Some scaling methods for infinite tensor network calculations @ Toulouse 2022

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University of Vienna

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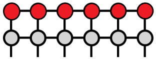
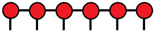
  - $1/\Lambda \quad \rightarrow \quad s/\Lambda$ (a cut-off or perturbation of dimension 1)
- Exponents ν_g and β_O are determined by the CFT
- Not clear tensor network approximations have an effective Λ description

MPS



$\approx \lambda$



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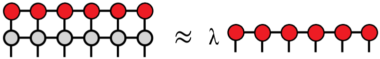

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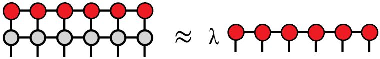

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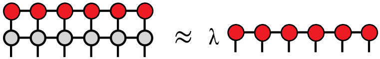

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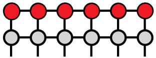
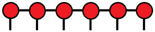
 $\approx \lambda$ 

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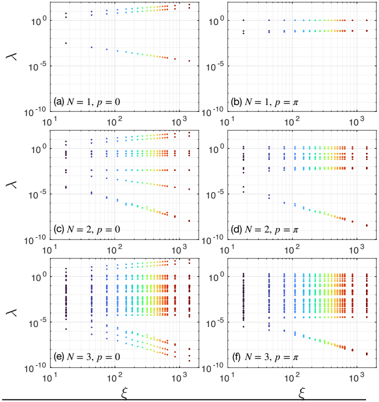

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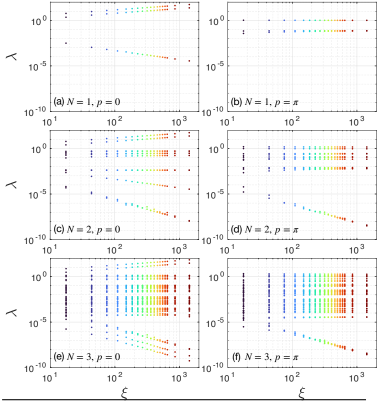

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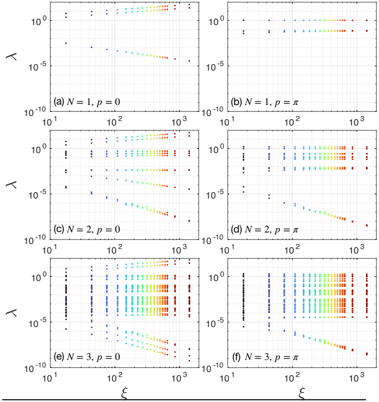

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- $\lambda = ||\sum_i G_i\,|\psi\rangle\,||_2^2$

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Off criticality - interacting dimers

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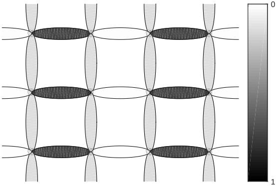

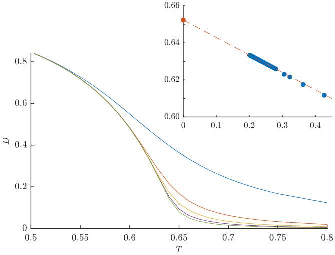

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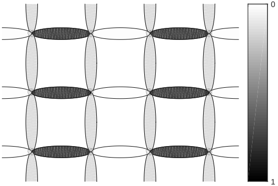

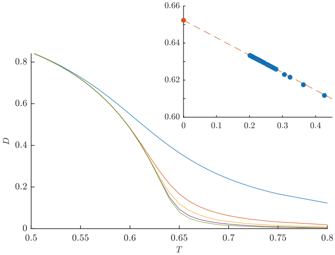

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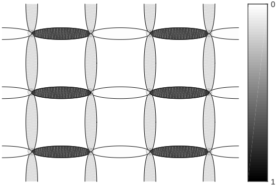

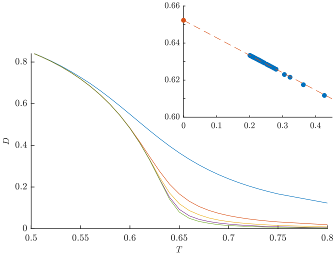

- KT transition
- Remarkably accurate relation - no theoretical explanation

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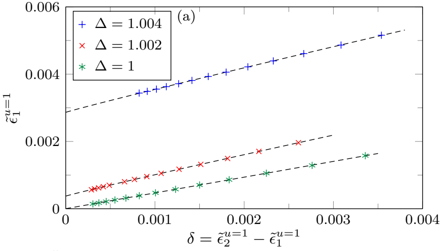

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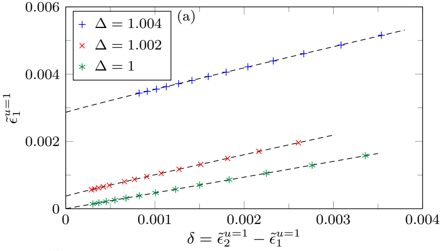

- $1/\delta$ should thus fit our purpose

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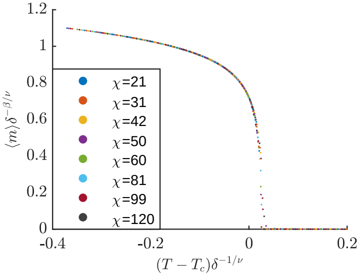

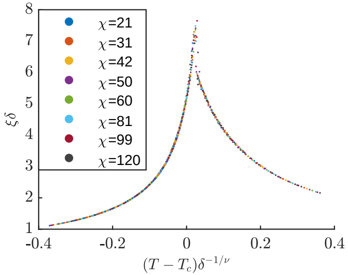

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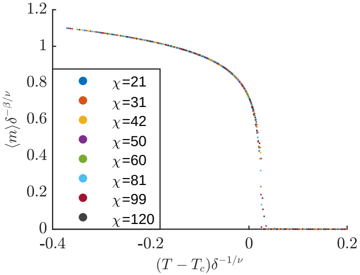

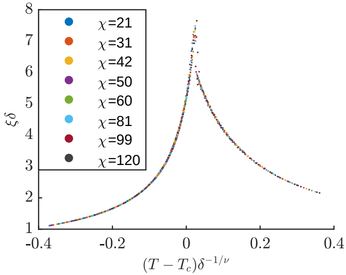

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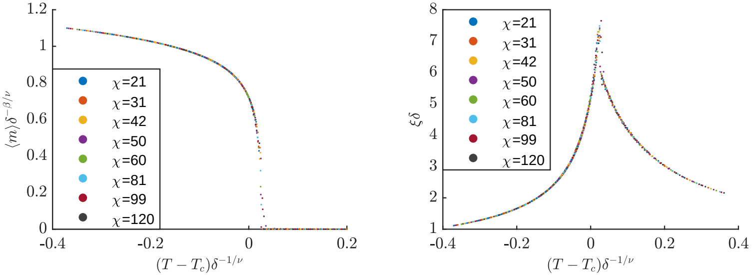

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QFT - even more scaling properties

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- A QFT with a critical point is thus subject to two independent scalings[5]:

	UV dimension	IR dimension
λ	2	0
$\alpha - \alpha_c$	0	$1/\nu = 1$
L^{-1}, δ	1	1
$l^{-1} = L^{-1}/\sqrt{\lambda}$, $\Delta = \delta/\sqrt{\lambda}$	0	1
ξ	-1	-1
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- One may use a double collapse

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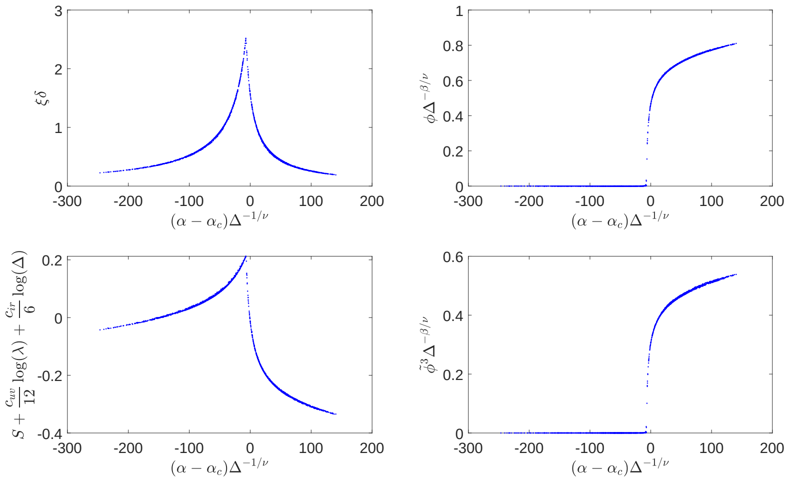

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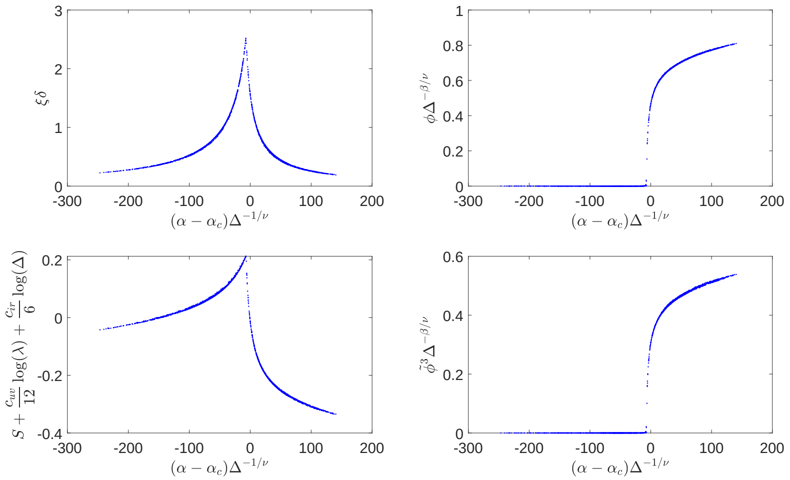

- Proved extremely effective at determining the critical point

PEPS

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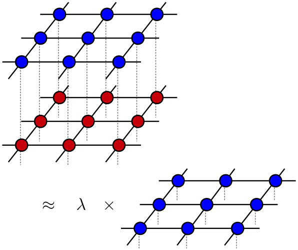

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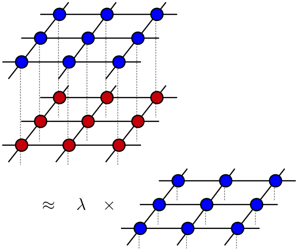

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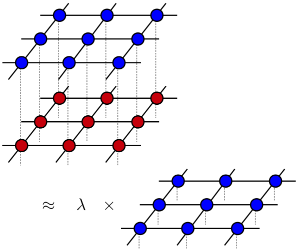

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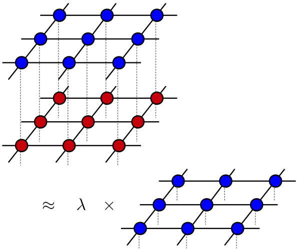

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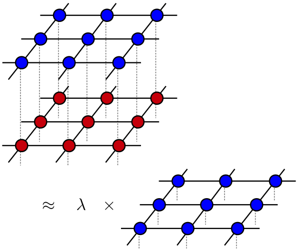

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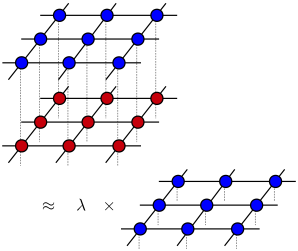

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Extrapolate in D

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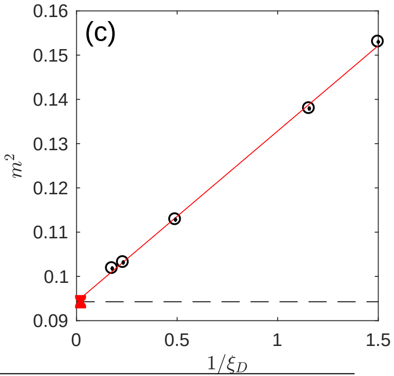
- Then only one approximation parameter -D- remains
- One can then extrapolate in ξ (correlation length of the boundary MPS)
- Example: 2D Heisenberg model[6]



[6] PRX P. Corboz, P. Czarnik, G.Kapteijns, and L. Tagliacozzo Finite Correlation Length Scaling with Infinite Projected Entangled-Pair States

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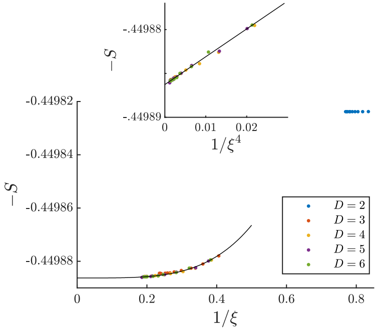

- Example: 3D dimer model

[7] PRL B. V., J. Hasik, F. Verstraete, L. Vanderstraeten

PEPS scaling hypothesis

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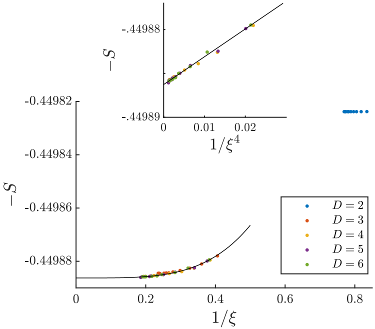

- Example: 3D dimer model

[7] PRL B. V., J. Hasik, F. Verstraete, L. Vanderstraeten

bPEPS scaling hypothesis

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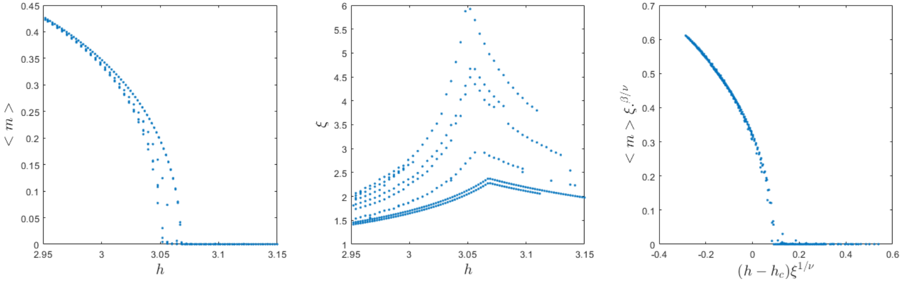
- PEPS scaling hypothesis also applies to bPEPS

bPEPS scaling hypothesis

bPEPS scaling hypothesis



Collaborators

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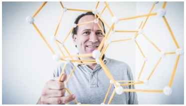

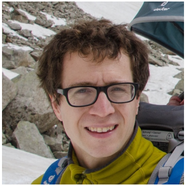

Thank you!

Questions?