Deterministic Dynamics of initially random systems

Vincent Pasquier

IPhT, Saclay

Entanglement Scaling and Criticality with Tensor Networks

Collaboration

- Ziga Krajnick, Enej Ilievski, Tomaz Prosen,
  Lubljana
- Johannes Schmidt,
  T.U. Berlin
- Help of Jean-Marc Luck
  IPhT

Plan

- Motivation

- I – Motivation, XXZ, Landau Lifshitz
- II – Large deviations in simple models
- III – Landau Lifshitz

- Conclusion

I - Motivation

- Transport out of equilibrium
- Spin transport in XXZ spin chain
- intégrable $\Rightarrow$ analytic results? difference with nonintegrable?

interesting quantity:
Large deviation of current.

transport

[Marco Znidaric **PRL**, 2011]

Transport of spin current in XXZ chain

Infinite temperature transport:

- Ballistic for $\Delta < 1$
- Anomalous for $\Delta = 1$
- Diffusive for $\Delta < 1$

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Infinite temperature transport of magnetization in Landau Lifshitz: the same.

[Ziga Krajnick, Tomaz Prosen **PRL**, 2019]

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- For $X = x\sqrt{t}$ with $x = O(1)$, we have in the large t limit

  The probability of Gain S_T at typical scale is the normal law:

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- Central limit theorem: Moivre, the probability is Gaussian.

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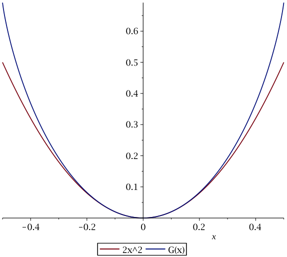
- Ask for the probability of a gain of order t:
  - $P(S_t/t \in [x, x+dx]) = \exp(-tI(x))dx$



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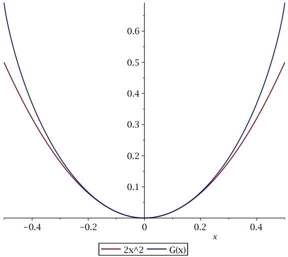

Rate function: $I(x)$. Its explicit expression follows from Stirling formula.

I - The coin problem,Cumulants

- Large deviation principle: The probability decays as e^{-TG}
- Cumulant Generating Function (CGF) behaves similarly:
$$\langle e^{\lambda S_t}\rangle = \int dx e^{-t(I(x)-\lambda x)} = e^{-tF(\lambda)}$$
- F=Legendre transform of rate function I:
$$F(\lambda) = \min_x (I(x) - \lambda x)$$
- Gartner Ellis Thm: It can apply with non independant variables under some conditions on MGF: $F(\lambda)$ differentiable? complex analytic?
- Consequence, all cumulants scale as t.

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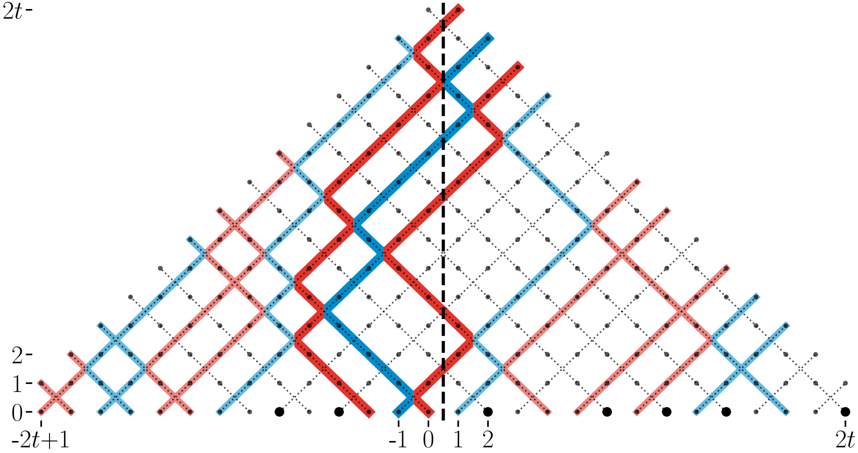
- Some discrepancy with the standard large deviation theory should be expected in the presence of dynamical phase transitions as free energy is not always extensive in the vicinity of thermodynamical phase transitions.

A simple model



Rate function

- uncharged model Two players R and L toss a coin t times independently, G_L and G_R their respective gains. The number of particles which have passed through the origin at large time coincides with the difference of their respective gains:

$$J_P = G_L - G_R$$

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- charged model Color independently blue (plus) or red (minus) each particle trajectory and ask the amount of charge J_c that has passed through the origin.

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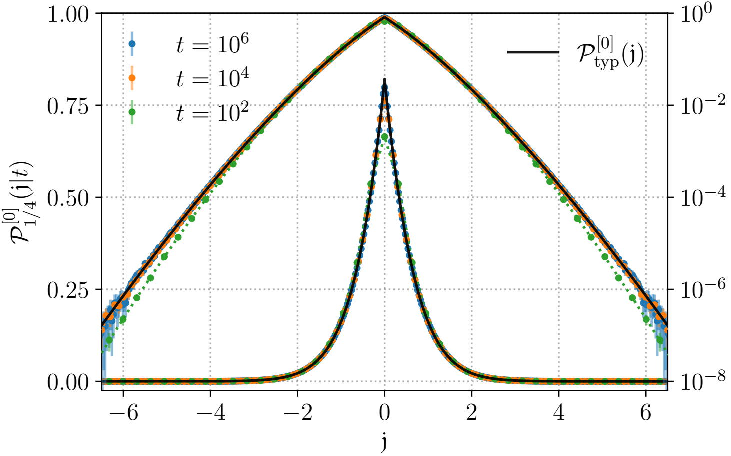
- explicitly:

$$P(J_c) = \frac{1}{2^{2N}} \sum_{G_L - G_R = J_P} \binom{t}{G_R} \binom{t}{G_L} \binom{|J_P|}{|J_P|/2 + J_c}$$

- Gaussian approximation $J_P = t^{1/2} x$, $J_c = t^{1/4} j$

$$P(j \in dj) = dj \frac{2}{\pi} \int_0^\infty \frac{dx}{\sqrt{x}} e^{-2(x^2 + \frac{j^2}{x})}$$

Non Gaussian profile



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- expect non analyticity of CGF in the vicinity of $\lambda = 0$.

Phase transition

- CGF= partition function with t the size of the system:

$$Z(\lambda) = \frac{1}{2^{2N}} \sum_{G_L - G_R = J_P} \binom{t}{G_R} \binom{t}{G_L} \cosh(\lambda)^{|J_P|} e^{hJ_P}$$

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- λ real double well,
  λ imaginary single well,
  $\lambda = 0$ Dynamical critical point signals breaking of large deviation principle.

zeros of partition function

- CGF using saddle point approximation:

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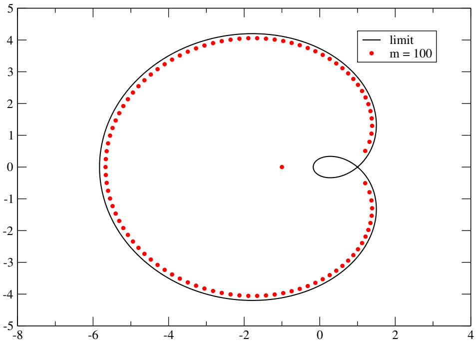
Zeros in complex c plane



Bernoulli curve !

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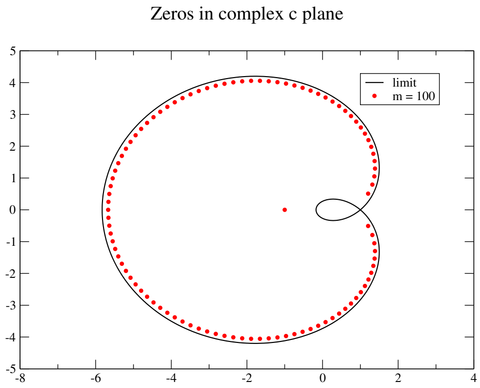

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  On each lattice site j, $S_j(t = 0)$ is a random vector on the unit sphere.

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  On each lattice site j, $S_j(t = 0)$ is a random vector on the unit sphere.
- exchange relation imply discrete integrability:

$$L(u, S_1)L(v, S_2) = L(v, S_2')L(u, S_1')$$

Discrete time evolution

- $J = 0$ dynamics:

$$S_{t+1} = \frac{1}{\sigma^2+\tau^2}(\sigma^2 S + \tau^2 V + \tau S \times V)$$
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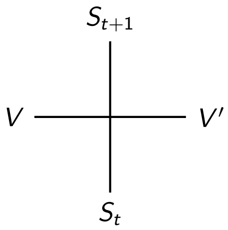
- The vertex



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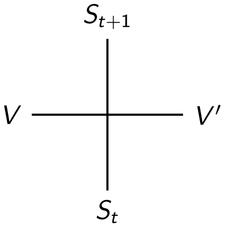
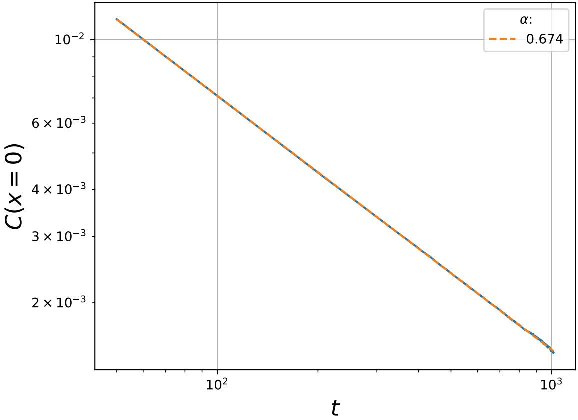


Figure: Time evolution of the spin

- Yang Baxter map

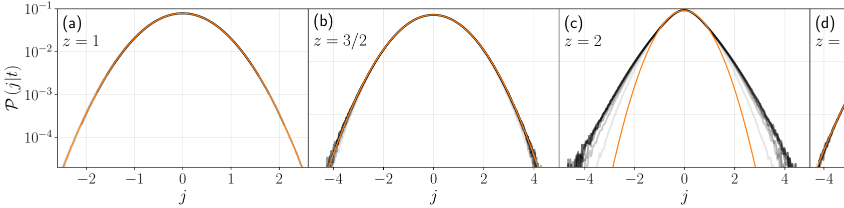
Dynamical exponent 3/2

- Dynamical exponent



Large deviation of magnetization current

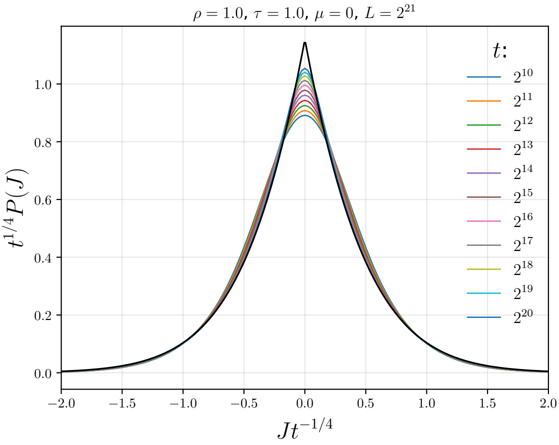
- Rate Function



- easy plane, isotropic, easy axis

Non Gaussianity

- Easy axis rate Function



Not Gaussian, again cusp

Cumulants

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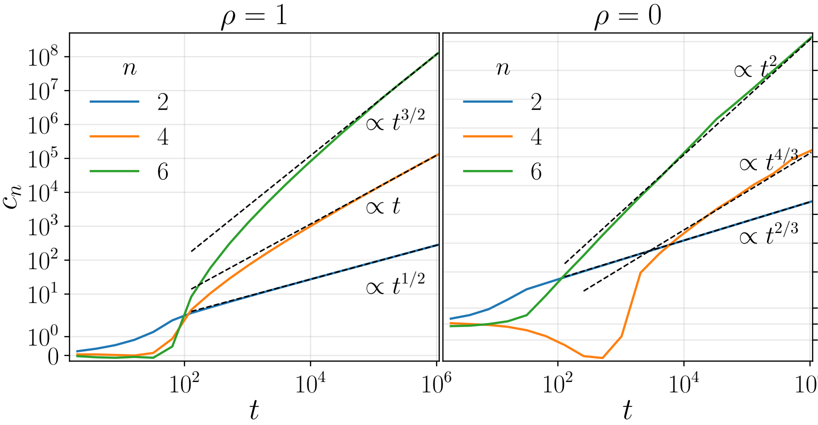

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- Is the $3/2$ dynamical exponent related to KPZ universality class?