Hyperfine Structure in Muonic Systems and Conceivable Signs of New Physics

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Abstract

Recently, experiments at the ATOMKI laboratory, located in Debrecen, Hungary, point to the possible existence of a low-energy addition to the Standard Model. They have generated widespread attention. While the experimental findings lack an independent confirmation, they give incentive, from a more general point of view, to investigate the possible detection of new particles with a rest mass of 15 to 20 MeV, in atomic physics high-precision experiments. We find that, in this mass range, the effects will be most pronounced in the spectra of muonic (not electronic) bound systems, and, most notably, in the hyperfine structure. While it is a challenge to separate the effects from nuclear-structure corrections to the hyperfine splitting, this endeavor is definitely not hopeless. In general, in our work [Phys.Rev.A 101 (2020) 062503], we derive the effective potentials corresponding to the exchange of a pseudoscalar particle, and a new vector particle, in two-body bound systems. Comparison with the literature reveals that the corresponding effective potentials may not have been treated consistently in the past. In our analysis of conceivable effects to be seen in true muonium (bound system of oppositely charged muons), we compare the magnitude of the new effects to the uncertainty due to hadronic vacuum polarization. An interesting theoretical question, of some general importance, concerns the use of the Coulomb gauge for the massive vector propagator and its suitability for bound-state calculations. The project has been supported by the NSF (grants PHY-1710856 and PHY-2011762).

Outline

First, Some "Commercial" on a Recent Book

Then, Some Words on X17 and Muonic Systems

Finally, Some Dedicated Statements on Hadronic VP

Upcoming Book

For more than 60 years, the book of BETHE and SALPETER has been a cornerstone in the description of few-body atomic systems. An update may be indicated. Starting from the basic nonrelativistic and relativistic formulas for the hydrogen bound and continuum states, we proceed to discuss Green functions (including Schwinger's momentum representation of the Schrödinger-Coulomb Green function and its derivation) in detail. The calculation of Bethe logarithms is discussed. interaction potentials are derived by matching scattering amplitudes and effective Hamiltonians. Higher-order effects are an integral part of the treatment, and so is the calculation of the helium spectrum, including relativistic and radiative corrections. Relativistic recoil corrections are discussed in detail, using various methods, i.e., the Bethe-Salpeter equation and nonrelativistic quantum electrodynamics (NRQED). Further topics include the renormalization group, field-theory methods, and vacuum-mediated corrections to photon propagators. The book serves both as a textbook as well as a monograph and has meanwhile appeared in print (July 2022).

Title Page

Quantum Electrodynamics Atoms, Lasers and Gravity

This book introduces readers to a variety of topics surrounding quantum field theory, notably its role in bound states, laser physics, and the gravitational coupling of Dirac particles. It discusses some rather sophisticated concepts based on detailed derivations which cannot be found elsewhere in the ilterature.

It is suitable for undergraduates, graduates, and researchers working on general relativity, relativistic atomic physics, quantum electrodynamics, as well as theoretical laser physics.

Quantum Electrodynamic Atoms, Lasers and Gravi

Jentschu Adkins Quantum Electrodynamics Atoms, Lasers and Gravity

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Table of Contents (770 Pages, 19 Chapters, Some Examples Next)

- 1. Introduction
- 2. From Unit Systems for the Microworld to Field Quantization
- 3. Time–Ordered Perturbations
- 4. Bound–Electron Self–Energy and Bethe Logarithm
- 5. Interatomic and Atom–Surface Interactions
- 6. Racah–Wigner Algebra
- 7. Free Dirac Equation
- 8. Dirac Equation for Bound States, Lasers and Gravity
- 9. Electromagnetic Field and Photon Propagators
- 10. Tree–Level and Loop Diagrams, and Renormalization
- 11. Foldy–Wouthuysen Transformation and Lamb Shift
- 12. Relativistic Interactions for Many–Particle and Compound Systems
- 13. Fully Correlated Basis Sets and Helium
- 14. Relativistic Many–Particle Calculations
- 15. Beyond Breit Hamiltonian and On-Shell Form Factors
- 16. Bethe–Salpeter Equation
- 17. NRQED: An Effective Field Theory for Atomic Physics
- 18. Fermionic Determinants and Effective Lagrangians
- 19. Renormalization–Group Equations

General Results for the Eighth–Order Foldy–Wouthuysen Transformation (Surprise!)

Chapter 15: Calculation of Binding Corrections to the Lamb Shift





▶ The original result was due to Bethe, Baranger and Feynman,

$$\Delta E = \frac{\alpha}{\pi} \frac{(Z\alpha)^5 m}{n^3} A_{50}(nL_j), \qquad A_{50}(nL_j) = 4\pi \,\delta_{L0} \left(\frac{139}{128} - \frac{1}{2}\ln(2)\right).$$

▶ Modern Approach: Dispersion Relation! Idea: Let the incoming Coulomb momentum, which is initially space-like, $q^2 = -\vec{q}^2 \leq 0$, continue into the time-like domain, $q^2 \rightarrow Q^2 > 4m^2$, where the expression for the energy correction develops a branch cut in the complex plane. Then, write a dispersion relation which connects the cut to real energy shift. This simplifies the calculation dramatically.

Chapters 15, 16, 17: Three Ways to the Relativistic Recoil Correction (Salpeter)

The derivation of the so-called "Relativistic Recoil Correction" (correction to bound-state energy levels beyond the Breit Hamiltonian) is presented, for a general atomic reference state and for arbitrary mass ratios, in three different ways:

- ▶ Way I (Chapter 15): Ad hoc approach, matching scattering amplitudes
- ▶ Way II (Chapter 16): Ab initio, relativistic Bethe–Salpeter equation

G = S + S K G



 Way III (Chapter 17): From an Effective Field Theory (NRQED) (There is a Bethe–Salpeter Equation of NRQED!) (It corresponds to the Schrödinger equation!)

Summary of Book Project

- ► World Scientific will be the publisher
- Textbook on advanced quantum mechanics; textbook on quantum field theory with an emphasis on renormalization, and the renormalization group; monograph on the gravitational coupling of relativistic quantum mechanical spin-1/2 particles; monograph on the laser-dressed relativistic electron propagator; monograph on higher-order binding corrections to QED effects in atoms; monograph on the Bethe–Salpeter equation and Nonrelativistic QED (NRQED)

Hungarian Experimental Results (ATOMKI Laboratory, Debrecen)

Excess of electron-positron pairs in the bombardement of ^{7}Li by protons in a distinct angular region, consistent with competing reactions (standard theory with a virtual photon)

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{}^{7}\mathrm{Li} + p \to {}^{8}\mathrm{Be}^{*} \to {}^{8}\mathrm{Be} + \gamma \to {}^{8}\mathrm{Be} + e^{+}e^{-} \qquad (1^{+} \to 0^{+})
```

and transition via an intermediate $X^{(17\,{\rm MeV})}$ bosonic virtual particle, termed the X17 boson,

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<sup>7</sup>Li + p \rightarrow {}^{8}\text{Be}^{*} \rightarrow {}^{8}\text{Be} + X^{(17 \text{ MeV})} \rightarrow {}^{8}\text{Be} + e^{+}e^{-} \qquad (1^{+} \rightarrow 0^{+})
                           PRI. 116. 042501 (2016)
                                                                 PHYSICAL REVIEW LETTERS.
                                                                                                                                       29 JANUARY 2014
                                Observation of Anomalous Internal Pair Creation in 8Be: A Possible Indication of a Light,
                                                                                Neutral Boson
                           A.J. Krasznahorkay, M. Csatlós, L. Csige, Z. Gácsi, J. Gulvás, M. Hunvadi, I. Kuti, B. M. Nvakó, L. Stuhl, J. Timár,
                                                                           T.G. Tomyi and Zs. Vaita
                               Institute for Nuclear Research, Hungarian Academy of Sciences (MTA Atomki), P.O. Box 51, H-4001 Debrecen, Hungary
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                                                                     P.O. Box 51, H-4001 Debrecen, Hungary
                                                                (Received 7 April 2015; published 26 January 2016)
                                           Electron-positron angular correlations were measured for the isovector magnetic dipole 17.6 MeV
                                        (J^{s} = 1^{+}, T = 1) state \rightarrow ground state (J^{s} = 0^{+}, T = 0) and the isoscalar magnetic dipole 18.15 MeV
                                        (J^a = 1^+, T = 0) state \rightarrow ground state transitions in <sup>8</sup>Be. Significant enhancement relative to the internal
                                        pair creation was observed at large angles in the angular correlation for the isoscalar transition with a
                                        confidence level of > 5\sigma. This observation could possibly be due to nuclear reaction interference effects or
                                        might indicate that, in an intermediate step, a neutral isoscalar particle with a mass of
                                        16.70 \pm 0.35(\text{stat}) \pm 0.5(\text{syst}) \text{ MeV}/c^2 and J^{\pi} = 1^+ was created.
                                        DOI: 10.1103/PhysRevLett.116.042501
```

Experimental results remain to be confirmed by other groups. (New 2019 paper by the Hungarian group on helium nuclei seems to confirm their observations. Reaction: ${}^{3}\text{H} + p \rightarrow {}^{4}\text{He}^{*} \rightarrow {}^{4}\text{He} + X^{(17\,\text{MeV})} \rightarrow {}^{4}\text{He} + e^{+}e^{-}$.) X17 has small coupling strengths to protons (in particular) and neutrons.

Motivation (Group of Attila Krasznahorkay, ATOMKI)

Beryllium (2016):

⁷Li +
$$p \rightarrow {}^{8}\text{Be}^{*} \rightarrow {}^{8}\text{Be} + \gamma \rightarrow {}^{8}\text{Be} + e^{+}e^{-} \qquad (1^{+} \rightarrow 0^{+})$$

Helium (2019):

$${}^{3}\mathrm{H} + p \rightarrow {}^{4}\mathrm{He}^{*} \rightarrow {}^{4}\mathrm{He} + \gamma \rightarrow {}^{4}\mathrm{He} + e^{+}e^{-} \qquad (0^{-} \rightarrow 0^{+})$$

From arXiv:1910.10459:



Caricature



Mass of new particle: about 17 MeV.

Possible Theoretical Explanations

Group of Jonathan Feng (PRL, 2016): "The X17 might be a protophobic vector boson."

Paper of Ellwanger and Moretti (JHEP, 2016): "The X17 might be a light pseudoscalar boson."

Let us remember that in atomic physics precision experiments, we would actually like to see deviations of experimental observations from experiments attributable to "new physics". This has been a significant motivation pushing the theoretical and experimental efforts for the last couple of decades.

Recent attempts at alternative explanations for the Hungarian observations, but noone has carried out any experiment. Room for improvement: Angular resolution, in Hungary!

Light Vector and Pseudoscalar Particles and Atomic Physics

We investigate, **irrespective of the Hungarian experimental results**, what the effect of a light (mass in the approximate range from 10 MeV to 100 MeV) vector or pseudoscalar new particle is for atomic-physics experiments.



Here, f denotes the bound fermion (typically, an electron or a muon) and N denotes the atomic nucleus. (Inspired by the mentioned theoretical papers of Feng *et al.*, and of Ellwanger and Moretti.) Unfortunately, the mass range of 17 MeV (give or take) is quite problematic for atomic-physics studies, because the Yukawa potentials are almost indistiguishable from a nuclear-size effect for electronic bound states. Way out: study muonic systems.

X17 Lagrangian

Vector or pseudoscalar? Vector hypothesis (Jonathan Feng's group, PRL, 2016):

$$\mathcal{L}_{X,V} = -\sum_f h'_f \,\bar{\psi}_f \,\gamma^\mu \,\psi_f X_\mu - \sum_N h'_N \,\bar{\psi}_N \,\gamma^\mu \,\psi_N X_\mu \,,$$

Parameterization:

$$h'_f = \varepsilon_f e, \qquad h'_N = \varepsilon_N e,$$

 $\varepsilon_p = 2\varepsilon_u + \varepsilon_d, \qquad \varepsilon_n = \varepsilon_u + 2\varepsilon_d.$

Available parameter space (electron, neutron, proton):

$$2 \times 10^{-4} < \varepsilon_e < 1.4 \times 10^{-3} ,$$
$$|\varepsilon_n| = |\varepsilon_u + 2\varepsilon_d| \approx \left| \frac{3}{2} \varepsilon_d \right| \approx \frac{1}{100} ,$$
$$|\varepsilon_p| = |2\varepsilon_u + \varepsilon_d| \lesssim 8 \times 10^{-4} .$$

Second equation ("conjecture"): we need this coupling in order to explain the ATOMKI experimental results! Latter bound: we assume a "protophobic" interaction!

X17 Lagrangian

Pseudoscalar hypothesis (Ellwanger and Moretti, JHEP, 2016):

$$\mathcal{L}_{X,A} = -\sum_{f} \hbar_f \, \bar{\psi}_f \, \mathrm{i} \, \gamma^5 \, \psi_f \, A - \sum_{N} \hbar_N \, \bar{\psi}_N \, \mathrm{i} \, \gamma^5 \, \psi_N \, A \, .$$

Parameterization:

$$h_f = \xi_f \frac{m_f}{v}, \qquad h_N = \xi_N \frac{m_N}{v}.$$

Ellwanger and Moretti obtain:

$$h_p = \frac{m_p}{v} \left(-0.40 \,\xi_u - 1.71 \,\xi_d \right) \approx -2.4 \times 10^{-3} \,,$$

$$h_n = \frac{m_n}{v} \left(-0.40 \,\xi_u + 0.85 \,\xi_d \right) \approx 5.1 \times 10^{-4} \,.$$

Available parameter space (electron):

$$4 < \xi_e < 500 \,,$$
$$8.13 \times 10^{-6} < h_e < 10^{-3} \,.$$

Effective Hamiltonian for Vector Boson Exchange

Vector exchange leads to the following contribution to HFS:

$$\begin{split} H_{\rm HFS,V} &= \frac{\hbar'_f \, \hbar'_N}{16 \, \pi \, m_f \, m_N} \, \left[-\frac{8\pi}{3} \delta^{(3)}(\vec{r}) \, \vec{\sigma}_f \cdot \vec{\sigma}_N \right. \\ & \left. - \frac{m_X^2 \, \left(\vec{\sigma}_f \cdot \vec{r} \, \vec{\sigma}_N \cdot \vec{r} - r^2 \, \vec{\sigma}_f \cdot \vec{\sigma}_N \right)}{r^3} \, \mathrm{e}^{-m_X \, r} \right. \\ & \left. - \left(1 + m_X \, r \right) \frac{3 \, \vec{\sigma}_f \cdot \vec{r} \, \vec{\sigma}_N \cdot \vec{r} - r^2 \, \vec{\sigma}_f \cdot \vec{\sigma}_N}{r^5} \, \mathrm{e}^{-m_X \, r} \right. \\ & \left. - \left(2 + \frac{m_f}{m_N} \right) \, \left(1 + m_X \, r \right) \, \frac{\vec{\sigma}_N \cdot \vec{L}}{r^3} \mathrm{e}^{-m_X \, r} \right] \, . \end{split}$$

Derivation: [Phys. Rev. A **101**, 062503 (2020)]

Effective Hamiltonian for Pseudoscalar Boson Exchange

Pseudoscalar exchange exclusively contributes to the HFS:

$$\begin{split} H_{\rm HFS,A} &= \frac{\hbar_f \, \hbar_N}{16 \, \pi \, m_f \, m_N} \, \left[\frac{4\pi}{3} \delta^{(3)}(\vec{r}) \, \vec{\sigma}_f \cdot \vec{\sigma}_N \right. \\ & \left. - \frac{m_X^2 \, \vec{\sigma}_f \cdot \vec{r} \, \vec{\sigma}_N \cdot \vec{r}}{r^3} \, {\rm e}^{-m_X \, r} \right. \\ & \left. + \left(1 + m_X \, r \right) \frac{3 \, \vec{\sigma}_f \cdot \vec{r} \, \vec{\sigma}_N \cdot \vec{r} - \vec{\sigma}_f \cdot \vec{\sigma}_N \, r^2}{r^5} \, {\rm e}^{-m_X \, r} \right] \, . \end{split}$$

Leaves Lamb shift invariant! [Phys. Rev. A **101**, 062503 (2020)]

Bound on the Muon Coupling Parameter



Vector model:

$$h'_{\mu} = (h'_{\mu})_{\text{opt}} = 5.6 \times 10^{-4}$$
.

Pseudoscalar model:

$$h_{\mu} = (h_{\mu})_{\text{max}} = 3.8 \times 10^{-4}$$
.

Enhancement of X17 Effects in Muonic Systems

Example: Relative correction to the S state splitting is

 $\frac{E_{X,V}(nS_{1/2})}{E_F(nS_{1/2})} \approx -\frac{2\hbar'_f \hbar'_N}{g_N \pi} \frac{Z m_r}{m_X} ,$ $\frac{E_{X,A}(nS_{1/2})}{E_F(nS_{1/2})} \approx \frac{\hbar_f \hbar_N}{g_N \pi} \frac{Z m_r}{m_X} .$

Have the reduced mass m_r in the numerator after dividing by the leading-order Fermi splitting.

(Electronic systems: relative corrections to HFS of order 10^{-9} .)

(So: Concentrate on muonic systems)

Just to clarify:

The nuclear g factor g_N is used in a specific normalization [Phys. Rev. A **101**, 062503 (2020)].

Predictions for Muonic Deuterium

S states (with realistic estimates for coupling parameters):

$$\frac{E_{X,V}^{(\mu d)}(nS_{1/2})}{E_F(nS_{1/2})} \approx 3.8 \times 10^{-6} ,$$

$$\frac{E_{X,A}^{(\mu d)}(nS_{1/2})}{E_F(nS_{1/2})} \approx -1.0 \times 10^{-6} .$$

P states:

$$\frac{E_{X,V}^{(\mu d)}(nP_{1/2})}{E_F(nP_{1/2})} \approx 2.5 \times 10^{-7} \left(1 - \frac{1}{n^2}\right),$$
$$\frac{E_{X,A}^{(\mu d)}(nP_{1/2})}{E_F(nP_{1/2})} \approx 6.6 \times 10^{-8} \left(1 - \frac{1}{n^2}\right).$$

This could be measurable but an enhanced understanding of nuclear polarization effects might be required for S states. For P states, nuclear effects are strongly suppressed. Predictions for True Muonium $(\mu^+\mu^-)$

Define

$$\chi_V(nS) = \frac{4}{7} \frac{E_{X,V}(nS)}{E_F(nS)} + \frac{3}{7} \frac{E_{\text{ANN},V}(nS)}{E_{\text{ANN},\gamma}(nS)} ,$$

$$\chi_A(nS) = \frac{4}{7} \frac{E_{X,A}(nS)}{E_F(nS)} + \frac{3}{7} \frac{E_{\text{ANN},A}(nS)}{E_{\text{ANN},\gamma}(nS)} .$$

Obtain the estimates

 $\chi_V(nS) \approx 1.3 \times 10^{-6},$ $\chi_A(nS) \approx 2.1 \times 10^{-6}.$

This could very well be measurable; only a very moderate improvement of the accuracy of the predictions for hadronic vacuum polarization is required.

Summary on the X17 and Atomic Physics

- ▶ X17 effects have to be confirmed in other nuclear transitions.
- ▶ Somewhat unfortunate energy range for atomic physics.
- ▶ Drastic enhancement of X17 effects in muonic systems.
- ► Look at the hyperfine splitting.
- ▶ Pseudoscalar hypothesis leads to "wrong sign" in regard to a muon g-2 "remedy".
- Estimates for X17-mediated effects for a number of atomic systems: [Phys. Rev. A 101, 062503 (2020)].
- ▶ Most promising candidates: Muonic deuterium and true muonium.

Hadronic Vacuum Polarization

Justified to dedicate additional attention to hadronic vacuum polarization because it limits the accuracy to which we can test QED effects. Inaccuracies in hadonic VP calculations could shadow new physics effects in low-energy precision tests of the Standard Model.

Example Feynman diagram: Contribution of hadronic VP to the anomalous magnetic moment of an electron (muon)



Hadronic Vacuum Polarization

Contribution of hadronic VP to a (mainly) QED observable X_i :

$$\Delta X(\text{hVP}) = C \int_{s=s_{\text{th}}}^{\infty} \mathrm{d}s \,\rho(s) \,K_X(s,m) \tag{5}$$

One integrates from pion pair production threshold $s_{\rm th} = 4m_{\pi}^2$ up to infinity. K_i is a specific kernel, C_i is a coefficient, and K is a kernel, while m is a mass scale external to the kernel.

Drell ratio R = R(s) enters the density

$$\rho(s) = \frac{R(s)}{3s}, \qquad R(s) = \frac{\sigma(e^+ e^- \to h)}{\sigma(e^+ e^- \to \mu^+ \mu^-)}$$
(6)

Example of the kernel K for $X = \kappa$ [anomalous magnetic moment of an electron (muon)] with $g = 2(1 + \kappa)$:

$$K_{\kappa}(s) = -\left(1 - \frac{4m^2}{s}\right)^{-1/2} \left(\frac{s^2}{2m^4} - \frac{2s}{m^2} + 1\right) \ln\left(\frac{1 + \sqrt{1 - \frac{4m^2}{s}}}{1 - \sqrt{1 - \frac{4m^2}{s}}}\right) + \left(\frac{s^2}{2m^4} - \frac{2s}{m^2}\right) \ln\left(\frac{s}{m^2}\right) - \frac{s}{m^2} + \frac{1}{2}.$$
(7)

Here, $m = m_e$ or $m = m_{\mu}$.

Fitting the Drell Ratio

From H. Lamm, Hadronic vacuum polarization in true muonium, Phys. Rev. A 95, 012505 (2017):



FIG. 1. R(s) vs. s. The solid line indicates the experimental results from the compilation used by alphaQED [40–43] and rhad [44]. The dotted line are the estimates used in the JSIK calculations of $C_{1,>}$ [2], and the dashed line indicates the estimates of this work.

Example: Paper on the Muon Anomalous Magnetic Moment



DOI: 10.1103/PhysRevD.97.114025

Accuracy of hVP contribution: 3 permille

Example: Paper on the Hadronic VP in True Muonium

PHYSICAL REVIEW A 95, 012505 (2017)

Hadronic vacuum polarization in true muonium

Henry Lamm*

Department of Physics, Arizona State University, Tempe, Arizona 85287-1504, USA (Received 14 November 2016; published 23 January 2017)

In order to reduce the theoretical uncertainty in the prediction, the leading-order hadronic vacuum polarization contribution to the hyperfine splitting of true muonium is reevaluated in two ways. A more complex pionic form factor and better estimates of the perturbative QCD contributions are used to study the model dependence of the previous calculation. The second, more accurate method directly integrates the Drell ratio R(s) to obtain $C_{1.HVP} = -0.04874(9)$. This corresponds to an energy shift in the hyperfine splitting (HFS) of $\Delta E_{HSHVP}^{\mu} = -8202(16)$ MHz and represents a factor-of-50 reduction in the theoretical uncertainty from hadronic sources. We also compute the contribution in positronium, which is too small at present to detect.

DOI: 10.1103/PhysRevA.95.012505

Accuracy of hVP contribution: 2 permille

Recent Measurements at VEPP–4M



Phys. Lett. B **788**, 42 (2019) and Phys. Lett. B **770**, 174 (2017) by the same authors We need the Drell ratio in a kinematic region where QCD is highly nonperturbative.

For the perturbative regime, see R. V. Harlander and M. Steinhauser, Comput. Phys. Commun. **153**, 244 (2003).

One notes the inverse power of s in the expression $\rho(s) = R(s)/(3s)$, which stresses the importance of the nonperturbative region.

It may be difficult to get better than the permille range for the accuracy of $\Delta X(hVP)$ in typical cases.

Conclusions

- ▶ Book: QED and low-energy field theory revisited, in many ways.
- ▶ X17: A candidate for new physics, nothing more and nothing less.
- ▶ X17: We learned something about pseudoscalar particles and atomic physics.
- ▶ hVP: Interesting effect, very interesting.
- ▶ hVP: May be difficult to get better than the permille range for hVP.