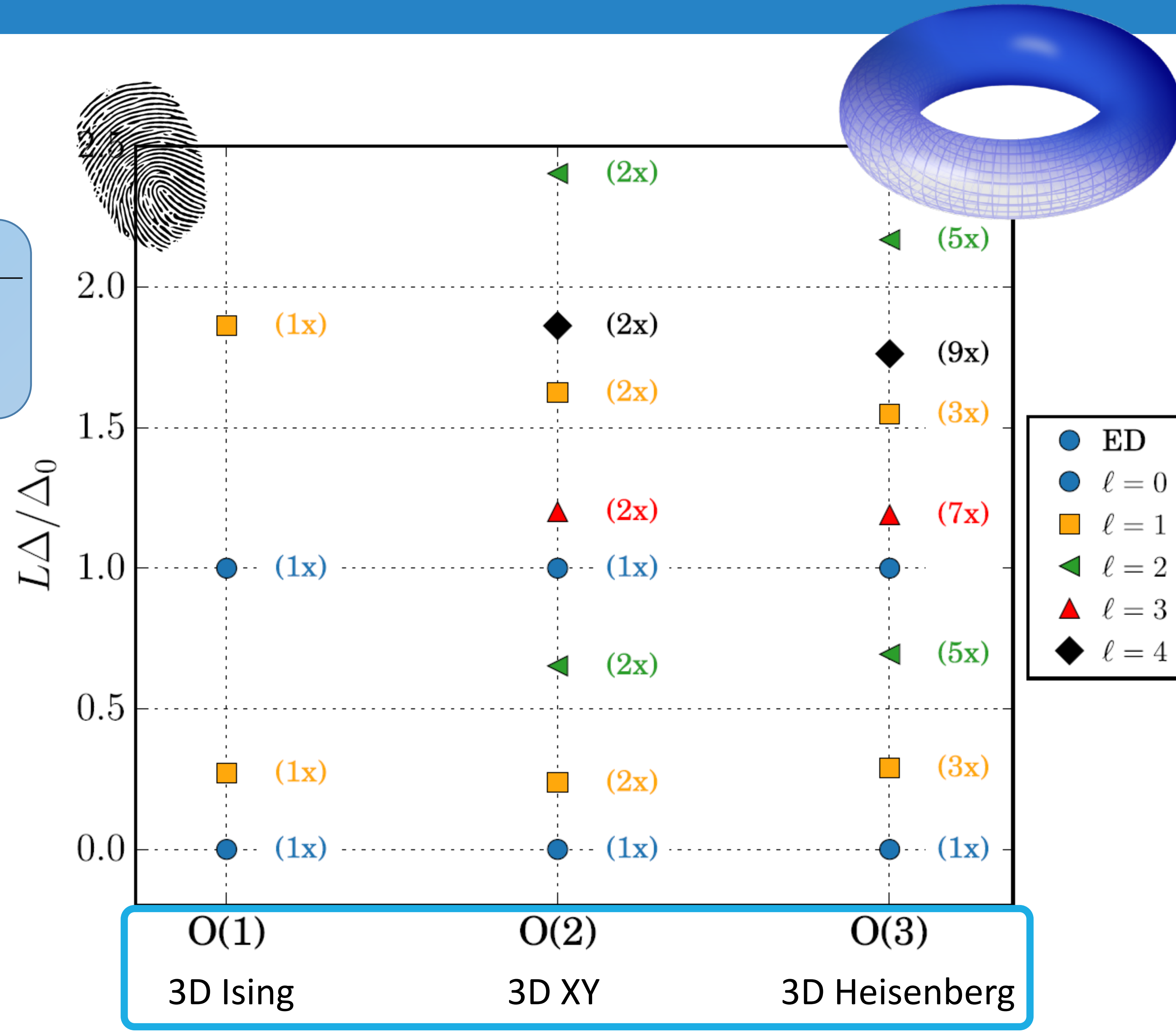
S. Whitsitt, MS, L.-P. Henry, A. M. Läuchli, S. Sachdev; PRB **96**, 035142 (2017)

Critical torus energy spectra for different universality classes

Standard approach:
Critical exponents can be very similar

	ν	η
$O(1) \equiv \mathbb{Z}_2$	0.629988(65)	0.03631(5)
$O(2)$	0.6719(7)	0.0381(2)
$O(3)$	0.7113(16)	0.0375(5)

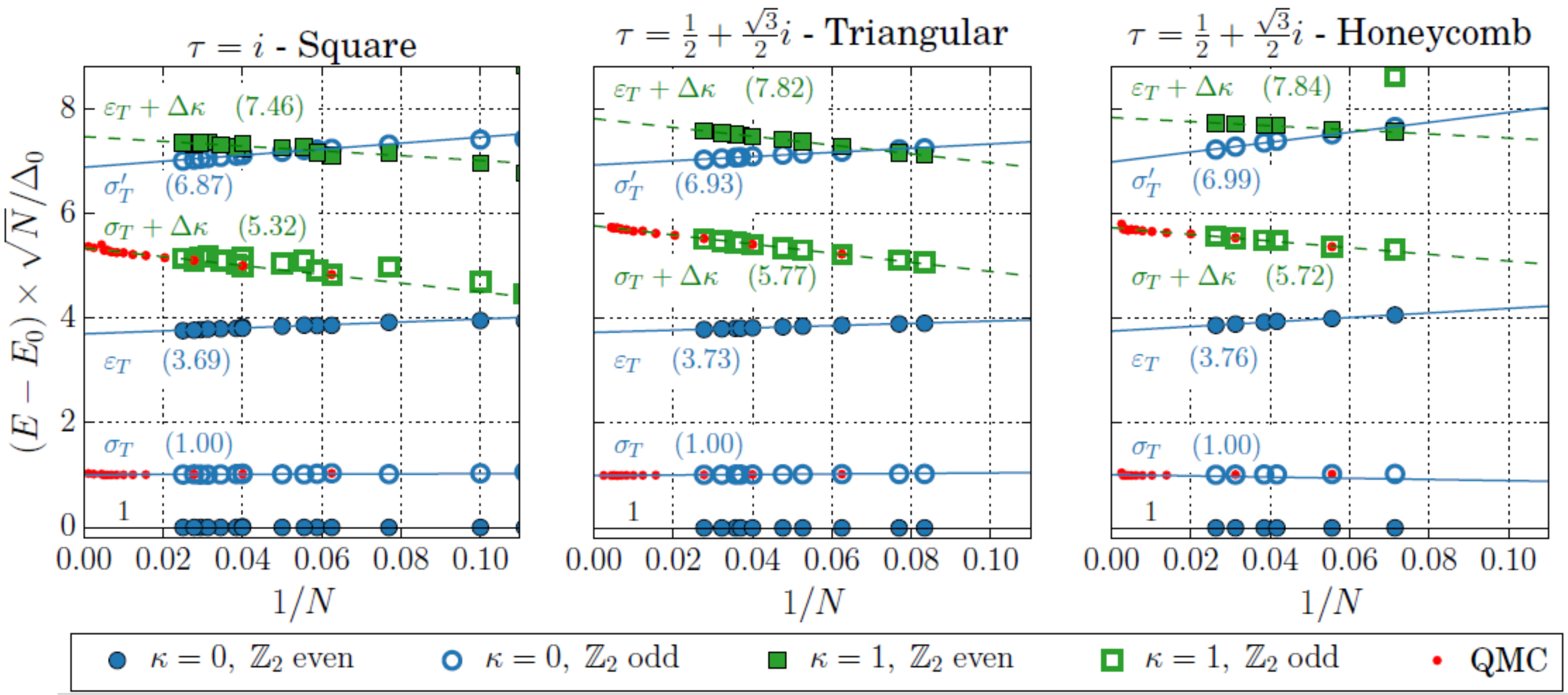
Spectral approach:
Qualitatively different structure
Multiplicities and quantum numbers provide very valuable data to differentiate universality classes

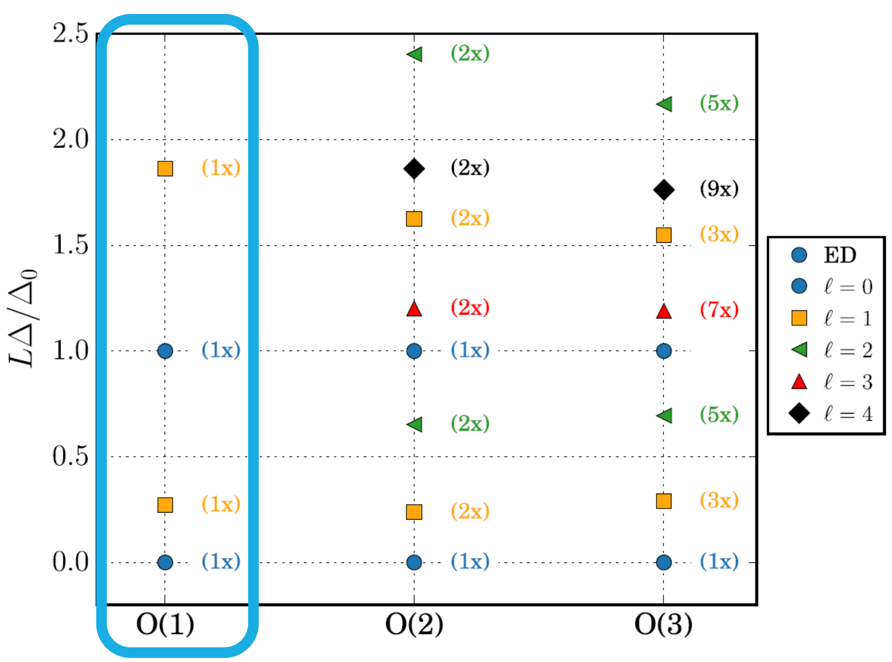


Different Wilson-Fisher critical points / Universality classes

S. Whitsitt, MS, L.-P. Henry, A. M. Läuchli, S. Sachdev; PRB **96**, 035142 (2017)

Computing the critical torus energy spectrum





- Example: Transverse field Ising model at criticality on spatial torus

$$H = -\sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h_c \sum_i \sigma_i^x$$

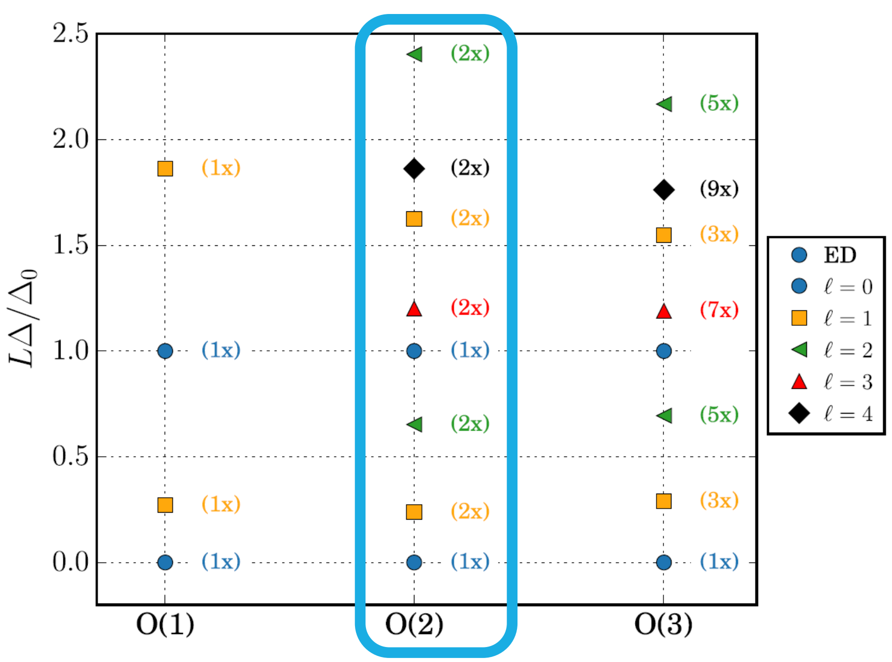
- Measure finite-size torus spectra $\Delta \times L$ at critical point
- Extrapolation to thermodynamic limit to obtain critical torus energy spectrum
- Accessible with many techniques (ED, QMC, ε-expansion, Tensor Networks?)
- Intermediate system sizes typically sufficient – Qualitative structure of critical spectrum matters
- Slight infrared-cutoff dependence (e.g. Square vs. Triangular / Honeycomb)

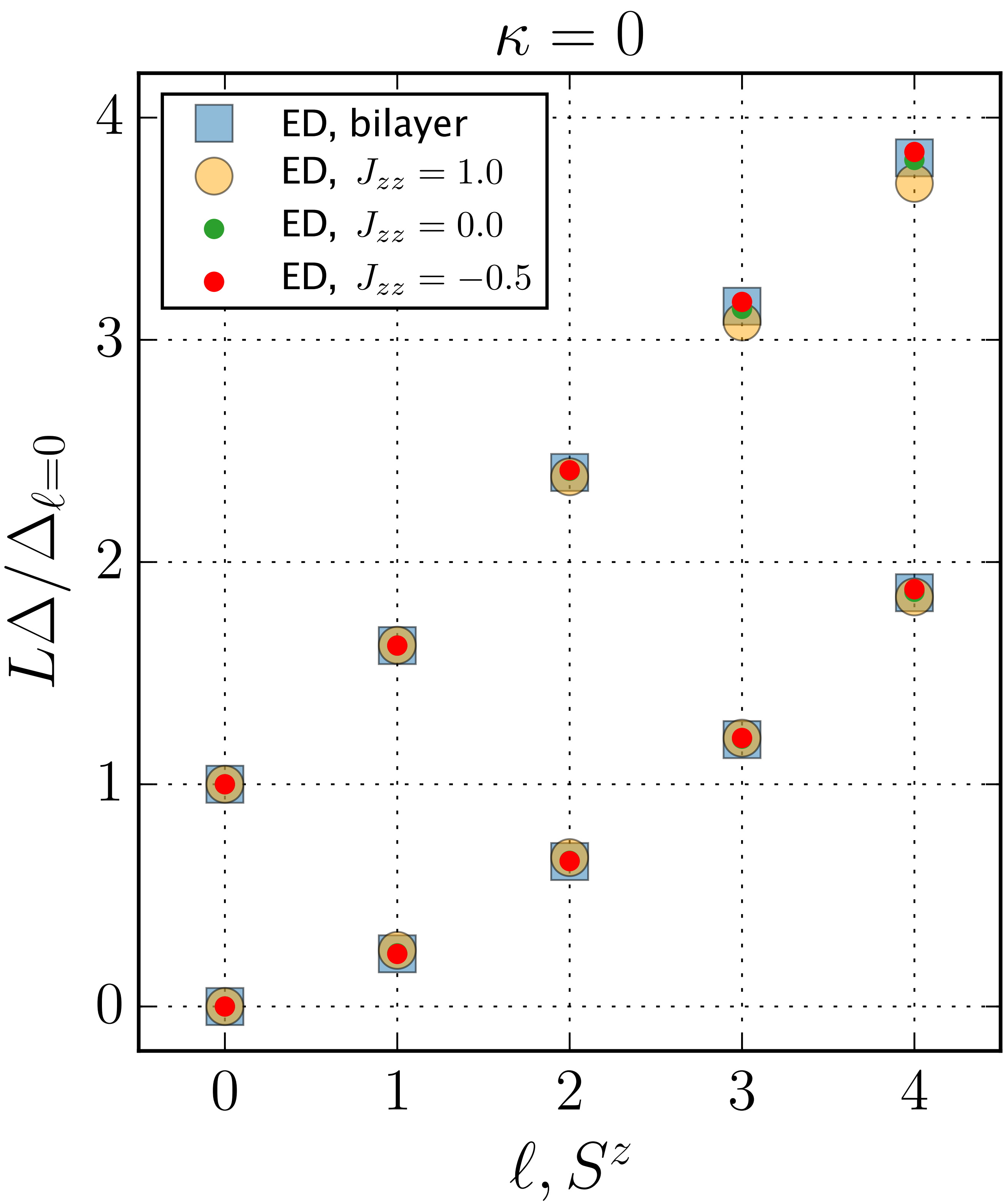
MS, S. Whitsitt, L.-P. Henry, S. Sachdev, A. M. Läuchli; PRL **117**, 210401 (2016)

Universality ?

- Different microscopic models exhibit identical spectra

$$\Delta_i = \frac{v}{L}\xi_i$$



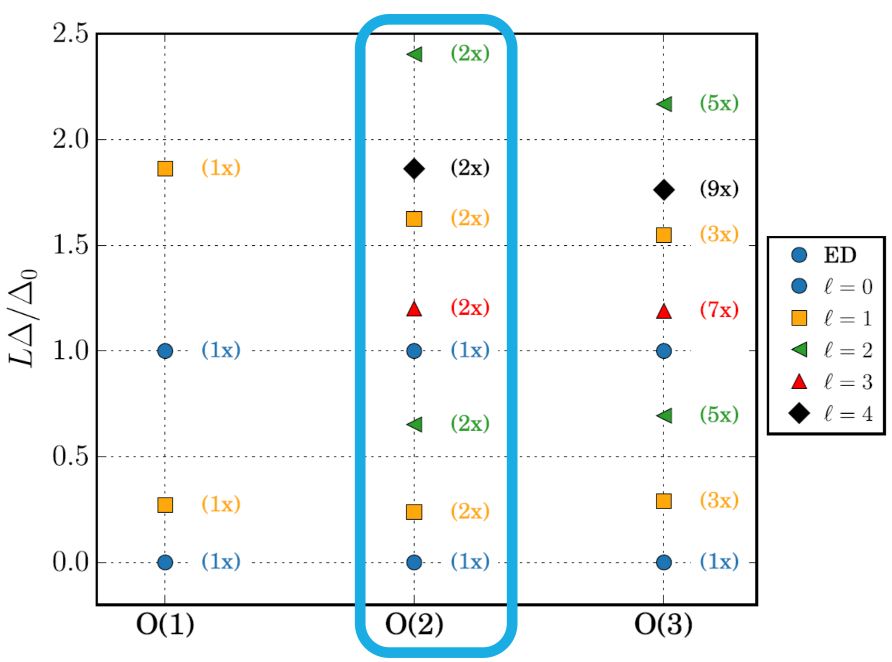


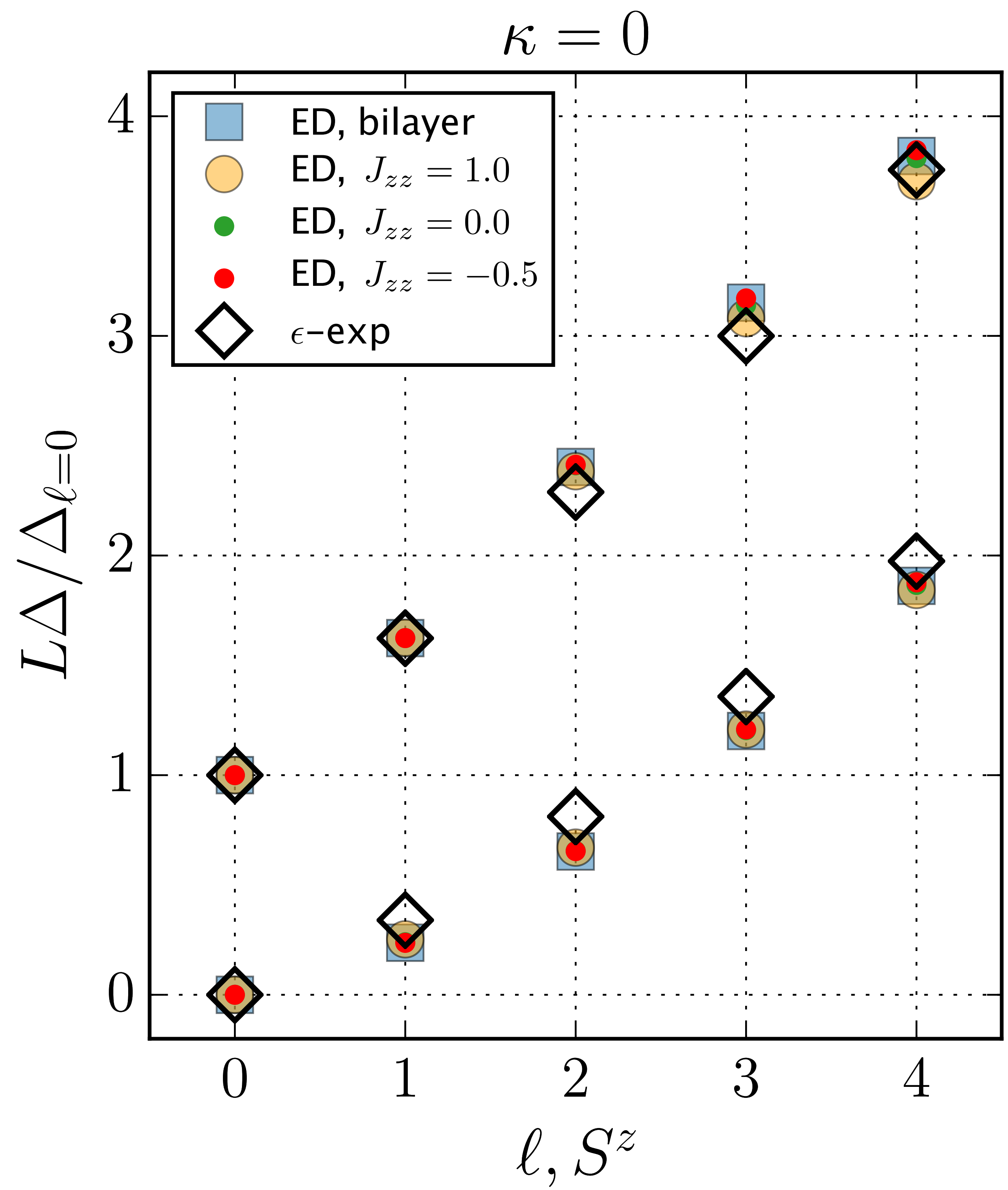
S. Whitsitt, MS, L.-P. Henry, A. M. Läuchli, S Sachdev; PRB **96**, 035142 (2017)

Universal !

- Different microscopic models exhibit identical spectra
- Field theory (ε – expansion) shows excellent agreement

$$\Delta_i = \frac{v}{L}\xi_i$$





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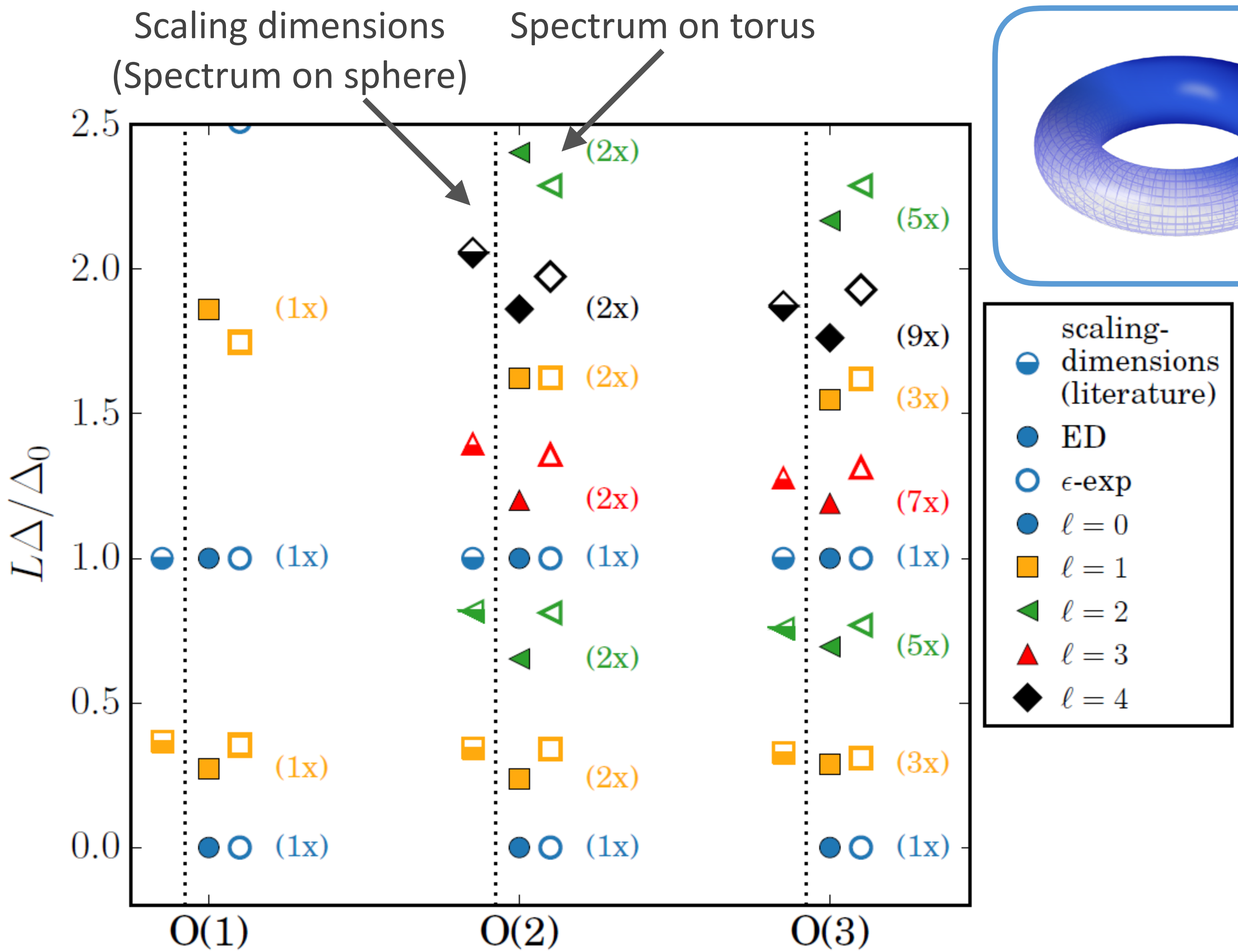
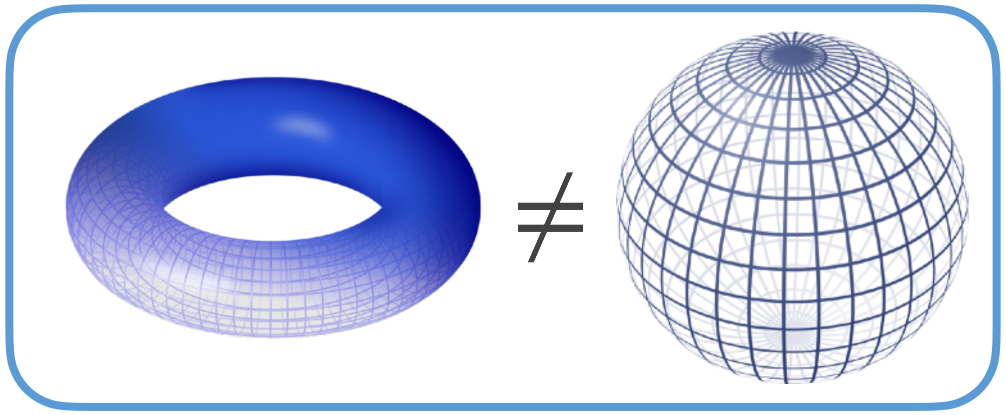
Comparison with CFT operator content





$$\eta = 2\Delta_\sigma^{\text{scaledim}} - 1$$
$$\nu^{-1} = 3 - \Delta_\epsilon^{\text{scaledim}}$$

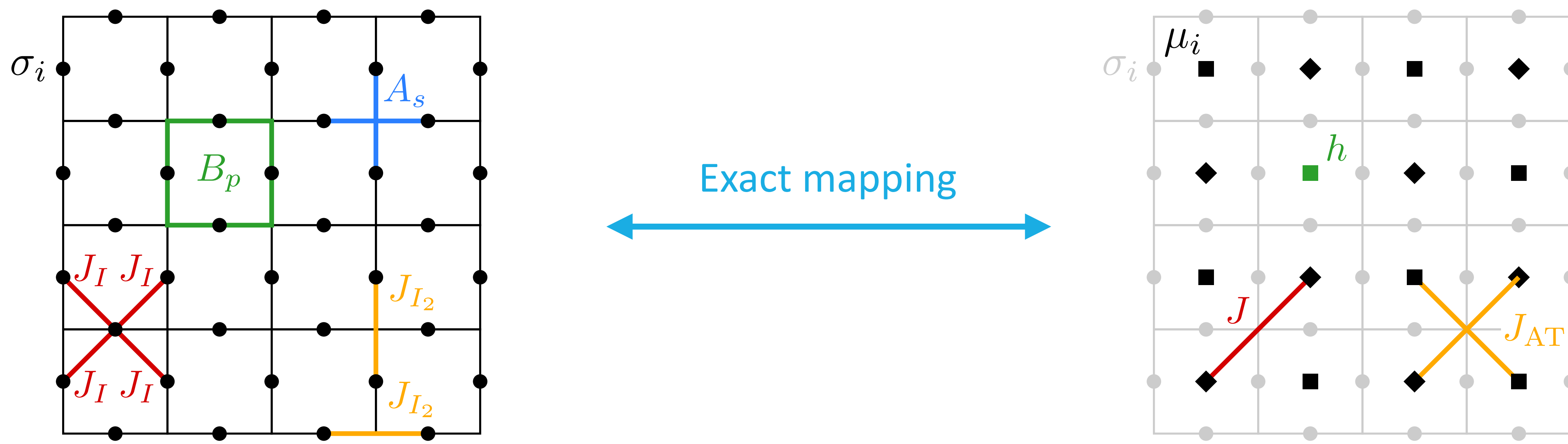
- Qualitatively similar structure for Wilson-Fisher field theories
- Similar sequence of low-lying levels
- Advocate phenomenological picture that gains insight into the CFT operator content

S. Whitsitt, MS, L.-P. Henry, A. M. Läuchli, S Sachdev; PRB **96**, 035142 (2017)

Characterization of QCPs in the Toric Code with Ising interactions

AN APPLICATION FOR CRITICAL TORUS SPECTROSCOPY

Toric Code with Ising interactions



Toric Code + Ising (TCI)

Ashkin-Teller Transverse Field Ising (AT-TFI)

$$H = -J_e \sum_s A_s - J_m \sum_p B_p - J_I \sum_{\langle i,j \rangle} \sigma_i^x \sigma_j^x - J_{I_2} \sum_{\langle\langle i,j \rangle\rangle} \sigma_i^x \sigma_j^x$$

$$A_s = \prod_{i \in s} \sigma_i^x, \; B_p = \prod_{i \in p} \sigma_i^z$$

$$H_{\mathrm{AT}} = -h \sum_i \mu_i^z - J \sum_{\langle\langle i,j \rangle\rangle} \mu_i^x \mu_j^x + J_{\mathrm{AT}} \sum_i \mu_i^x \mu_{i+\hat{\mathbf{x}}}^x \mu_{i+\hat{\mathbf{y}}}^x \mu_{i+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^x$$

+ BC: periodic & antiperiodic

+ constraint: even Ising ($\mathbb{Z}_2$) symmetry sector

- Topological toy model — Toric Code — $\mathbb{Z}_2$ topological order
- Add Ising interactions — drive phase transition to $\mathbb{Z}_2$ symmetry broken phase
- Phase diagram, characterization of phase transitions
- Exact mapping to a non-topological model (AT-TFI) with constraints and BC peculiarities
- First, consider AT-TFI model. Then, infer from that many properties of the TCI model

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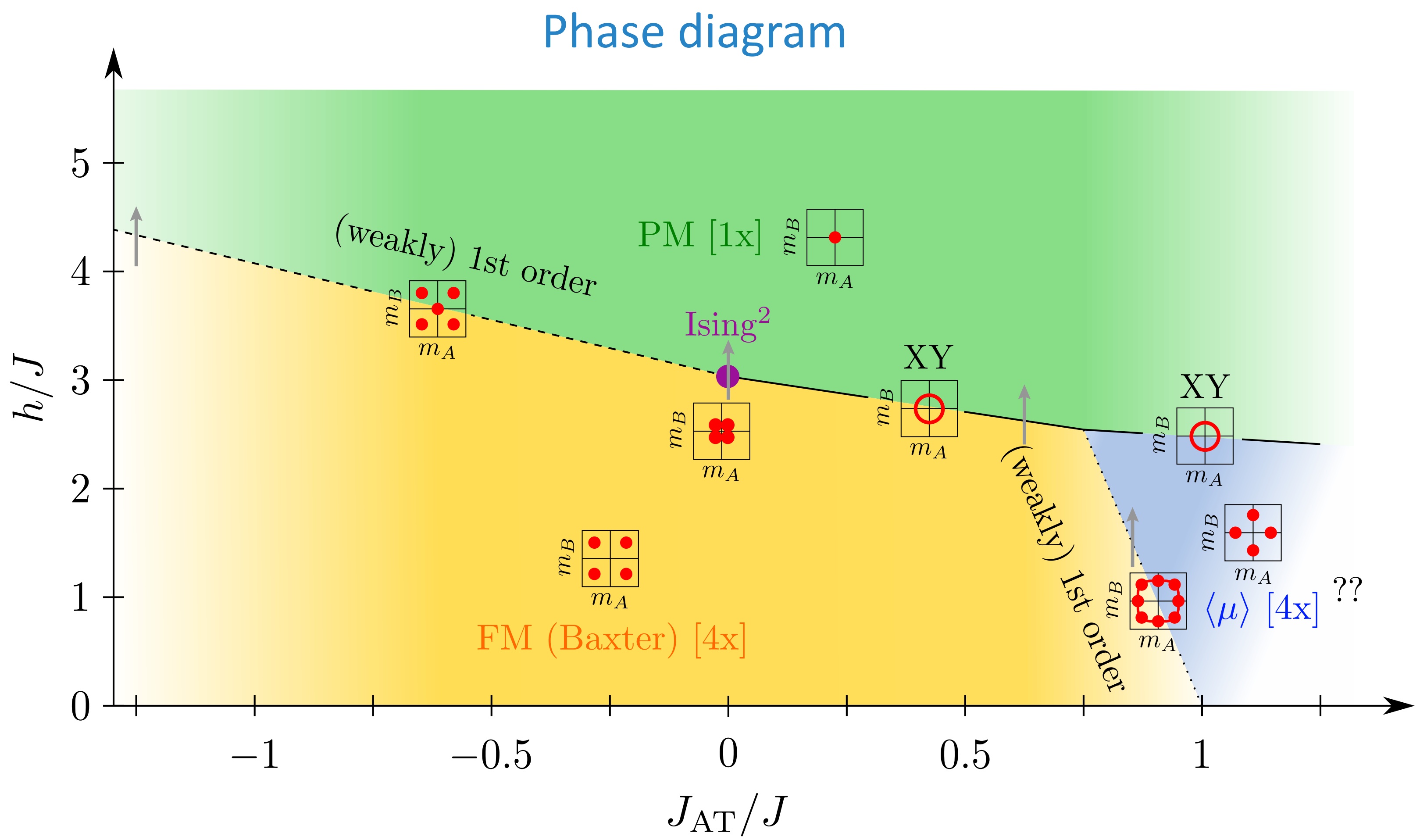
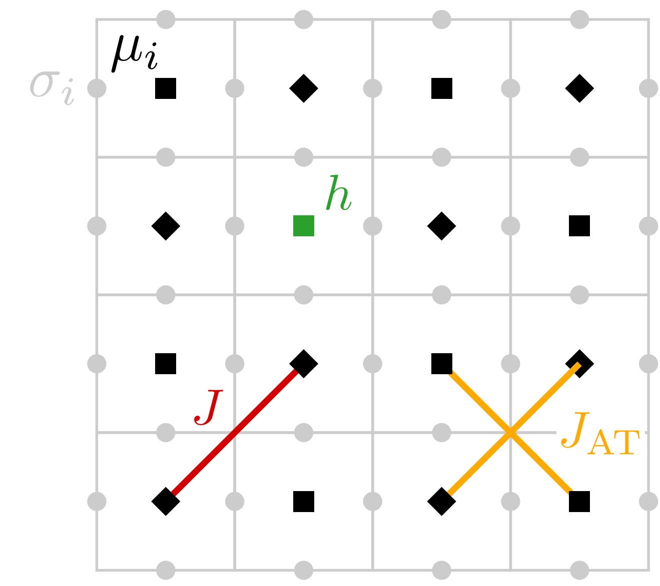
AT-TFI model





- Rich phase diagram, obtained from ED and QMC calculations
- First-order and continuous phase transition lines

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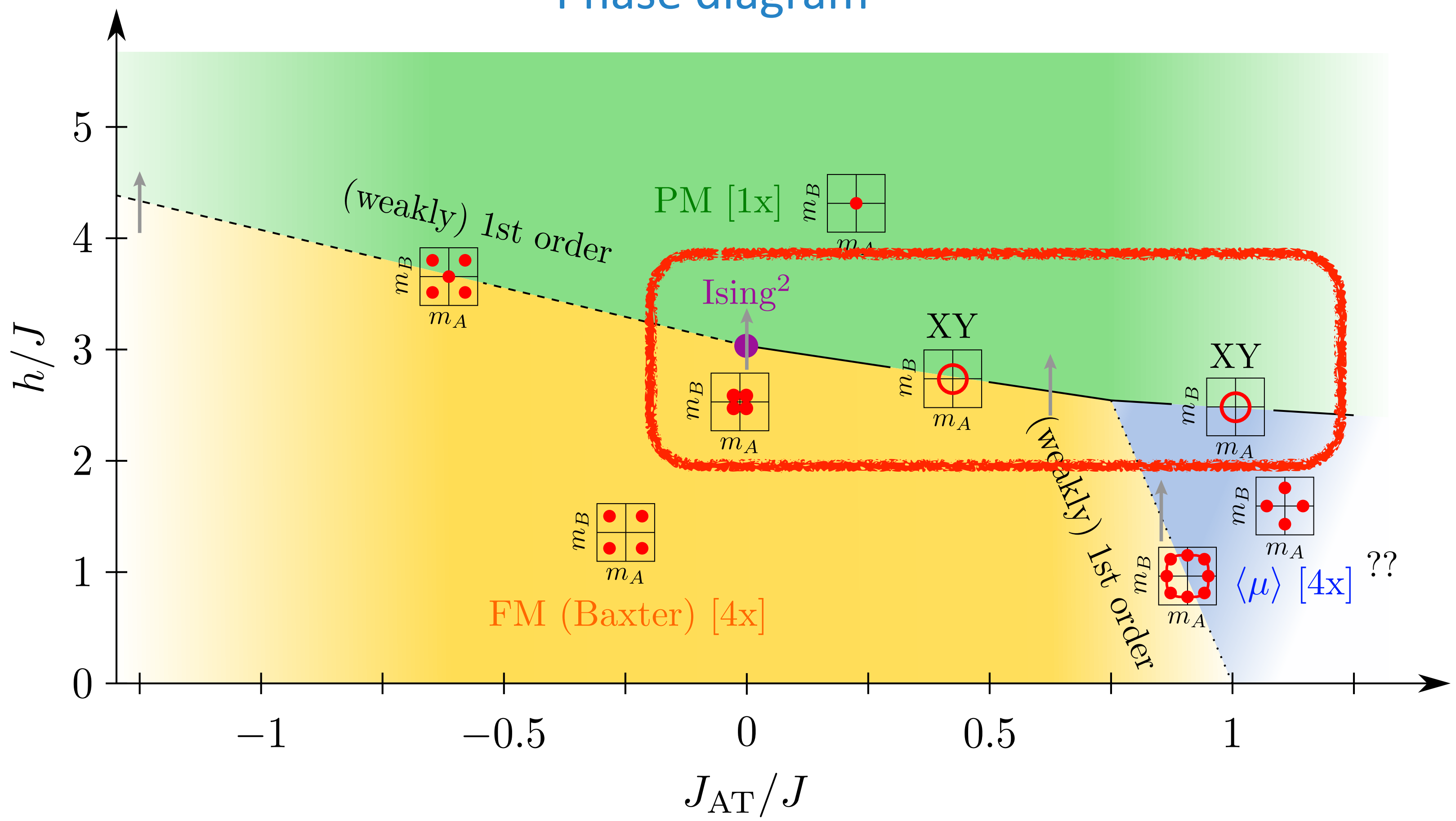
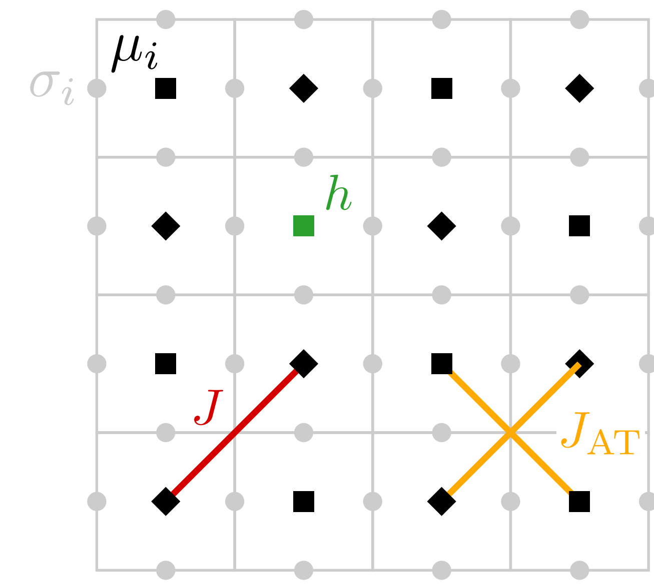
AT-TFI model

Phase diagram





- Rich phase diagram, obtained from ED and QMC calculations
- First-order and continuous phase transition lines
- Universality class of continuous transitions?

Critical torus energy spectroscopy

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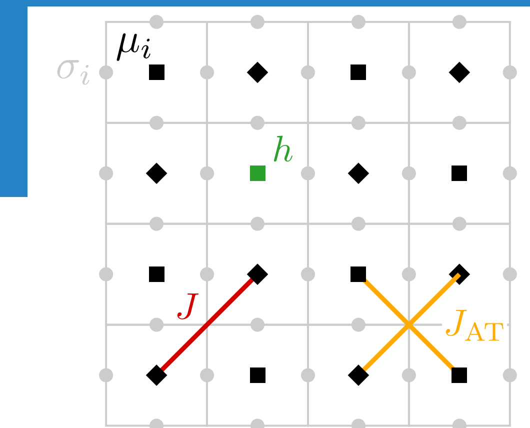
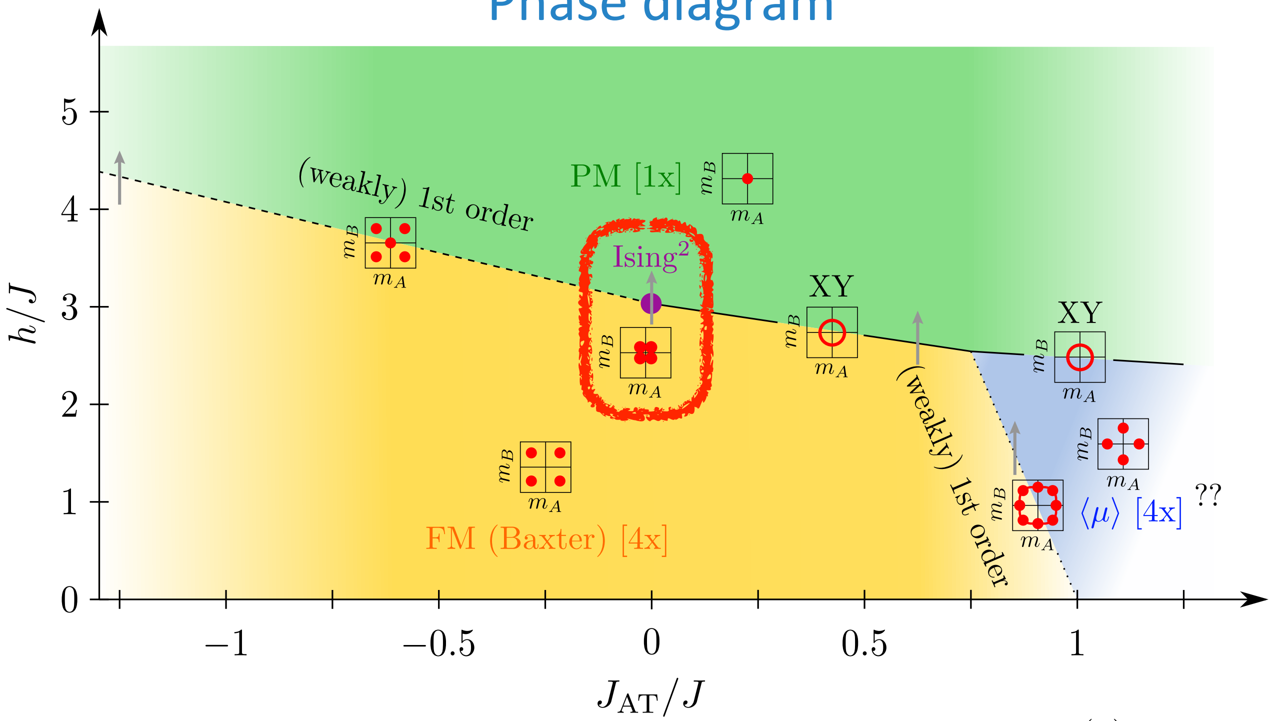
AT-TFI model

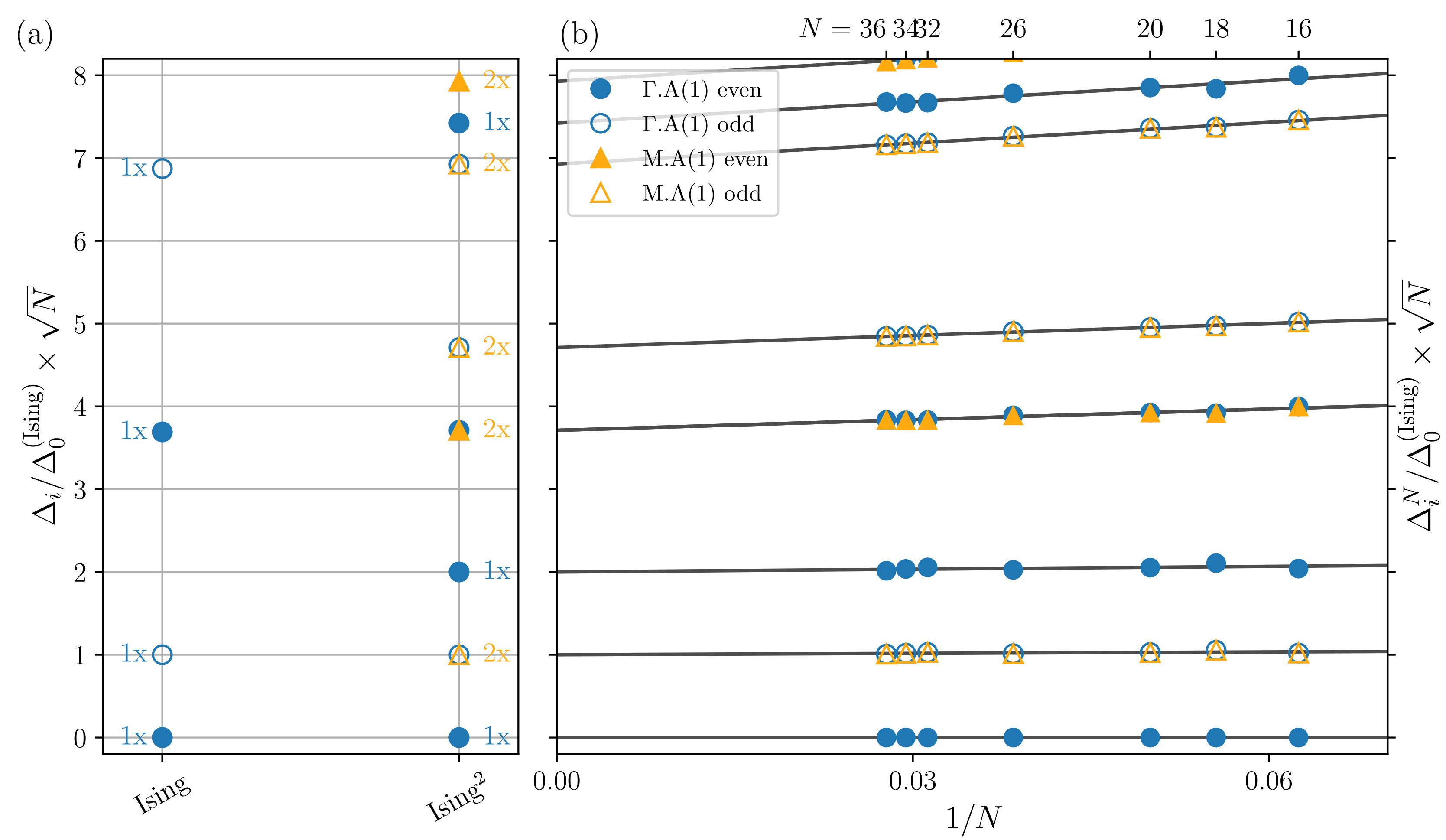


Phase diagram



- First $J_{\mathrm{AT}} = 0$ — Two decoupled transverse field Ising models
- CTES consists of two copies of the Ising CTES (obtained above)

➡ Ising² transition



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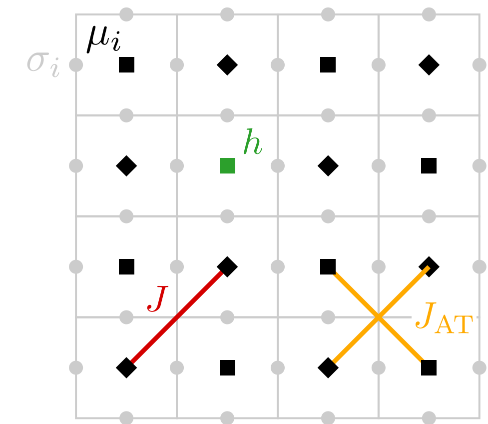
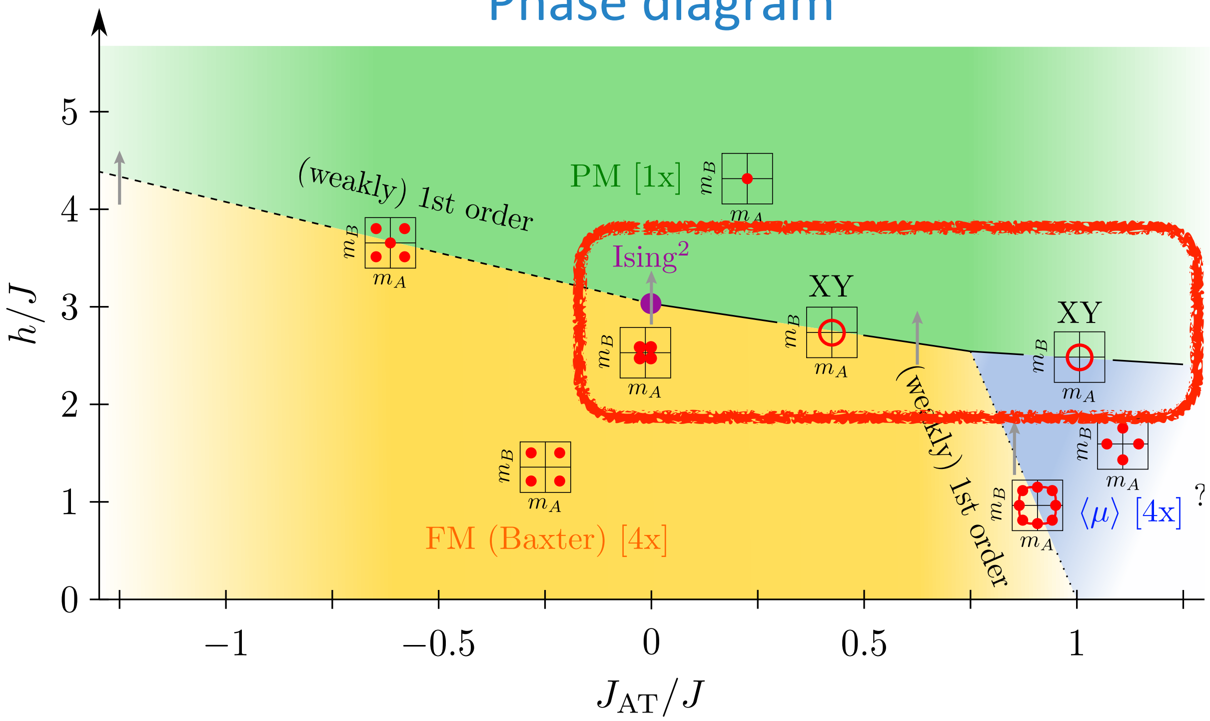
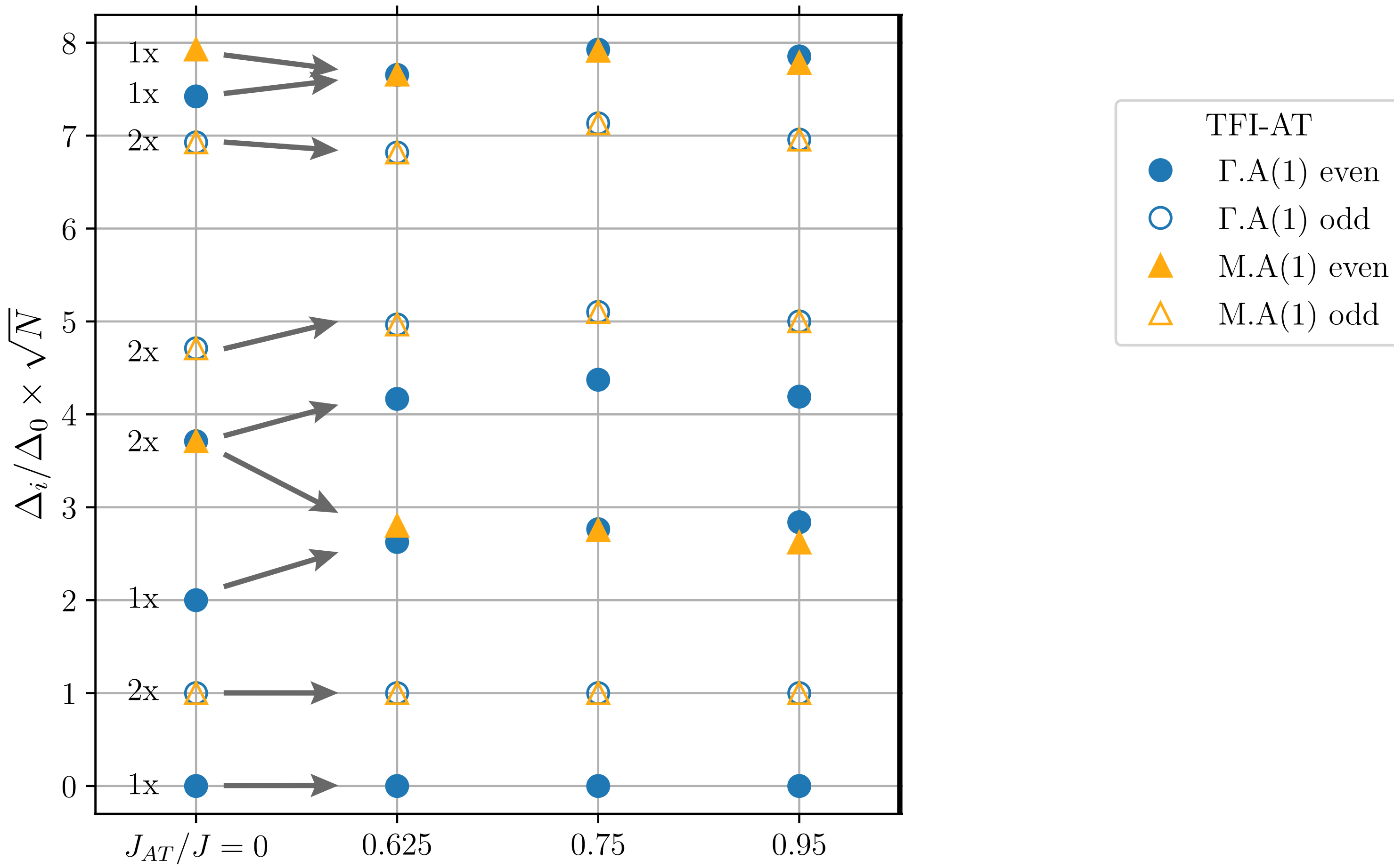
AT-TFI model



Phase diagram



- Now critical line for $J_{AT} > 0$
- Pronounced recombinations and shifting of CTES levels indicate change of universality class



Ising^2 QCPs to analyze

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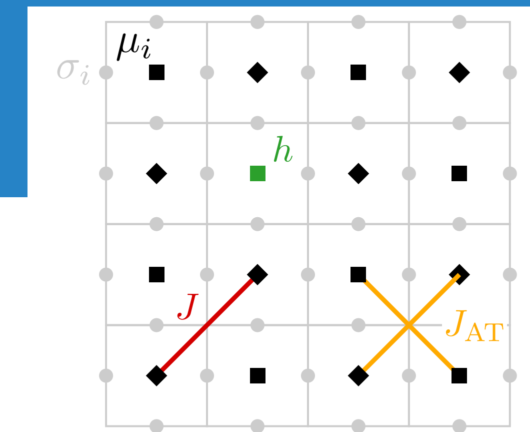
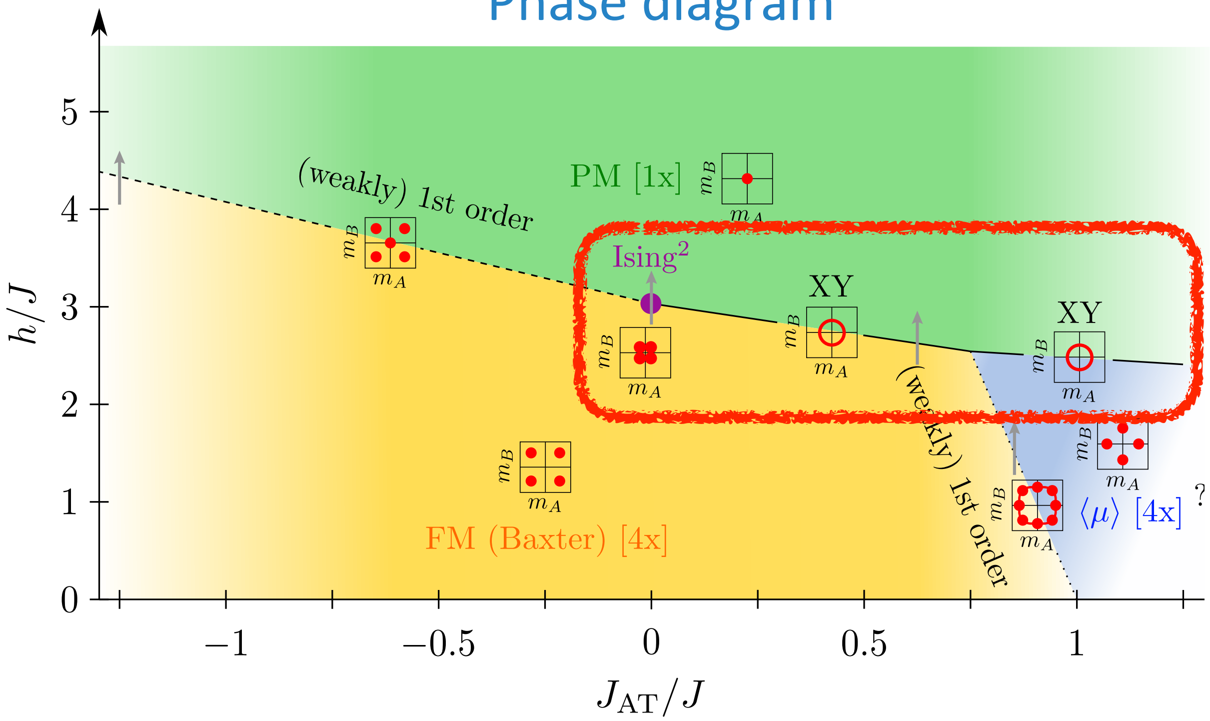
AT-TFI model



Phase diagram



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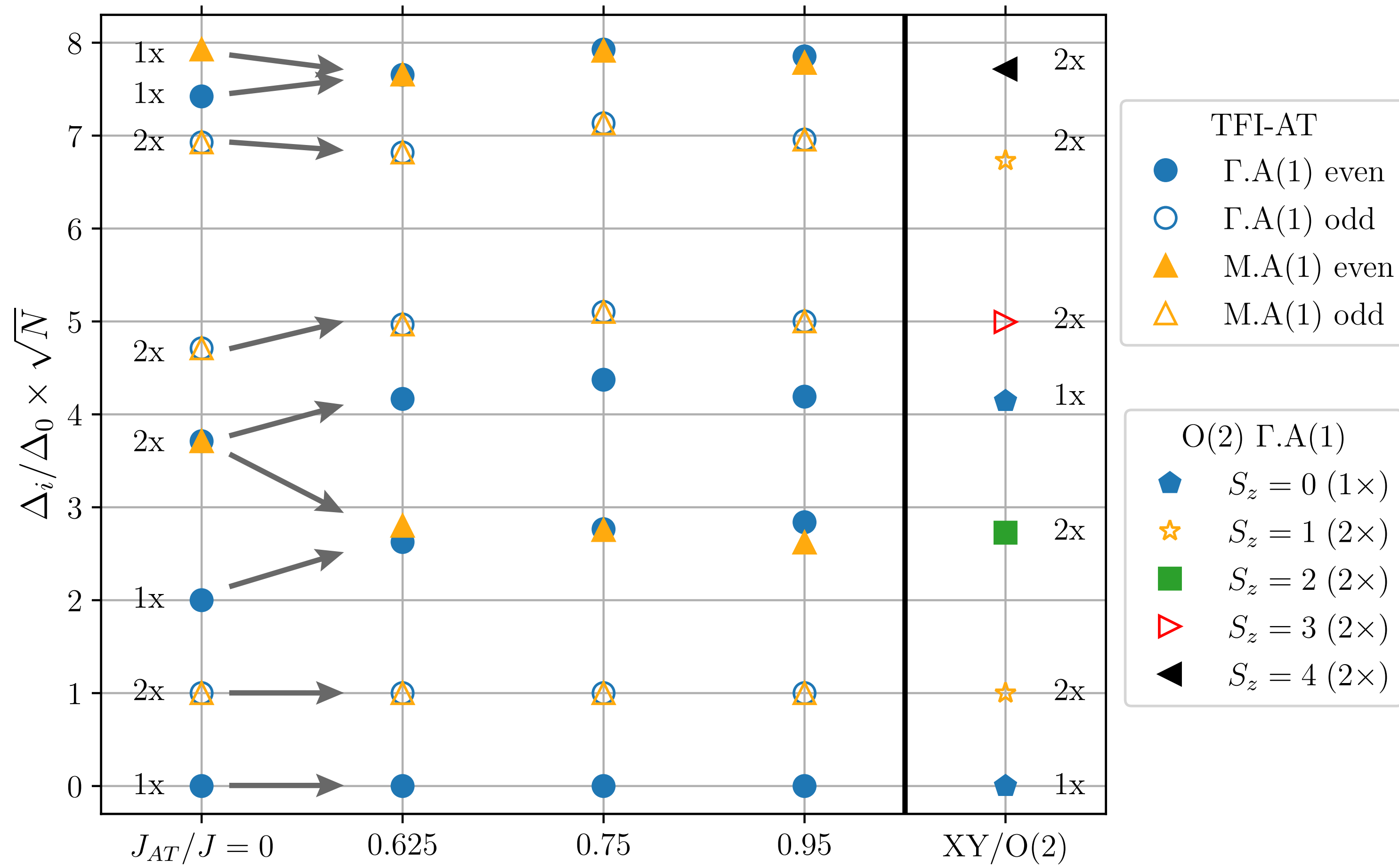

Ising^2 QCPs to analyze Tabulated data

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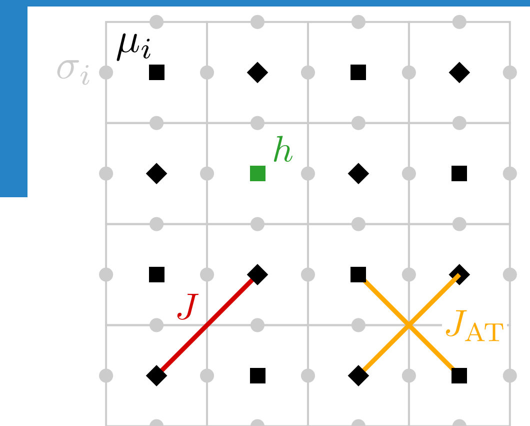
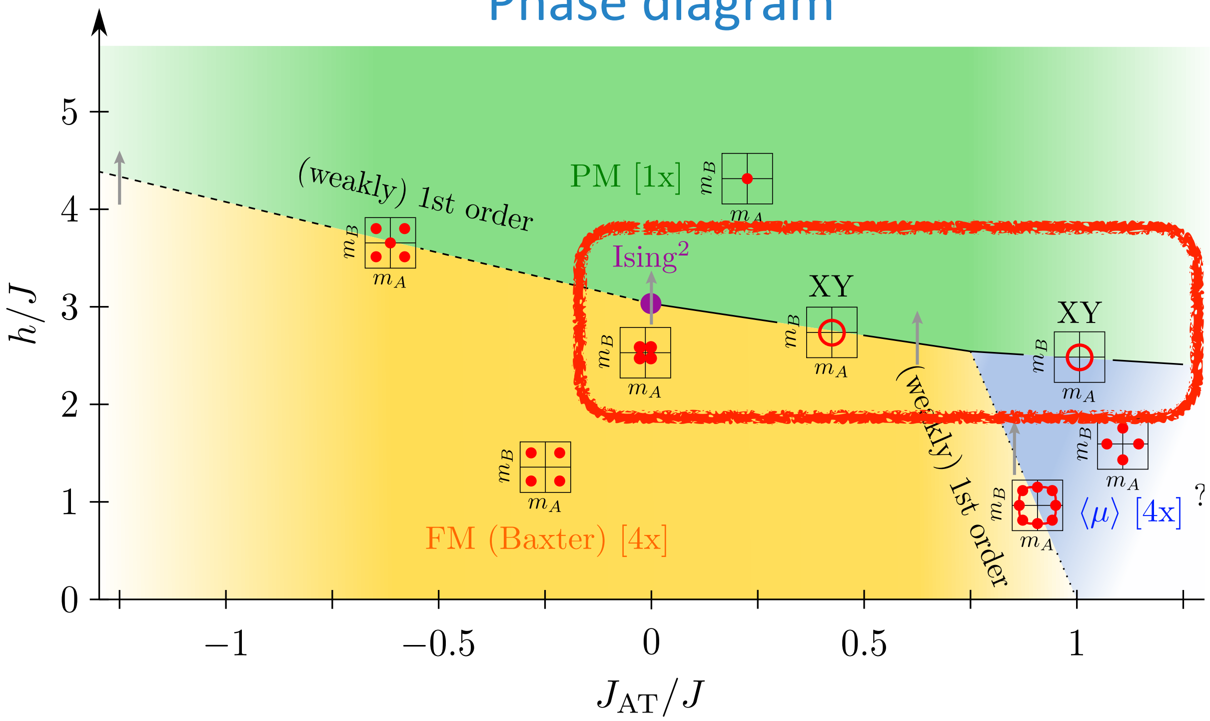
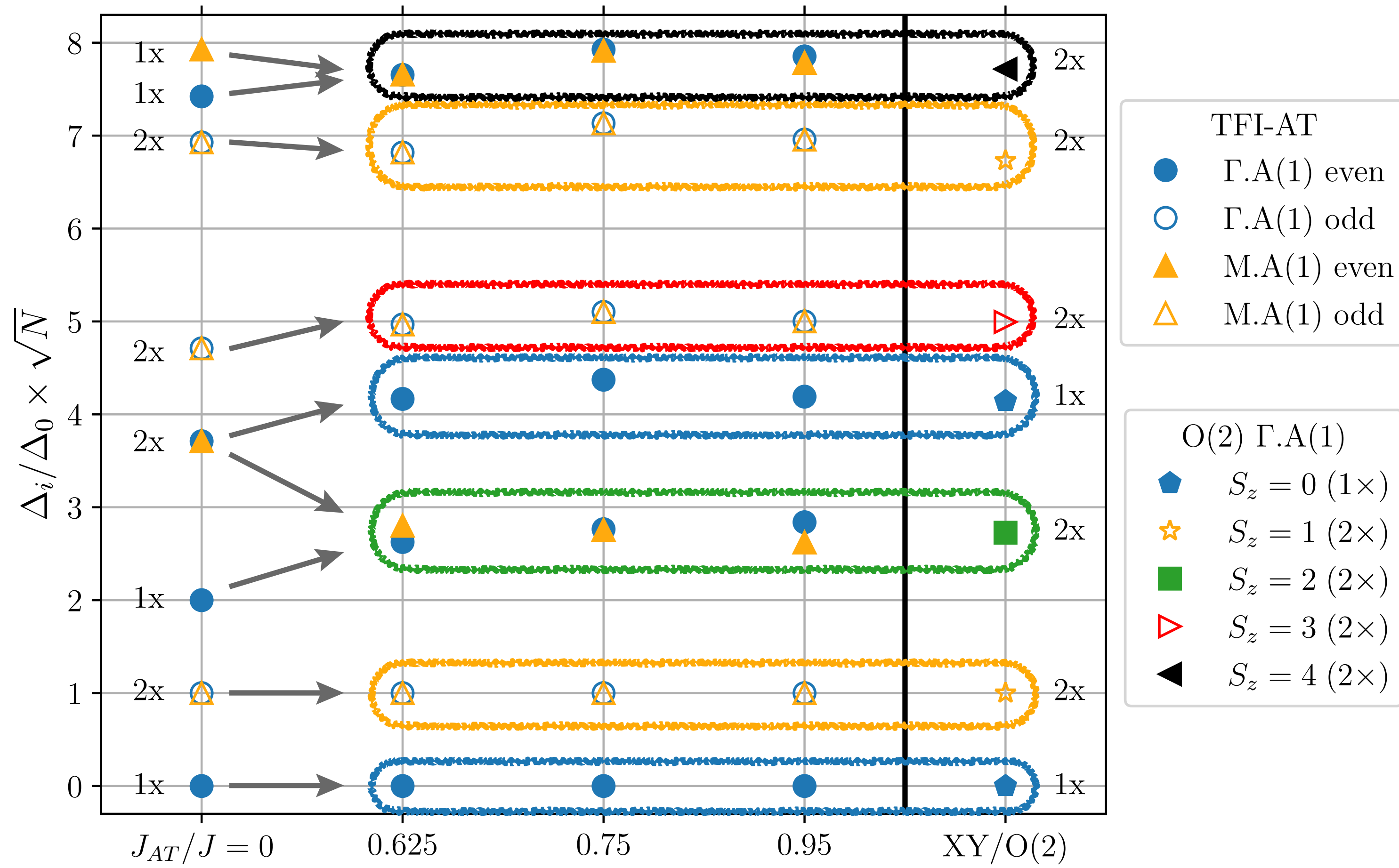
AT-TFI model



Phase diagram





Ising^2 QCPs to analyze Tabulated data

→ Emergent XY/O(2)

- Now critical line for $J_{\text{AT}} > 0$
- Pronounced recombinations and shifting of CTES levels indicate change of universality class
- Excellent agreement with previously charted XY/O(2) CTES

➡ Emergent O(2) universality class

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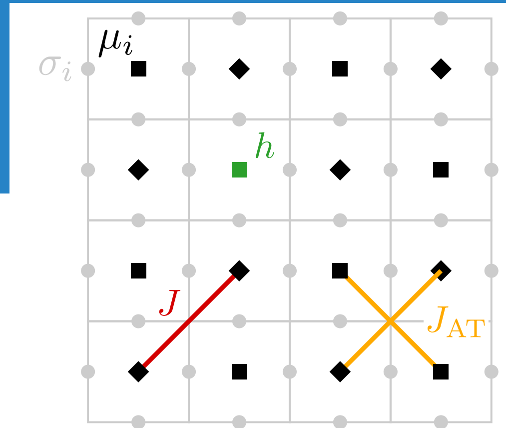
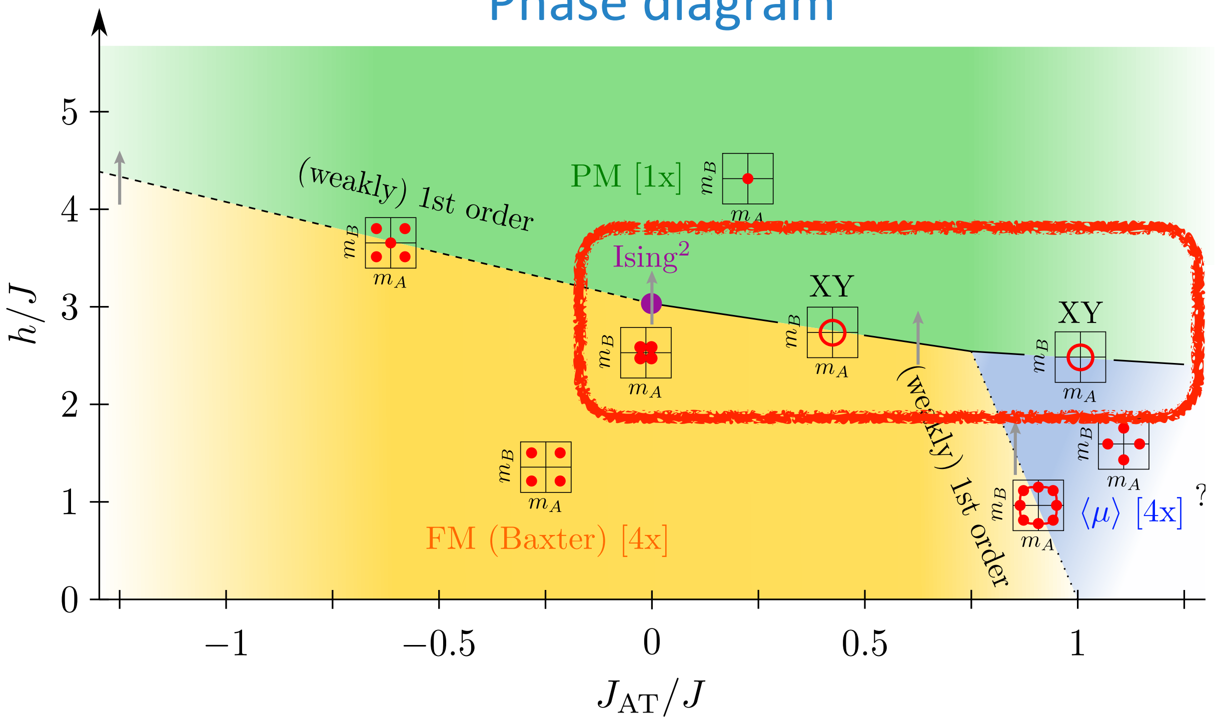
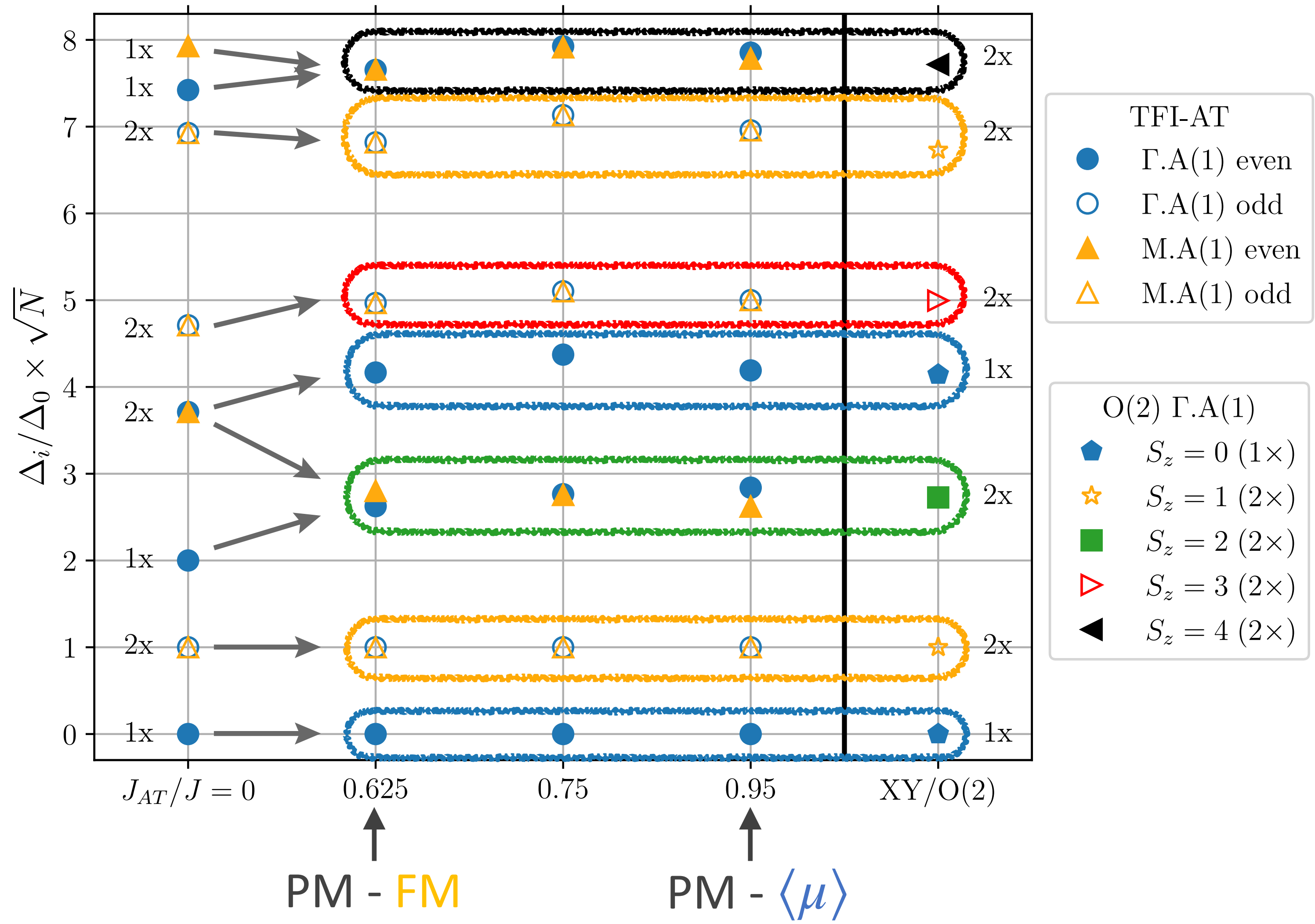
AT-TFI model



Phase diagram



- Closer look at $S_z = 2$ levels reveals a small splitting



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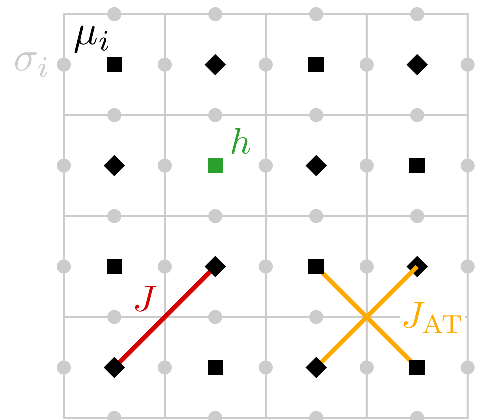
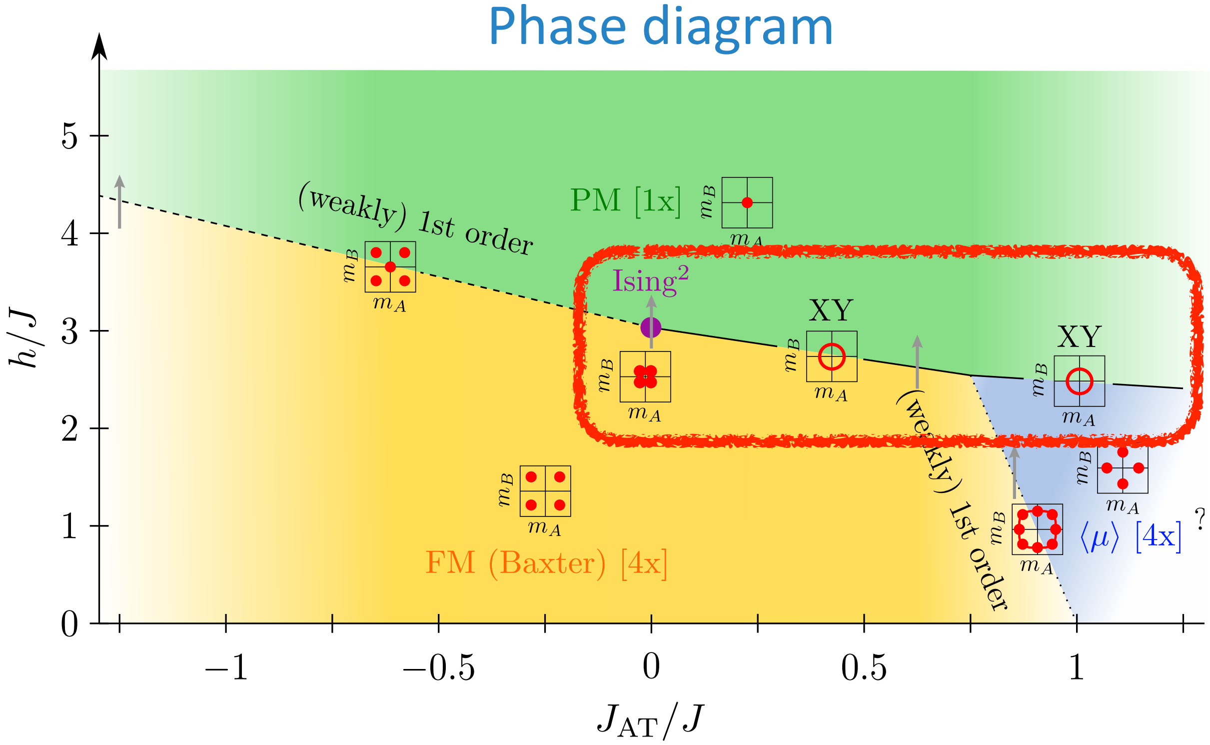
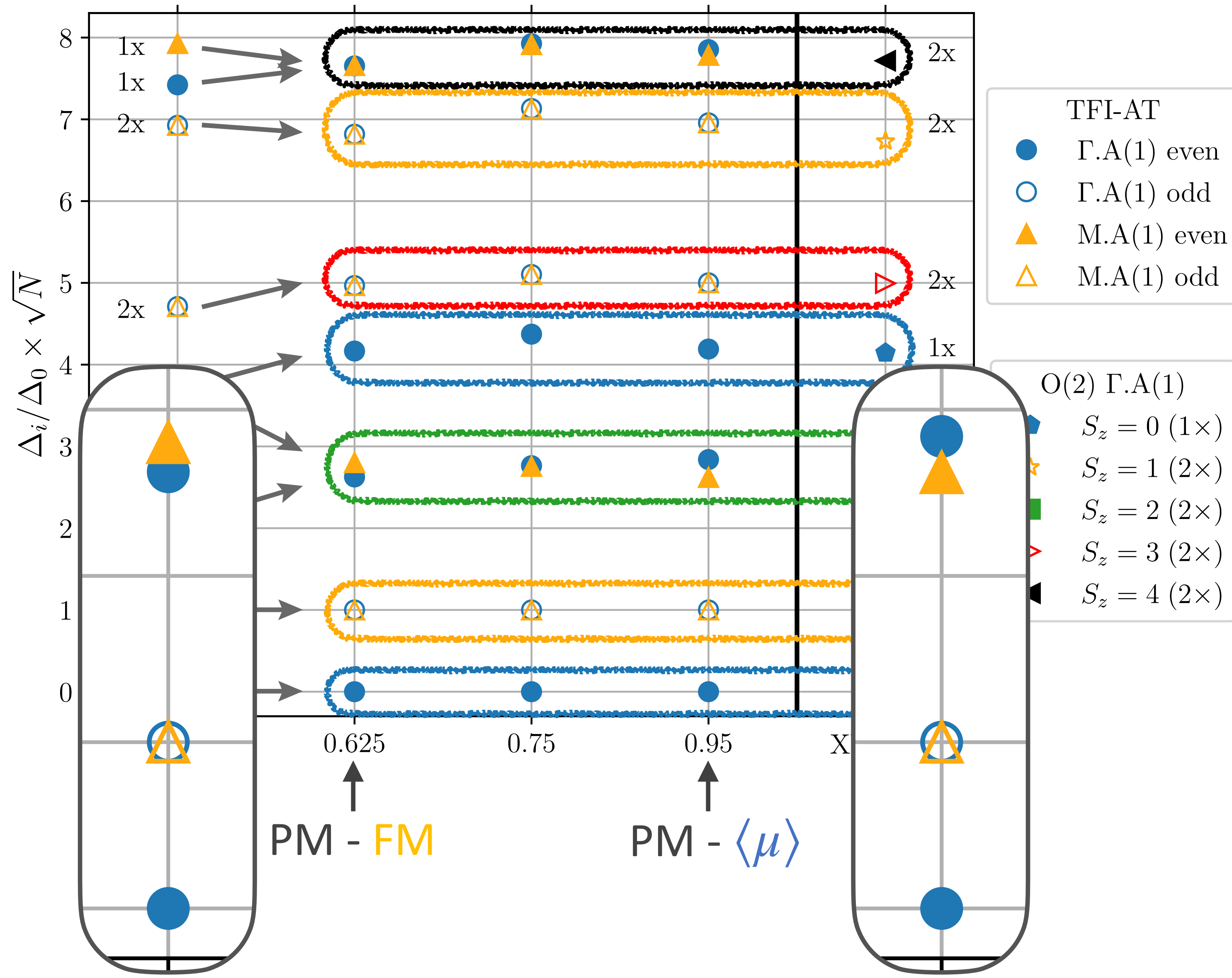
AT-TFI model



Phase diagram



- Closer look at $S_z = 2$ levels reveals a small splitting
- Order of levels swaps for the two distinctly ordered phases (FM, $\langle\mu\rangle$)



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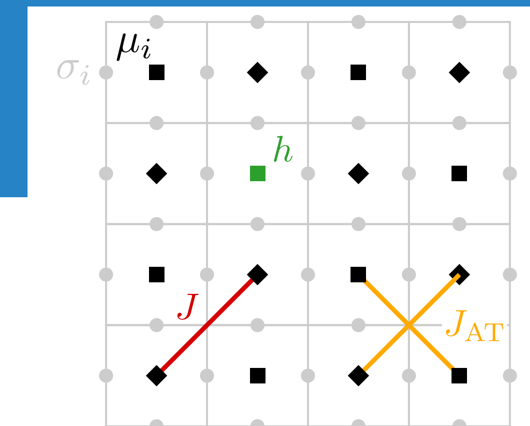
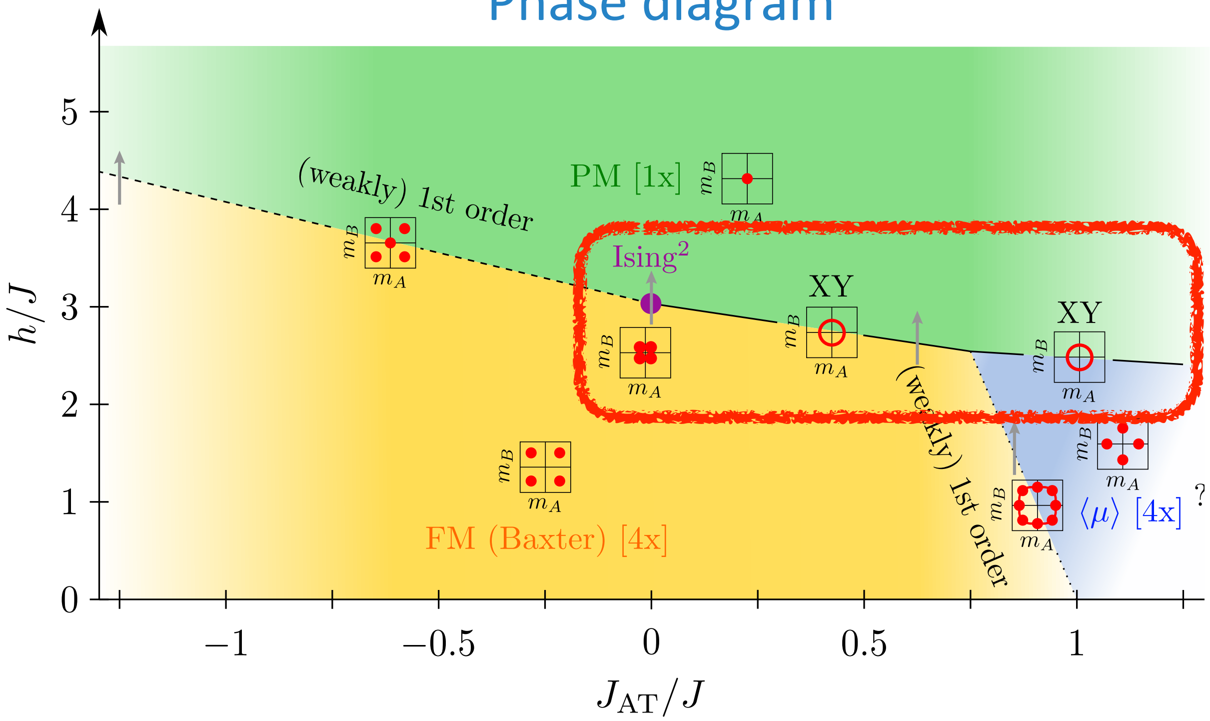
AT-TFI model

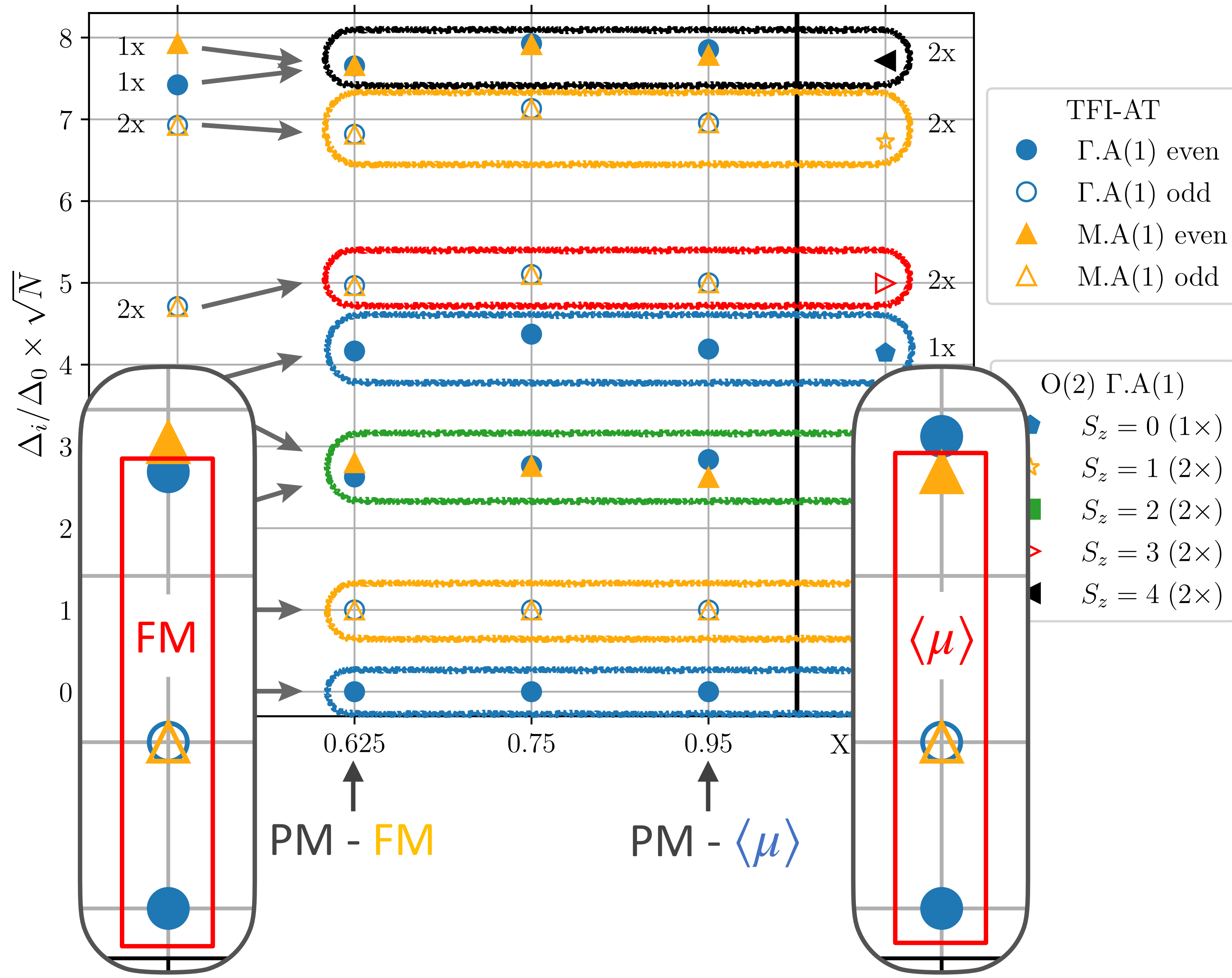


Phase diagram



- Closer look at $S_z = 2$ levels reveals a small splitting
- Order of levels swaps for the two distinctly ordered phases (FM, $\langle\mu\rangle$)
- Four lowest levels form the ground state manifold for the adjacent ordered phases
- Interpretation: Splitting from dangerously irrelevant terms in critical 3D XY theory

➡ Read off sign of $q = 4$ monopole operator coupling constant for critical 3D XY fixed point



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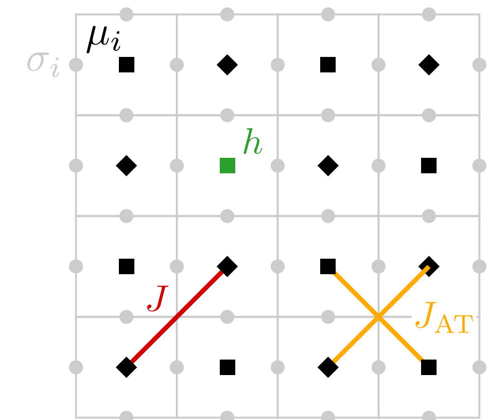

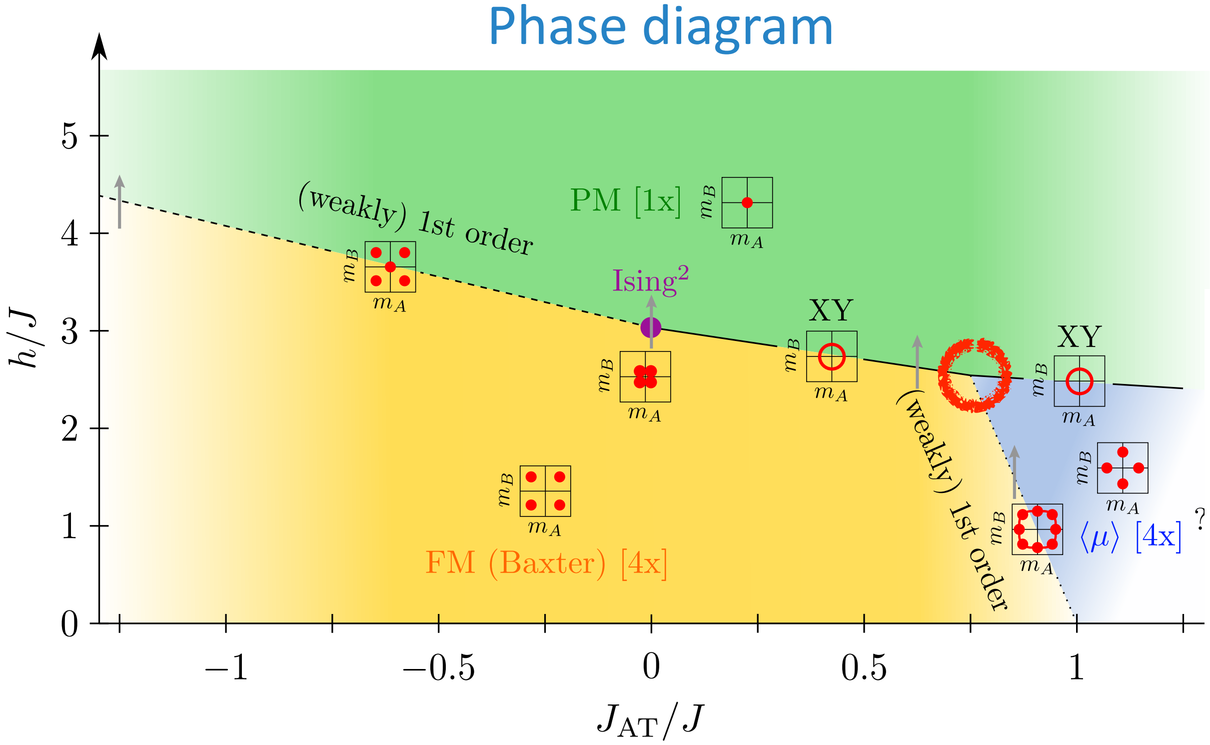

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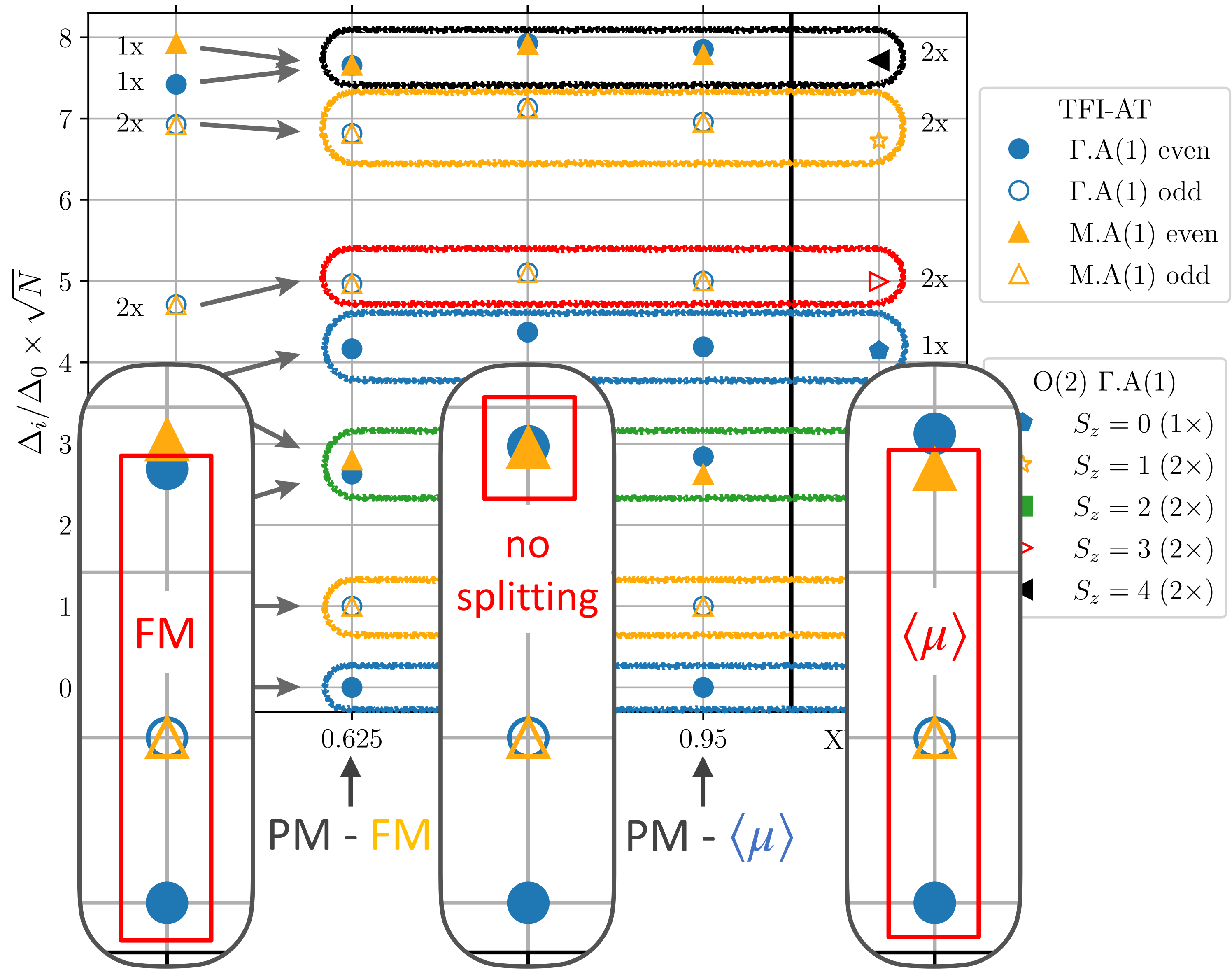
➡ Read off sign of $q = 4$ monopole operator coupling constant for critical 3D XY fixed point



- Point between PM, FM, $\langle\mu\rangle$ not tricritical, but 3D XY
- Somewhat special: $q = 4$ monopole operator vanishes

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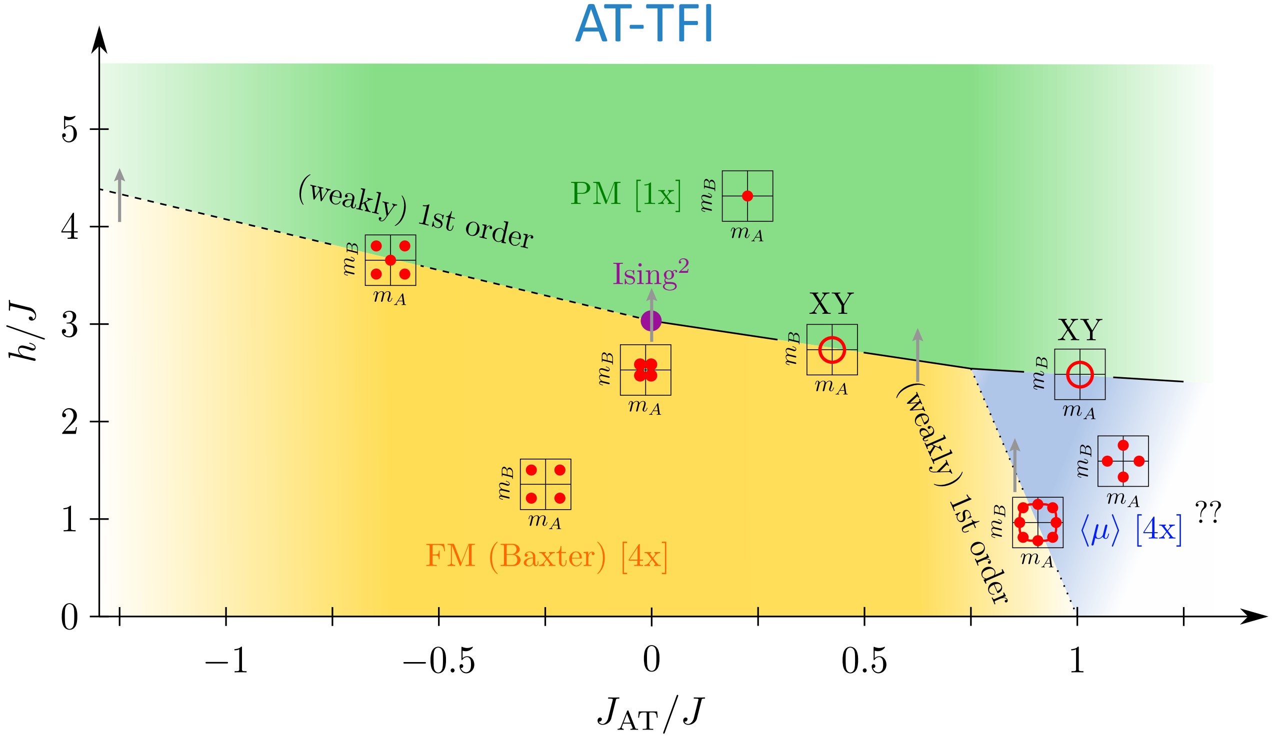
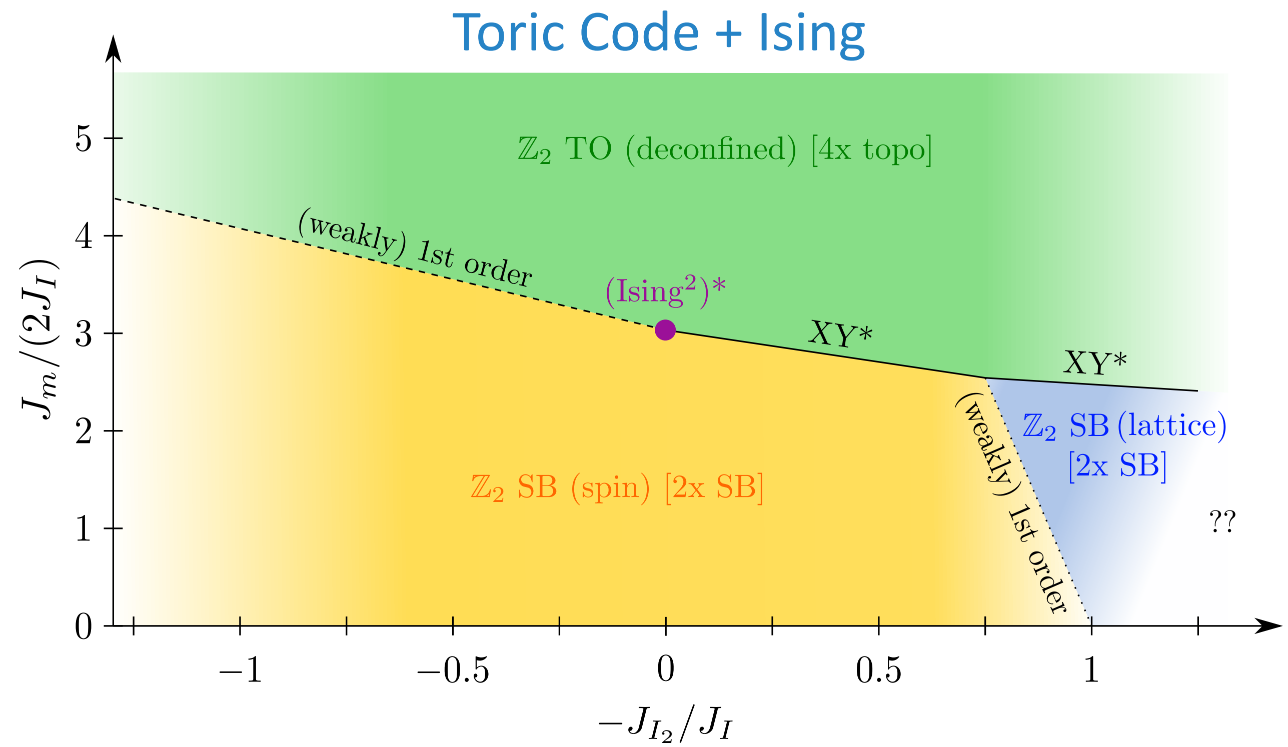
Back to our original goal: Toric Code Ising model





- Use exact mapping to obtain phase diagram, consider constraint ($\mathbb{Z}_2$ even) & different BCs
- Fractional excitations (m anyons) undergo corresponding conventional phase transitions
  - ➡ *Starred* universality classes: (Ising²)*, XY*
- General expectation: $\mathbb{Z}_2$ topological order — $\mathbb{Z}_2$ symmetry broken transition is XY* or first order
- (Ising²)* needs fine-tuning

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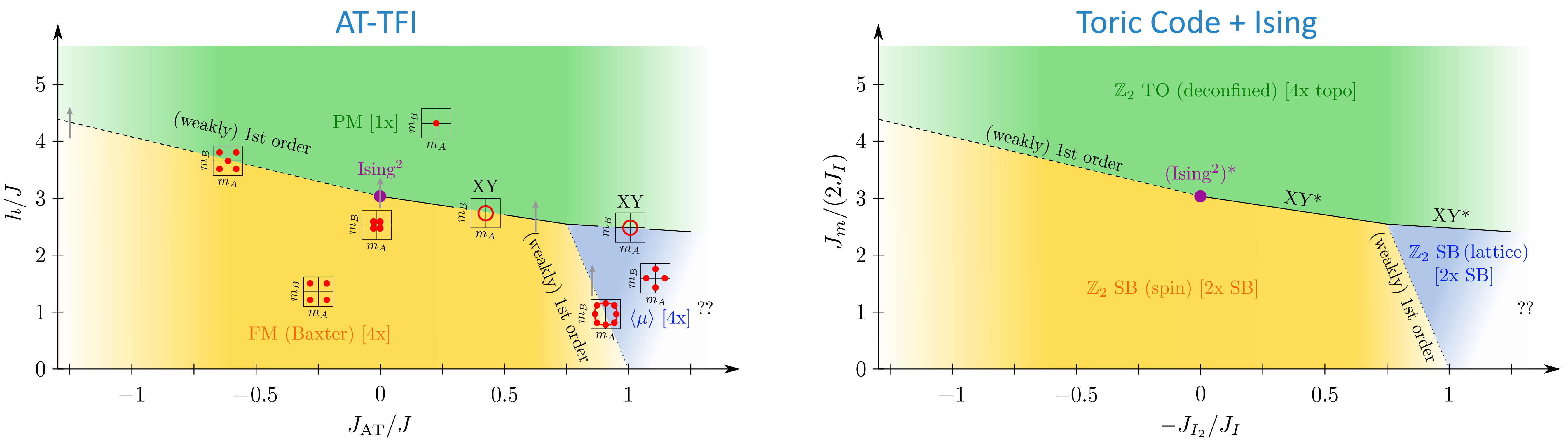
Back to our original goal: Toric Code Ising model



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  - ➡ *Starred* universality classes: (Ising²)*, XY*
- General expectation: $\mathbb{Z}_2$ topological order — $\mathbb{Z}_2$ symmetry broken transition is XY* or first order
- (Ising²)* needs fine-tuning
- Condensing anyons m are symmetry fractionalized (non-trivial projective representation of the symmetry group)
  - Has strong consequences on fate of confined phases
  - Here: Either spin-inversion symmetry or space group symmetry spontaneously broken
  - Observe examples for both cases in our phase diagram

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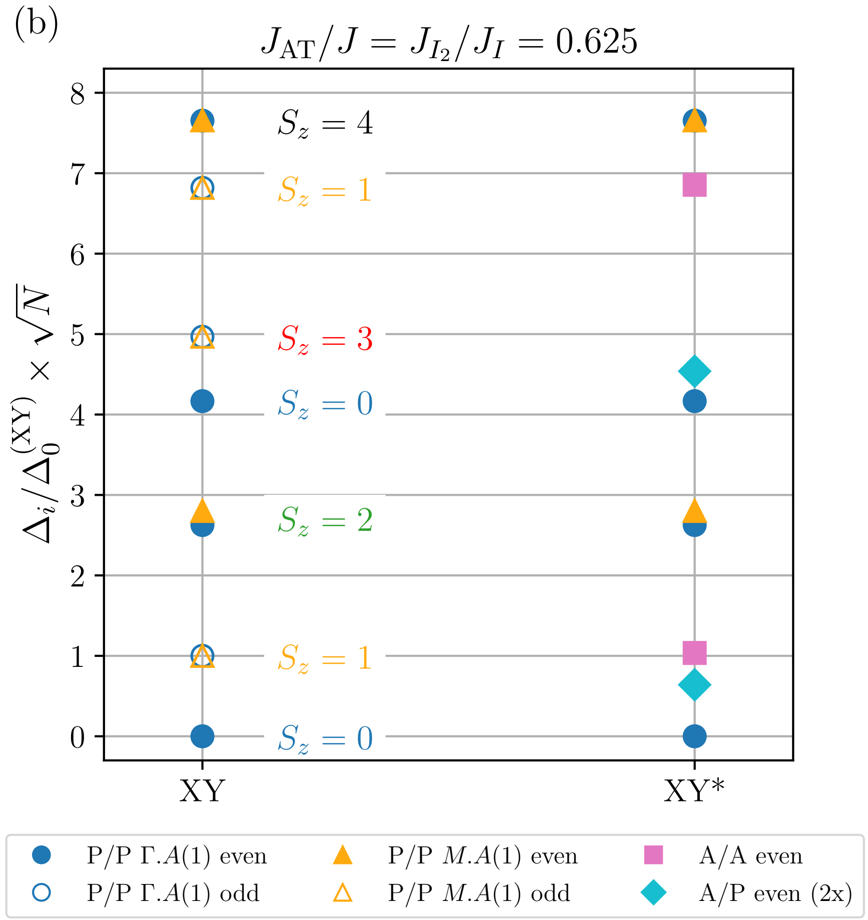
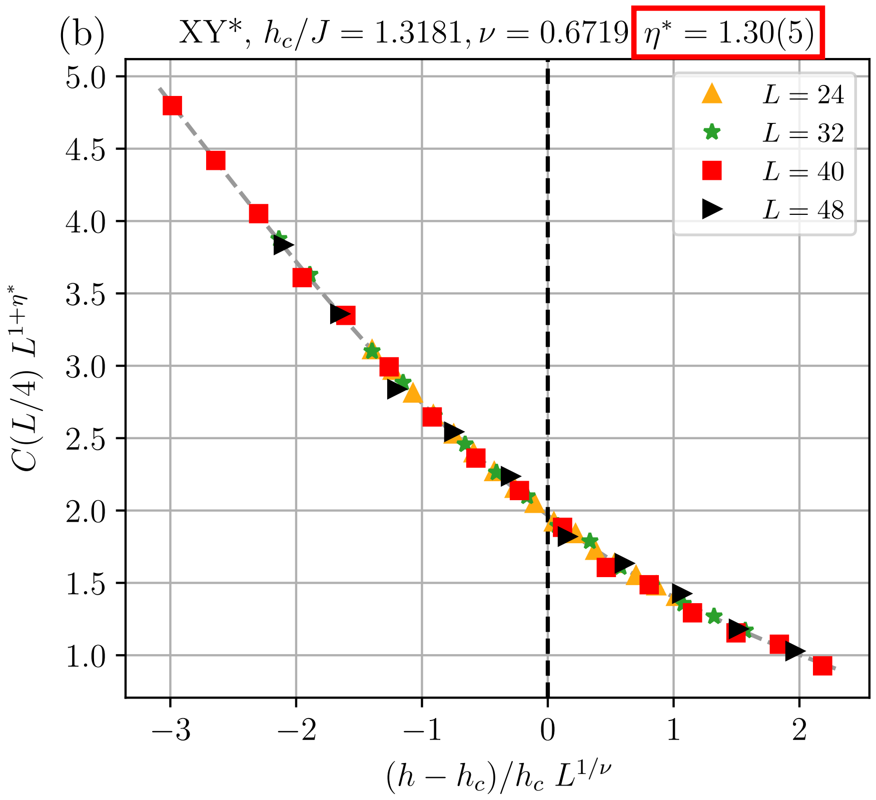
XY* transition

CTES



Standard approach:
Unusually large η



- XY* CTES by removing spin-inversion ($\mathbb{Z}_2$) odd levels and adding A/A, A/P BCs
- Features characteristic low-lying A/A, A/P levels, similar to Ising* and (Ising²)*

- Standard approach: We observe an unusually large critical exponent η^* due to the fractionalisation of the quasi-particles
- Value of η^* a bit below expectation ($\eta^* \approx 1.47$), possibly due to emergent nature of phase transition?
- Standard approach needs much larger system sizes than CTES

Summary & Outlook

Summary & Outlook

$$\Delta_i = \frac{v}{L}\xi_i$$

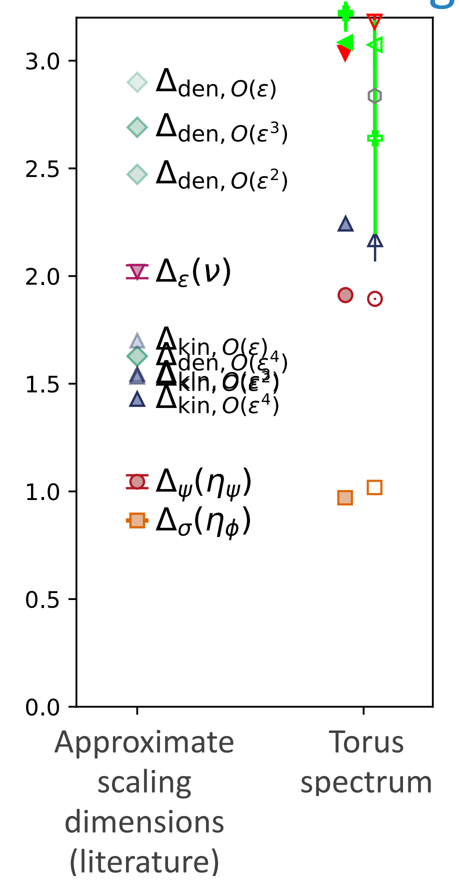
Summary

- Critical torus spectrum = Fingerprint of QCP (c.f. mass spectroscopy)
- Given by universal numbers times $1/L$
- Qualitatively distinct → Great tool to identify QCPs
- Different methods: Microscopic models, field theory
- Similarity between torus spectrum and CFT operator content (at least for Wilson Fisher QCPs)
- Applicable to fermionic QCP (Chiral QCPs)

Outlook

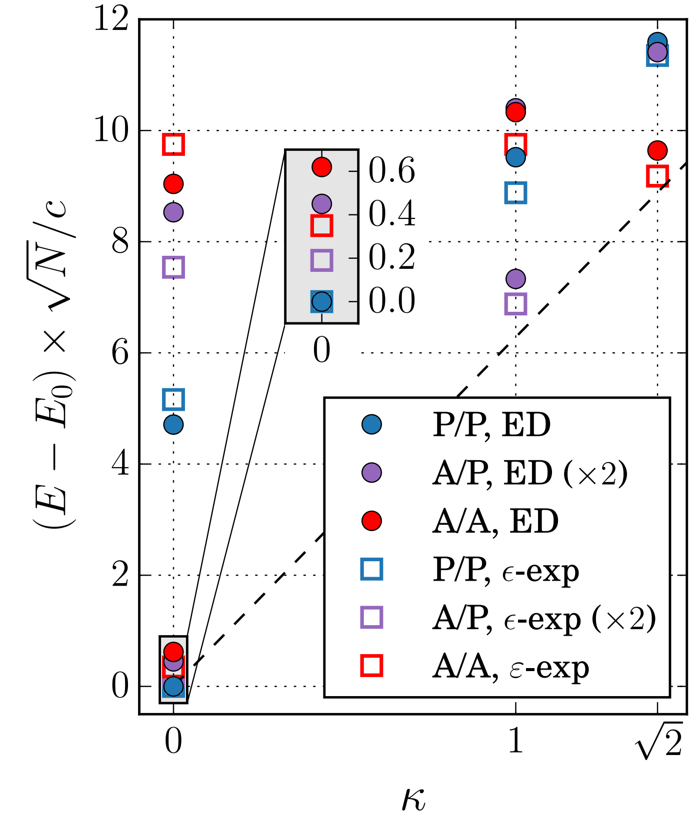
- Catalogue of critical torus spectra
- Deconfined QCPs — Traces of emergent symmetries?
- Dirac fermions — Chiral XY, Chiral Heisenberg (critical exponents under debate)
- Torus spectrum for fermions coupled to gauge fields (e.g. QED_3)
- Tensor network methods to compute torus spectrum?
- (i)MPS — Local Hamiltonian spectrum, correlation lengths spectrum?
- Entanglement spectrum vs. torus energy spectrum - Similarities, differences?

Chiral Heisenberg

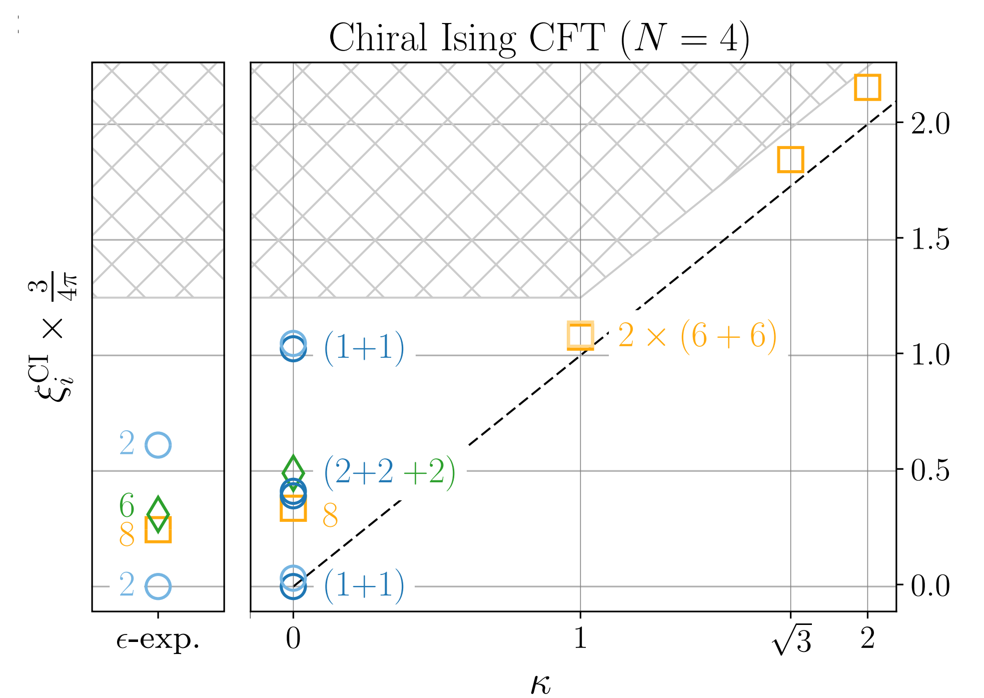


To be published

Ising*



MS et.al.; PRL **117**, 210401 (2016)



MS et.al.; PRB **103**, 125128 (2021)

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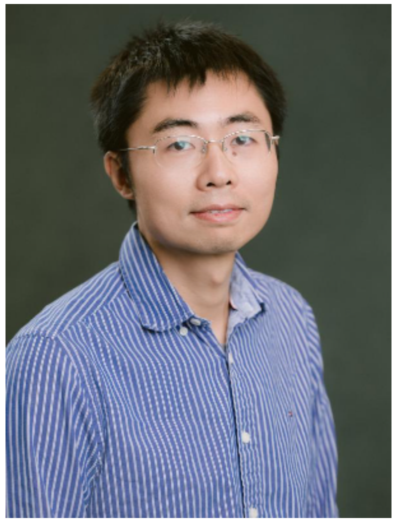

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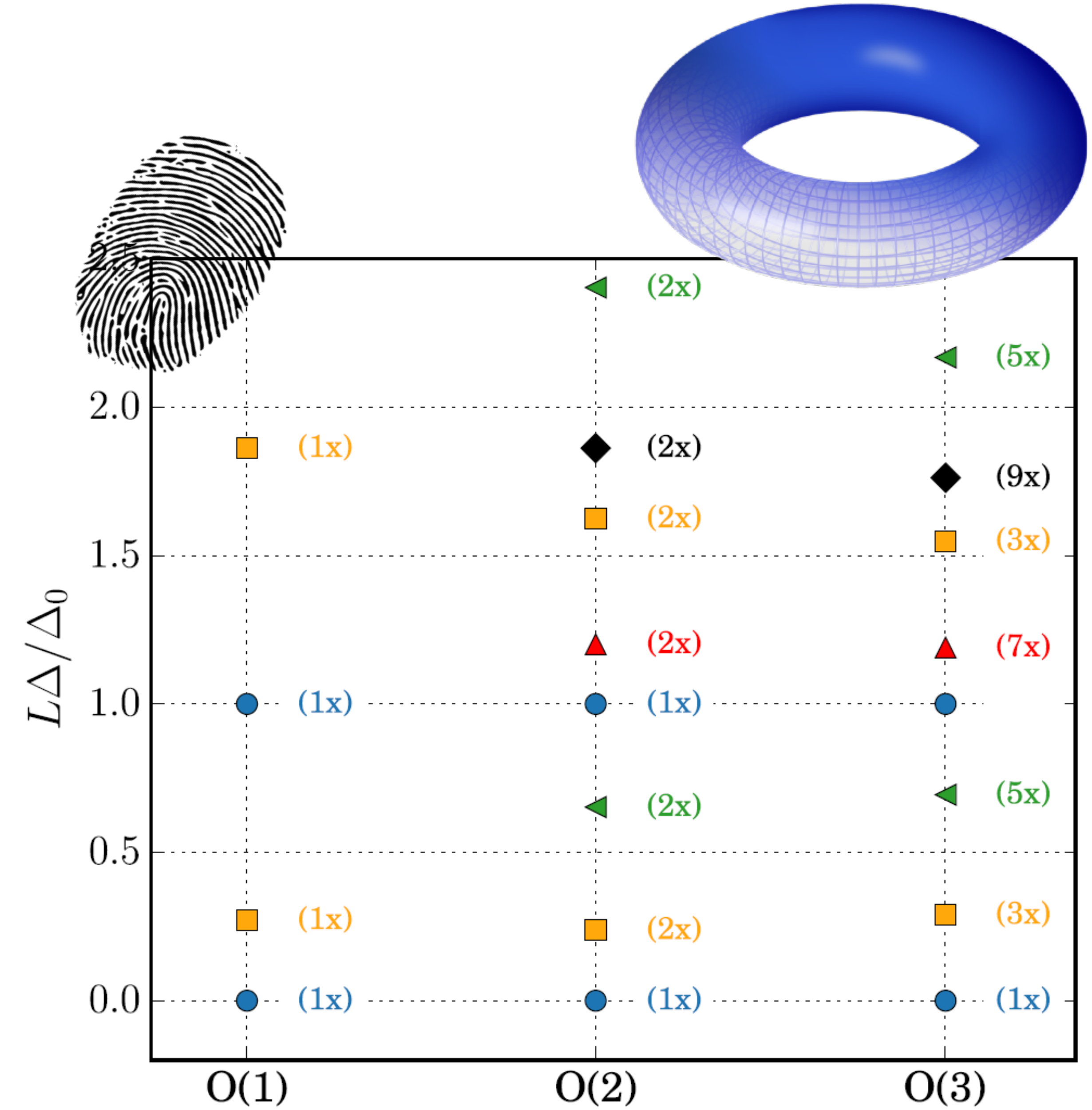
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Thank you!