Critical properties of the interacting Majorana chain
The power of Friedel oscillations

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Delft Technology Fellowship

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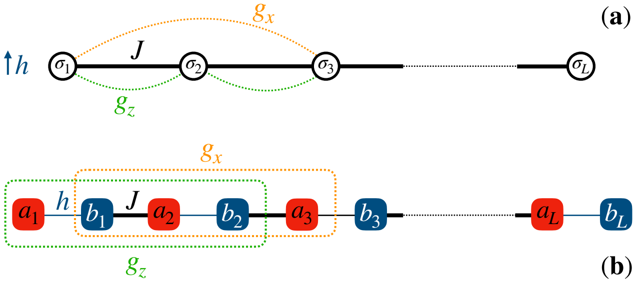
The model

Spin chain:

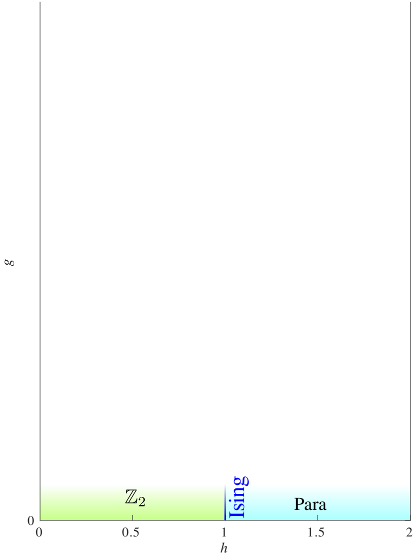
$$H = \sum_j J\sigma_j^x\sigma_{j+1}^x - h\sigma_j^z + g\sigma_j^z\sigma_{j+1}^z + g\sigma_j^x\sigma_{j+2}^x$$

Majorana chain:

$$H = \sum_j -it_{\text{even,odd}}\gamma_j\gamma_{j+1} - g\gamma_j\gamma_{j+1}\gamma_{j+2}\gamma_{j+3}$$



Starting point



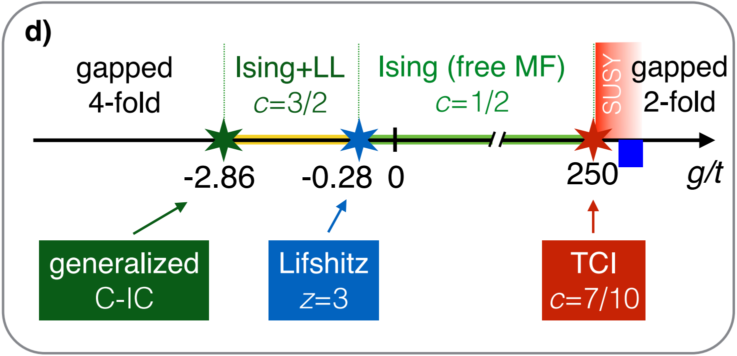
$$H = J\sum_i \sigma_i^x \sigma_{i+1}^x + h\sum_i \sigma_i^z$$

$$+g\sum_i(\sigma_i^x\sigma_{i+2}^x + \sigma_i^z\sigma_{i+1}^z)$$

- Ising transition at $h = J$

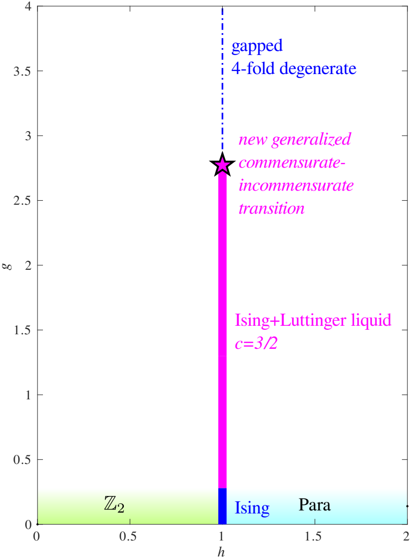
Previous results

$$H = J\sum_i \sigma_i^x\sigma_{i+1}^x - h\sum_i \sigma_i^z - g\sum_i(\sigma_i^x\sigma_{i+2}^x + \sigma_i^z\sigma_{i+1}^z)$$



Rahmani, Zhu, Franz, Affleck, Phys. Rev. B 92, 235123 (2015);

Starting point

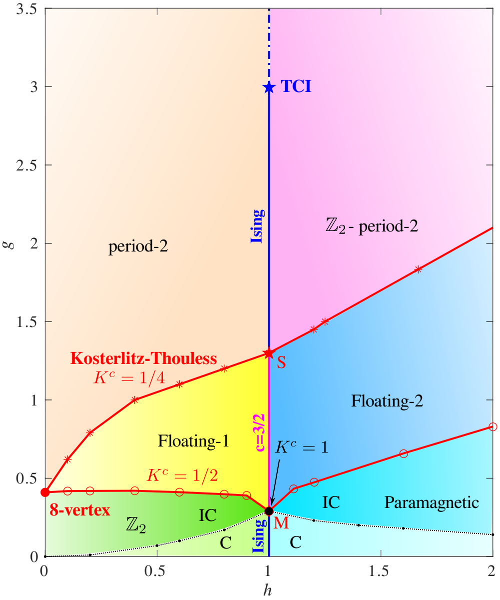


$$H = J\sum_i \sigma_i^x\sigma_{i+1}^x - h\sum_i \sigma_i^z + g\sum_i(\sigma_i^x\sigma_{i+2}^x + \sigma_i^z\sigma_{i+1}^z)$$

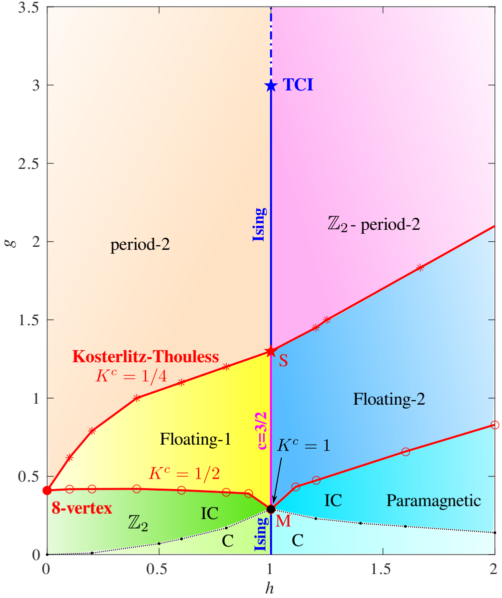
$h = J$

- $g < 0.28$ Ising phase
- $0.28 < g < 2.86$ Ising+LL
- $g > 2.86$ gapped, 4-fold degenerate

Rahmani, Zhu, Franz, Affleck, Phys. Rev. B 92, 235123 (2015)
see also Milsted, Seabra, Fulga, Beenakker, Cobanera Phys. Rev. B 92, 085139 (2015) with $g^c \approx 5$.



NC, Laflorencie, arXiv:2211.15598



Scope

- Self-duality
- Exact zero modes
- Floating phases
  - Boundaries
  - Floating-1 vs Floating-2
- $h = 1$ line
  - No generalized C-IC transition
  - Tri-critical Ising
  - Multicritical points
- Outlook & perspective

NC, Laflorencie, arXiv:2211.15598

Duality

$$H = \sum_i \sigma_i^x \sigma_{i+1}^x + h\sigma_i^z + g(\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

Kramers–Wannier duality

$$\sigma_i^x \sigma_{i+1}^x = \tilde{\sigma}_i^z; \qquad \sigma_i^z = \tilde{\sigma}_i^x \tilde{\sigma}_{i+1}^x$$

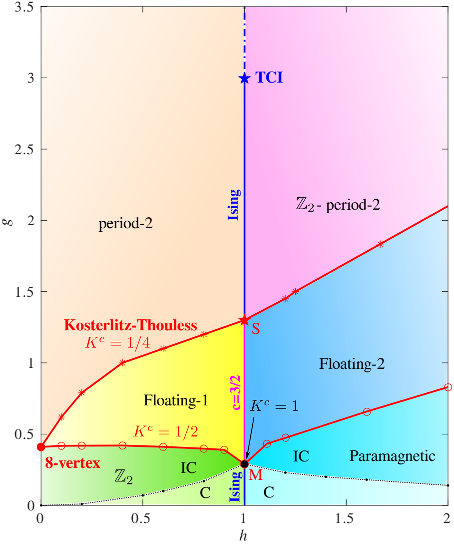
$$H = h\sum_i \tilde{\sigma}_i^x \tilde{\sigma}_{i+1}^x + \frac{1}{h}\tilde{\sigma}_i^z + \frac{g}{h}(\tilde{\sigma}_i^x \tilde{\sigma}_{i+2}^x + \tilde{\sigma}_i^z \tilde{\sigma}_{i+1}^z)$$

- Duality: $h \to h^{-1}$ and $g \to g/h$
- $h = 1$ self-dual line

$\mathbb{Z}_2$ phases

Exact zero modes

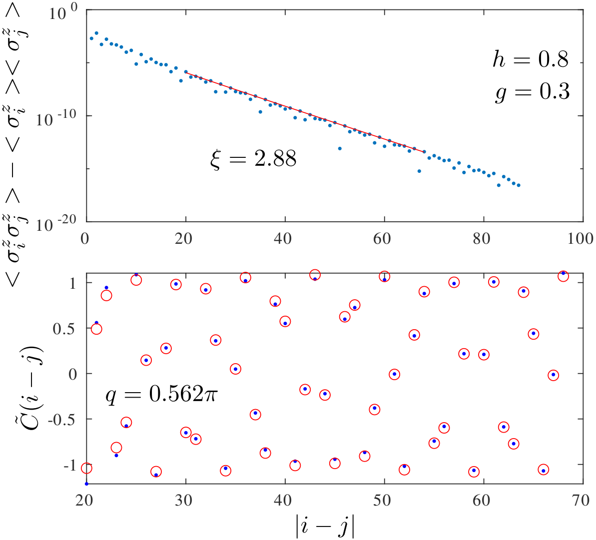
Disorder line



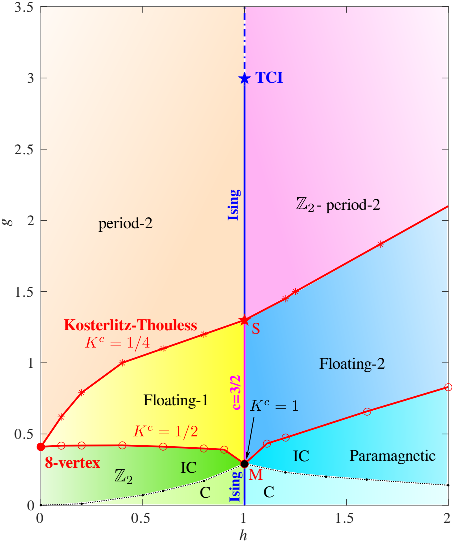
$$H = \sum_i \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z$$
$$+ g \sum_i (\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

- Beyond **the disorder line** correlations are incommensurate

Incommensurate correlations

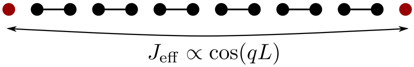


Exact zero modes



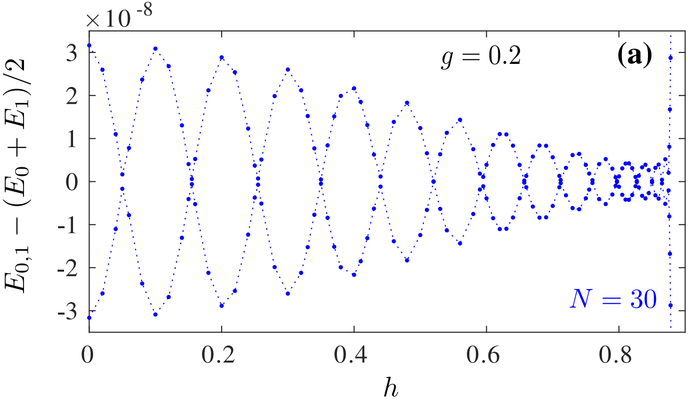
$$H = \sum_i \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z + g \sum_i (\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

- Beyond **the disorder line** correlations are incommensurate
- IC + egde states = Exact zero modes

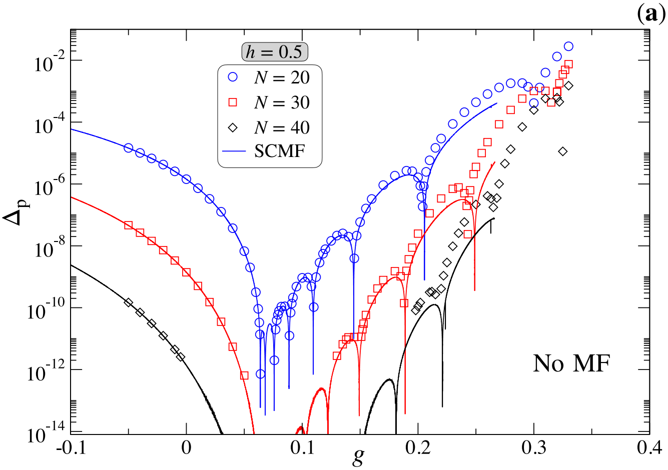


Toskovic et al. Nat. Phys. 12, 656 (2016);
Vionnet, Kumar, Mila, PRB 95, 174404 (2017);
NC, Mila, PRB 96, 060409 (2017);
NC, Mila, PRB 97, 174434 (2018)

Exact zero modes: $g = 0.2$



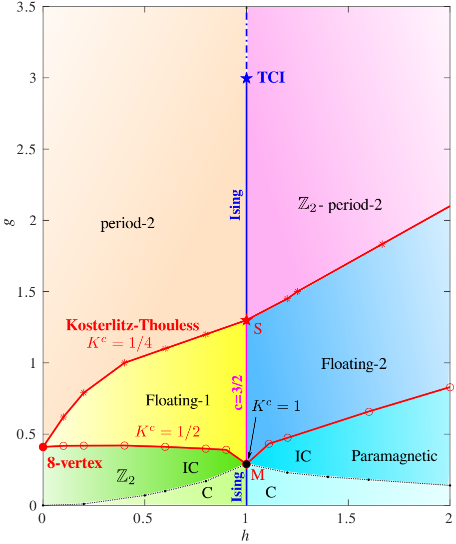
Exact zero modes: $h = 0.5$



Period-2-$\mathbb{Z}_2$ phase

Exact zero modes

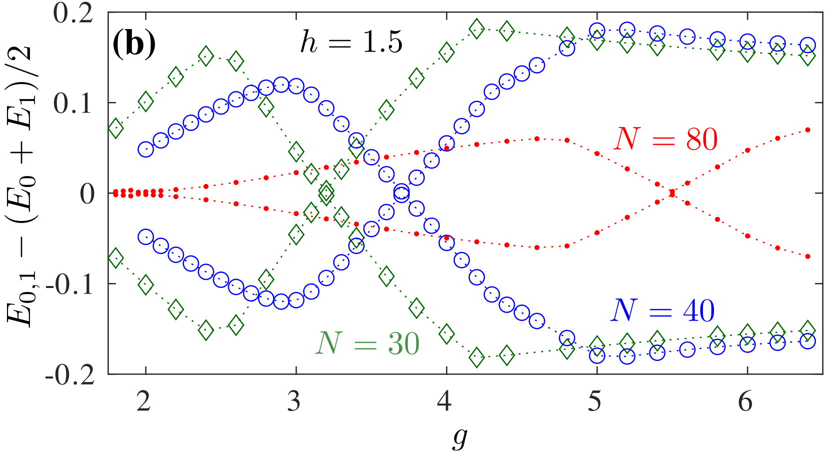
Phase diagram



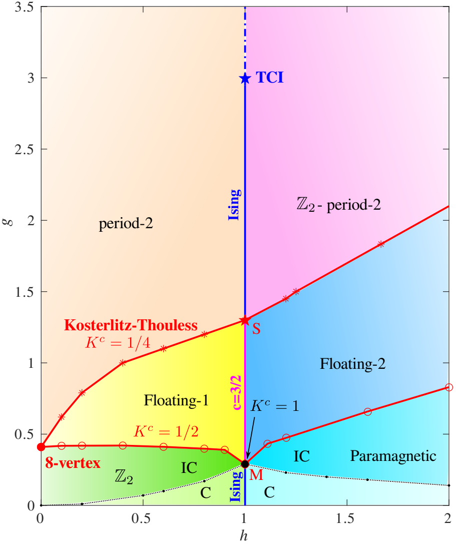
$$H = \sum_i \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z$$
$$+ g \sum_i (\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

- $\mathbb{Z}_2$ phases: exact zero modes

Exact zero modes in period-2-$\mathbb{Z}_2$



Phase diagram



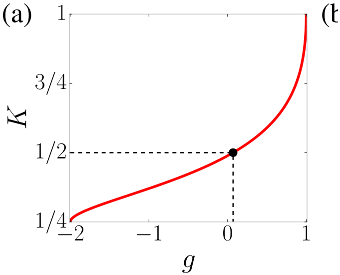
$$H = \sum_i \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z + g \sum_i (\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

- $\mathbb{Z}_2$ phases: exact zero modes
- **Floating-1 phase**
- **Kosterlitz-Thouless transitions**

Floating phase

Kosterlitz-Thouless transitions with Friedel oscillations

Stability of the Luttinger liquid

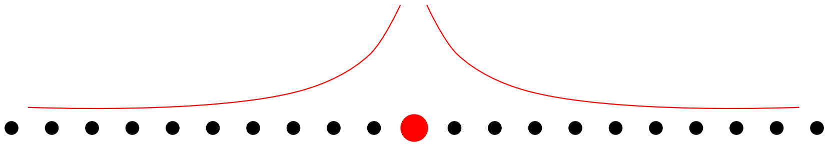


- Superconducting instability: $p^2/(4K)$ then $K^c = 1/2$
- Density wave: $K^c = (1-\rho_0)^2 = 1/4$
- Stable Luttinger liquid for $1/4 < K < 1/2$
- Emergent U(1) symmetry

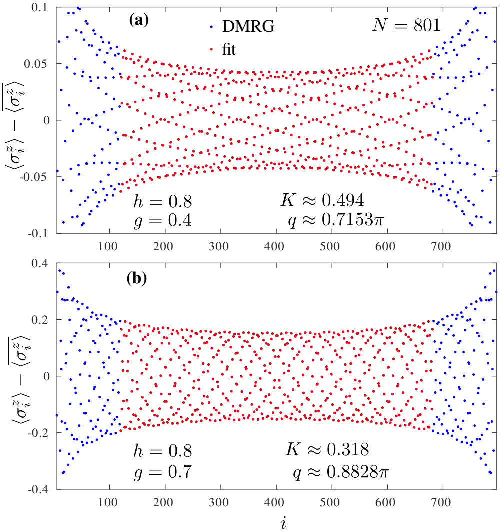
Verresen, Vishwanath, Pollmann, arXiv:1903.09179

Friedel oscillations

- Response of the system to an impurity
- In the gapped phase it decays exponentially
- At the critical point - with the corresponding critical exponent
- Open boundary conditions = impurity
- Prediction by boundary-CFT

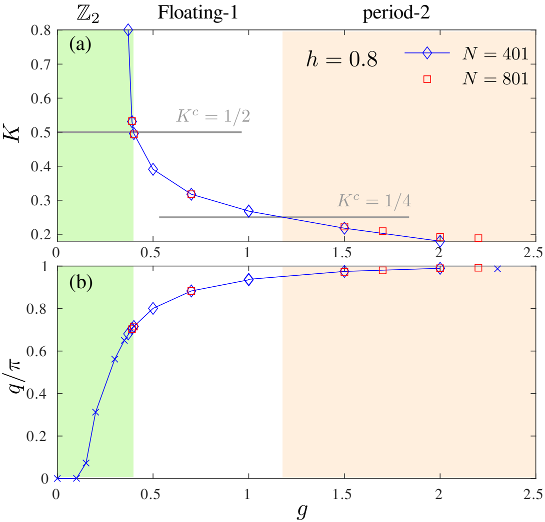


Friedel oscillations inside the floating phase



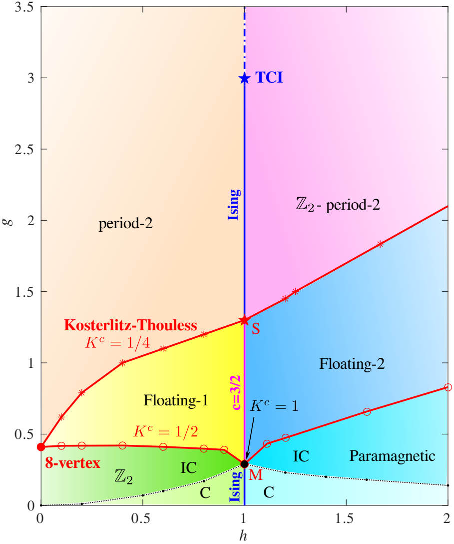
- Edges polarized along z!
- bCFT: $\sigma_i^z \propto \frac{\cos(qj)}{[(N/\pi)\sin(\pi j/N)]^K}$
- Scaling dimension = Luttinger liquid parameter K

Luttinger liquid exponent K & wave-vector q



- Stable Luttinger liquid for $1/4 < K < 1/2$
- Incommensurate in both $\mathbb{Z}_2$ and period-2 phases
- IC-IC Kosterlitz-Thouless transition

Phase diagram



$$H = \sum_i \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z + g \sum_i (\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

- $\mathbb{Z}_2$ phase: exact zero modes
- Kosterlitz-Thouless transitions
- **8-vertex critical point**

Multicritical point in the

8-vertex universality class

Eight-vertex criticality. Integrable model

$$H = \sum_i J_x \sigma_i^x \sigma_{i+1}^x + J_y \sigma_i^y \sigma_{i+1}^y + J_z \sigma_i^z \sigma_{i+1}^z - B\sigma_i^z,$$

- Critical, in 8-vertex universality class at $J_x = -J_z$ and $B = 0$
- Control parameter:

$$\rho = \mathrm{acos}[J_y/J_x].$$

- Critical exponents

$$\nu = \pi/(2\rho), \quad \beta = (\pi - \rho)/(4\rho),$$

- Scaling dimenision

$$d = \beta/\nu = \pi - \rho/(2\pi)$$

Eight-vertex criticality. Integrable model

$$H = \sum_i J_x \sigma_i^x \sigma_{i+1}^x + J_y \sigma_i^y \sigma_{i+1}^y + J_z \sigma_i^z \sigma_{i+1}^z - B\sigma_i^z$$

$$H_{\mathrm{NN}} = \sum_i -t(d_i^\dagger d_{i+1} + \mathrm{h.c.}) + \lambda(d_i^\dagger d_{i+1}^\dagger + \mathrm{h.c.}) - \mu n_i + V n_i n_{i+1}$$

- Critical, in 8-vertex universality class at $\lambda = (V-2t)/2$ and $V = \mu$
- Control parameter:

$$\rho = \mathrm{acos}[(1-\lambda)/(1+\lambda)].$$

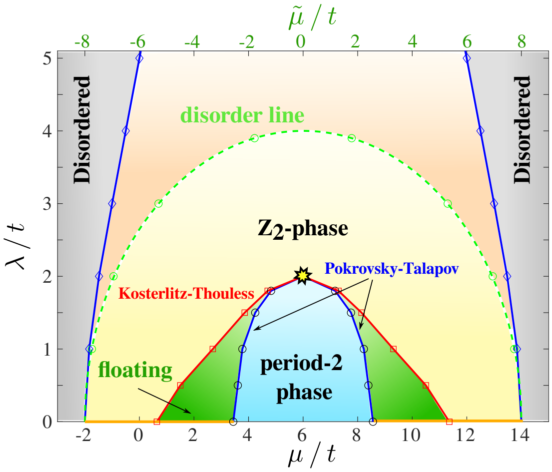
- Critical exponents

$$\nu = \pi/(2\rho), \quad \beta = (\pi - \rho)/(4\rho),$$

- Scaling dimenision

$$d = \beta/\nu = \pi - \rho/(2\pi)$$

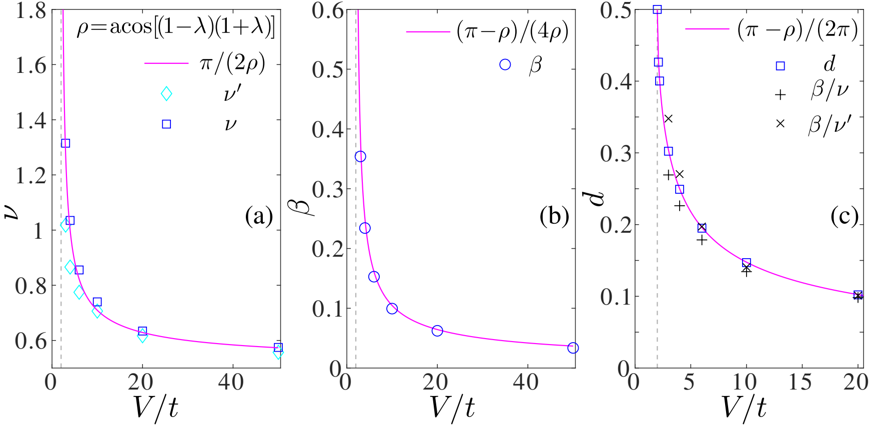
Eight-vertex criticality. Integrable model



- Multicritical point at the particle-hole symmetry line

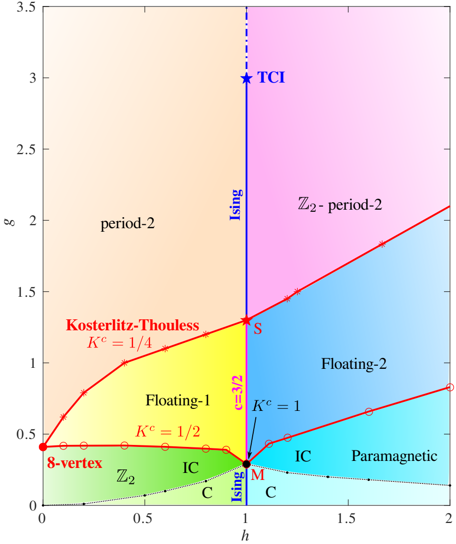
NC, Mila, arXiv:2206.11754

Eight-vertex criticality. Integrable model



NC, Mila, arXiv:2206.11754

Phase diagram



$$H = \sum_i \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z$$
$$+ g \sum_i (\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

- $\mathbb{Z}_2$ phase: exact zero modes
- Kosterlitz-Thouless transitions
- **8-vertex critical point**

Eight-vertex criticality. Majorana chain

$$H = \sum_i \sigma_i^x \sigma_{i+1}^x \cancel{+ J_y \sigma_i^y \sigma_{i+1}^y} + g\sigma_i^x \sigma_{i+2}^x + g\sigma_i^z \sigma_{i+1}^z - h\sigma_i^z$$

- Multicritical point: $\mathbb{Z}_2$, Floating-1, period-2
- Particle-hole symmetry at $h = 0$
- 8-vertex universality class?
- Control parameter: no prediction!

$$\cancel{\rho = \mathrm{acos}[(1-\lambda)/(1+\lambda)]}$$

- Critical exponents: still valid

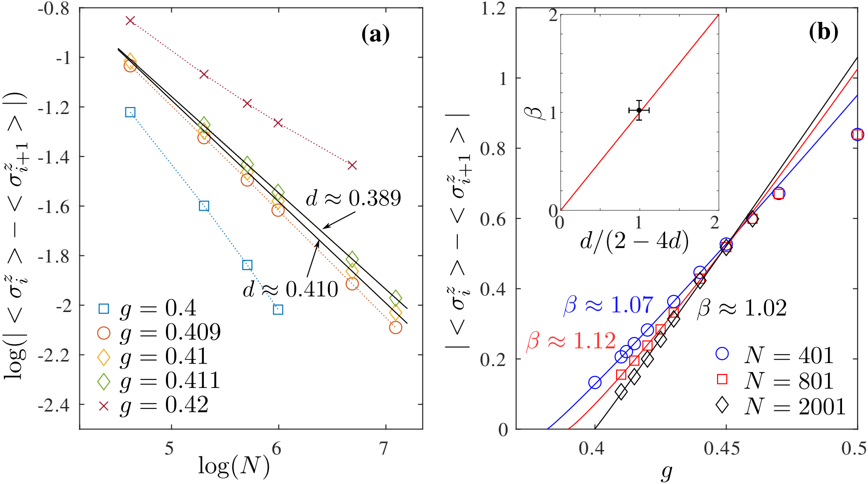
$$\nu = \pi/(2\rho), \quad \beta = (\pi - \rho)/(4\rho),$$

$$d = \pi - \rho/(2\pi)$$

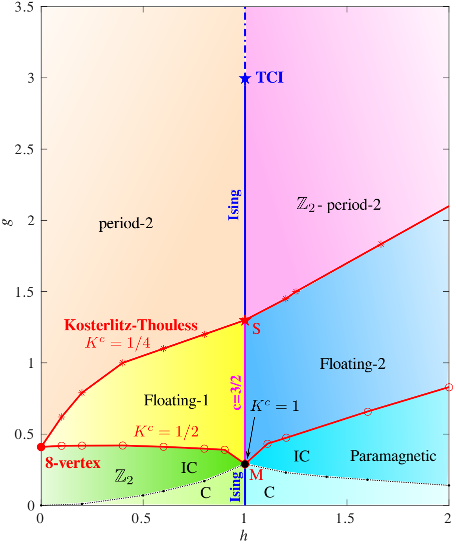
- Express one exponent in terms of another:

$$\beta = d/(2 - 4d)$$

Eight-vertex criticality. Majorana chain



Phase diagram



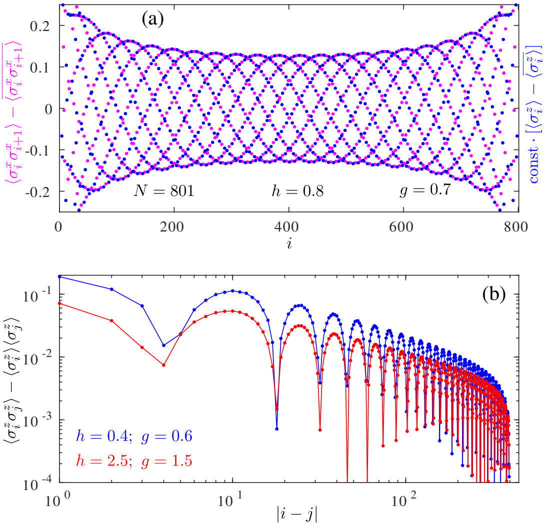
$$H = \sum_i \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z + g \sum_i (\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

- $\mathbb{Z}_2$ phase: exact zero modes
- Kosterlitz-Thouless transitions
- 8-vertex critical point
- **Floating-1 vs Floating-2**

Floating-1 vs Floating-2

Broken $\mathbb{Z}_2$ symmetry

Floating phases: more than just dual?



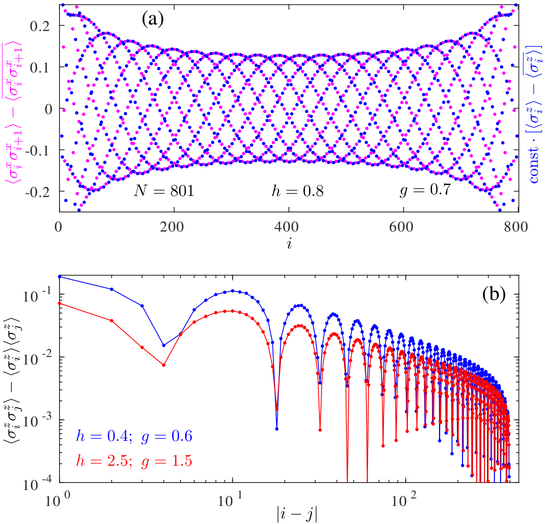
Duality implies:

- $\langle\sigma_i^z\sigma_j^z\rangle_c$ at (h, g) scale as $\langle\sigma_i^x\sigma_{i+1}^x\sigma_j^x\sigma_{j+1}^x\rangle_c$ at $(1/h, g/h)$

But it is not the end of the story:

- $\langle\sigma_i^z\sigma_j^z\rangle_c$ scale the same in dual points
- And σ_i^z and $\sigma_i^x\sigma_{i+1}^x$ scale the same at a single point

Floating phases: more than just dual?



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Translation symmetry in Majorana fermions: $\langle a_i b_i a_j b_j \rangle \propto \langle b_i a_{i+1} b_j a_{j+1} \rangle$

The model

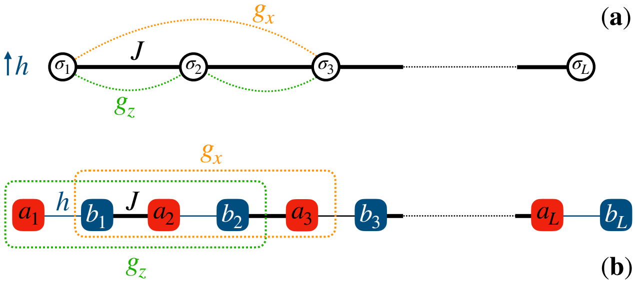
Spin chain:

$$H = \sum_j J\sigma_j^x\sigma_{j+1}^x - h\sigma_j^z + g\sigma_j^z\sigma_{j+1}^z + g\sigma_j^x\sigma_{j+2}^x$$

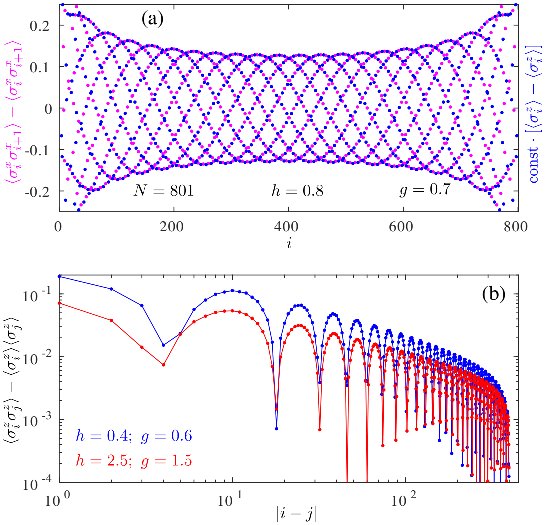
Majorana chain:

$$H = \sum_j it_{\text{even,odd}}\gamma_j\gamma_{j+1} - g\gamma_j\gamma_{j+1}\gamma_{j+2}\gamma_{j+3}$$

But we do break translation symmetry!



Floating phases: more than just dual?



Duality implies:

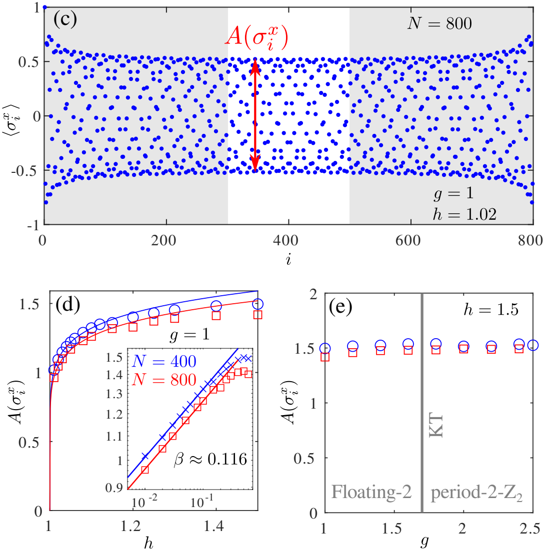
- $\langle \sigma_i^z \sigma_j^z \rangle_c$ at (h, g) scale as $\langle \sigma_i^x \sigma_{i+1}^x \sigma_j^x \sigma_{j+1}^x \rangle_c$ at $(1/h, g/h)$

But it is not the end of the story:

- $\langle \sigma_i^z \sigma_j^z \rangle_c$ scale the same in dual points
- And σ_i^z and $\sigma_i^x \sigma_{i+1}^x$ scale the same at a single point

Emergent Translation symmetry in Majorana fermions: $\langle a_i b_i a_j b_j \rangle \propto \langle b_i a_{i+1} b_j a_{j+1} \rangle$

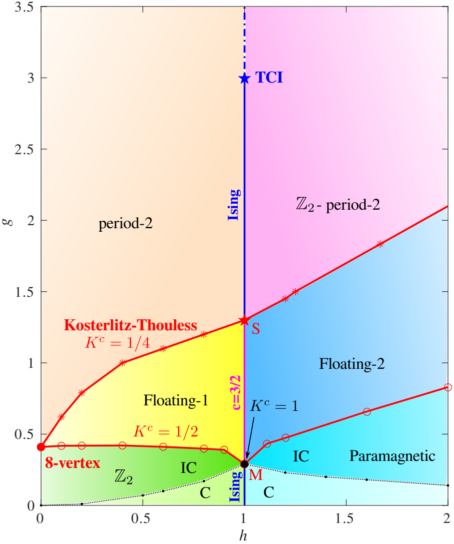
Floating-2 with broken $\mathbb{Z}_2$ symmetry



Order parameter

- Commensurate Ising: $|\sigma_i^x - \sigma_{i+1}^x|$
- Generalization to IC: $A(\sigma_i^x)$
- Decay towards $h = 1$ with $\beta \approx 0.116$
- Ising $\beta = 1/8$
- Insensitive to Kosterlitz-Thouless transition

Phase diagram



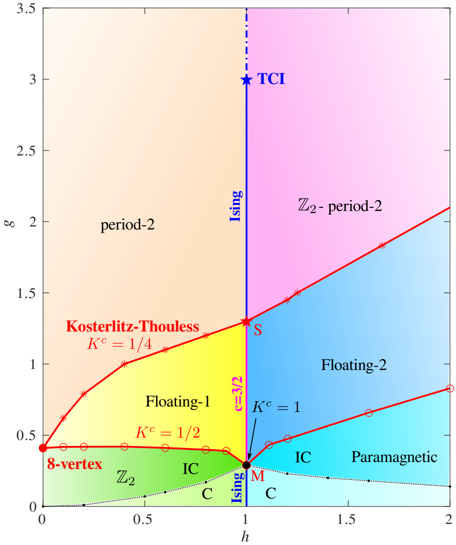
$$H = \sum_i \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z + g \sum_i (\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

- $\mathbb{Z}_2$ phase: exact zero modes
- Kosterlitz-Thouless transitions
- 8-vertex critical point
- Period-2-$\mathbb{Z}_2$ phase
- Floating-1 vs Floating-2
- $h = 1$ line
  - Multicritical points
  - Ising transition and TCI
  - No generalized C-IC transition

$h = 1$ line

Multicritical points
Ising transition and TCI end point
No generalized C-IC transition

Phase diagram



$$H = \sum_i \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z + g \sum_i (\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z)$$

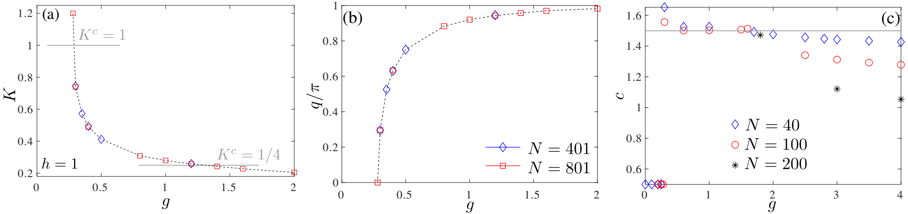
Point M:

- Pairing instability is relevant for $K > 1/2$
- In paramagnet spin flip creates a pair of domain walls
- ... thus $K^c = 1/2$
- BUT none of the two are relevant perturbations along $h = 1$
- $K^c = 1$ is in the free-fermion theory

Point S:

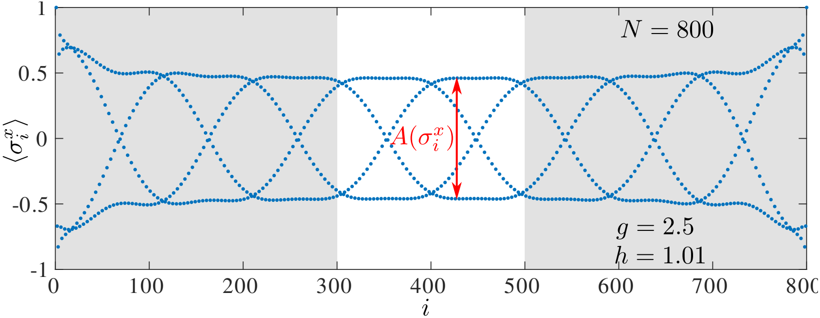
- translation is broken everywhere $K^c = 1/4$

Boundaries of the Luttinger liquid



- Luttinger liquid for $1/4 < K < 1 \Leftrightarrow 0.29 \lesssim g \lesssim 1.3$
- Central charge
  - $c = 1/2$ for $g \lesssim 0.29$
  - $c = 3/2$ $0.29 \lesssim g \lesssim 1.8$
  - systematically deviates from $c = 3/2$ for $g \gtrsim 1.8$
  - ... well below the end point $g \approx 3$

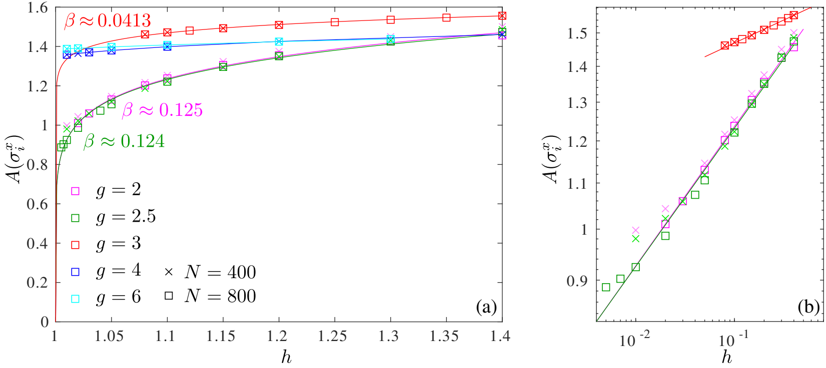
Friedel oscillations in period-2-$\mathbb{Z}_2$ phase



Commensurate Ising transition: $|\sigma_i^x - \sigma_{i+1}^x|$

Incommensurate Ising transition $\rightarrow A(\sigma_i^x)$

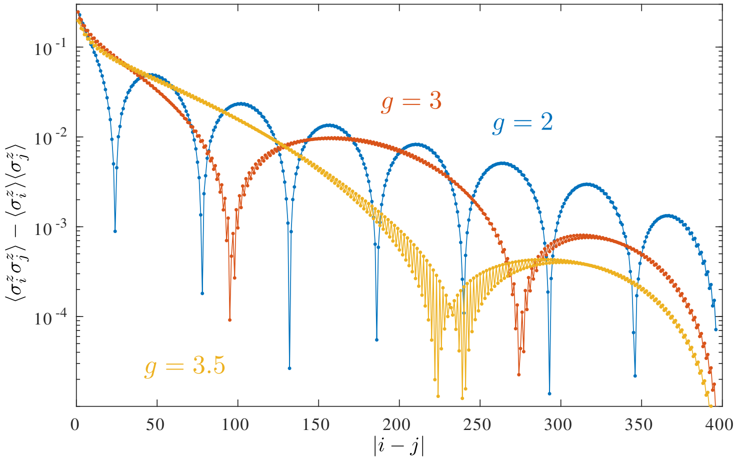
Transition between period-2 and period-2-$\mathbb{Z}_2$ phases



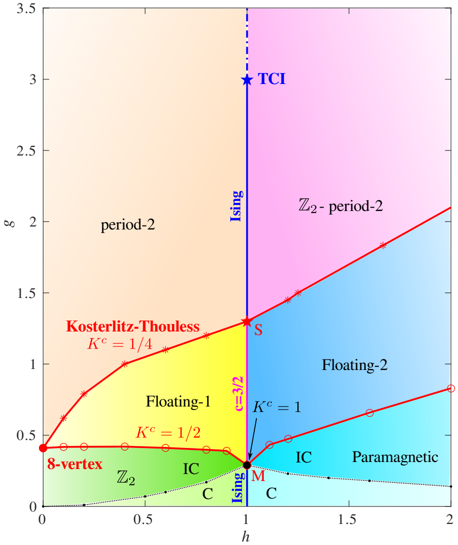
Ising: $\beta = 1/8 = 0.125$

Tri-critical Ising: $\beta = 1/24 \approx 0.0417$

The persistence of incommensurability

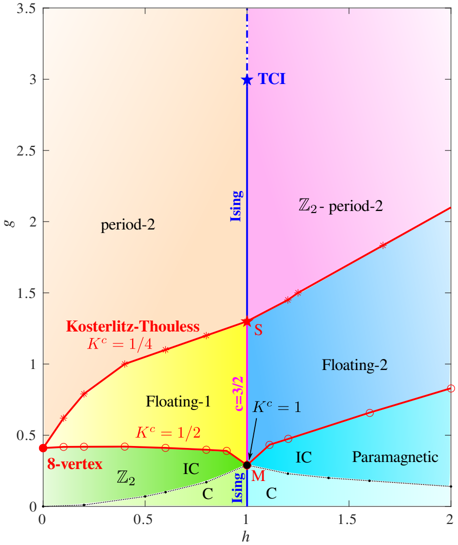


Summary



- Rich phase diagram
- Two floating phases separated by an Ising transition
- Extended phase diagram shed a light on a symmetric model
- No generalized C-IC transition
  - LL terminates at $g \approx 1.3$
  - Ising transition continues and...
  - terminates at the TCI at $g \approx 3$
  - IC persists beyond it

Outlook

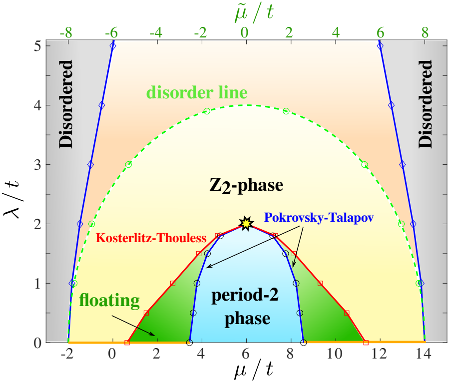


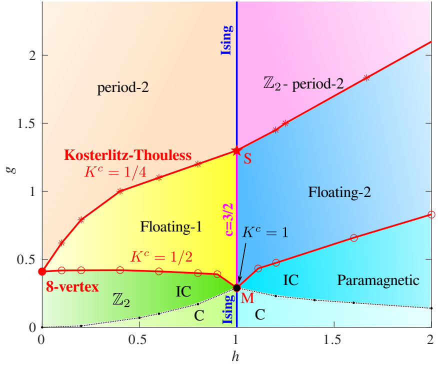
- Dual 8-vertex point
  - Symmetries are broken on one side of the transition

Pokrovsky-Talapov vs Kosterlitz-Thouless

$$H_{\rm NN} = \sum_i -t(d_i^\dagger d_{i+1} + \text{h.c.}) - \mu n_i + \lambda(d_i^\dagger d_{i+1}^\dagger + \text{h.c.}) + V n_i n_{i+1}$$

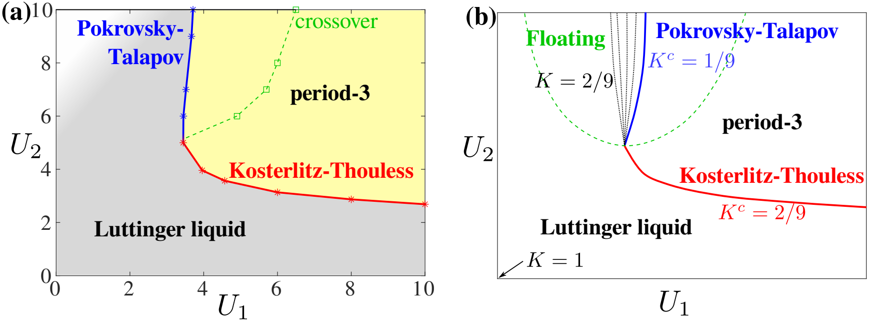
$$H = \sum_i J(d_i^\dagger d_{i+1} + d_i^\dagger d_{i+1}^\dagger + \text{h.c.}) - (2h+4g)n_i + 4g n_i n_{i+1} + g(d_i^\dagger d_{i+2} + d_i^\dagger d_{i+2}^\dagger + \text{h.c.})$$





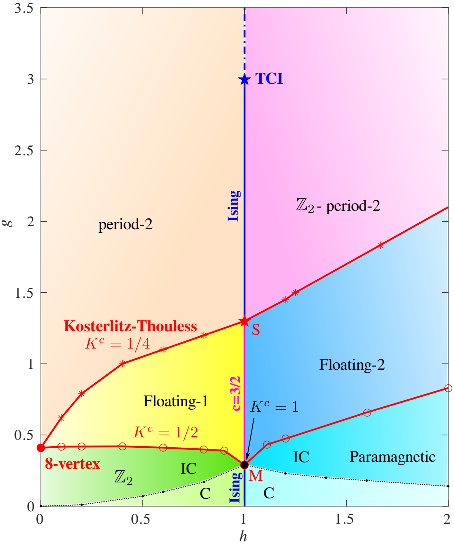
Switch from Kosterlitz-Thouless to Pokrovsky-Talapov

$$H_{\text{ferm}} = \sum_i -t(c_i^\dagger c_{i+1} + \text{h.c.}) + U_1 n_i n_{i+1} + U_2 n_i n_{i+2},$$



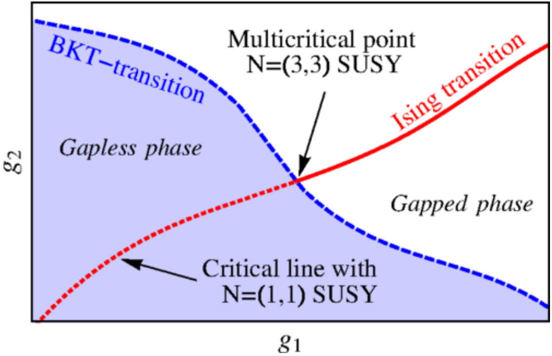
NC, arXiv:2209.10390

Outlook



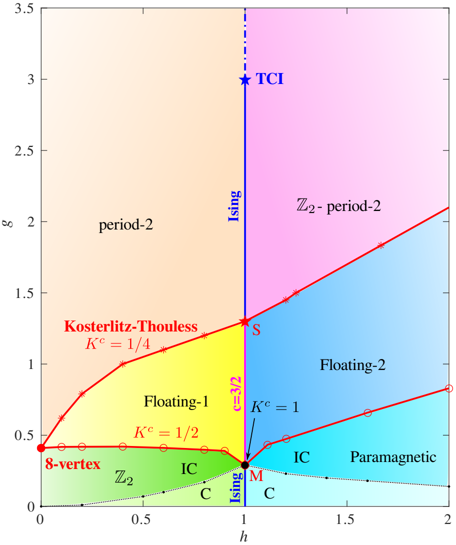
- Dual 8-vertex point
    - Symmetries are broken on one side of the transition
- Kosterlitz-Thouless vs Pokrovsky-Talapov
    - Can we interpolate between the two?
- Emergent supersymmetry

Phase diagram



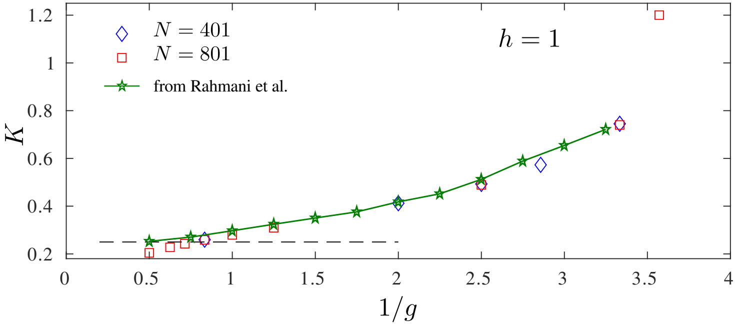
Huijse, Bauer, Berg, PRL 114, 090404; Sitte, Rosch, Meyer, Matveev, Garst, PRL 102, 176404 (2009)

Outlook



- Dual 8-vertex point
    - Symmetries are broken on one side of the transition
- Kosterlitz-Thouless vs Pokrovsky-Talapov
    - Can we interpolate between the two?
- Emergent supersymmetry
    - Fermion vs boson velocities

Similar results... Different conclusions



Rahmani, Zhu, Franz, Affleck, Phys. Rev. B 92, 235123 (2015);