Entanglement, Symmetries, and Asymmetry

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Entanglement and symmetries

Let's assume that $|\psi\rangle$ is symmetric under the action of a charge Q, i.e $[\rho, Q] = 0$
The charge is local: $Q = Q_A + Q_B$

$$[\rho, Q] = 0 \xrightarrow{\mathrm{Tr}_B} [\rho_A, Q_A] = 0$$

$\Rightarrow$ ρ_A has a block diagonal form:

$$\rho_A = \begin{pmatrix} q_1 & & & \\ & q_2 & & \\ & & q_3 & \\ & & & \ddots \end{pmatrix}$$

$$\rho_A = \oplus_q \Pi_q \rho_A = \oplus_q \left[p(q)\rho_A(q)\right] \quad \text{with} \quad p(q) = \mathrm{Tr}\left(\Pi_q \rho_A\right)$$

probability of being in the sector q

<u>Symmetry resolved entanglement entropy:</u>

$$\boxed{S(q) = -\,\mathrm{Tr}[\rho_A(q) \log \rho_A(q)]}$$

Entanglement and symmetries II

The symmetry resolved entanglement satisfies the sum rule

$$S = \sum_q p(q)S(q) - \sum_q p(q)\log(p(q)) \equiv S^c + S^n$$

S^c: Configurational entropy

S^n: Number entropy

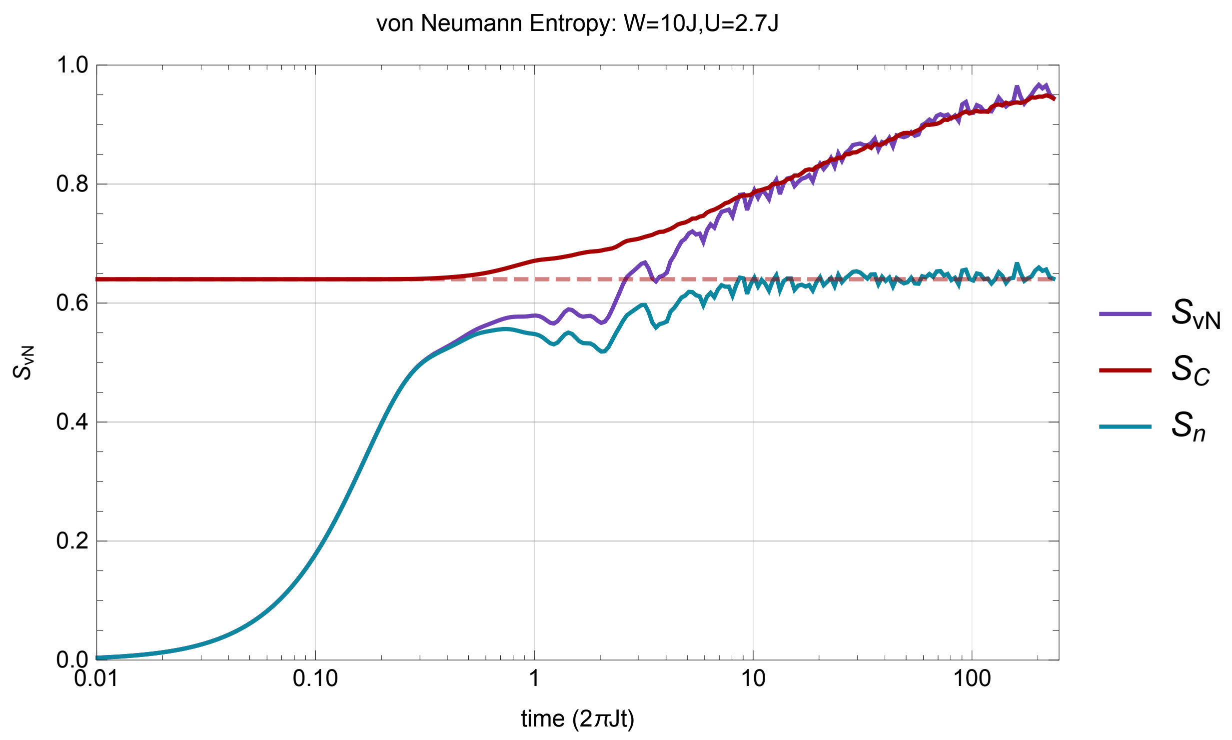
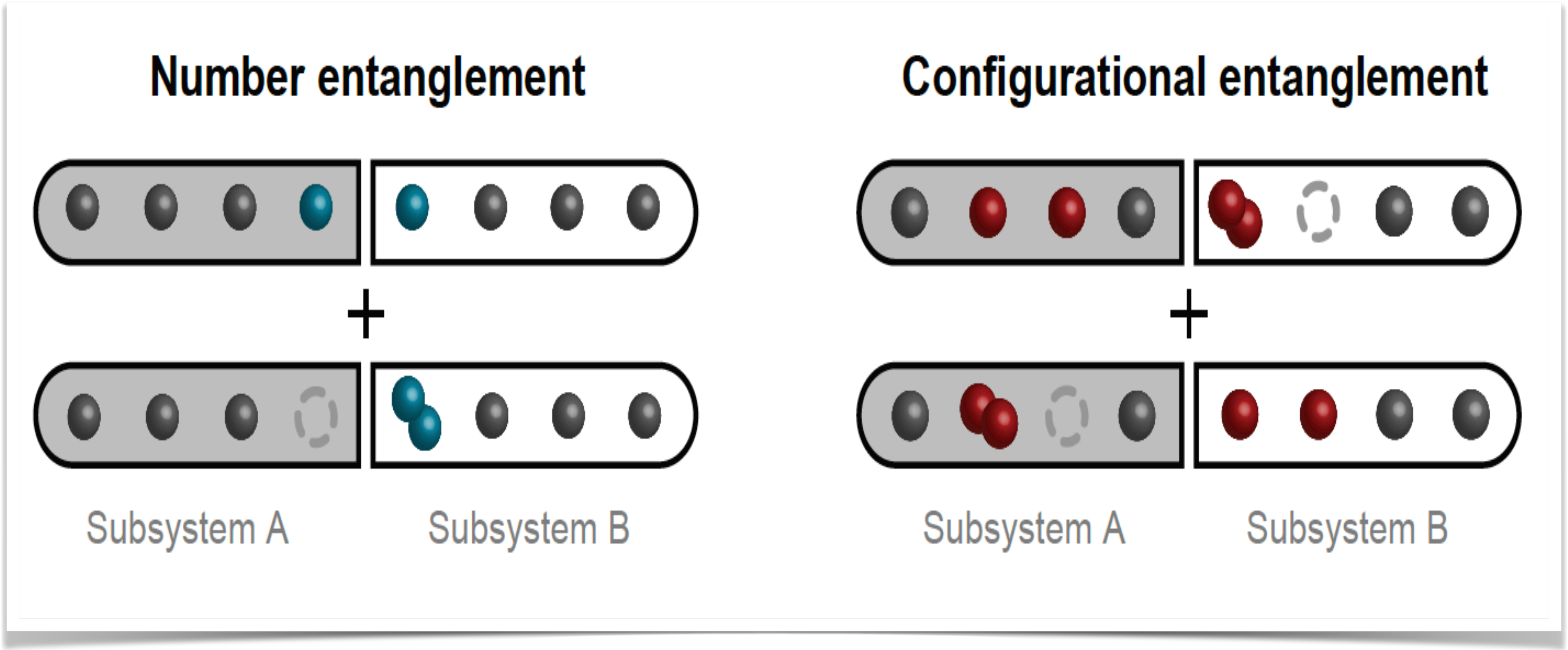


FIG. S8. **Total entropy partitioned** The total von Neumann entanglement entropy $S_{\rm vN}$ for the half-system is shown as a function of time in an interacting system at strong disorder. The entropy is split up into $S_{\rm n}$ and $S_{\rm c}$. For visual



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Entanglement and symmetries: results

Early work

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SRE in CFT

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B. Estienne et al, SciPost Phys. 10, 54 (2021)
S. Murciano, J. Dubail, P. Calabrese, JHEP 10 (2021) 067
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Relative entropy and distances:
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Free QFT

S. Murciano, G. Di Giulio, and P. Calabrese, JHEP 08 (2020) 073
L. Capizzi, O. Castro-Alvaredo, C. De Fazio, M. Mazzoni, and L. Santamaria-Sanz, arXiv:2203.12556

Disorder Systems

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M. Kiefer-Emmanouilidis, R. Unanyan, J. Sirker, and M. Fleischhauer, PRL 124, 243601 (2020).

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Lattice free fermions

R. Bonsignori, P. Ruggiero, and P. Calabrese, JPA 52, 475302 (2019)
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S. Fraenkel and M. Goldstein, JSTAT 033106 (2020)
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F. Ares, S. Murciano, and P. Calabrese, ArXiv:2202.05874
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Non-equilibrium and quantum quenches

N. Feldman and M. Goldstein, PRB 100, 235146 (2019)
G. Parez, R. Bonsignori and P. Calabrese, PRB 103, L041104 (2020)
V. Vitale, A. Elben, R. Kueng, A. Neven, J. Carrasco, B. Kraus, P. Zoller, P. Calabrese, B. Vermersch, and M. Dalmonte, SciPost Phys. 12, 106 (2022)
S. Fraenkel and M. Goldstein, SciPost Phys. 11, 085 (2021).
G. Parez, R. Bonsignori and P. Calabrese, PRB 103, L041104 (2021); J. Stat. Mech. (2021) 093102; ArXiv:2202.05309
S. Scopa and D. Horvath, ArXiv:2205.02924

Integrability

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U(1) Symmetry resolution in CFT

M. Goldstein and E. Sela, PRL 120, 200602 (2018)
J.C. Xavier, F.C. Alcaraz, and G. Sierra, PRB 98, 041106 (2018)

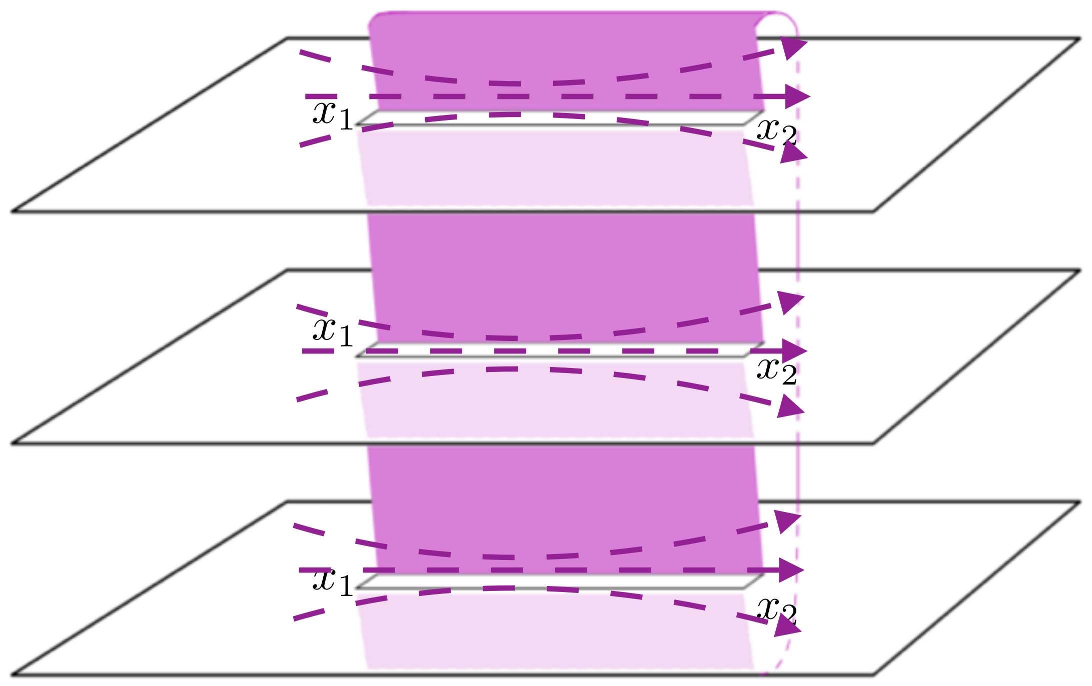
Symmetry resolved Renyi: $S_n(q) \equiv \dfrac{1}{1-n} \log \mathrm{Tr} \rho_A^n(q)$

It requires the resolution of the spectrum in Q

Introduce the charged moments: $\boxed{Z_n(\alpha) \equiv \mathrm{Tr} \rho_A^n e^{iQ_A \alpha}}$ $\Rightarrow$ $\mathcal{Z}_n(q) \equiv \mathrm{Tr}(\Pi_q \, \rho_A^n) = \displaystyle\int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{-iq\alpha} Z_n(\alpha)$

$p(q) = \mathcal{Z}_1(q)$

$\Rightarrow$ $S_n(q) = \dfrac{1}{1-n} \log \left[\dfrac{\mathcal{Z}_n(q)}{\mathcal{Z}_1(q)^n} \right], \qquad S_1(q) = -\partial_n \left[\dfrac{\mathcal{Z}_n(q)}{\mathcal{Z}_1(q)^n} \right]_{n=1}$



$Z_n(\alpha)$ is the partition function in the presence of a charge flux.
The field takes a total phase α going through $\mathscr{R}_n$.
In CFT it is placed all in one sheet introducing the composite field:

$$\mathcal{T}_{n,\alpha}(x,\tau)\phi_i(x',\tau) = \begin{cases} \phi_{i+1}(x',\tau) e^{i\alpha\delta_{i,n}} \mathcal{T}_{n,\alpha}(x,\tau) & (x < x'), \\ \phi_i(x',\tau)\mathcal{T}_{n,\alpha}(x,\tau) & \text{otherwise.} \end{cases}$$

$\Rightarrow$ $Z_n(\alpha) = \langle \mathcal{T}_{n,\alpha}(\ell,0)\tilde{\mathcal{T}}_{n,\alpha}(0,0)\rangle$ $\Rightarrow$ $h_{n,\alpha} = h_n + \dfrac{h_\alpha}{n}$ $\Rightarrow$ $Z_n(\alpha) = c_{n,\alpha} \ell^{-\frac{c}{6}\left(n-\frac{1}{n}\right)-2\frac{h_\alpha+\bar{h}_\alpha}{n}}$

Symmetry resolution in CFT: Luttinger liquid

Action: $\mathcal{S}_E[\varphi] = \frac{1}{8\pi K}\int d\tau dx\ \partial_\mu\varphi\partial^\mu\varphi$ Conserved charge $Q_A = \frac{1}{2\pi}\int_A \partial\varphi(x,0)dx$ $\Rightarrow$ $e^{i\alpha Q_A} = e^{i\frac{\alpha}{2\pi}\varphi(u,0)}e^{-i\frac{\alpha}{2\pi}\varphi(v,0)}$

$\Rightarrow$ $h_\alpha = \bar{h}_\alpha = \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2 K$ $\Rightarrow$ $Z_n(\alpha) = c_{n,\alpha}\ell^{-\frac{c}{6}(n-\frac{1}{n})-\frac{2K}{n}\left(\frac{\alpha}{2\pi}\right)^2}$

Fourier transform using saddle point

$$\Rightarrow \mathcal{Z}_n(q) = c_n\ell^{-\frac{c}{6}(n-\frac{1}{n})}\sqrt{\frac{n\pi}{2K\ln\ell+\gamma_n}}e^{\frac{n\pi^2(q-\langle Q_A\rangle)^2}{2K\ln\ell+\gamma_n}}$$

$$\Rightarrow \boxed{S_n(q) = S_n - \frac{1}{2}\log\left(\frac{2K}{\pi}\log\ell\right) + \frac{\log n}{2(1-n)} + o(\ell^0)}$$

Entanglement equipartition: up to order $o(1)$, the SR entanglement does not depend on the symmetry sector

J.C. Xavier, F.C. Alcaraz, and G. Sierra, PRB 98, 041106 (2018)

Symmetry resolution in CFT: compact boson II

R. Bonsignori, P. Ruggiero, and P. Calabrese, JPA 52, 475302 (2019)

$$S_n(q) = S_n - \frac{1}{2}\log\left(\frac{2K}{\pi}\log\ell\right) + \frac{\log n}{2(1-n)} + o(\ell^0)$$

Q: Where the log log term ends up in the total entropy?

A: It is exactly canceled by the number entropy:

$$S = \sum_q p(q)S(q) - \sum_q p(q)\log(p(q)) \equiv S^c + S^n$$

$$S^n = \frac{1}{2} + \frac{1}{2}\ln\left(\frac{2K}{\pi}\ln\ell\right) + o(1)$$

Note: The number entropy satisfies $S^n \ll S \sim S(q)$, a fact valid much more generally

Lattice free fermions

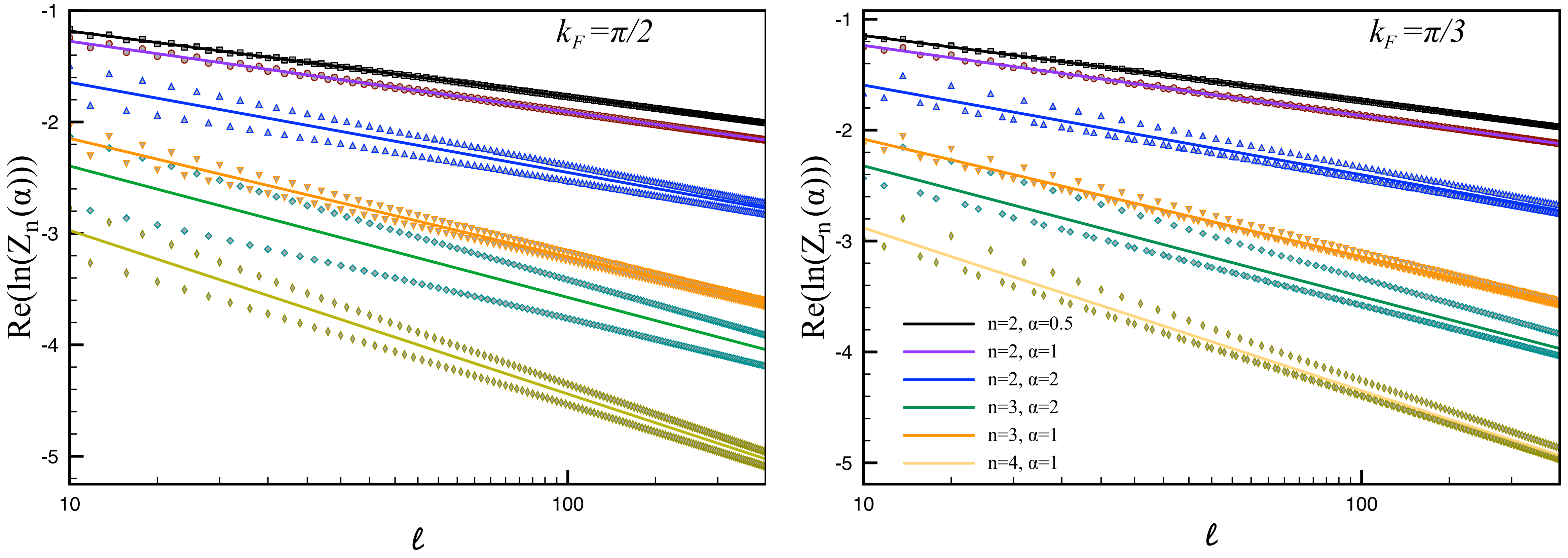
R. Bonsignori, P. Ruggiero, and P. Calabrese, JPA 52, 475302 (2019)

$$H = -\sum_{i=-\infty}^{\infty}\left[c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i - 2h\left(c_i^\dagger c_i - \frac{1}{2}\right)\right]$$

Using Fisher Hartwig techniques:

$$\ln Z_n^{(0)}(\alpha) = i\alpha\frac{k_F\ell}{\pi} - \left[\frac{1}{6}\left(n-\frac{1}{n}\right) + \frac{2}{n}\left(\frac{\alpha}{2\pi}\right)^2\right]\ln L_k + \Upsilon(n,\alpha)$$

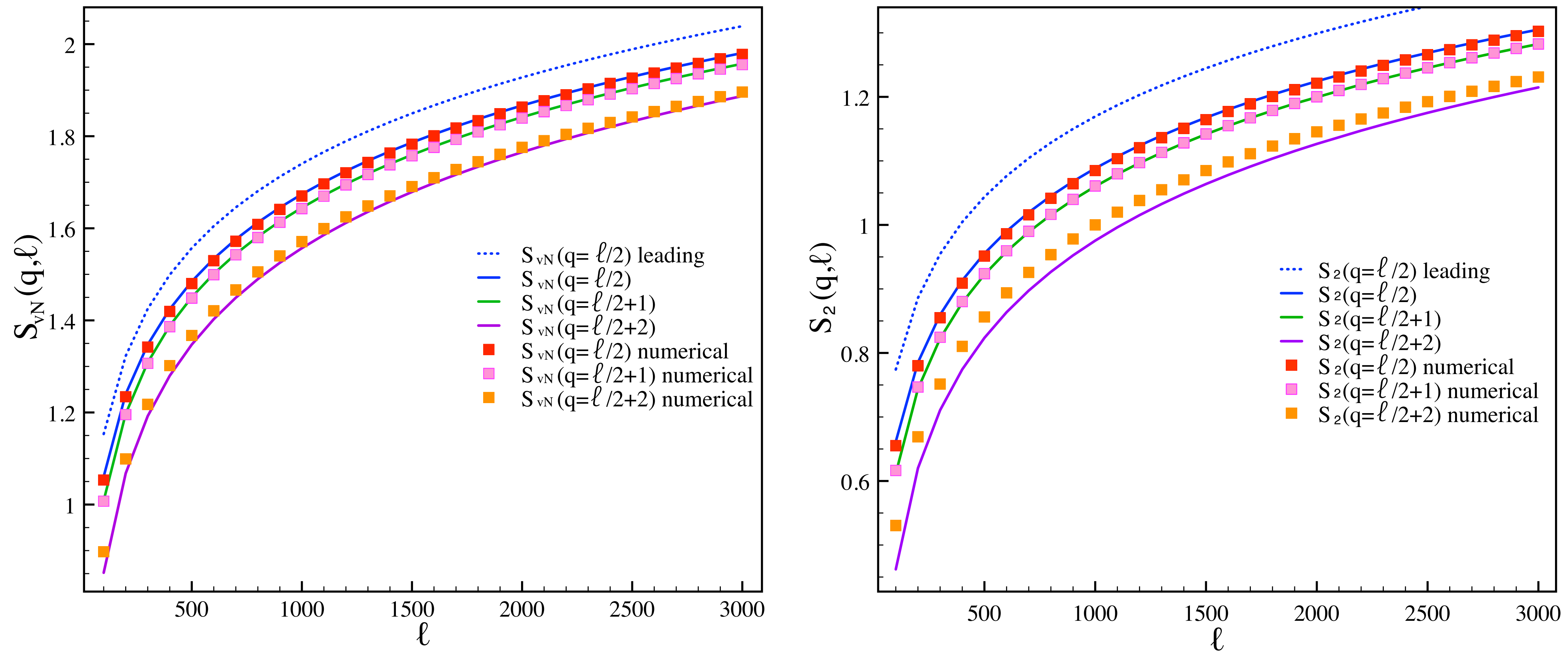
$$\Upsilon(n,\alpha) = ni\int_{-\infty}^{\infty} dw[\tanh(\pi w) - \tanh(\pi n w + i\alpha/2)]\ln\frac{\Gamma(\frac{1}{2}+iw)}{\Gamma(\frac{1}{2}-iw)}$$



Fourier tranform + ratios for entropies

$$S_n(q) = S_n - \frac{1}{2}\ln\left(\frac{2}{\pi}\ln\delta_n L_k\right) + \frac{\ln n}{2(1-n)} + (q-\bar{q})^2\,\pi^4\frac{n(\gamma_2(1)-n\gamma_2(n))}{1-n}\frac{1}{\ln^2\kappa_n L_k} + \cdots$$

Equipartition is broken at order $(\log\ell)^{-2}$



Resolution of non-abelian symmetries: WZW models

S. Murciano, J. Dubail, P. Calabrese, JHEP 10 (2021) 067

Consider a general non-abelian group G (of dimension *d* and volume *Vol(G)*) and the corresponding WZW model

$\rho_A = \oplus_r \, [p(r)\rho_A(r)]$ $\quad$ r labels the irreducible representations of G, $\dim(r)$ its dimension

SU(2) done by Goldstein and Sela in 2018 paper using *SU(2)* algebra

Our Strategy (without mentioning many highly non trivial points and assumptions)

- Write the charged moments as a linear combination of the unspecialised characters
- Use their modular properties to compute the resolved partition functions, by identifying all states in a given representation of the group.
- The SR entropies are obtained integrating the group characters around all saddles (that are the elements of the center *Z(G)* (of order |Z(G)|)

Final Result

$$S_n^r(L) = S_n(L) - \frac{d}{2}\log(\log L) + 2\log\dim(r) - \log\frac{\mathrm{Vol}(G)}{|Z(G)|} + \frac{d}{2}\left(-\log k + \frac{\log n}{1-n} + \log(2\pi^3)\right) + o(L^0)$$

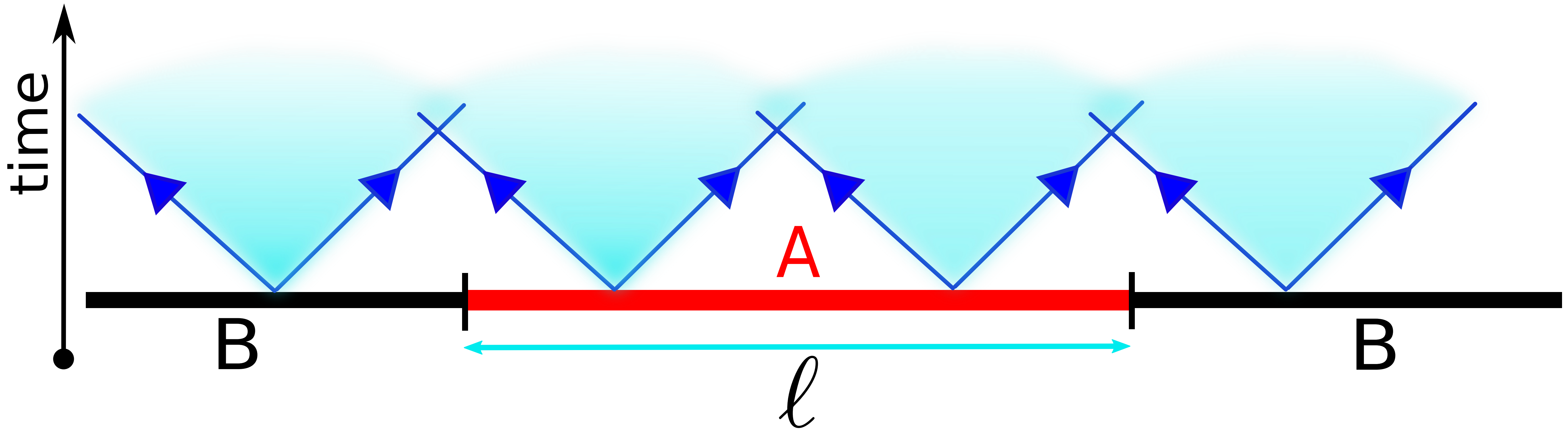
Equipartition broken at order O(1)!!

SRE after a quantum quench

Prepare a system in a low-entangled initial state $|\psi_0\rangle$ and let it evolve unitarity $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$

Long story short: In integrable models the entanglement dynamics is captured by the **quasiparticle picture**

PC & Cardy, 2005 + Alba & PC 2017



$$S = \int \frac{dk}{2\pi} h(k) \min[2v_k t, \ell]$$

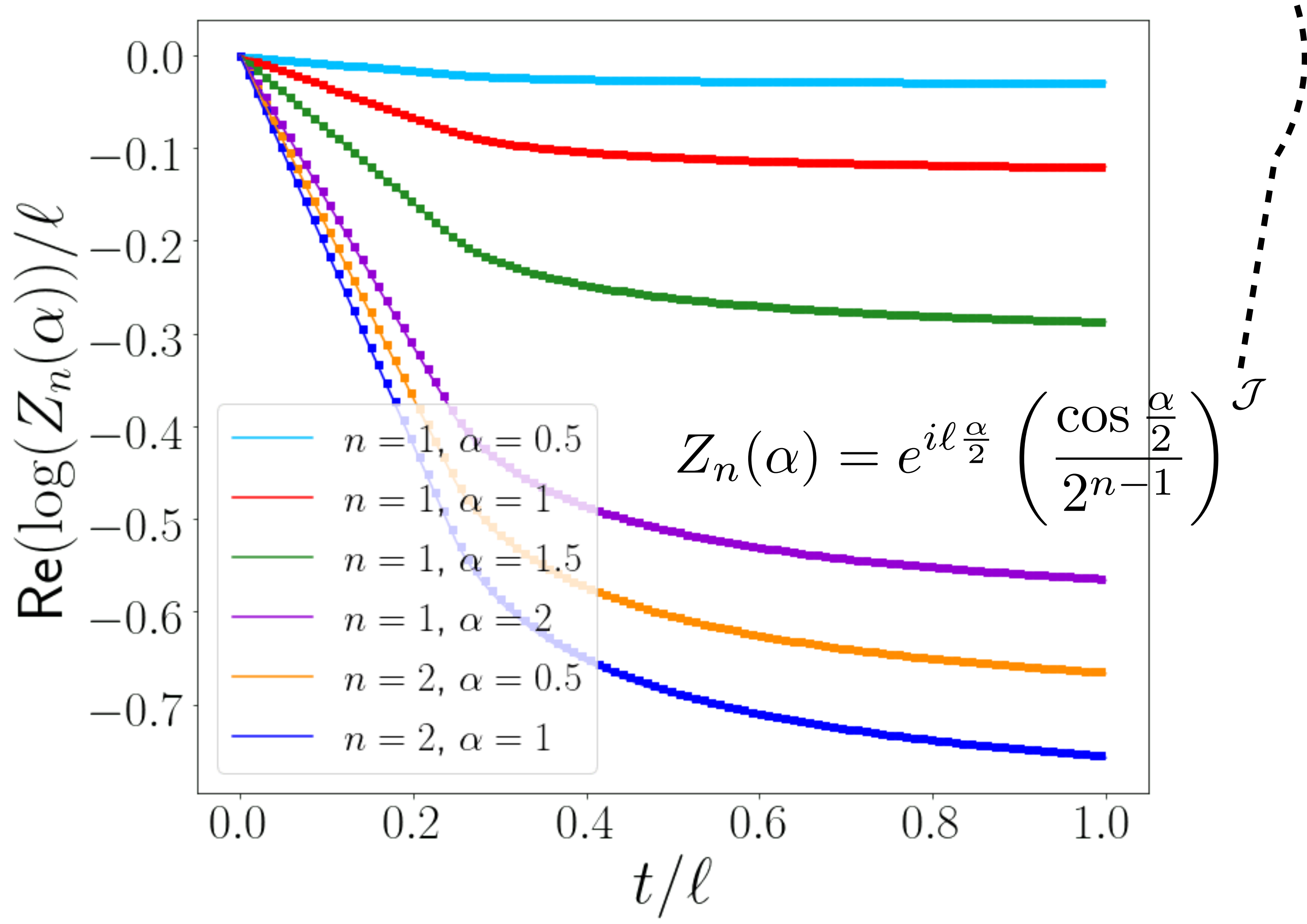


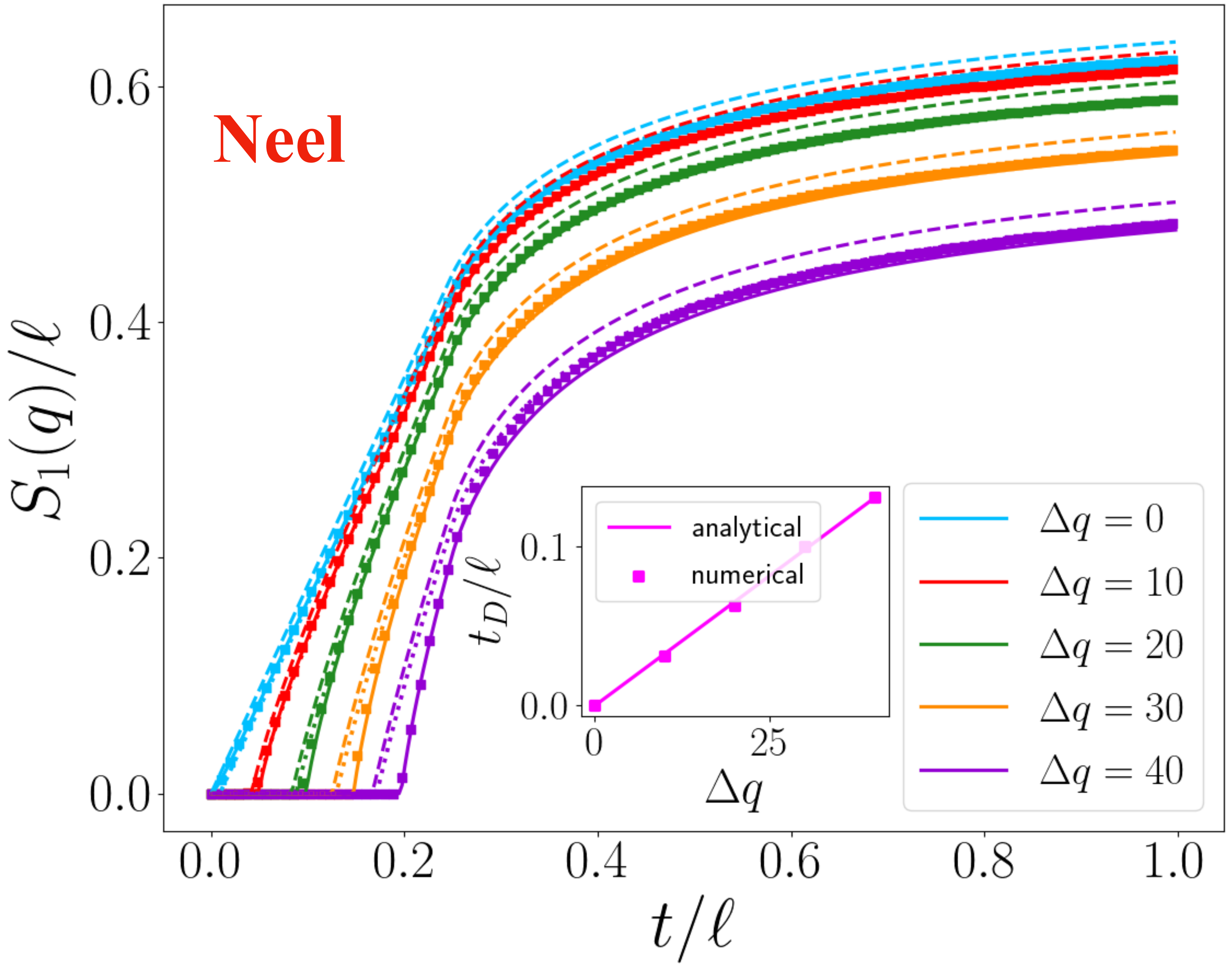
FIG. 1. The time evolution of the charged moments $Z_n(\alpha)$ after a quench from the Néel state in the free fermion model

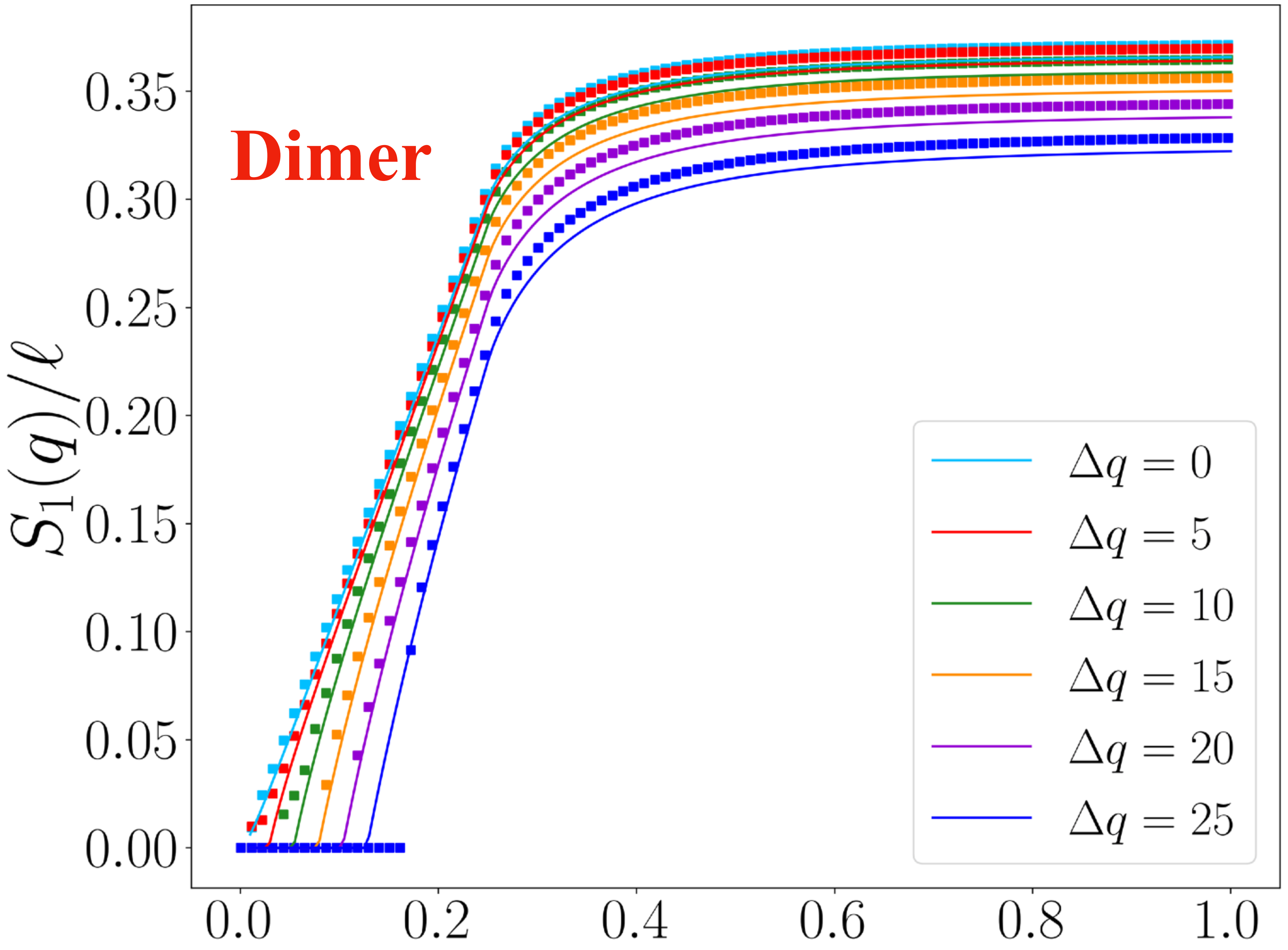
Adapting the QP picture to the charged moments, we conjecture

G. Parez, R. Bonsignori and P. Calabrese, PRB 103, L041104 (2020)

$$\log Z_n(\alpha) = i\langle Q_A\rangle\alpha + \int \frac{dk}{2\pi} f_{n,\alpha}(k) \min[2v_k t, \ell],$$

but the kernel $f_{n,\alpha}(k)$ is difficult to compute for generic model, while free is possible





SRE after a quantum quench II

G. Parez, R. Bonsignori and P. Calabrese, PRB 103, L041104 (2020)

Some general features in charge space:

- Delay time $t_D \propto |\Delta q|$

$$t_D = \pi \frac{|\Delta q|}{4} \quad \text{for free fermions}$$

The time needed to change the charge of an amount $|\Delta q|$ within A

- c for small $|\Delta q|$

$$S_n(q) = S_n - \frac{\Delta q^2}{4(1-n)} \left\{ \frac{1}{\mathcal{J}_n} - \frac{n}{\mathcal{J}_1} \right\}$$

- Number entropy

$$S^n \simeq \frac{1}{2} \log t$$

Application to ion-trap experiment: SR dynamical purification

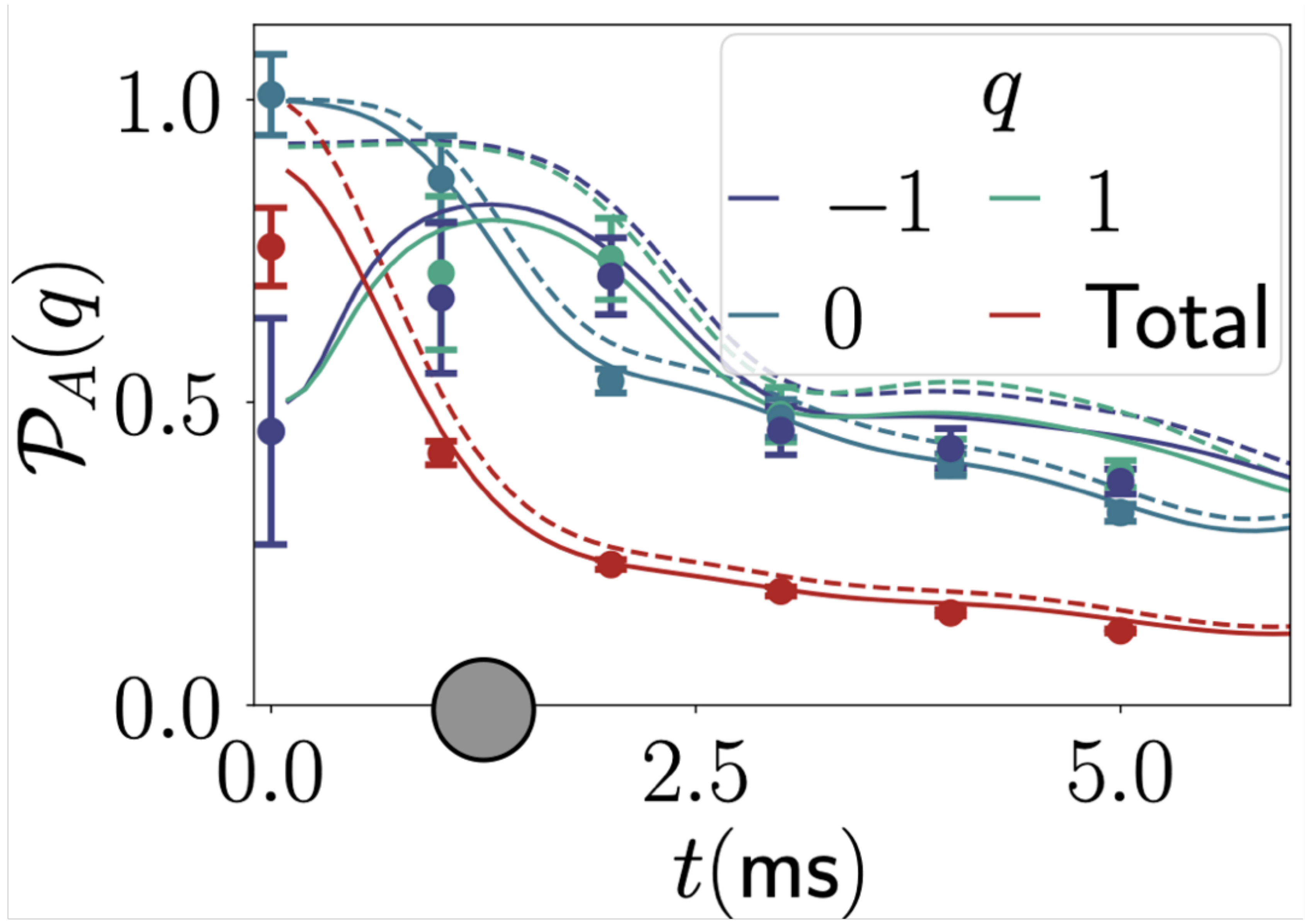
V. Vitale, A. Elben, R. Kueng, A. Neven, J. Carrasco, B. Kraus, P. Zoller, P. Calabrese, B. Vermersch, and M. Dalmonte, ArXiv:2101.07814

Hamiltonian + dissipative dynamics

$$\partial_t \rho = -\frac{i}{\hbar}[H,\rho] + \sum_j \gamma \left[b_j \rho b_j^\dagger + b_j^\dagger \rho b_j - \frac{1}{2}\{b_j b_j^\dagger + n_j, \rho\} \right]$$

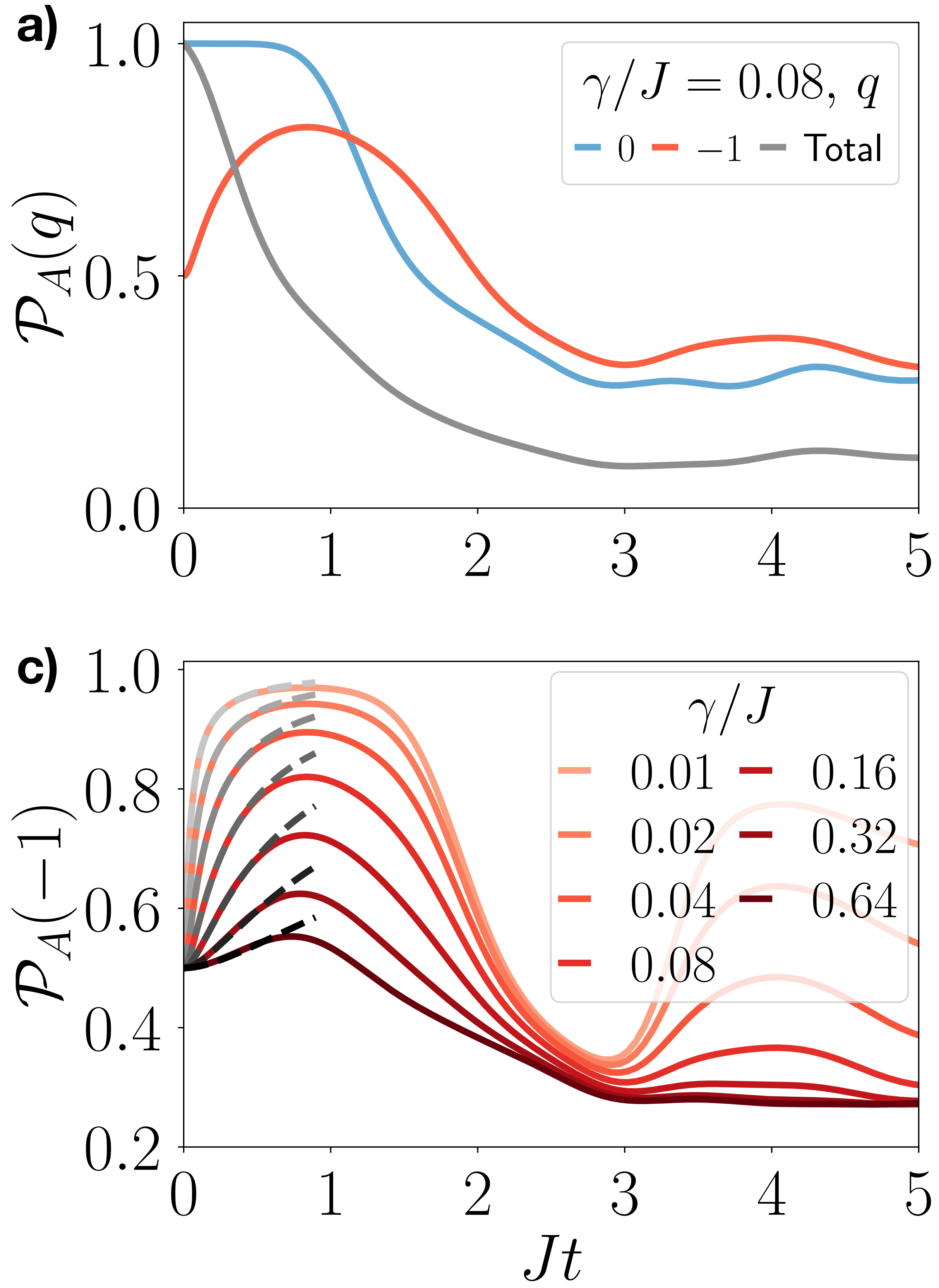
Recap:
- Both dynamics leads to entropy growth (entanglement and total)
- The total entropy grows, purity reduces

Analysis of experimental results:



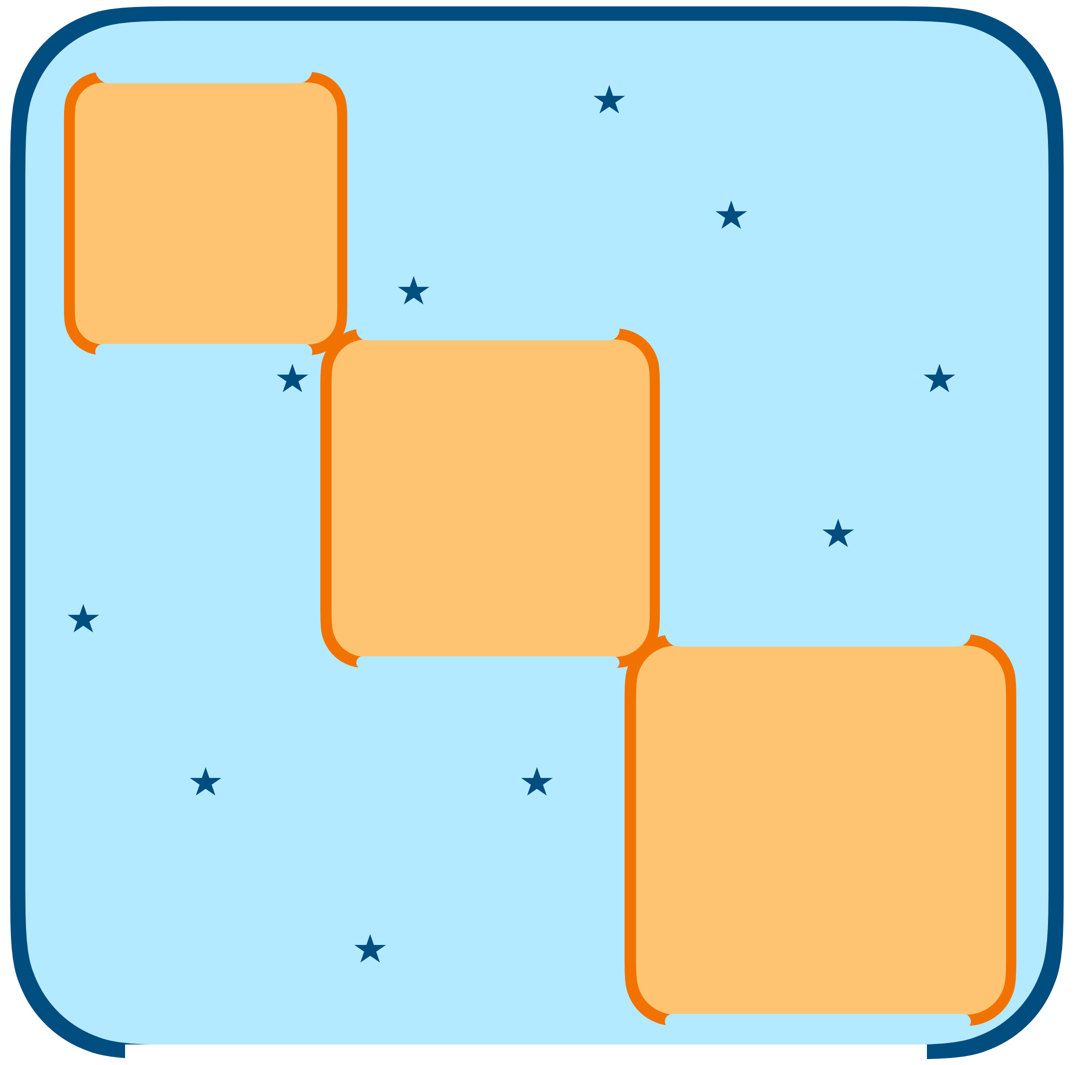
Some sectors purifies at intermediate times

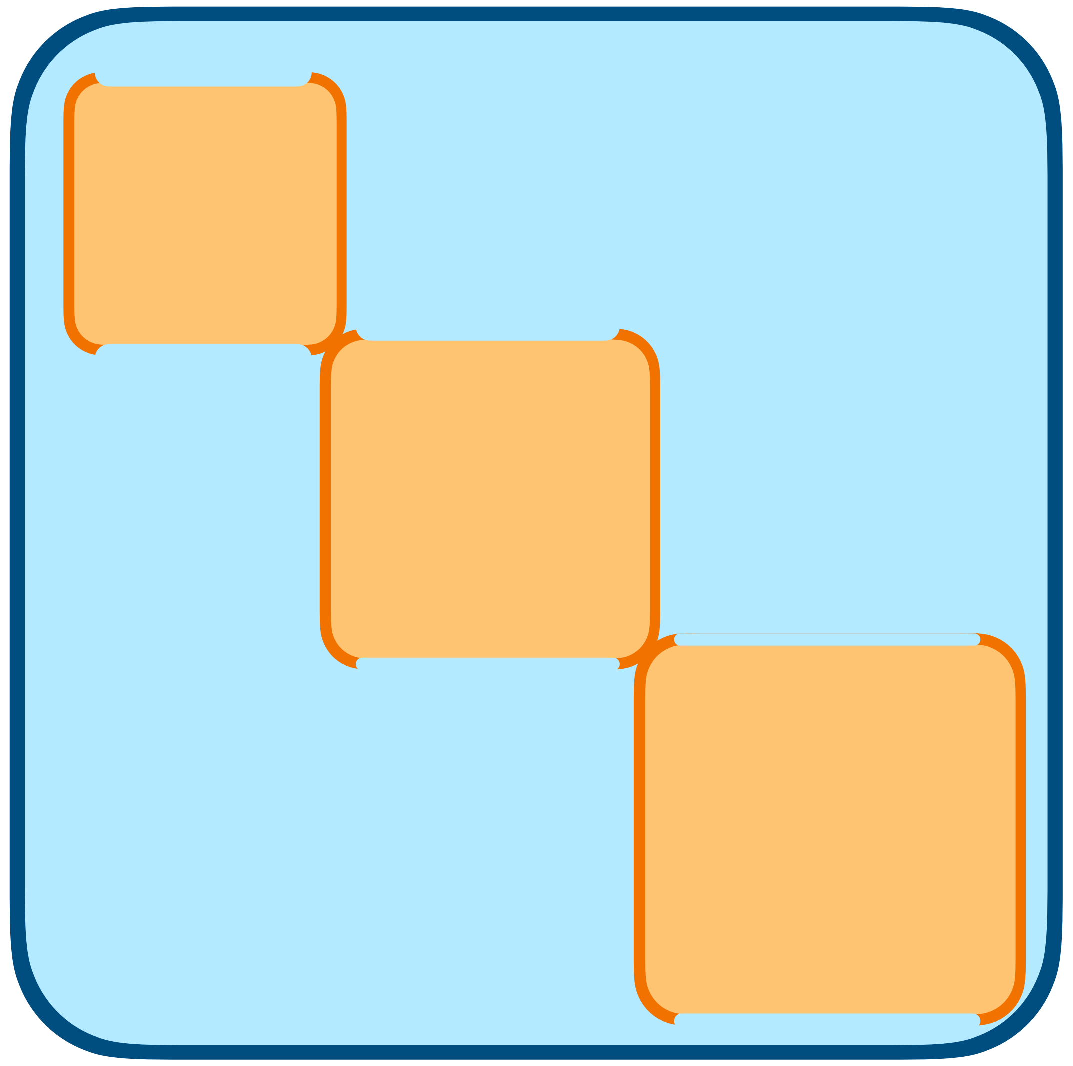
A general phenomenon that can be easily shown in perturbation theory in γ



Entanglement asymmetry: a probe of symmetry breaking

F. Ares, S.Murciano P. Calabrese,, ArXiv:2207.14693

$\rho_A =$ 

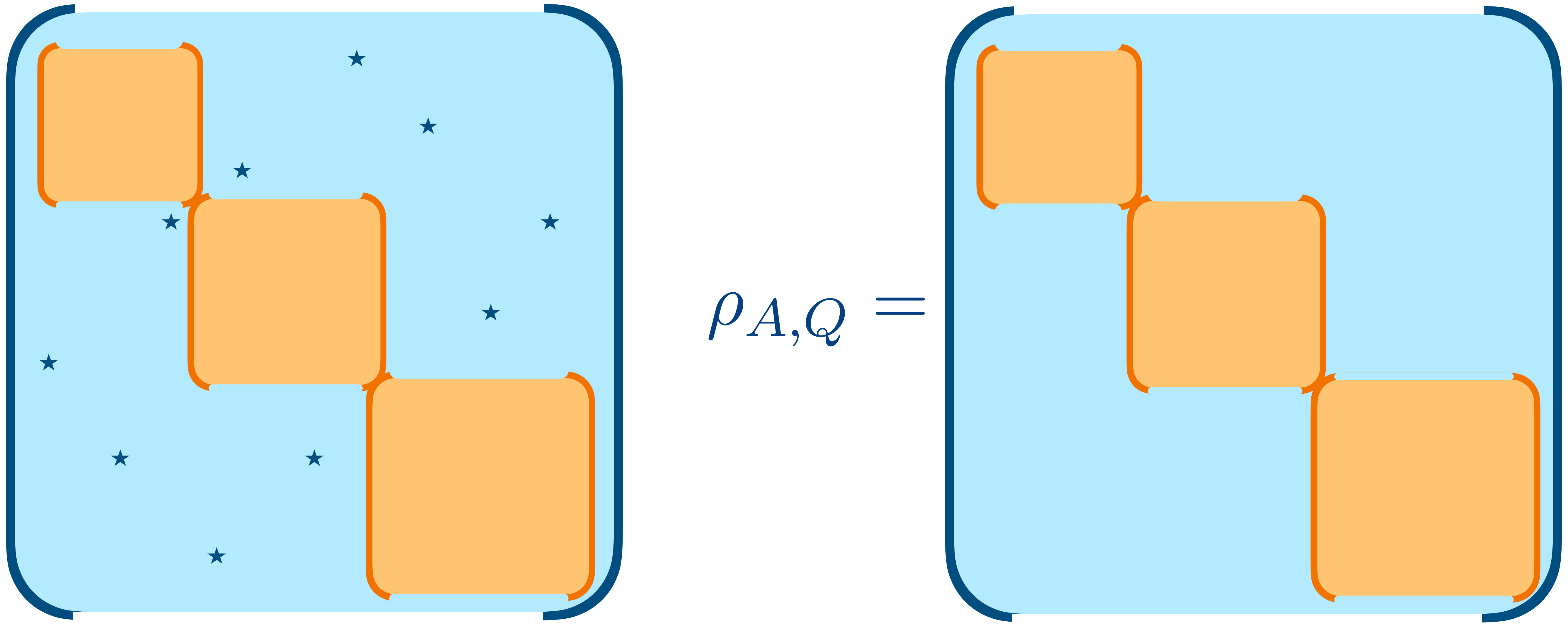
$\rho_{A,Q} =$ 

Entanglement asymmetry: $$\Delta S_A = S(\rho_{A,Q}) - S(\rho_A) \begin{cases} \geq 0; \\ 0 \iff \rho_A = \rho_{A,Q} \end{cases}$$

Replica trick for asymmetry

F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

$\rho_A =$ $\rho_{A,Q} =$



$$\Delta S_A^{(n)} = \frac{1}{1-n}\left[\log \mathrm{Tr}(\rho_{A,Q}^n) - \log \mathrm{Tr}(\rho_A^n)\right]$$

$$\mathrm{Tr}(\rho_{A,Q}^n) = \int_{-\pi}^{\pi} \frac{\mathrm{d}\alpha_1 \ldots \mathrm{d}\alpha_n}{(2\pi)^n} Z_n(\boldsymbol{\alpha})$$

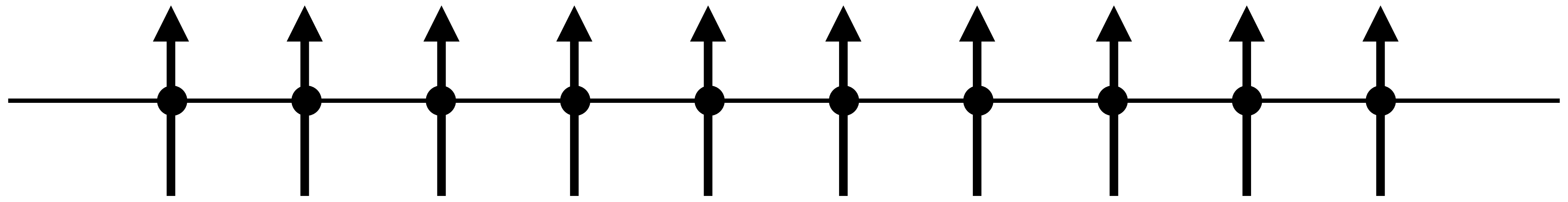
In terms of the generalized charged moments

$$Z_n(\boldsymbol{\alpha}) = \mathrm{Tr}\left[\prod_{j=1}^n \rho_A e^{i\alpha_{j,j+1} Q_A}\right]$$

A case study

F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

Ferromagnet

$|\Psi\rangle =$ 

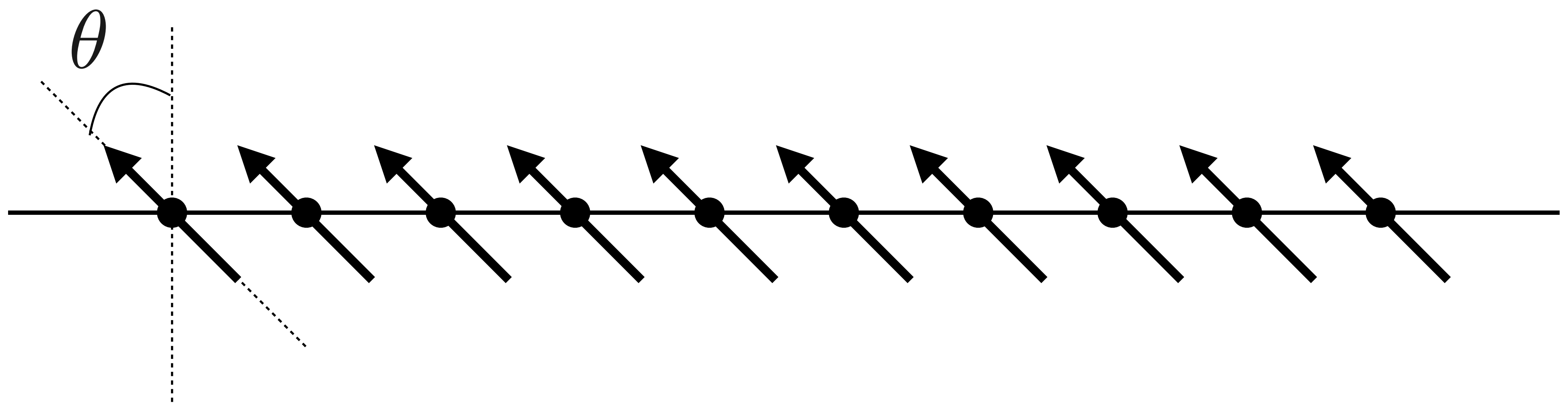
$Q = \frac{1}{2}\sum_j \sigma_j^z \qquad [|\Psi\rangle\langle\Psi|, Q] = 0$

$$\Delta S_A = 0$$

A case study

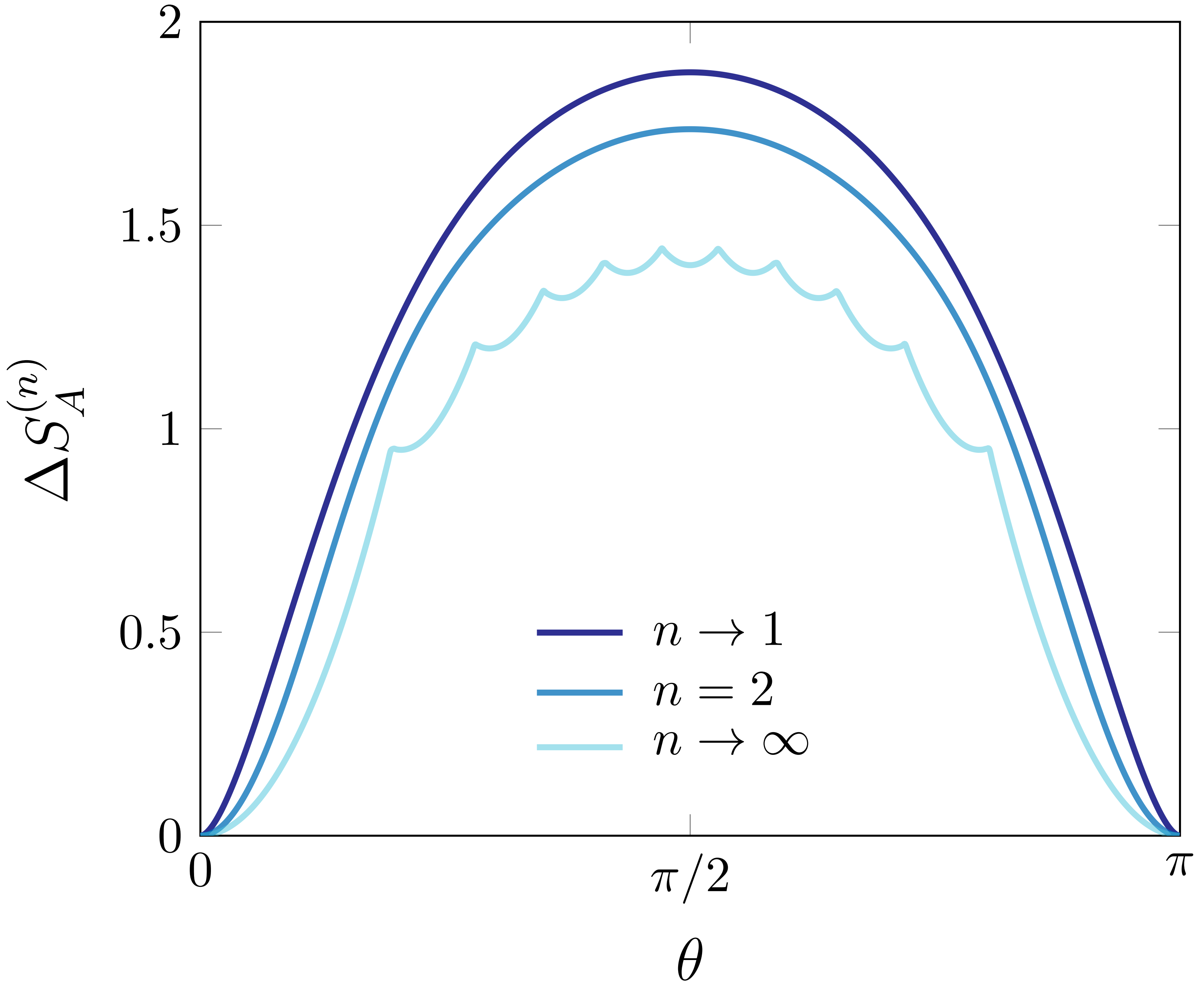
Tilted Ferromagnet

$$|\Psi\rangle =$$



$$Q = \frac{1}{2}\sum_j \sigma_j^z \qquad [|\Psi\rangle\langle\Psi|, Q] \neq 0$$

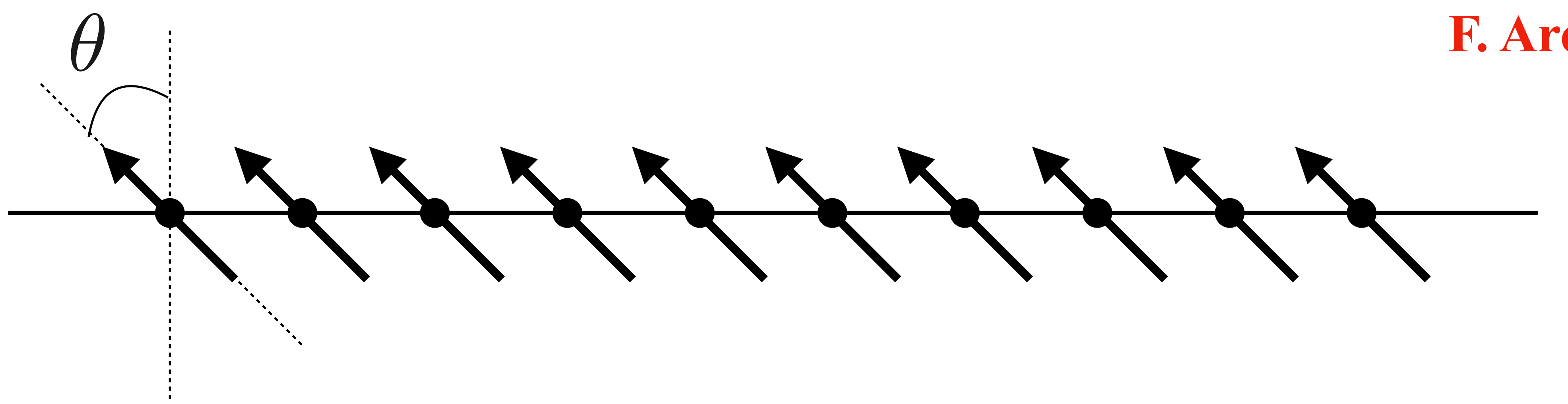
$$\Delta S_A \neq 0$$



A case study: asymmetry after a quench

F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

$t = 0$ $\qquad |\Psi\rangle =$



$$[|\Psi\rangle\langle\Psi|, Q] \neq 0$$

$t > 0$ $\qquad |\Psi(t)\rangle = e^{-itH}|\Psi(0)\rangle \qquad H = -\frac{1}{4}\sum_{j=-\infty}^{\infty}\left[\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y\right]$

A lot of algebra!

$$Z_n(\boldsymbol{\alpha}, t) = Z_n(\mathbf{0}, t)e^{\ell(A(\boldsymbol{\alpha})+B(\boldsymbol{\alpha},\zeta))}$$

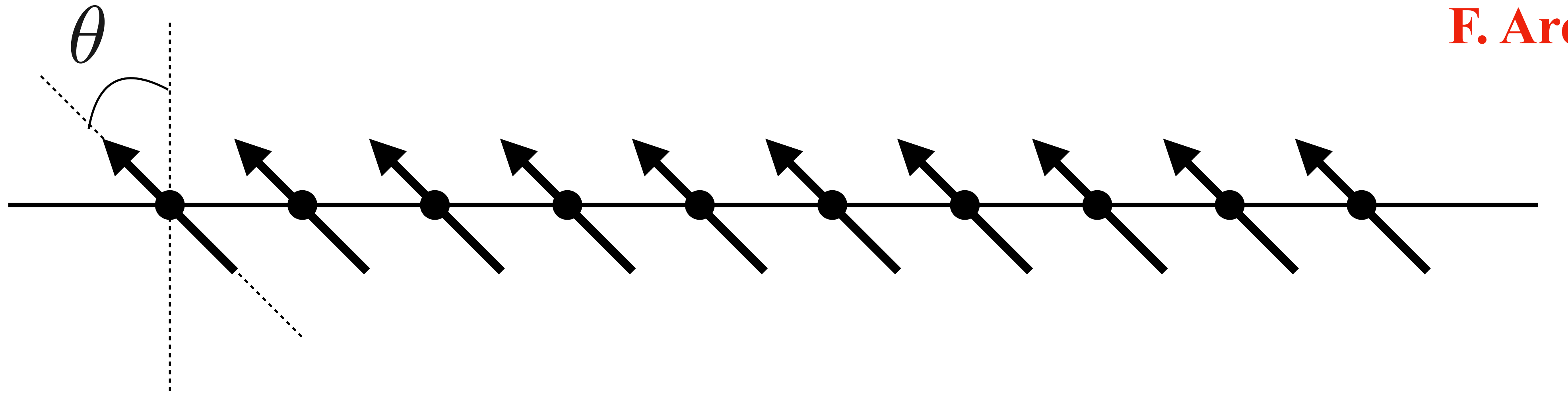
$$B(\boldsymbol{\alpha}, \zeta) \to -A(\boldsymbol{\alpha}) \text{ as } \zeta \equiv t/\ell \to \infty$$

$$B(\boldsymbol{\alpha}, \zeta) = -\int_0^{2\pi} \frac{\mathrm{d}k}{2\pi}\min(2\zeta|\epsilon'(k)|, 1)\log\prod_{j=1}^{n} f(e^{i\Delta_k}, \alpha_{j,j+1})$$

$$\text{with } f(\lambda, \alpha) = i\lambda\sin\left(\frac{\alpha}{2}\right) + \cos\left(\frac{\alpha}{2}\right)$$

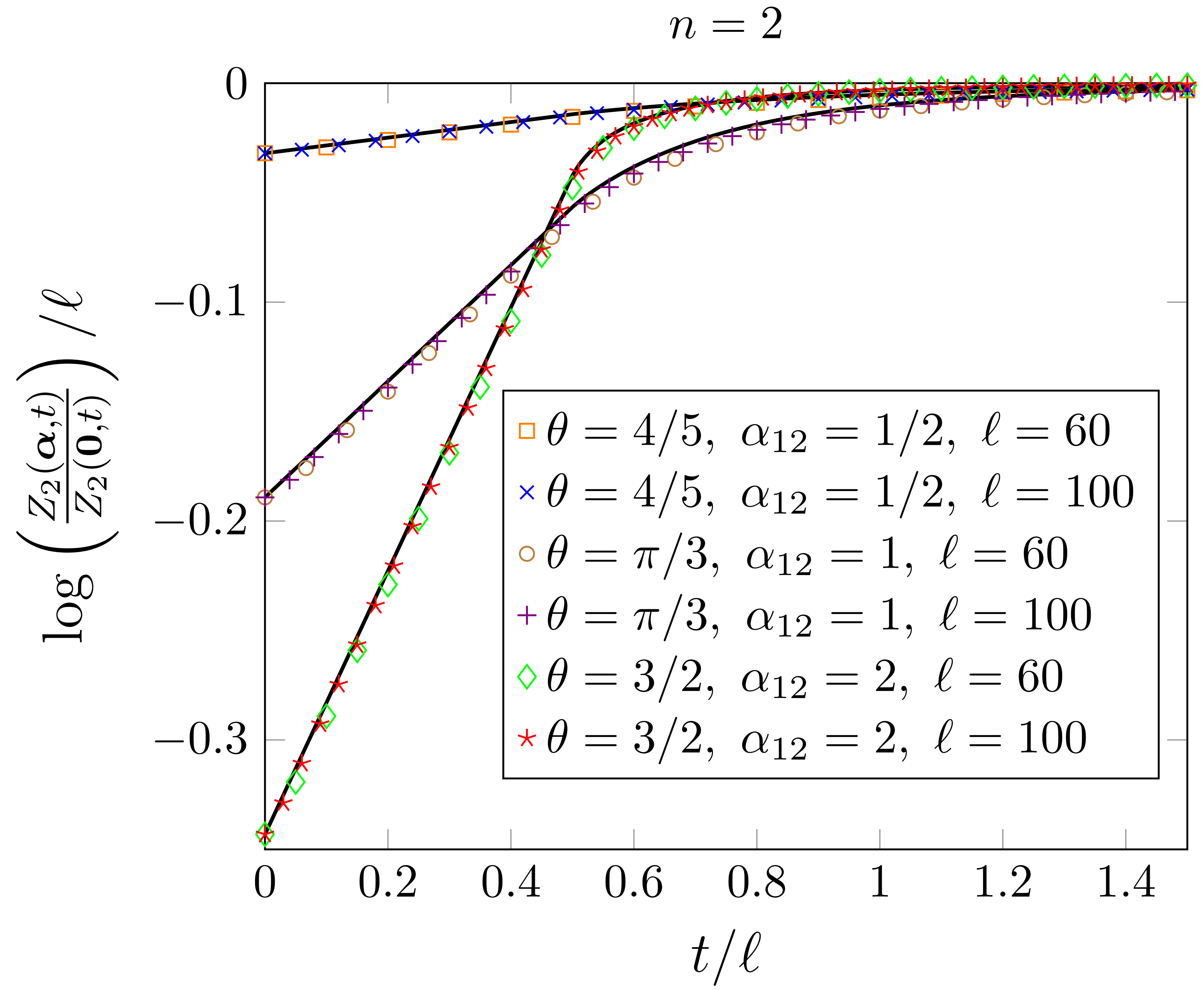
A case study: asymmetry after a quench

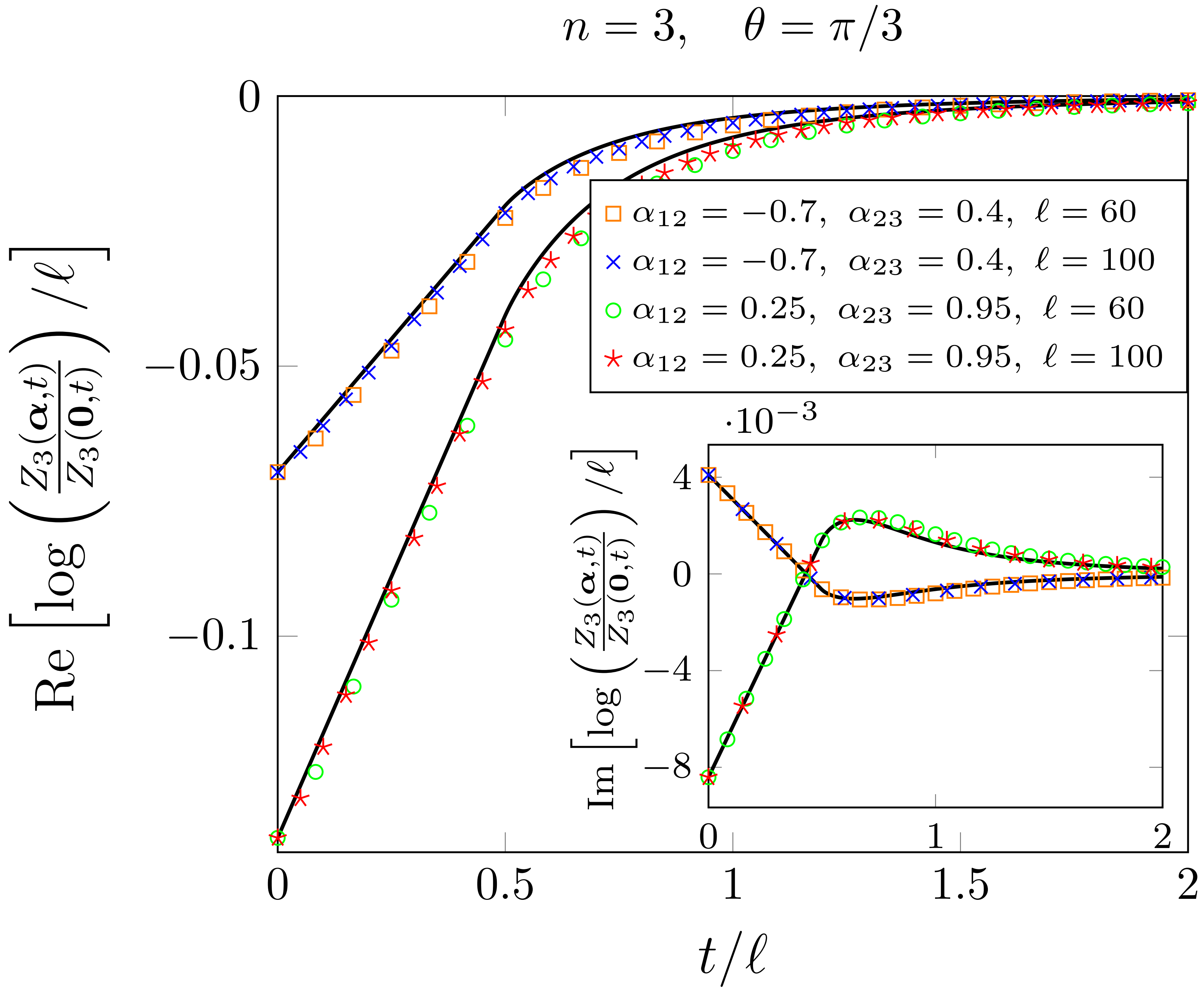
F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

$t=0$ $\quad |\Psi\rangle =$ 

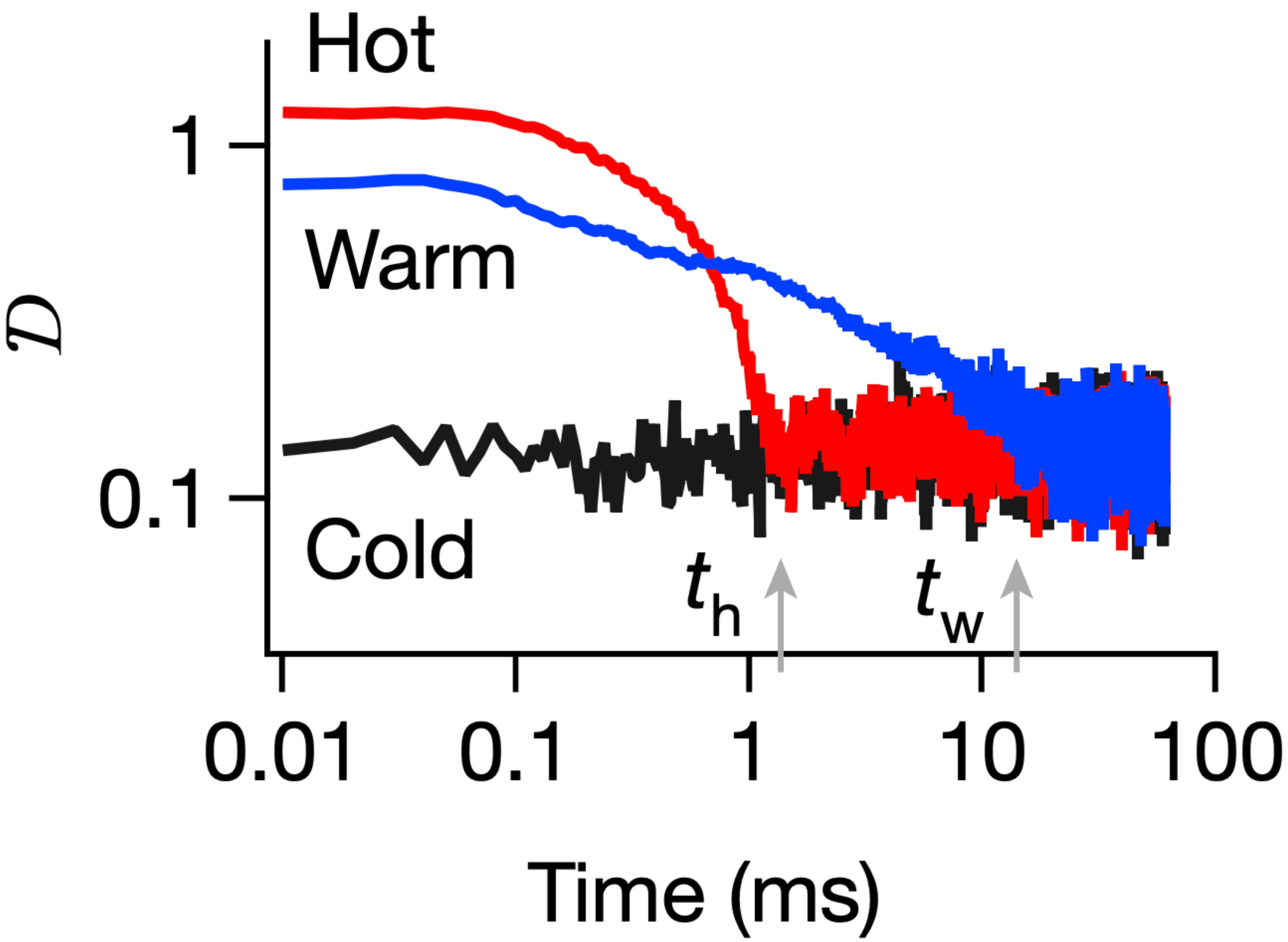
$\quad [|\Psi\rangle\langle\Psi|, Q] \neq 0$

$t>0$ $\quad |\Psi(t)\rangle = e^{-itH}|\Psi(0)\rangle \qquad H = -\frac{1}{4}\sum_{j=-\infty}^{\infty}\left[\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y\right]$





Mpemba effect

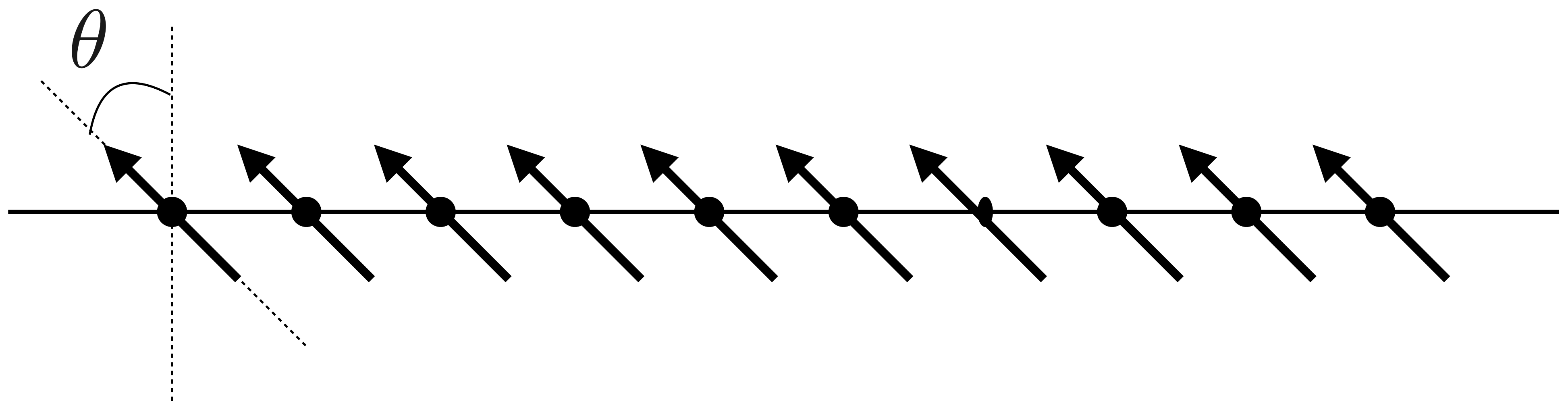


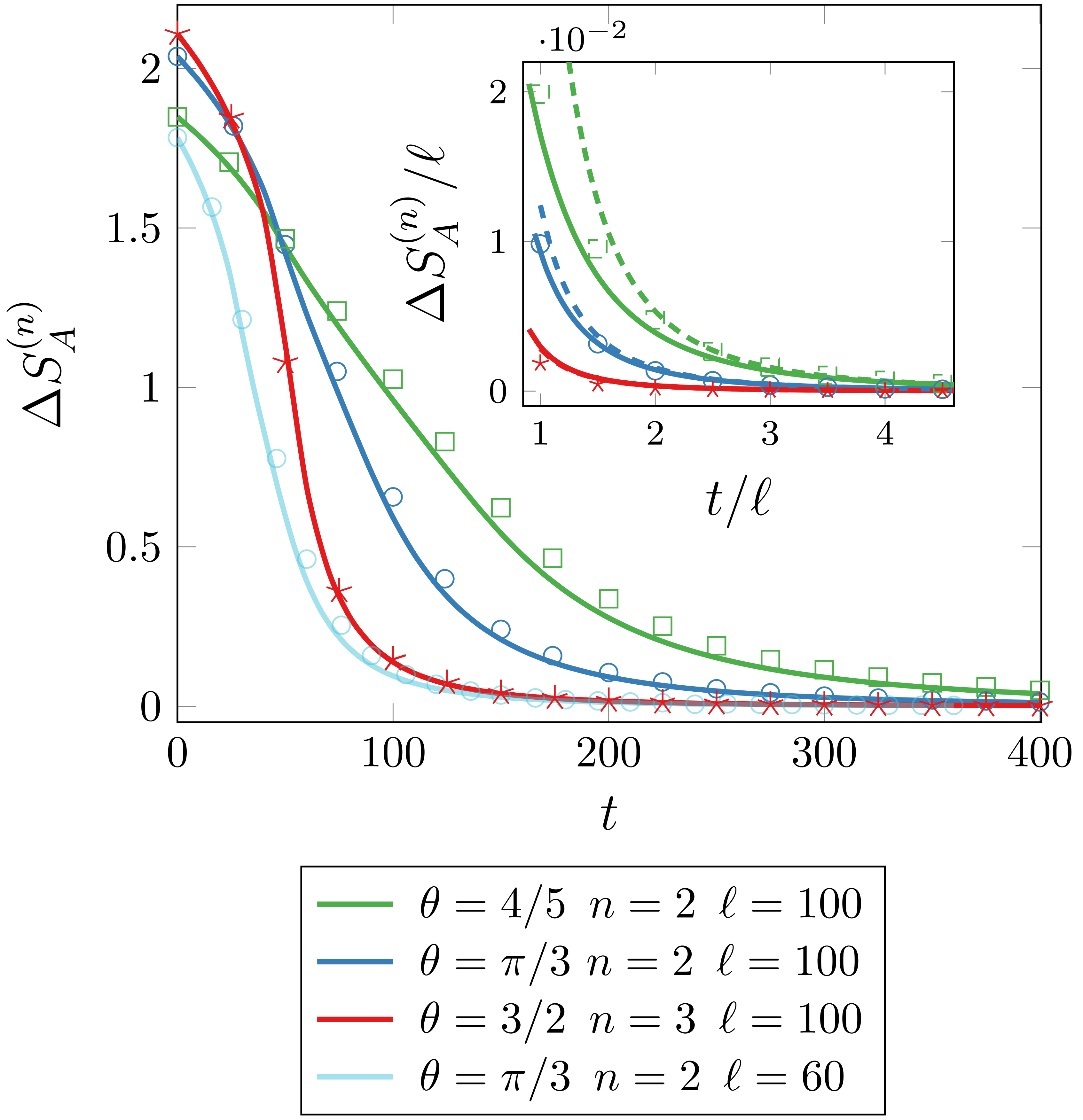
A. Kumar, J. Bechhoefer, Nature 584 (2020)

Quantum Mpemba effect

F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

$t = 0$ $\quad |\Psi\rangle =$





- $\Delta S_A^{(n)}(t)$ tends to zero for large t and the $\mathrm{U}(1)$ symmetry is restored

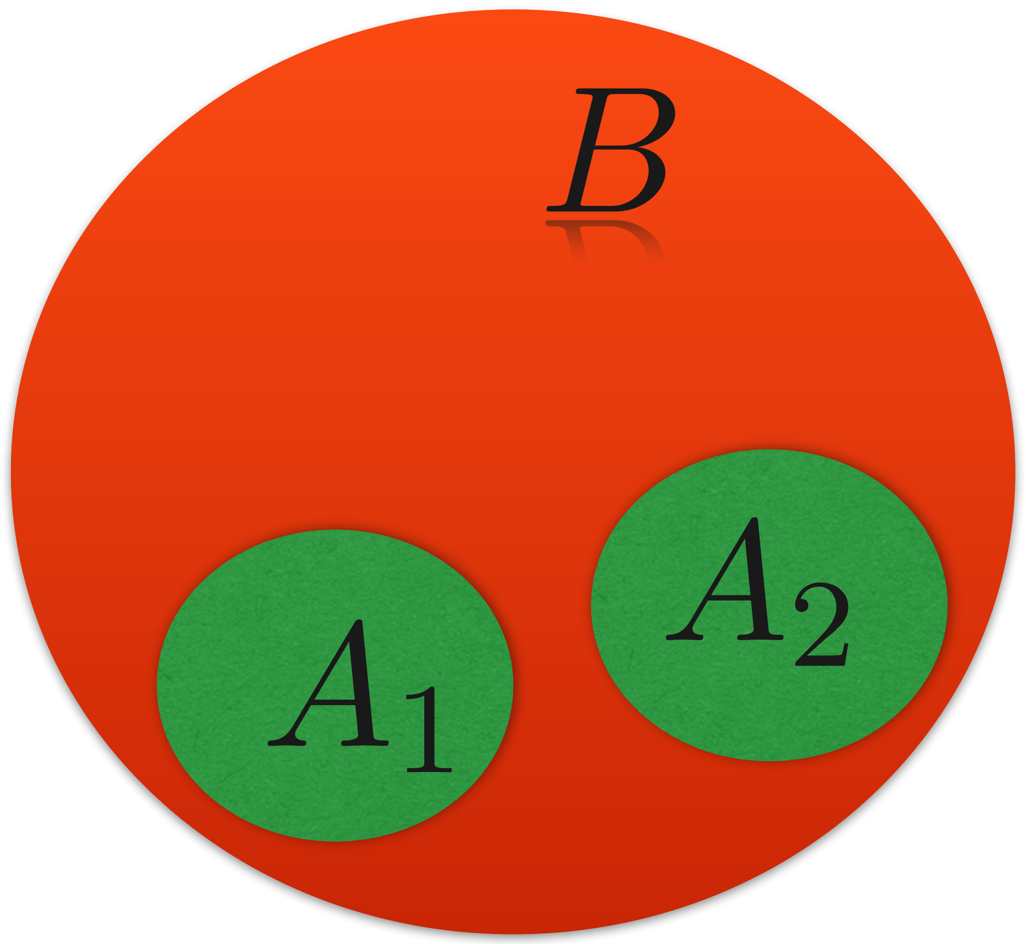
$$\Delta S_A^{(n)}(t) \simeq \frac{\pi}{1152}\left(1 + 8\frac{\cos^2\theta}{\sin^4\theta}\right)\frac{\ell}{\zeta^3}$$

- Larger subsystems require more time to recover the symmetry

- Quantum Mpemba effect: The more the symmetry is initially broken, shorter is the time to restore it

Mixed state entanglement: Partial transpose and negativity

Q: what is the entanglement in a mixed state?



$|e_k^1\rangle$ and $|e_l^2\rangle$ bases of A_1 and A_2

$$(|e_i^1, e_j^2\rangle\langle e_k^1, e_l^2|)^{T_1} \equiv |e_k^1, e_j^2\rangle\langle e_i^1, e_l^2|$$

$$\rho_A = \sum_{ijkl} \langle e_i^1, e_j^2|\rho_A|e_k^1, e_l^2\rangle |e_i^1, e_j^2\rangle\langle e_k^1, e_l^2|$$

$$\rho_A^{T_1} = \sum_{ijkl} \langle e_k^1, e_j^2|\rho_A|e_i^1, e_l^2\rangle |e_k^1, e_j^2\rangle\langle e_i^1, e_l^2|$$

PPT criterion:

If $\rho_A^{T_1}$ has negative eigenvalues ρ_A is entangled

Peres, 1996

The Negativity $= \mathcal{N} = \dfrac{\mathrm{Tr}|\rho_A^{T_1}| - 1}{2}$ measure how much the eigenvalues of $\rho_A^{T_1}$ are negative and it is an entanglement monotone

Vidal Werner 2002

Replica trick: $\mathrm{Tr}|\rho_A^{T_1}| = \lim_{n\to 1/2} \mathrm{Tr}(\rho_A^{T_1})^{2n}$

P. Calabrese, J. Cardy, E. Tonni 2012

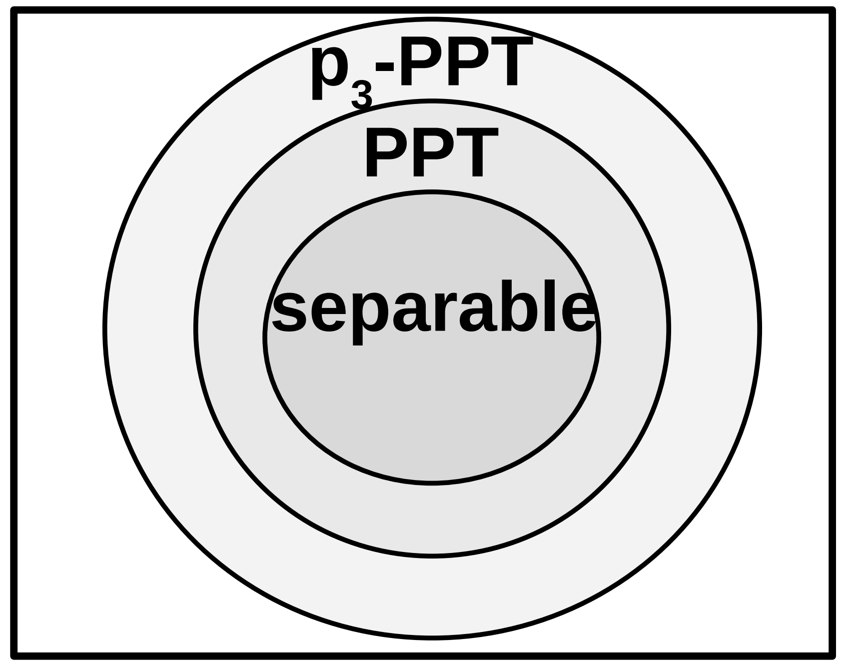
Intermezzo: "Negativity" in experiments

The negativity is difficult to measure experimentally, but the moments of the partial transpose p_n can

E. Cornfeld, M. Goldstein, and E. Sela, PRA 98, 032302 (2018)
J. Gray, L. Banchi, A. Bayat, and S. Bose, Phys. Rev. Lett. 121, 150503 (2018)
A. Elben, R. Kueng, H.-Y. Huang, R. van Bijnen, C. Kokail, M. Dalmonte, P. Calabrese, B. Kraus, J. Preskill, P. Zoller, and B. Vermersch, PRL 125, 200501 (2020)

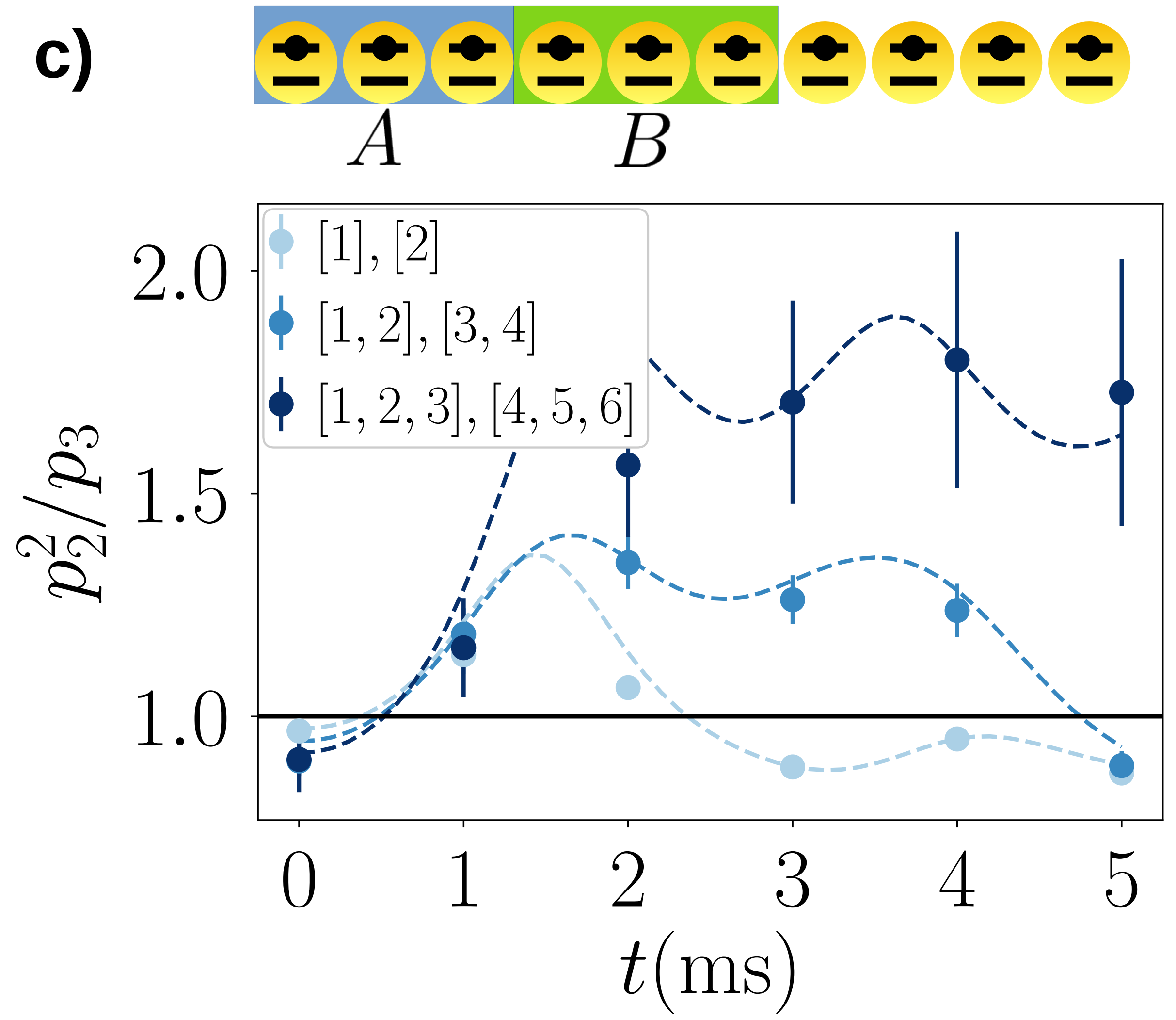
In Elben et al, PRL 2020 p_n are obtained by performing local random measurements and post-processing using the classical shadows framework

a)



p_3-PPT condition: if $p_3 < p_2^2$, then PPT is violated and there is entanglement

c)



Quench from Néel state in a long range XX

Generalizations

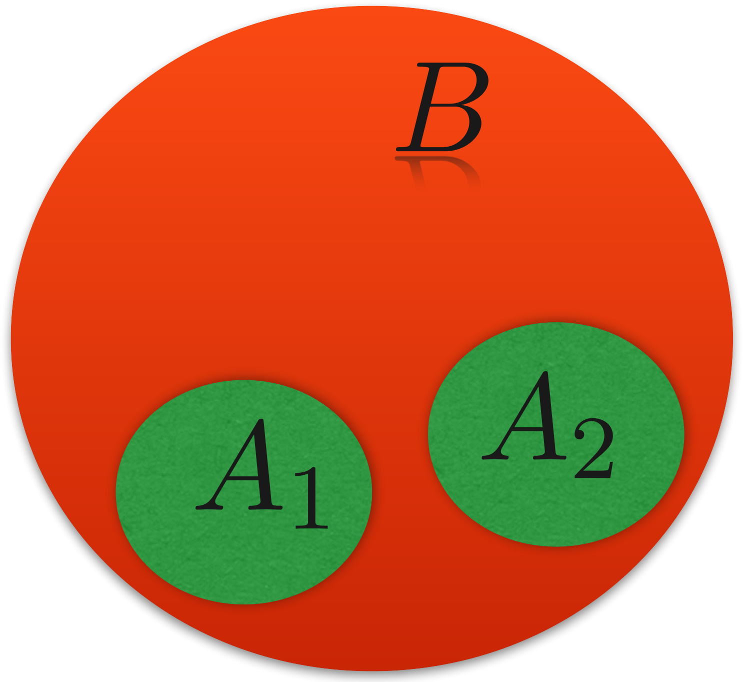
A.Neven, J. Carrasco, V. Vitale, C. Kokail, A. Elben, M. Dalmonte, P. Calabrese, P. Zoller, B. Vermersch, R. Kueng, and B. Kraus, ArXiv:2103.07443

- D_n conditions: generalized conditions, involving higher moments
- Symmetry resolution of **p_3-PPT:**
  - Allow to understand in which sector negative eigenvalues are
  - More sensitive to small negative eigenvalues

Fermionic partial transpose

H. Shapourian, K. Shiozaki, S. Ryu, PRB 95, 165101 (2017)

Occupation number basis: $|\{n_j\}_{j\in A_1},\{n_j\}_{j\in A_2}\rangle = (f^\dagger_{m_1})^{n_{m_1}}\dots(f^\dagger_{m_{\ell_1}})^{n_{m_{\ell_1}}}(f^\dagger_{m'_1})^{n_{m'_1}}\dots(f^\dagger_{m'_{\ell_2}})^{n_{m'_{\ell_2}}}|0\rangle$



Fermionic partial transpose: $$(|\{n_j\}_{A_1},\{n_j\}_{A_2}\rangle\langle\{\bar n_j\}_{A_1},\{\bar n_j\}_{A_2}|)^{R_1} = (-1)^{\phi(\{n_j\},\{\bar n_j\})}(|\{\bar n_j\}_{A_1},\{n_j\}_{A_2}\rangle\langle\{n_j\}_{A_1},\{\bar n_j\}_{A_2}|)$$

$\rho_A^{R_1}$ non hermitian

$$\mathcal{N} = \frac{\mathrm{Tr}|\rho_A^{R_1}|-1}{2} = \frac{\mathrm{Tr}\sqrt{\rho_A^{R_1}(\rho_A^{R_1})^\dagger}-1}{2} = \lim_{n\to 1/2}\frac{\mathrm{Tr}(\rho_A^{R_1}(\rho_A^{R_1})^\dagger)^n-1}{2}$$

Fermionic negativity (no negative eigenvalues, but entanglement monotone)

$$\rho_A^{T_1} = \frac{e^{i\pi/4}\rho_A^{R_1}+e^{-i\pi/4}(\rho_A^{R_1})^\dagger}{\sqrt{2}}$$

$\mathrm{Tr}(\rho_A^{T_1})^{2n}$

$\sum$ all spin structures

$\rho_A^{T_1}$ sum of 2 Gaussian

$\mathrm{Tr}(\rho_A^{R_1}(\rho_A^{R_1})^\dagger)^n$

only 1 cycle

$\rho_A^{R_1}$ Gaussian

Symmetry Resolution: example

A particle in one out of three boxes, $A = A_1 \cup A_2 \cup B$

$$|\Psi\rangle = \alpha\,|100\rangle + \beta\,|010\rangle + \gamma\,|001\rangle$$

$$\begin{array}{c} \\ \rho_A = \end{array} \begin{array}{c} \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} \\ \begin{pmatrix} |\gamma|^2 & 0 & 0 & 0 \\ 0 & |\beta|^2 & \alpha^*\beta & 0 \\ 0 & \beta^*\alpha & |\alpha|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

block-diagonal structure

$$\rho_A \cong \left(|\gamma|^2\right)_{\tilde{q}=0} \oplus \begin{pmatrix} |\beta|^2 & \alpha\beta^* \\ \beta\alpha^* & |\alpha|^2 \end{pmatrix}_{\tilde{q}=1} \oplus (0)_{\tilde{q}=2}$$

E. Cornfeld, M. Goldstein, and E. Sela, PRA 98, 032302 (2018)

(fermionic) partial transpose

$$\rho_A^{R_1} = \begin{pmatrix} |\gamma|^2 & 0 & 0 & i\alpha\beta^* \\ 0 & |\beta|^2 & 0 & 0 \\ 0 & 0 & |\alpha|^2 & 0 \\ i\beta\alpha^* & 0 & 0 & 0 \end{pmatrix}$$

block-diagonal structure

$$\rho_A^{R_1} \cong \left(|\alpha|^2\right)_{q=-1} \oplus \begin{pmatrix} |\gamma|^2 & i\alpha\beta^* \\ i\beta\alpha^* & 0 \end{pmatrix}_{q=0} \oplus \left(|\beta|^2\right)_{q=1}$$

$$Q = Q_{A_2} - Q_{A_1}^{R_1}$$

charge imbalance resolved negativity

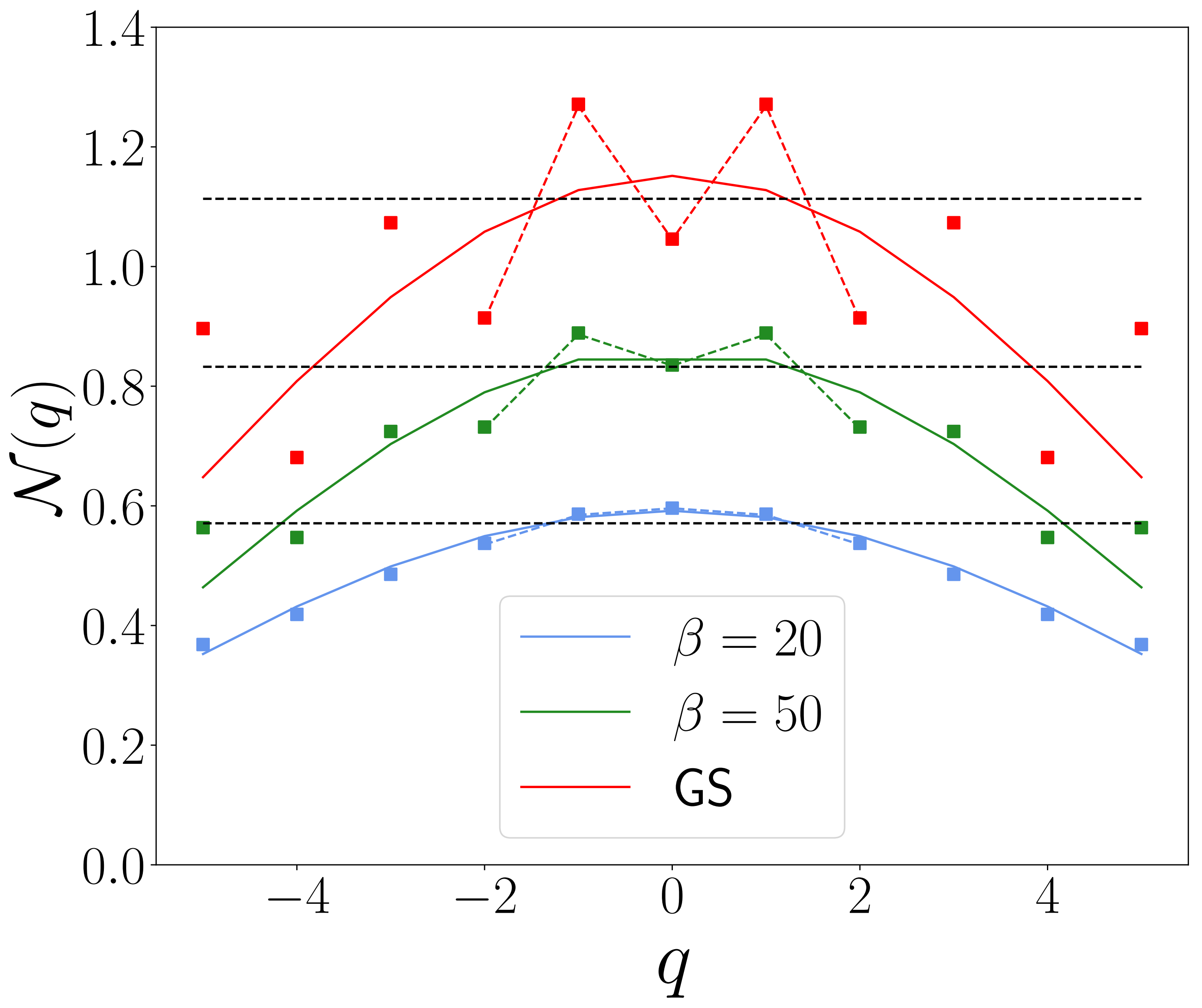
$$\mathcal{N}(q) = \frac{\mathrm{Tr}|(\rho_A^{R_1}(q))| - 1}{2}, \qquad \rho_A^{R_1}(q) = \frac{\mathcal{P}_q \rho_A^{R_1} \mathcal{P}_q}{\mathrm{Tr}(\mathcal{P}_q \rho_A^{R_1})}$$

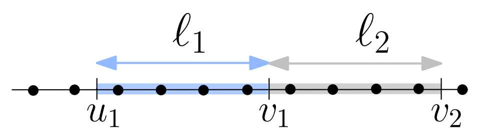
$$\mathcal{N} = \sum_q p(q)\mathcal{N}(q), \qquad p(q) = \mathrm{Tr}(\mathcal{P}_q \rho_A^{R_1})$$

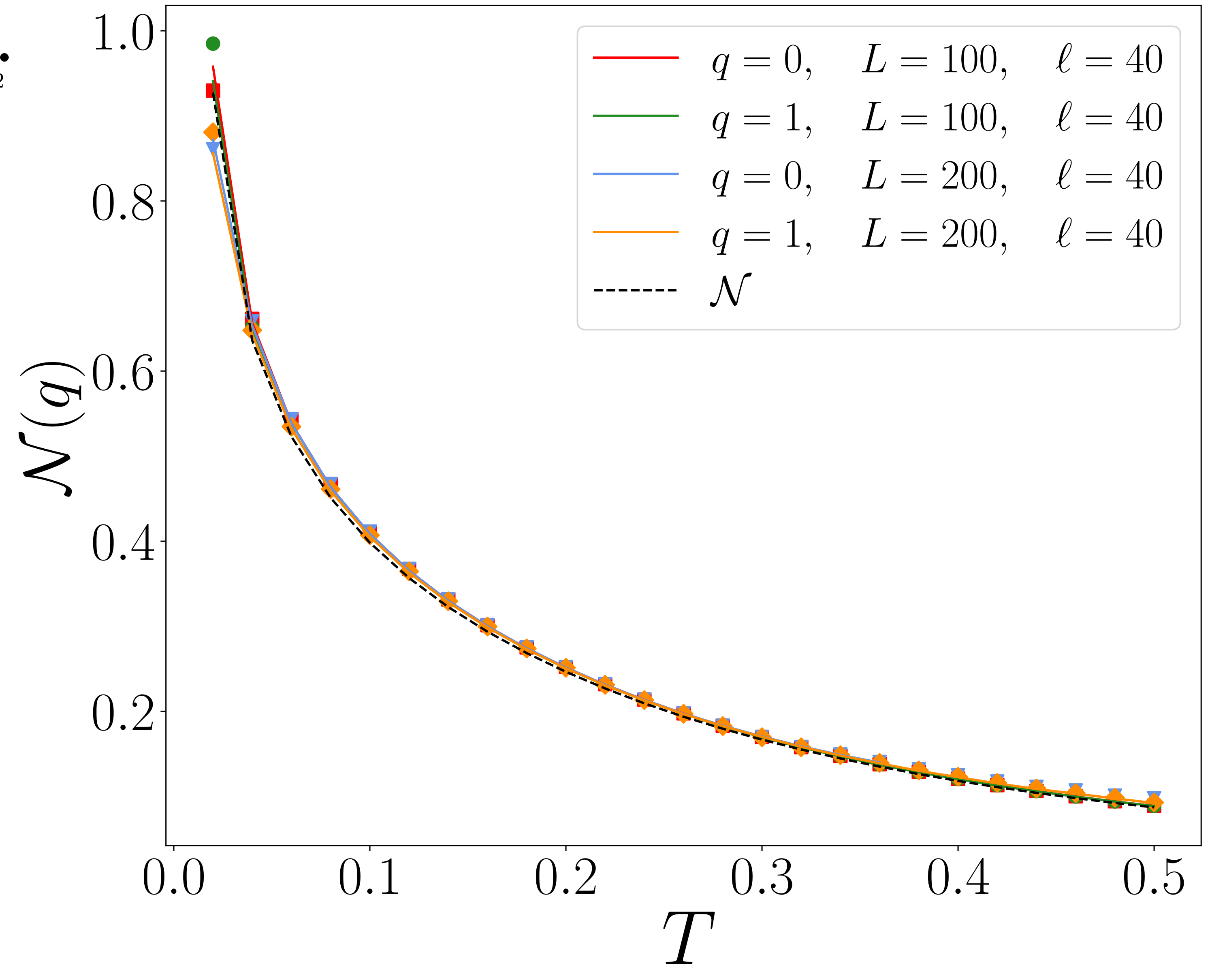
S. Murciano, R. Bonsignori, P. Calabrese, SciPost Phys 10, 111(2021)

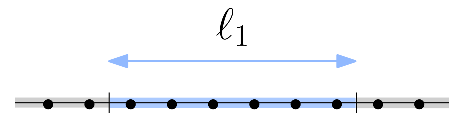
Some results: Negativity equipartition

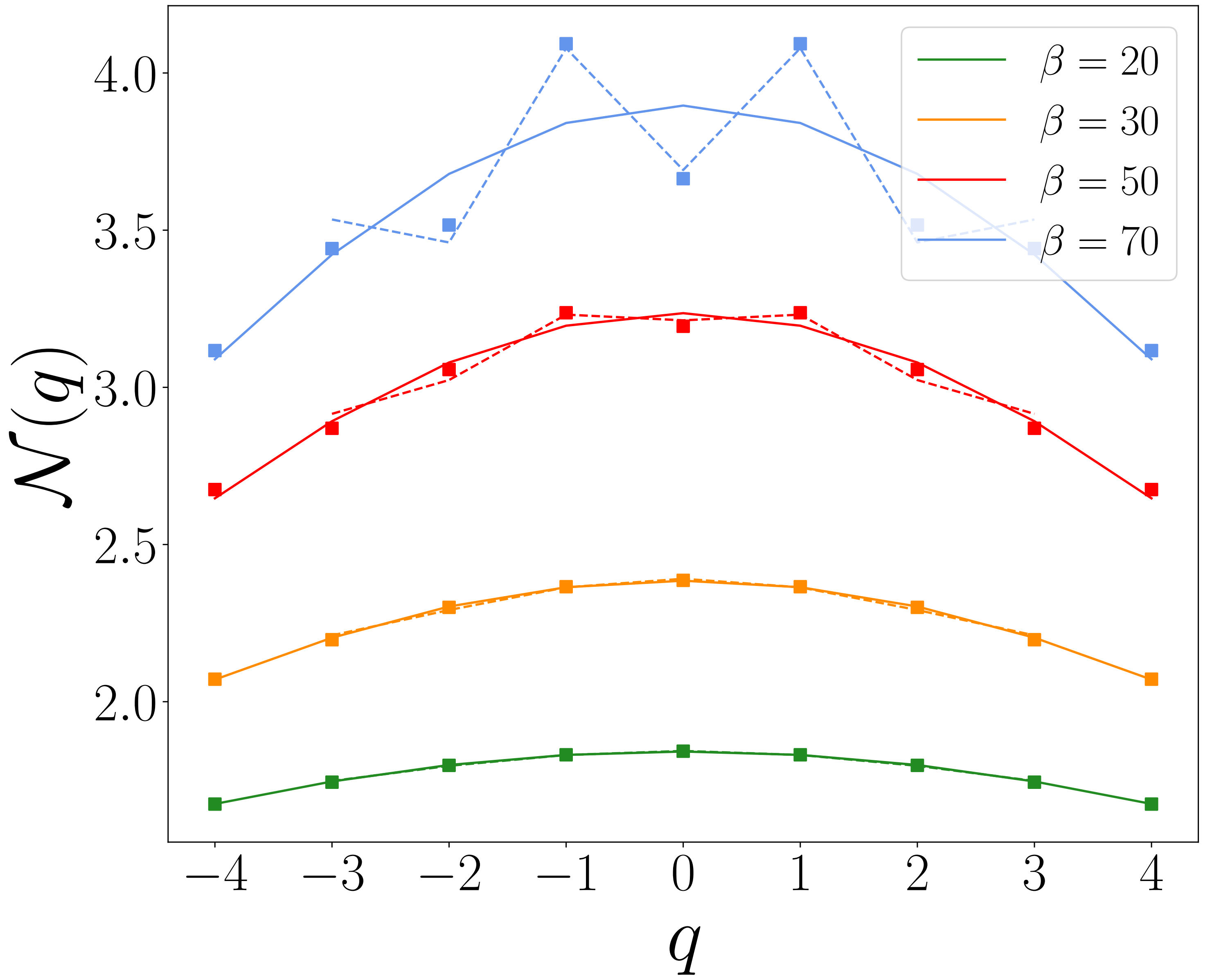
S. Murciano, R. Bonsignori, P. Calabrese, SciPost Phys 10, 111(2021)

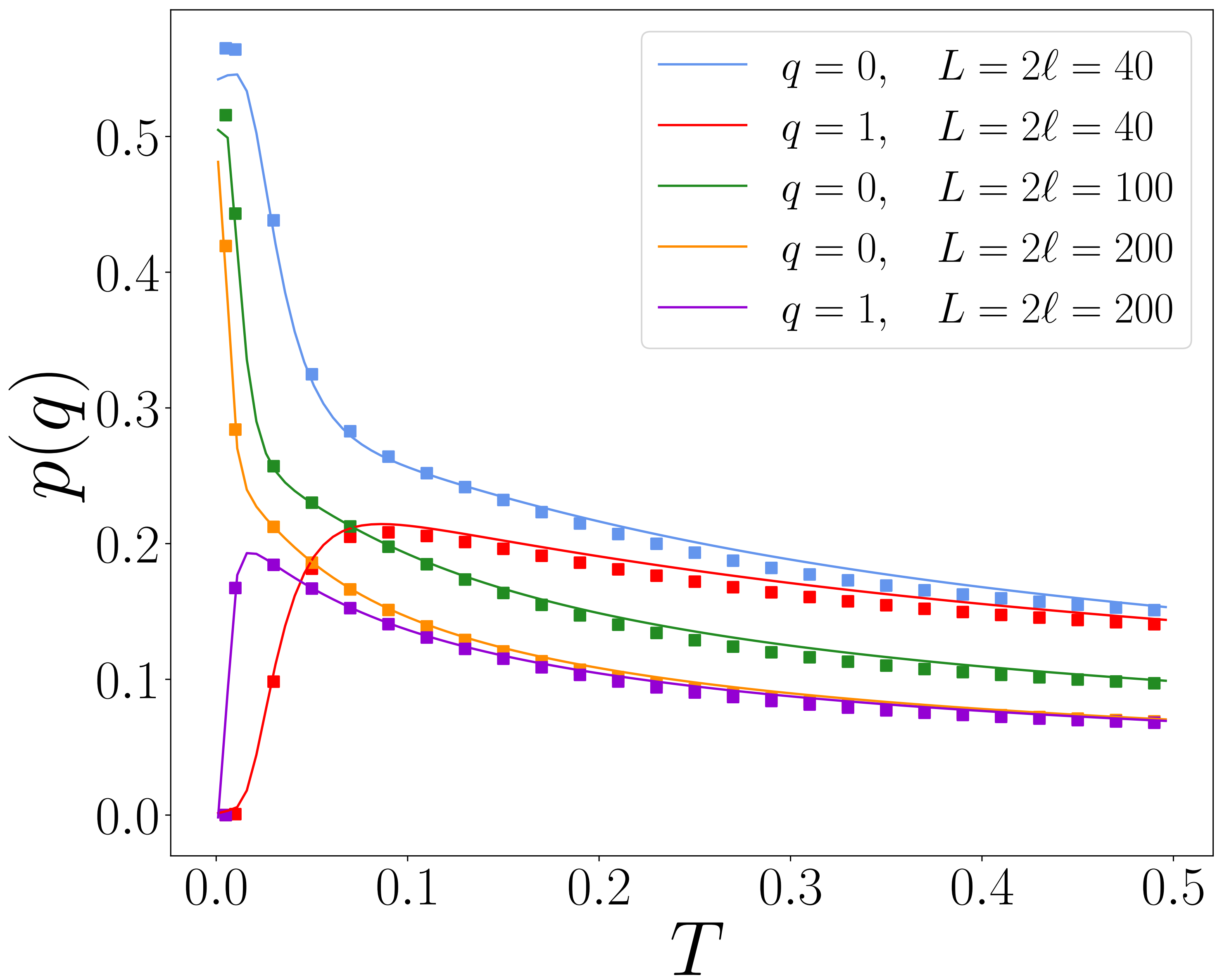












Main message:

The symmetry resolution of entanglement measures provides a fine structure of the entanglement content of physical states of extended quantum systems that is not accessible from the measure of the total entanglement

Some features:

- Measurable experimentally (actually already measured!)
- Easy to compute via charged moments
- Relation to charge statistics, entanglement Hamiltonian, ….