A Tutorial on Control Methods for Low Level RF Accelerator Applications

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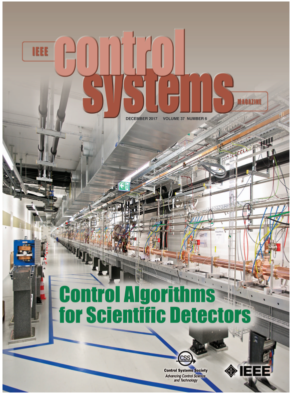
Outline

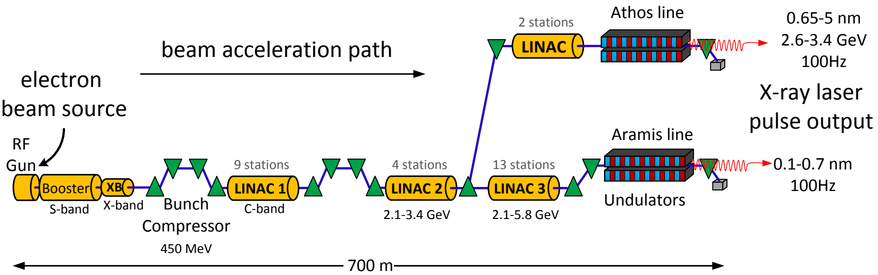
- Iterative learning control
  - Repetitive control tasks
  - Example: RF pulse shaping
- Robust control
  - Modeling and handling system uncertainty
  - Robust control design
  - Example: Beam injector control
- Adaptive robust methods
  - Online estimation of uncertainty
  - Example: Laser pulse stacking (automated tuning)
  - Example: Beam orbit correction
- Data-driven control
  - Dynamic models
  - Integration with predictive control methods



A. Rezaeizadeh, T. Schilcher, R.S. Smith, "Control of the Swiss Free Electron Laser," *IEEE Control Systems Magazine*, **37**(6), pp. 30–51, 2017.

Swiss Free Electron Laser

Schematic



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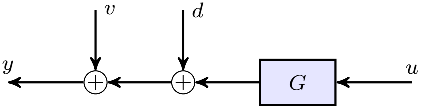
Problem configuration



Linear system description:

$$y_k \;=\; G(z)\,u_k \;+\; d_k \;+\; v_k, \quad \text{for time steps } k = 1, \ldots, N.$$

or, in matrix form,

$$\underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}}_{y} \;=\; G \underbrace{\begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix}}_{u} + \underbrace{\begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}}_{d} + \underbrace{\begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix}}_{v},$$

where:

d_k	disturbance
$v_k \sim \mathcal{N}(0, \sigma_v^2)$	(stochastic noise)

Iterative learning control

Iterative operation

The system operates repetitively. Each repetition is described by:

$$y = Gu + d + v, \qquad y, u, v, d \in \mathbb{R}^N, \quad G \in \mathbb{R}^{N \times N},$$

Repetitions usually have a significant uncontrolled time interval between them.

The unknown disturbance signal, $d \in \mathbb{R}^N$, is constant over all repetitions.
For example: unknown, but constant, initial conditions.

Design problem (feedforward)

Design the input signal, $u \in \mathbb{R}^N$ to achieve:

- Reference tracking: $y = y^{\text{ref}} \in \mathbb{R}^N$.
- Information (u and y) from past repetitions is available.

Potential difficulties

- Uncertainty about the true dynamics:

$$y = G\,u + \Delta_G(u) + d + v, \qquad \text{(errors caused by } d \text{ and } \Delta_G(u) \text{ can't be disambiguated)}$$

- Measurement noise: $v_k \sim \mathcal{N}(0, \sigma_v^2)$

Iterative learning control approach

- Iteratively update the next input: $u^i \in \mathbb{R}^N$ (i is the iteration index)
- If y^{ref} is the same for all repetitions then we want u^i to converge so that:

$$\mathbf{E}\left\{y^{\text{ref}} + v\right\} = y^{\text{ref}} = G\,u^i + \Delta_G(u^i) + d, \qquad \text{(proceed cautiously)}$$

- Only update u^i to u^{i+1} every L repetitions of the signal. Estimate y^i by averaging,

$$\hat{y}^i = \frac{1}{L}\sum_{l=1}^{L} y^{i,l} \qquad (u^i \text{ is constant for all } L \text{ repetitions})$$

Iterative learning control

Cost function

At each update of u (from u^i to u^{i+1}) we will minimise,

$$\underbrace{\|y^{\text{ref}} - y^{i+1}\|_Y^2}_{\substack{\text{tracking}\\\text{error}}} \;+\; \underbrace{\|u^{i+1} - u^i\|_R^2}_{\substack{\text{input}\\\text{change}}} \;= (y^{\text{ref}} - y^{i+1})^T Y (y^{\text{ref}} - y^{i+1}) + (u^{i+1} - u^i)^T R (u^{i+1} - u^i).$$

Tracking error dynamics

Neglecting noise,

$$y^{\text{ref}} - y^{i+1} = y^{\text{ref}} - G u^{i+1} - \Delta_G(u^{i+1}) - d^{i+1} \qquad \text{(and similarly for } y^{\text{ref}} - y^i\text{)}$$

If, as assumed, $\Delta_G(u^{i+1}) + d^{i+1} = \Delta_G(u^i) + d^i$ then,

$$\underbrace{(y^{\text{ref}} - y^{i+1})}_{\text{next error}} = \underbrace{(y^{\text{ref}} - y^i)}_{\text{prior error}} - G(u^{i+1} - u^i)$$

Updating the input signal

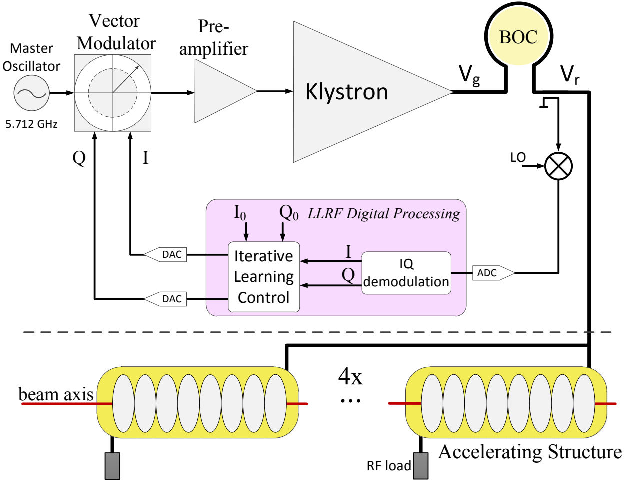
$$\begin{aligned} \underset{u^{i+1}}{\text{minimise}} \quad & \|y^{\text{ref}} - y^{i+1}\|_Y^2 \; + \; \|u^{i+1} - u^i\|_R^2 \\ \text{subject to:} \quad & (y^{\text{ref}} - y^{i+1}) \; = \; (y^{\text{ref}} - y^i) \; - \; G(u^{i+1} - u^i) \end{aligned}$$

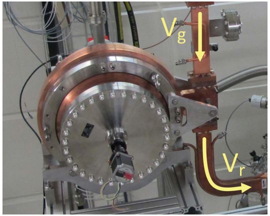
Thiis has a closed-form solution:

$$u^{i+1} \; = \; u^i + \; \underbrace{(R + G^T Y G)^{-1} G^T Y}_{\text{constant matrix}} \; (y^{\text{ref}} - y^i).$$

- We could also include input constraints (e.g. $u^{\text{min}} \leq u^{i+1} \leq u^{\text{max}}$)
- In practice we will use the averaged estimate of the tracking error: $y^{\text{ref}} - \hat{y}^i$.

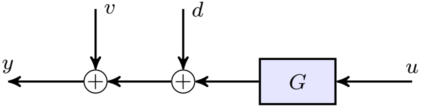
Iterative Learning Control:





Control problem

Inputs:	u	Vector modulator input $(u_I + j u_Q)$
Outputs:	y	Barrel Open Cavity (BOC) reflected output $(y_I + j y_Q)$
Plant:	$G(z)$	Vector modulator and BOC dynamics



Plant model

$$G_{\mathsf{BOC}}(z) \;=\; \frac{V_r(z)}{V_g(z)} \;=\; \frac{T_s(\alpha - 1) - \tau - jT_s\tau\Delta_\omega + \tau z^{-1}}{T_s + \tau + jT_s\tau\Delta_\omega - \tau z^{-1}}.$$

(first order, complex-valued)

$$G(z) \;=\; K \frac{1-\gamma}{1-\gamma z^{-1}}\, G_{\mathsf{BOC}}(z).$$

T_s: sample period, τ: filling time, Δ_ω: detuning shift, α: coupling term.

Nominal plant model: G

$$G_{IQ} = \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ h_2 & h_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ h_N & \cdots & \cdots & h_1 \end{bmatrix}$$

Toeplitz matrix of impulse response coefficients (complex-valued)

Real and complex part decomposition: $G_{IQ} = G_{\text{real}} + jG_{\text{imag}}$

Real-valued model:

$$\underbrace{\begin{bmatrix} y_I \\ y_Q \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} G_{\text{real}} & -G_{\text{imag}} \\ G_{\text{imag}} & G_{\text{real}} \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} u_I \\ u_Q \end{bmatrix}}_{u}$$

Operating modes: input signal

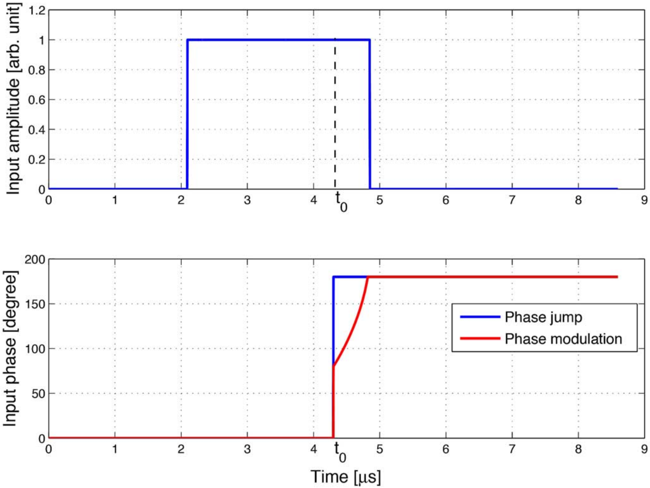
Control interval: $N = 2048$

Phase jump:

1. Constant amplitude "filling"
2. 180° phase jump
3. Output reflected voltage jump
4. Output reflected voltage decay

Phase modulation:

1. Constant amplitude "filling"
2. Smaller phase jump
3. Lower output voltage
4. Phase modulated to maintain constant voltage.



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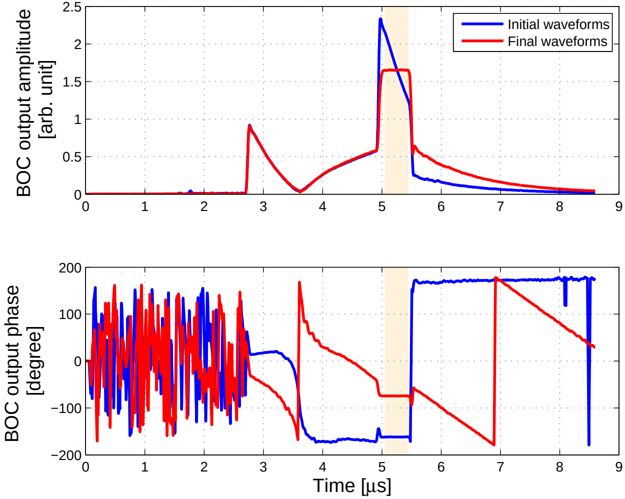
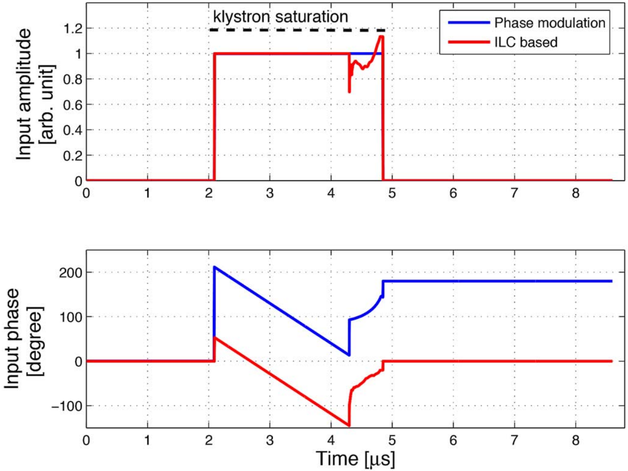


ILC results: Comparison with phase modulation

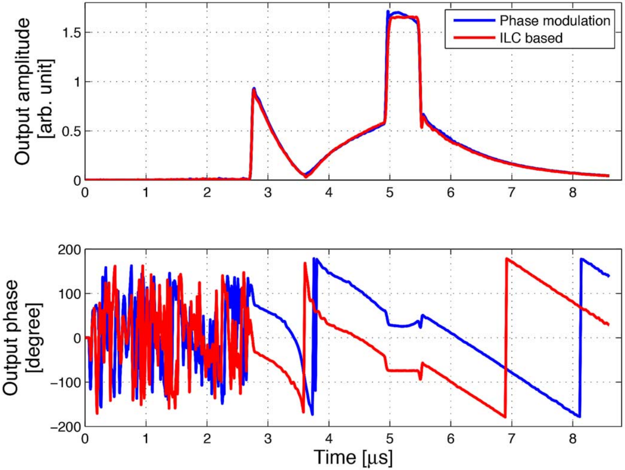
Input signal:



Iterative Learning Control

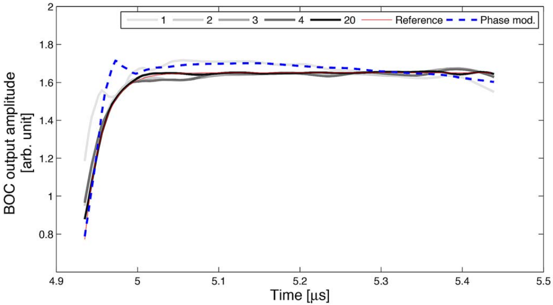
ILC results: Comparison with phase modulation

Output signal:

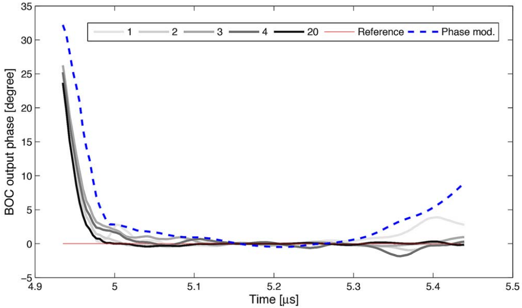


ILC results: Output voltage detail

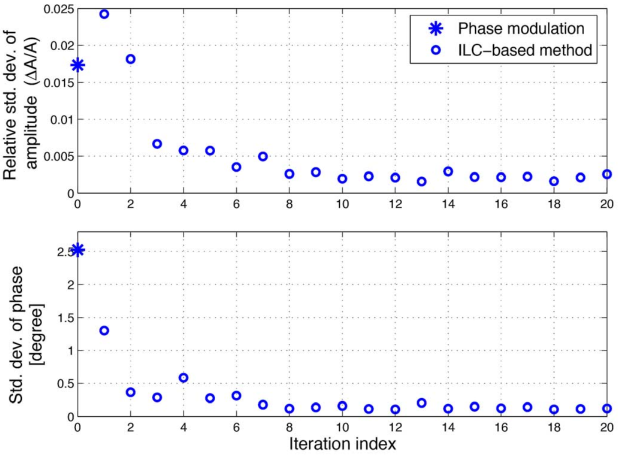
Amplitude



Phase



ILC results: convergence



Summary

- Applicable to repetitive problems, particularly pulsed-mode accelerators.
- Effectively removes the effect of unknown but constant disturbances and initial conditions
- Works well for feedforward reference signal generation (can be used in combination with feedback)
- Trade-off between rate of input signal updates and tracking error convergence.

Robust control

Sources of our uncertainty

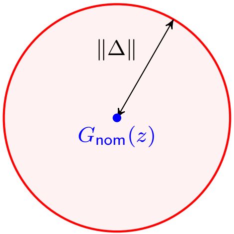
- Environmental variations (e.g. temperature)
- Component ageing
- Unmodeled dynamics (too complex, no easy model)
- Neglected dynamics (accuracy is too expensive)
- Operation over a range of operating points.
- Non-repeatable dynamic behaviour

Modeling uncertainty: set-based description

$$G(z) \in \{\, G_{\text{nom}}(z) + \Delta \mid \|\Delta\| \leq \gamma \,\}$$

$G_{\text{nom}}(z)$ = Nominal plant

Δ = unknown, but bounded, perturbation (i/o operator)



Robustness

Stability and performance for all $G(z)$ in the set.

Uncertain static systems

Basic model (response matrix, R):

$$y = f(u), \qquad \text{linearised as} \quad y - y_0 = R_{\text{nom}}(u - u_0), \quad R_{\text{nom}} \in \mathbb{R}^{n \times m},$$

Assume that $n \geq m$.

Perturbation model:

$$R \in \mathcal{R}, \quad \mathcal{R} = \{ R \mid R = R_{\text{nom}}(I + \Delta), \ \Delta \text{ block structured}, \|\Delta\|_2 \leq \gamma < 1 \}.$$

Note that in the static case, if we don't have $\|\Delta\|_2 < 1$ then we do not know the sign of R_{nom} and control is not possible.

Simple (somewhat naïve and non-robust) approach

If $\Delta = 0$, then the predicted next output is $\hat{y}_{k+1} = y_k + R_{\text{nom}}(u_{k+1} - u_k)$.

Least-squares solution (assume for simplicity that R_{nom} has full column rank):

$$u_{k+1} = u_k + (R_{\text{nom}}^T R_{\text{nom}})^{-1} R_{\text{nom}}^T (y^{\text{ref}} - y_k).$$

Linearised error $(y^{\text{ref}} - y)$ propagation is,

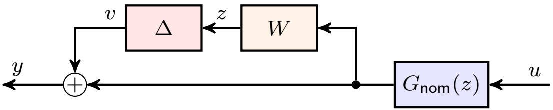
$$\begin{aligned} e_{k+1} &\approx (I - R_{\text{nom}}(I + \Delta)(R_{\text{nom}}^T R_{\text{nom}})^{-1} R_{\text{nom}}^T)\, e_k \\ &= -R_{\text{nom}} \Delta R_{\text{nom}}^{-1}\, e_k \qquad \text{(assume } R_{\text{nom}} \text{ is square)} \end{aligned}$$

If for any $\Delta \in \mathbf{\Delta}$, $\max \left| \text{eig}(R_{\text{nom}} \Delta R_{\text{nom}}^{-1}) \right| > 1$, then the error is unstable.

But $\displaystyle \max_{\Delta \in \mathbf{\Delta}} \max \left| \text{eig}(R_{\text{nom}} \Delta R_{\text{nom}}^{-1}) \right| = \gamma \|R_{\text{nom}}\|_2 \|R_{\text{nom}}^{-1}\|_2 = \gamma\kappa$ (κ is condition number of R_{nom})

Model structures

Multiplicative output perturbation

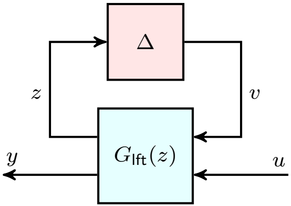


$$y = (I + \Delta W) G_{\mathsf{nom}}(z)\, u$$

$$\Delta \in \boldsymbol{\Delta},$$

$$\boldsymbol{\Delta} = \{\Delta \mid \Delta \text{ block structured, } \|\Delta\| \leq 1\}$$

Linear fractional perturbation



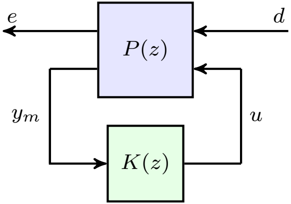
$$\begin{aligned} y &= \mathcal{F}_u\left(G_{\mathsf{lft}}, \Delta\right) u \\ &= \left(G_{2,2} + G_{2,1}\Delta(I - G_{1,1}\Delta)^{-1}G_{1,2}\right) u \end{aligned}$$

$$\Delta \in \boldsymbol{\Delta},$$

$$\boldsymbol{\Delta} = \{\Delta \mid \Delta \text{ block structured, } \|\Delta\| \leq 1\}$$

Robust control

Nominal design

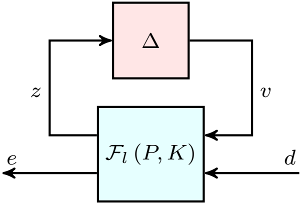


Design $K(z)$ so that:

Nominal closed-loop is stable.

$\|e\|_2 \leq \|d\|_2$.

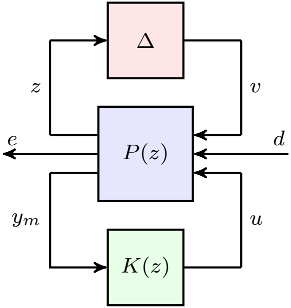
Robustness analysis



Closed-loop stable for all $\Delta \in \boldsymbol{\Delta}$?

$\|e\|_2 \leq \|d\|_2$ for all $\Delta \in \boldsymbol{\Delta}$?

Robust control design



Design $K(z)$ such that:

Closed-loop stable $\forall \Delta \in \boldsymbol{\Delta}$

$\|e\|_2 \leq \|d\|_2$ for all $\Delta \in \boldsymbol{\Delta}$

Robust control

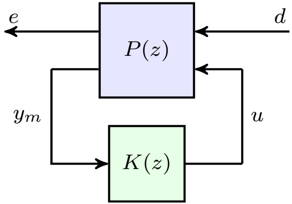
Nominal design: $\mathcal{H}_\infty$

$\mathcal{H}_\infty$ control objective:

$$\left\| \mathcal{F}_l\left(P(z), K(z)\right) \right\|_{\mathcal{H}_\infty} \quad = \quad \max_{\|d\|_2 \leq 1} \|e\|_2$$

$\mathcal{H}_\infty$ control design:

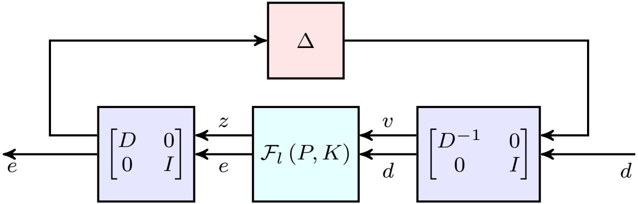
$$\min_{K(z),\text{stabilising}} \left\| \mathcal{F}_l\left(P(z), K(z)\right) \right\|_{\mathcal{H}_\infty}$$

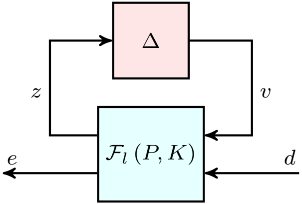


This is a convex problem, easily solved in Matlab (via Robust Control Toolbox).

Robust control

Robustness analysis (structured singular value)





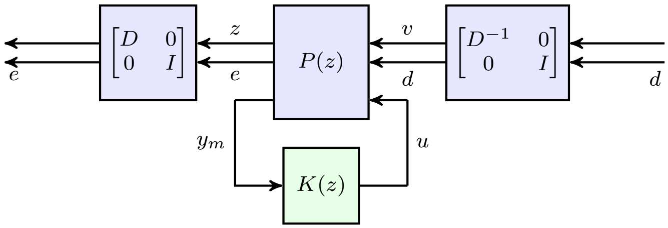
Define $\mathcal{D} = \left\{ D \;\middle|\; D\Delta D^{-1} \in \mathbf{\Delta},\ D = D^T,\ D > 0 \right\}$ (D commute with all $\mathbf{\Delta}$)

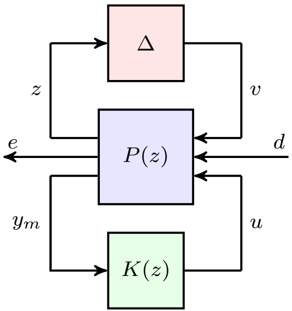
Sufficient condition

If there exists $D \in \mathcal{D}$ such that, $\overline{\sigma}\left(\begin{bmatrix} D & 0 \\ 0 & I \end{bmatrix} \mathcal{F}_l\left(P(z), K(z)\right) \begin{bmatrix} D^{-1} & 0 \\ 0 & I \end{bmatrix} \right) < 1,$ ($\overline{\sigma}$ () is maximum singular value)

then, the closed loop system is stable for all $\Delta \in \mathbf{\Delta}$, and $\|e\|_2 < \|d\|_2$ for all $\Delta \in \mathbf{\Delta}$.

Robust design

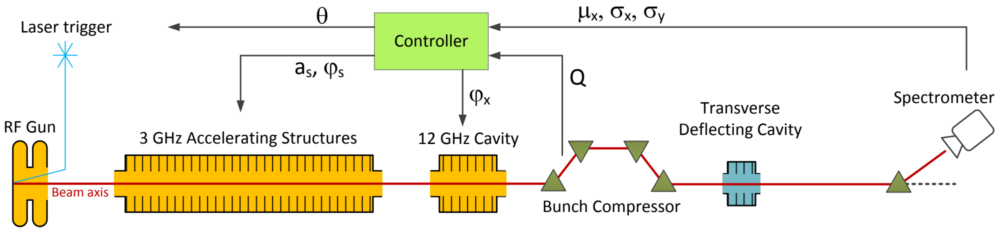




Iterative approach:

- Design a nominal $\mathcal{H}_\infty$ controller (convex)
- Calculate the robustness upper-bound ($D \in \mathcal{D}$, convex)
- Rescale design problem with D, D^{-1}matrices
- Iterate.

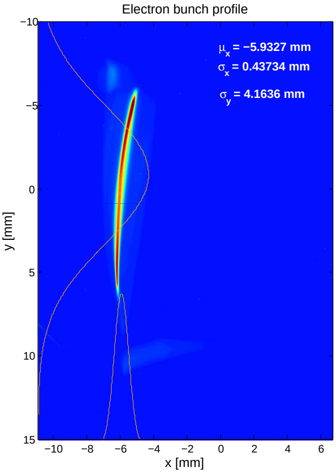
SwissFEL beam profile control



Inputs:	θ	laser intensity	Measurements:	Q	charge	
	a_s	S-band amplitude		μ_x	beam mean x posn.	energy change
	ϕ_s	S-band phase		σ_x	beam std. dev. x	energy spread
	ϕ_x	X-band phase.		σ_y	bean std. dev. y	bunch length

Robust control

Electron bunch profile



Response matrix (linearised model)

$$\delta y \;=\; R\,\delta u, \quad \text{where,}$$

$$y \;=\; \begin{bmatrix} Q \\ \mu_x \\ \sigma_x \\ \sigma_y \end{bmatrix} \qquad u \;=\; \begin{bmatrix} \theta \\ a_s \\ \phi_s \\ \phi_x \end{bmatrix}$$

Perturbation model

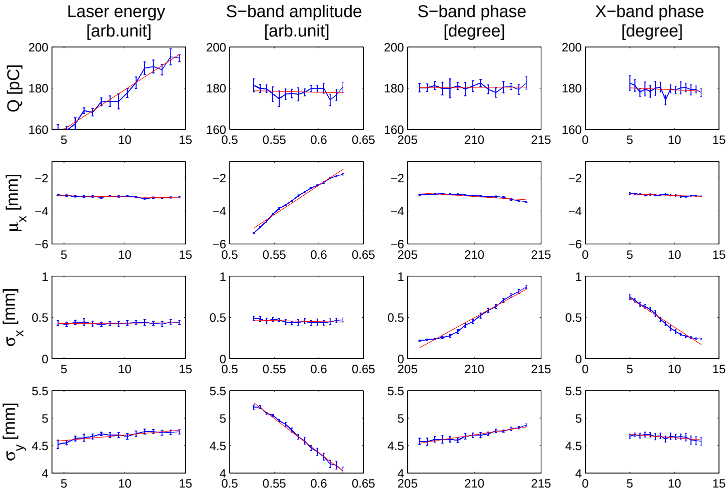
$$R \;=\; (I + W\Delta)\,R_{\text{nom}}$$

Nominal system, $R_{\text{nom}} \in \mathbb{R}^{4\times 4}$

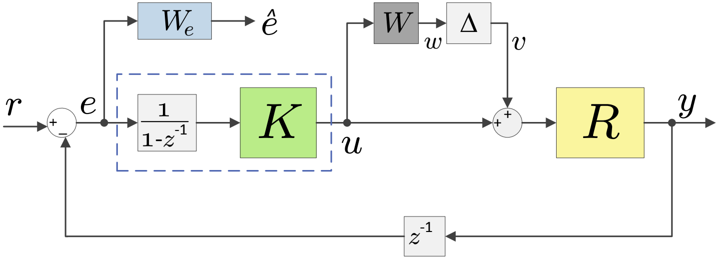
Perturbation, $\Delta \in \mathbb{R}^{4\times 4}$

Robust control

Experimental response matrix estimate



Robust control approach - weighted static design

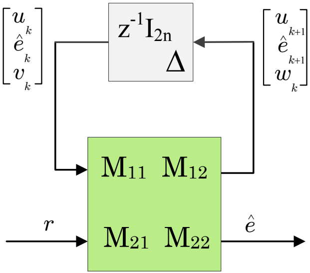


Controller: integrated error (4 channels) $\times$ constant matrix gain, K

Error/penalty ($\hat{e}$): low pass filtered tracking error, $W_e(z) = \alpha/(z - \gamma)$

Measurement delay in the feedback loop.

Closed-loop robust formulation



$$M = \left[\begin{array}{cc|cc} I & 0 & 0 & 0 \\ \alpha R & \gamma I & \alpha R & \alpha I \\ \hline W & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{array}\right] + \left[\begin{array}{c} I \\ 0 \\ \hline 0 \\ 0 \end{array}\right] K_{\text{rob}} \left[\begin{array}{cc|cc} R & 0 & R & I \end{array}\right]$$

$$= P + U K_{\text{rob}} V \qquad \text{(affine in } K_{\text{rob}}\text{)}$$

Define $\mathcal{D} = \operatorname{diag}(D_z, d_\Delta I_n, d_p I_n), \quad D_z \in \mathbb{C}^{2n \times 2n}$,

If there exists $D \in \mathcal{D}$ such that,

$$\overline{\sigma}\left(D(P + U K_{\text{rob}} V) D^{-1}\right) < 1, \qquad (\overline{\sigma}\,() \text{ is maximum singular value})$$

then the closed-loop system is stable and

$$\|\hat{e}\|_2 \leq \|r\|_2 \quad \text{for all} \quad \Delta \in \boldsymbol{\Delta}. \qquad \text{Robust performance}$$

Design: computational approach

Design objective:

$$\beta_{\text{opt}} = \inf_{\substack{K_{\text{rob}} \in \mathbb{R}^{4\times 4} \\ D \in \mathcal{D}}} \overline{\sigma}\left(D(P + UK_{\text{rob}}V)D^{-1}\right)$$

If $\beta_{\text{opt}} < 1$ then the robust performance objective is achieved.

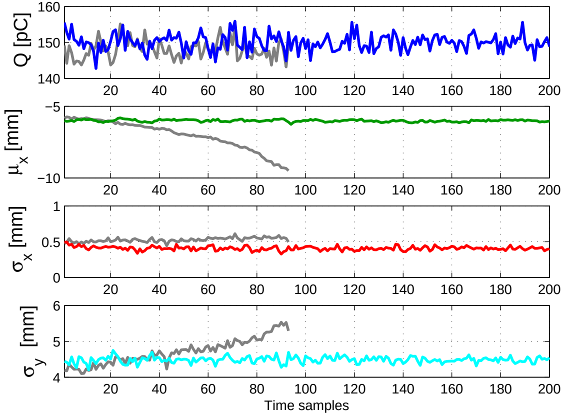
For fixed K_{rob}, $\quad \inf_{D \in \mathcal{D}} \overline{\sigma}\left(D(P + UK_{\text{rob}}V)D^{-1}\right) \quad$ is a convex problem

For fixed D, $\quad \inf_{K_{\text{rob}}} \overline{\sigma}\left(D(P + UK_{\text{rob}}V)D^{-1}\right) \quad$ is a convex problem

Computational approach: iterate between minimising D and minimising K_{rob}.

This gives an upper bound on β_{opt}.

Experimental results: comparison with inverse-based control



Inverse-based control:

$$u_{k+1} = u_k + \gamma R_{\text{nom}}^{-1} e_k, \quad \gamma = 0.005.$$

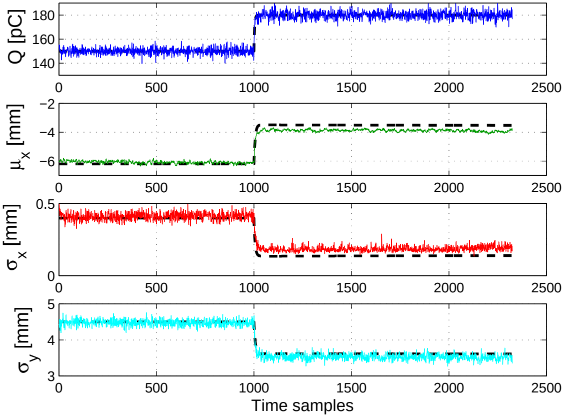
Robust control:

$$u_{k+1} = u_k + K_{\text{rob}} e_k.$$

The inverse-based controller (shown in grey) loses control of the beam.

The condition number of R_{nom} is $\approx 10^4$.

Experimental results: reference step change



Summary

- Uncertainty about the system (dynamic and static) is captured by a set description.
- Uncertain set is parametrised by perturbation Δ which can be:
  - Static (unknown parameters) or dynamic (unknown systems);
  - Structured (uncertainty at specific locations) or unstructured (cross-coupling).
- Design and analysis applicable to highly structured systems (e.g. high condition number plants).
- Stability and performance for all perturbations in a set is verified computationally.
- Robust design achieved by iterating between nominal design and robust analysis.
- Static controllers (and static plants) lead to simpler matrix problems.
- Typically there is a trade-off between robustness and performance.

Uncertain static systems

Basic model (response matrix, R):

$$y \;=\; f(u), \qquad \text{linearised as} \quad y - y_0 \;=\; R_{\textsf{nom}}(u - u_0), \quad R_{\textsf{nom}} \in \mathbb{R}^{n\times m},$$

Assume that $n \geq m$. (more measurements than controls)

Perturbation model:

$$R \in \mathcal{R}, \quad \mathcal{R} = \{ R \;|\; R = R_{\textsf{nom}}(I + \Delta),\; \Delta \text{ block structured}, \|\Delta\|_2 \leq \gamma < 1 \,\}\,.$$

Adaptive robust control

Overall structure: run-to-run

- Robust control design to calculate:
  - the next input: u_{k+1}, or
  - the next controller, K;
- Collect data over a "run" using the same controller or input calculation strategy;
- Use data from the last "run" to refine the estimate of the system by:
  - estimating Δ; or
  - refining both R_{nom} and γ
- Iterate until converged.

Adaptive robust control

Initialisation

Robust control calculation of u_1:

$$\begin{aligned} u_1 \;=\; \min_{u} \max_{\Delta \in \boldsymbol{\Delta}, \|\Delta\| \leq \gamma} & \quad \|y^{\textsf{ref}} - y_0 + R_{\textsf{nom}}(I + \Delta)u\|_2 \\ \text{subject to:} & \quad \|u\|_2 < u_{\textsf{max}}, \\ & \quad \|u - u_k\|_2 < \delta u_{\textsf{max}}. \end{aligned}$$

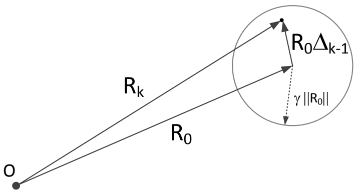
Simple case: Δ is unstructured

The maximisation over Δ simplifies the cost function to:

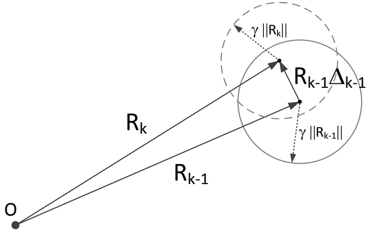
$$\|y^{\textsf{ref}} - y_0 + R_{\textsf{nom}} u\|_2 \;+\; \gamma \|R_{\textsf{nom}}\|_2 \|u\|_2$$

This gives a (potentially constrained) least squares problem.

Model refinement



(a) Nonadaptive



(b) Adaptive

Nonadaptive: optimistic

Estimate Δ_{k-1} from u_{k-1} and y_{k-1}

Design using $R_k \;=\; R_{\text{nom}} \;+\; R_{\text{nom}}\Delta_{k-1}$

Adaptive: conservative

Estimate Δ_{-1} from u_{k-1} and y_{k-1}

Update the nominal plant:
$R_k \;=\; R_{k-1} + R_{k-1}\Delta_{k-1}$

Adjust the perturbation bound to $\gamma\|\mathbb{R}_{k-1}\|_2$.

Model refinement: estimating Δ

Using the most recent data: u_{k-1}, y_{k-1},

$$\begin{aligned} \min_{\Delta} \quad & \|y_{k-1} - y_0 - R_{\mathsf{nom}}(I + \Delta)(u_{k-1} - u_0)\|_2 \\ \text{subject to:} \quad & \|\Delta\|_2 \leq \gamma. \end{aligned}$$

Practical issues

- Using the most recent past data allows us to track slow plant drifts.
- The solution to the optimisation problem is generally not unique and optimistic.
- Uniqueness may be regained (temporarily) if we use multiple (m^2/n) past u, y data points.
- As u_k, $k \longrightarrow \infty$, converges uniqueness is again lost (insufficiently exciting input).
- See A. Rezaeizadeh's Ph.D. thesis for more details.
- Strongly related to the field of dual control and exploration/exploitation trade-offs.

Adaptive robust control

Optimisitic design

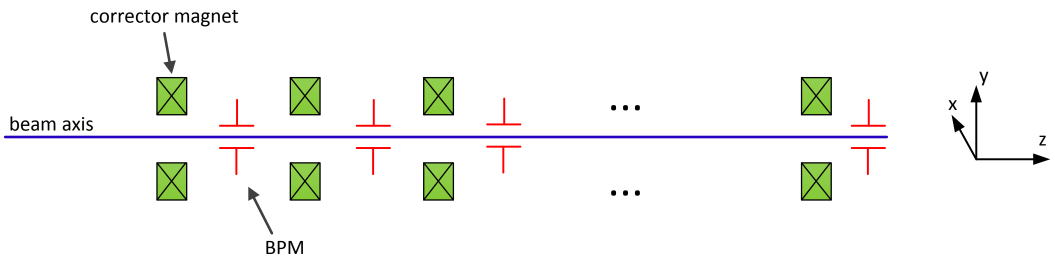
$$
\begin{aligned}
\delta u_{k+1} \;=\; \min_{\delta u} \quad & \|y^{\mathsf{ref}} - y_k + R_{\mathsf{nom}}(I + \Delta_k)\delta u\|_2 \\
\text{subject to:} \quad & |u_k + \delta u| < u_{\mathsf{max}} \\
& \|\delta u\|_2 < \delta u_{\mathsf{max}}.
\end{aligned}
$$

Robust (conservative) design

$$
\begin{aligned}
u_{k+1} \;=\; \min_{u} \; \max_{\Delta \in \mathbf{\Delta}, \|\Delta\| \leq \gamma} \quad & \|y^{\mathsf{ref}} - y_0 + R_k(I + \Delta)(u - u_0)\|_2 \\
\text{subject to:} \quad & \|u\|_2 < u_{\mathsf{max}}, \\
& \|u - u_k\|_2 < \delta u_{\mathsf{max}}.
\end{aligned}
$$

Adaptive robust control

Control of the electron beam orbit



Transverse beam position measured by monitors (BPM) and adjusted by corrector magnets.

Ten magnets and ten BPMs in each direction (x and y)

Static orbit control with adaptive updates at approx. 1 Hz.

Orbit control: static model

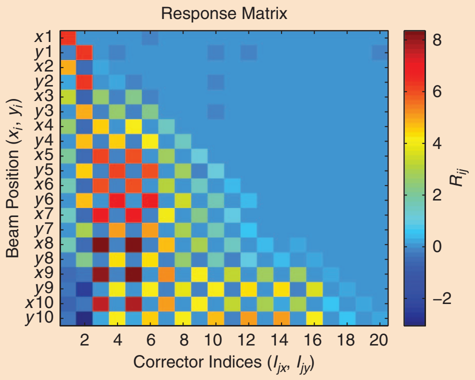
Incremental position change (x-direction)

$$\delta x_i \;=\; \sum_{j=1}^{N} r_{ix,jx}\,\delta I_{jx} \;+\; r_{ix,jy}\delta I_{jy}$$

$$\begin{bmatrix} \delta x_1 \\ \delta y_1 \\ \vdots \\ \delta x_N \\ \delta y_N \end{bmatrix} \;=\; \underbrace{\begin{bmatrix} r_{1x,1x} & r_{1x,1y} & \cdots & r_{1x,Nx} & r_{1x,Ny} \\ r_{1y,1x} & r_{1y,1y} & \cdots & r_{1y,Nx} & r_{1y,Ny} \\ \vdots & \vdots & & \vdots & \vdots \\ r_{Nx,1x} & r_{Nx,1y} & \cdots & r_{Nx,Nx} & r_{Nx,Ny} \\ r_{Ny,1x} & r_{Ny,1y} & \cdots & r_{Ny,Nx} & r_{Ny,Ny} \end{bmatrix}}_{R_{\text{nom}}} \begin{bmatrix} \delta I_{1x} \\ \delta I_{1y} \\ \vdots \\ \delta I_{Nx} \\ \delta I_{Ny} \end{bmatrix}$$

Perturbation model: $\quad R = R_{\text{nom}}(I + \Delta), \quad \Delta = \delta I, \quad$ (scalar $\times$ identity)

Orbit control: experimental response matrix



Negligible cross-coupling between x and y directions

Corrector magnets do not affect upstream positions

Significant differences between x and y responses

Condition number of R_{nom} is approx. 1.5×10^4

Significant actuation limits (corrector magnet saturation).

Orbit control: control algorithm

Calculation of the next corrector magnet input vector.

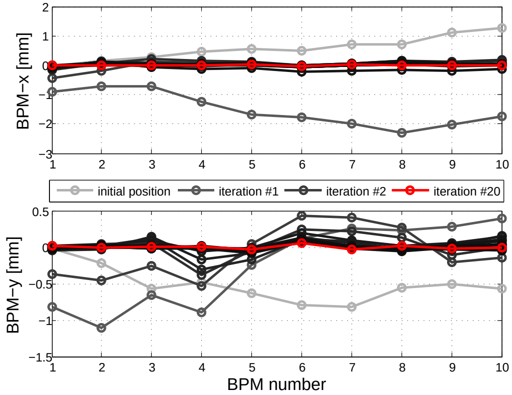
$$\begin{aligned} u_{k+1} \; = \; \operatorname*{argmin}_{u} & \quad \| y^{\mathsf{ref}} - R_{\mathsf{nom}}(I + \Delta_k) u \|_2 + p(u), \\ \text{subject to:} & \quad \|u\|_2 < u_{\mathsf{max}}, \\ & \quad \|u - u_k\|_2 < \delta u_{\mathsf{max}}. \end{aligned}$$

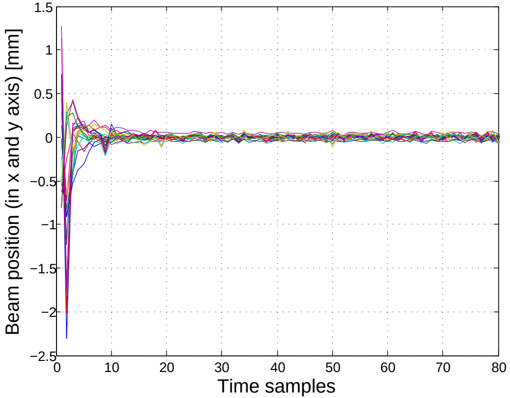
Penalty function: $\quad p(u) \; = \; \alpha \sum_{i=1}^{m} \left| \frac{u_i}{u_{\mathsf{max}}} \right|^{10}, \quad \alpha > 0.$

The optimisation problem is solved via the Matlab CVX toolbox.

Adaptive robust control

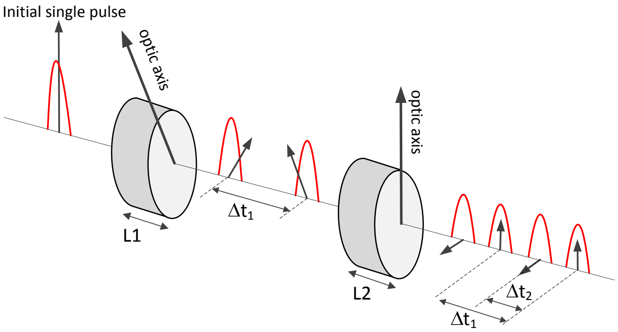
Orbit control: experimental results (25 times reduction in RMS error)





Laser pulse-stacking automation

- Single Gaussian laser pulse input
- N birefringent crystals
- Each crystal splits the pulse into two time-separated pulses.
- Δt determined by L and refractive index.
- Relative intensities determined by crystal optical axis rotation.
- 2^N Gaussian pulses superimpose to give a single, long pulse.
- Objective: adjust crystal angles to create a flat-topped output pulse.



Laser pulse-stacking automation

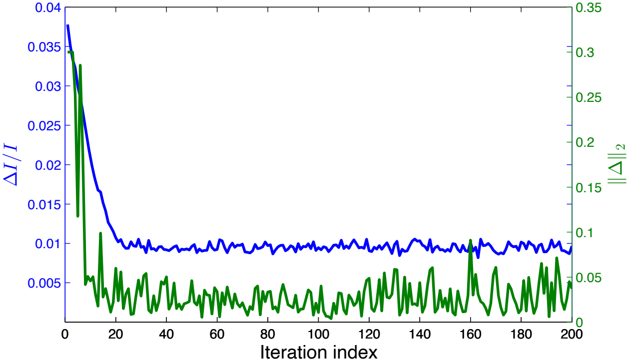
Model structure: $R = R_{\text{nom}}(I + \Delta)$

- 4 motor driven crystals ($R \in \mathbb{R}^{16\times 4}$)
- Δ is unstructured ($\Delta \in \mathbb{C}^{4\times 4}$)

Perturbation update:

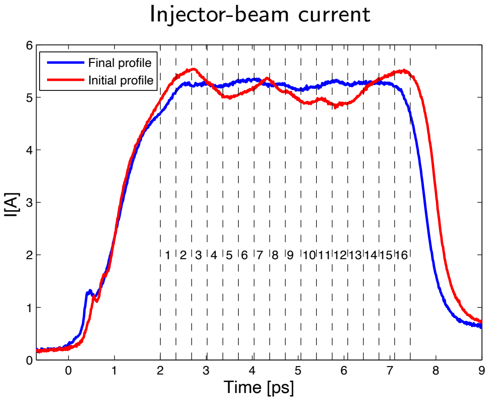
Response matrix update:

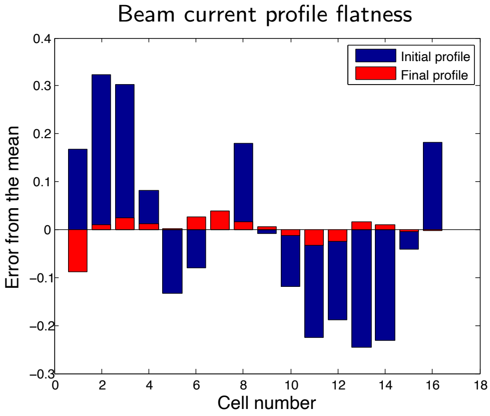
$$R_{k+1} \;=\; R_k \;+\; R_k\,\Delta_k.$$



Adaptive robust control

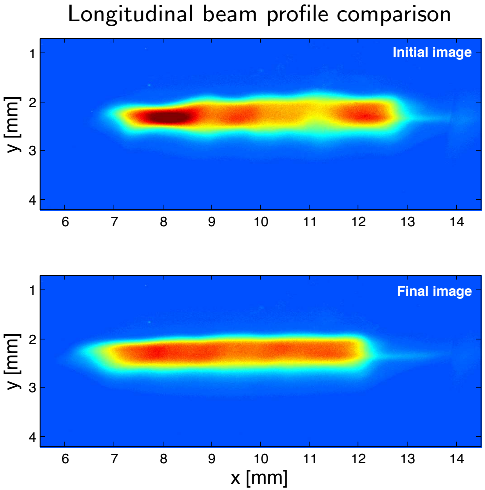
Laser pulse-stacking automation





Adaptive robust control

Laser pulse-stacking automation



Summary

- Control performance depends on model accuracy.
- Control can start from a poorly known model and improve as the model becomes more accurate.
- Model adaptation allows us to track slowly drifting plants.
- The speed of adaptation is often important.
- Fundamental loss of information as the adaptation (and control improvement) proceeds.
- Trade-off between exploration (for model improvement) and exploitation (tracking accuracy).

Willems' fundamental lemma

Given a sufficiently exciting input data, u^{d}_t, $t = 0, \ldots, T-1$ and a corresponding system output data, y^{d}_t, $t = 0, \ldots, T-1$,

Form the Hankel matrices,

$$U^{\mathsf{d}} = \begin{bmatrix} u^{\mathsf{d}}_0 & u^{\mathsf{d}}_1 & u^{\mathsf{d}}_2 & \cdots & u^{\mathsf{d}}_{T-L} \\ u^{\mathsf{d}}_1 & u^{\mathsf{d}}_2 & \cdots & & \\ u^{\mathsf{d}}_2 & \cdots & & & \\ \vdots & & & & \\ u^{\mathsf{d}}_{L-1} & u^{\mathsf{d}}_L & \cdots & & u^{\mathsf{d}}_{T-1} \end{bmatrix} \quad Y^{\mathsf{d}} = \begin{bmatrix} y^{\mathsf{d}}_0 & y^{\mathsf{d}}_1 & y^{\mathsf{d}}_2 & \cdots & y^{\mathsf{d}}_{T-L} \\ y^{\mathsf{d}}_1 & y^{\mathsf{d}}_2 & \cdots & & \\ y^{\mathsf{d}}_2 & \cdots & & & \\ \vdots & & & & \\ y^{\mathsf{d}}_{L-1} & y^{\mathsf{d}}_L & \cdots & & y^{\mathsf{d}}_{T-1} \end{bmatrix}.$$

Then u, y is an input-output trajectory of the system if and only if there exists g such that,

$$\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} U^{\mathsf{d}} \\ Y^{\mathsf{d}} \end{bmatrix} g.$$

Data-driven simulation: using data as a model

Divide our data matrices into past & future blocks,

$$U^{\mathsf{d}} = \begin{bmatrix} U^{\mathsf{p}} \\ U^{\mathsf{f}} \end{bmatrix}, \quad Y^{\mathsf{d}} = \begin{bmatrix} Y^{\mathsf{p}} \\ Y^{\mathsf{f}} \end{bmatrix},$$

To specify the initial condition, suppose we have the input and output trajectory from the immediate past,

$$u^{\mathsf{ini}} = u_t, t = -N, \ldots, -1, \quad y^{\mathsf{ini}} = y_t, t = -N, \ldots, -1.$$

Then for any given $u_t, t = 0, \ldots, M$, we can calculate $y_t, t = 0, \ldots, M$ finding any g solving,

$$\begin{bmatrix} u^{\mathsf{ini}} \\ u \\ y^{\mathsf{ini}} \\ y \end{bmatrix} = \begin{bmatrix} U^{\mathsf{p}} \\ U^{\mathsf{f}} \\ Y^{\mathsf{p}} \\ Y^{\mathsf{f}} \end{bmatrix} g,$$

One solution: $$g = \begin{bmatrix} U^{\mathsf{p}} \\ U^{\mathsf{f}} \\ Y^{\mathsf{p}} \end{bmatrix}^{\dagger} \begin{bmatrix} u^{\mathsf{ini}} \\ u \\ y^{\mathsf{ini}} \end{bmatrix}, \quad y = Y^{\mathsf{f}} g.$$

Data-driven control

Model predictive control (DeePC)

Control objective:

$$\begin{aligned}
\underset{g,u,y}{\text{minimise}} \quad & (y - y^{\mathsf{ref}})^T Q (y - y^{\mathsf{ref}}) \, + \, u^T R u + \lambda \|g\|_2 \\
\text{subject to:} \; & \begin{bmatrix} u^{\mathsf{ini}} \\ u \\ y^{\mathsf{ini}} \\ y \end{bmatrix} \; = \; \begin{bmatrix} U^{\mathsf{p}} \\ U^{\mathsf{f}} \\ Y^{\mathsf{p}} \\ Y^{\mathsf{f}} \end{bmatrix} g \\
& u \in \mathcal{U}, \;\; y \in \mathcal{Y}.
\end{aligned}$$

Considerations

- Not necessary to specify or parametrise the state.
- The cost function must be regularised ($\lambda \|g\|_2$)
- Potential performance issues for large noise on y

Model predictive control (Signal Matrix Model)

Control objective:

$$\begin{aligned}\underset{u,y}{\text{minimise}} \quad & (y - y^{\mathsf{ref}})^T Q (y - y^{\mathsf{ref}}) + u^T R u \\ \text{subject to:} \quad & y = Y^{\mathsf{f}} g_{\mathsf{SMM}}(u, y, y^{\mathsf{ini}}), \\ & u \in \mathcal{U}, \;\; y \in \mathcal{Y},\end{aligned}$$

where $g = g_{\mathsf{SMM}}(u, y, y^{\mathsf{ini}})$ is based on a maximum likelihood solution.

Considerations

- Not necessary to specify or parametrise the state.
- Improved noise performance for noisy y.
- g_{SMM} requires an iterative algorithm for an exact solution.
- Updating g with one iteration from previous MPC solution works well in practice.

Summary

- Data-driven control methods are at the experimental stage of development.
- Capable of fitting calculating trajectories in the presence of large unknown transients.
- A single trajectory (or collection of trajectories) can replace the explicit model in predictive, or simulation-based, control.
- Noisy signals are embedded in the “model” giving complex noise effects.

Closing

Thank you.

Questions?

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