# Two-Photon Exchange in μH and μD: Baryon χEFT and pionless EFT

Vadim Lensky

In collaboration with A. Hiller Blin, F. Hagelstein, V. Pascalutsa, M. Vanderhaeghen

Muonic Atoms@PSI

Viligen, October 14-15, 2022

# Two-Photon Exchange in (Muonic) Atoms

Bohr radius

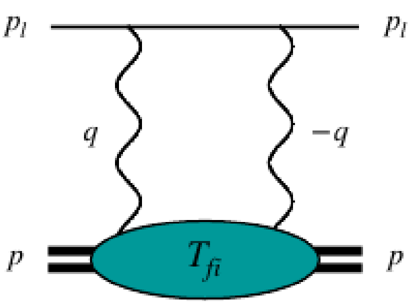
$$a = (Z\alpha m_r)^{-1}$$

- Muonic atoms: greater sensitivity to charge radii
- But also greater sensitivity to subleading nuclear response

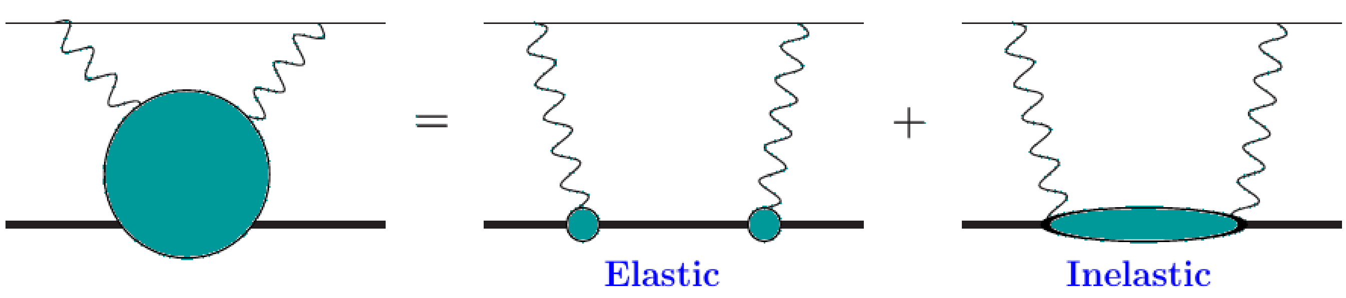
Lamb Shift: $$\Delta E_{nS} = \frac{2\pi Z\alpha}{3}\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R_F^3\right] + \dots$$

HFS: $$\Delta E_{nS} = \frac{8}{3}\frac{Z\alpha}{(an)^3}\frac{1+\kappa}{mM}\left[1 - 2Z\alpha m_r R_Z\right] + \dots$$

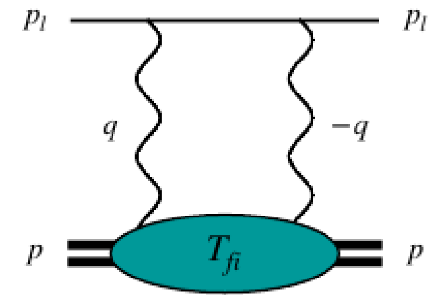
$\propto \alpha^5$




- Described in terms of (doubly virtual fwd) Compton scattering
- Elastic ( $\nu = \pm Q^2/2M_{\text{target}}$ , elastic e.m. form factors) and inelastic (~ nuclear generalised polarisabilities)



# VVCS and Structure Functions



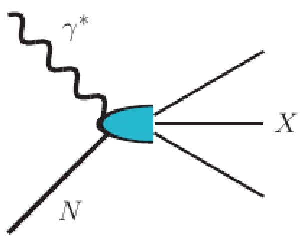

- Forward spin-1/2 VVCS amplitude

$$\alpha_{em} M^{\mu\nu}(\nu, Q^2) = -\Big\{\Big(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big) T_1(\nu, Q^2) + \frac{1}{M^2}\Big(p^\mu - \frac{p\cdot q}{q^2} q^\mu\Big)\Big(p^\nu - \frac{p\cdot q}{q^2} q^\nu\Big) T_2(\nu, Q^2)$$
$$+\frac{i}{M}\epsilon^{\nu\mu\alpha\beta} q_\alpha s_\beta S_1(\nu, Q^2) + \frac{i}{M^3}\epsilon^{\nu\mu\alpha\beta} q_\alpha (p\cdot q\, s_\beta - s\cdot q\, p_\beta) S_2(\nu, Q^2)\Big\}$$

Lamb Shift: $$E^{2\gamma}_{nS} = -8i\pi\alpha m\, [\phi_n(0)]^2 \int \frac{d^4 q}{(2\pi)^4} \frac{(Q^2 - 2\nu^2)\, T_1(\nu, Q^2) - (Q^2 + \nu^2)\, T_2(\nu, Q^2)}{Q^4(Q^4 - 4m^2\nu^2)}$$

- Unitarity and analyticity, data-driven: dispersive relations

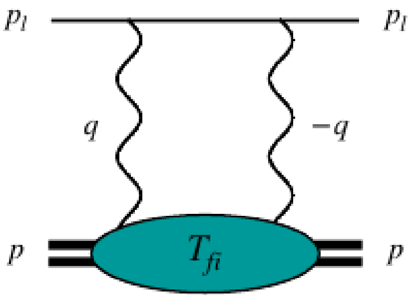
Structure functions $F_1(x, Q^2)$, $F_2(x, Q^2)$, $g_1(x, Q^2)$, $g_2(x, Q^2)$




$$T_1(\nu, Q^2) = T_1(0, Q^2) + \frac{32\pi M\nu^2}{Q^4} \int_0^1 dx\, \frac{x\, F_1(x, Q^2)}{1 - x^2(\nu/\nu_{el})^2 - i0^+},$$
$$T_2(\nu, Q^2) = \frac{16\pi M}{Q^2} \int_0^1 dx\, \frac{F_2(x, Q^2)}{1 - x^2(\nu/\nu_{el})^2 - i0^+}$$

- The subtraction function is not directly accessible in experiment
- Data on structure functions is sometimes deficient

# VVCS and Structure Functions

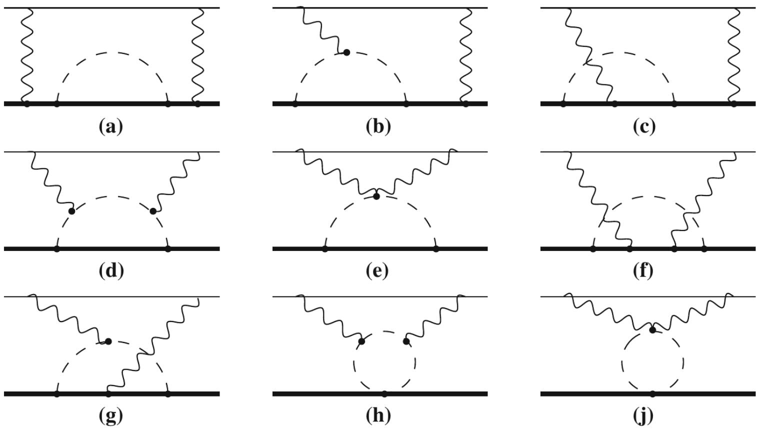



- Forward spin-1/2 VVCS amplitude

$$\alpha_{em} M^{\mu\nu}(\nu, Q^2) = -\Big\{\Big(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big) T_1(\nu, Q^2) + \frac{1}{M^2}\Big(p^\mu - \frac{p\cdot q}{q^2}\, q^\mu\Big)\Big(p^\nu - \frac{p\cdot q}{q^2}\, q^\nu\Big) T_2(\nu, Q^2)$$
$$+\frac{i}{M}\,\epsilon^{\nu\mu\alpha\beta}\, q_\alpha s_\beta\, S_1(\nu, Q^2) + \frac{i}{M^3}\,\epsilon^{\nu\mu\alpha\beta}\, q_\alpha (p\cdot q\, s_\beta - s\cdot q\, p_\beta)\, S_2(\nu, Q^2)\Big\}$$

Lamb Shift: $$E^{2\gamma}_{nS} = -8i\pi\alpha m\,[\phi_n(0)]^2 \int \frac{d^4q}{(2\pi)^4} \frac{(Q^2 - 2\nu^2)\, T_1(\nu, Q^2) - (Q^2 + \nu^2)\, T_2(\nu, Q^2)}{Q^4(Q^4 - 4m^2\nu^2)}$$

- Typical energies in (muonic) atoms are small: use effective field theories
  - chiral EFT (covariant, HB, ...)
  - or even pionless EFT for nuclear effects
  - expansion in powers of a small parameter
  - order-by-order uncertainty estimate



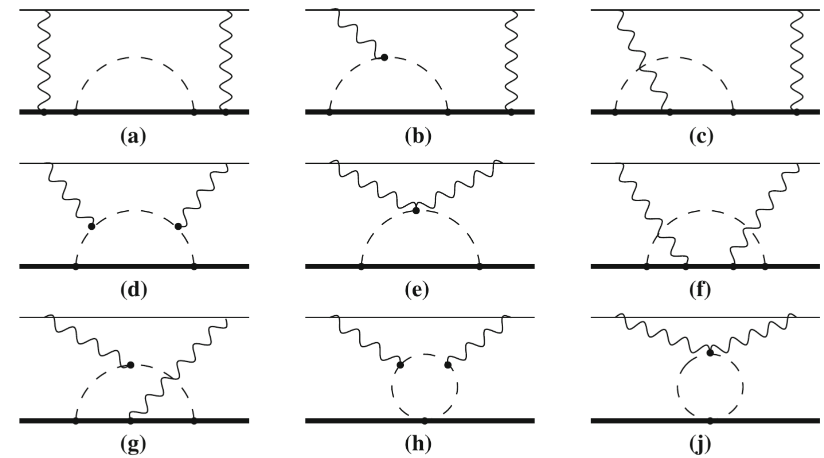

- Two cases:
  - μH using covariant baryon χEFT
  - μD using pionless EFT

# Lamb Shift of μH in Covariant BχPT

- Delta counting: $\Delta = M_\Delta - M \gg m_\pi$

Pascalutsa, Phillips (2003)

- The contributions of the Delta isobar are suppressed by powers of $m_\pi/\Delta$
- LO BχPT: pion-nucleon loops

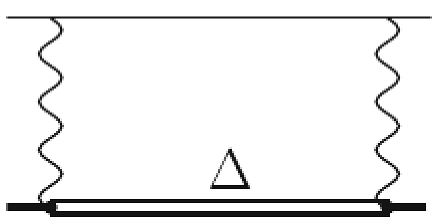



Alarcon, VL, Pascalutsa (2014)

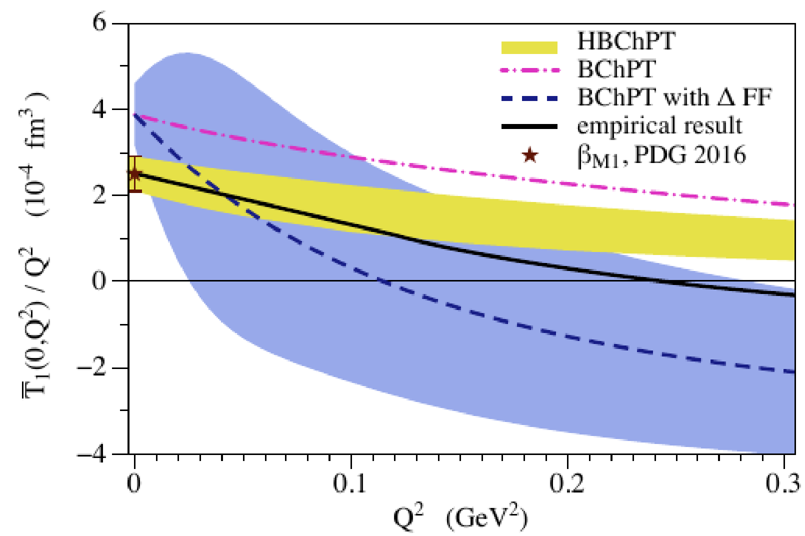
$$\Delta E_{2S}^{\text{LO, pol}} = -9.6^{+1.4}_{-2.9}\ \mu\text{eV}$$

- Delta exchange:
  - suppressed in $\Delta E_{2S}^{\text{pol}}$
  - insert transition form factors (Jones-Scadron)

$$\Delta E_{2S}^{\Delta-\text{pole}} = 0.95 \pm 0.95\ \mu\text{eV}$$




VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)



# Lamb Shift of μH in Covariant BχPT

**Table 1** **Forward 2γ-exchange contributions to the 2S-shift in μH, in units of μeV.**

| Reference | $E_{2S}^{(\text{subt})}$ | $E_{2S}^{(\text{inel})}$ | $E_{2S}^{(\text{pol})}$ | $E_{2S}^{(\text{el})}$ | $E_{2S}^{\langle 2\gamma\rangle}$ |
|---|---|---|---|---|---|
| DATA-DRIVEN DISPERSIVE EVALUATION | | | | | |
| (75) Pachucki '99 | 1.9 | −13.9 | −12(2) | −23.2(1.0) | −35.2(2.2) |
| (76) Martynenko '06 | 2.3 | −16.1 | −13.8(2.9) | | |
| (77) Carlson *et al.* '11 | 5.3(1.9) | −12.7(5) | −7.4(2.0) | | |
| (78) Birse and McGovern '12 | 4.2(1.0) | −12.7(5) | −8.5(1.1) | −24.7(1.6) | −33(2) |
| (79) Gorchtein *et al.*'13 [a] | −2.3(4.6) | −13.0(6) | −15.3(4.6) | −24.5(1.2) | −39.8(4.8) |
| (80) Hill and Paz '16 | | | | | −30(13) |
| (81) Tomalak'18 | 2.3(1.3) | | −10.3(1.4) | −18.6(1.6) | −29.0(2.1) |
| LEADING-ORDER BχPT | | | | | |
| (82) Alarcòn *et al.* '14 | | | $-9.6^{+1.4}_{-2.9}$ | | |
| (83) Lensky *et al.* '17 [b] | $3.5^{+0.5}_{-1.9}$ | −12.1(1.8) | $-8.6^{+1.3}_{-5.2}$ | | |
| LATTICE QCD | | | | | |
| (84) Fu *et al.* '22 | | | | | −37.4(4.9) |

[a] Adjusted values due to a different decomposition into the elastic and polarizability contributions.
[b] Partially includes the Δ(1232)-isobar contribution.

Antognini, Hagelstein, Pascalutsa (2022)

- Agreement with other calculations
- Also on the size of the subtraction contribution separately

# Subtraction Function: New Ideas

- The slope of the subtraction function $T_1(0, Q^2)$

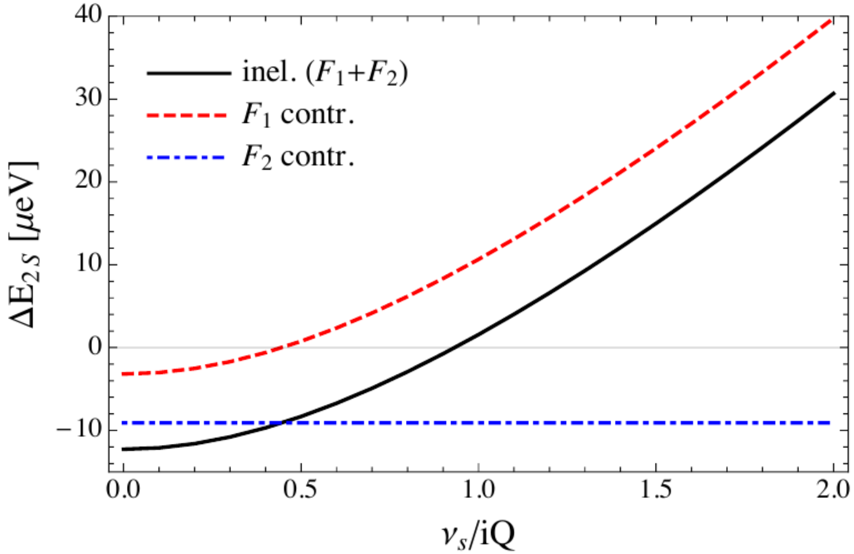
$$T_1(0,Q^2) \;=\; \beta_{M1}\,Q^2 + \left[\frac{1}{6}\beta_{M2} - \alpha_{\rm em}\sqrt{\frac{3}{2}}P'^{\,(M1,M1)0}(0) + \frac{1}{(2M)^2}\beta_{M1} + \alpha_{\rm em}\, b_{3,0}\right] Q^4 + \mathcal{O}(Q^6)$$

VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

- The knowledge of $b_{3,0}$ would constrain the slope, possibly reduce the uncertainty of the 2γ contribution to μH LS
- Might be possible to extract from dilepton electroproduction, $ep \to ep\ell^+\ell^-$

Pauk, Carlson, Vanderhaeghen (2020)

- A different subtraction point: $\nu_s = iQ$ instead of $\nu_s = 0$ Hagelstein, Pascalutsa (2021)
  - inelastic contribution suppressed
  - might be advantageous in a lattice QCD calculation
  - further EFT studies



# HFS of μH in Covariant BχPT

$$E_{\rm hfs}(nS) = \frac{8}{3}\frac{Z\alpha}{(an)^3}\frac{1+\kappa}{mM}\left(1+\Delta_{\rm QED}+\Delta_{\rm weak}+\Delta_{\rm strong}\right)$$

$$\Delta_{\rm strong} = \Delta_{\rm Z} + \Delta_{\rm recoil} + \Delta_{\rm pol}$$

$$\Delta_{\rm pol.} = \Delta_1 + \Delta_2 = \frac{Zm}{2\pi(1+\kappa)M}(\delta_1+\delta_2),$$

$$\delta_1 = 2\int_0^\infty \frac{dQ}{Q}\left\{\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)/Z^2 + F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0} dx\, x^2 g_1(x,Q^2)\right.$$
$$\left.\times\frac{1}{(v_l+v_x)(1+v_x)(1+v_l)}\left(4+\frac{1}{1+v_x}+\frac{1}{v_l+1}\right)\right\},$$

$$\delta_2 = 96M^2\int_0^\infty \frac{dQ}{Q^3}\int_0^{x_0} dx\, g_2(x,Q^2)\left(\frac{1}{v_l+v_x}-\frac{1}{v_l+1}\right)$$

$$I_1(Q^2) = \frac{2M^2Z^2}{Q^2}\int_0^{x_0} dx\, g_1(x,Q^2)$$ The generalised GDH integral

$v_l = \sqrt{1+1/\tau_l},\ v_x = \sqrt{1+x^2\tau^{-1}},\ \tau_l = Q^2/4m^2,\ \tau = Q^2/4M^2$ Kinematic functions

# HFS of μH in Covariant BχPT

$$E_{\rm hfs}(nS) = \frac{8}{3}\frac{Z\alpha}{(an)^3}\frac{1+\kappa}{mM}\left(1+\Delta_{\rm QED}+\Delta_{\rm weak}+\Delta_{\rm strong}\right)$$

$$\Delta_{\rm strong} = \Delta_{\rm Z}+\Delta_{\rm recoil}+\Delta_{\rm pol}$$

$$\Delta_{\rm pol.} = \Delta_{LT}+\Delta_{TT}+\Delta_{F_2} = \frac{m}{2\pi(1+\kappa)M}\left(\delta_{LT}+\delta_{TT}+\delta_{F_2}\right),$$

$$\delta_{LT} = \frac{4M}{\alpha\pi^2}\int_0^\infty {\rm d}Q\int_0^{x_0} dx\,\frac{1}{v_l+v_x}\frac{1}{x^2+\tau}\left[1-\frac{1}{(1+v_l)(1+v_x)}\right]\sigma_{LT}(x,Q^2),$$

$$\delta_{TT} = \frac{4M^2}{\alpha\pi^2}\int_0^\infty \frac{{\rm d}Q}{Q}\int_0^{x_0}\frac{{\rm d}x}{x}\frac{1}{1+v_l}\left[\frac{2\tau}{x^2+\tau}+\frac{1}{(v_l+v_x)(1+v_x)}\right]\sigma_{TT}(x,Q^2),$$

$$\delta_{F_2} = 2\int_0^\infty \frac{Q}{Q}\frac{5+4v_l}{(v_l+1)^2}F_2^2(Q^2)$$

$$v_l = \sqrt{1+1/\tau_l},\;\; v_x = \sqrt{1+x^2\tau^{-1}},\;\; \tau_l = Q^2/4m^2,\;\; \tau = Q^2/4M^2$$

Kinematic functions

- Rewritten in terms of scattering cross sections

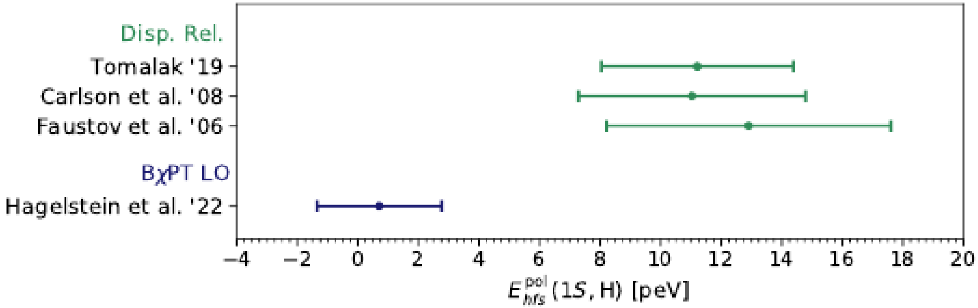
# HFS of μH in Covariant BχPT: Cancellations

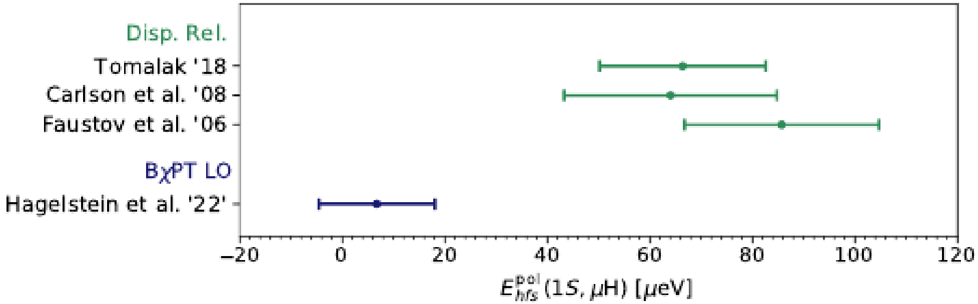
- LO BχPT result

$$E_{\text{hfs}}^{\langle\text{LO}\rangle\text{ pol.}}(1S,\text{H}) = 0.69(2.03)\,\text{peV}$$

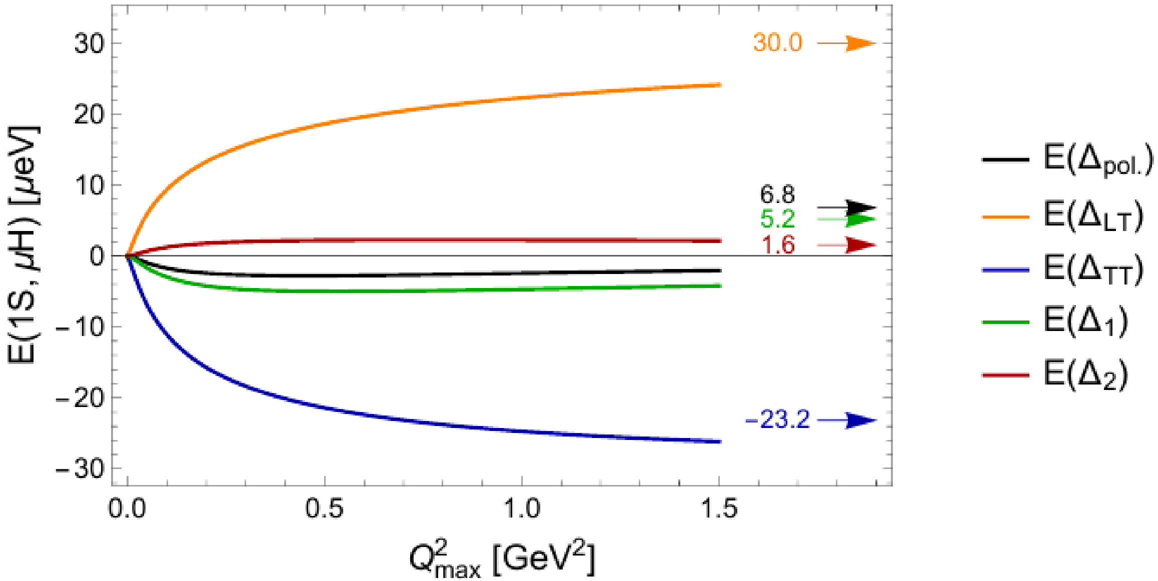
$$E_{\text{hfs}}^{\langle\text{LO}\rangle\text{ pol.}}(1S,\mu\text{H}) = 6.8(11.4)\,\mu\text{eV}$$

- Consistent with zero
- Cancellations!



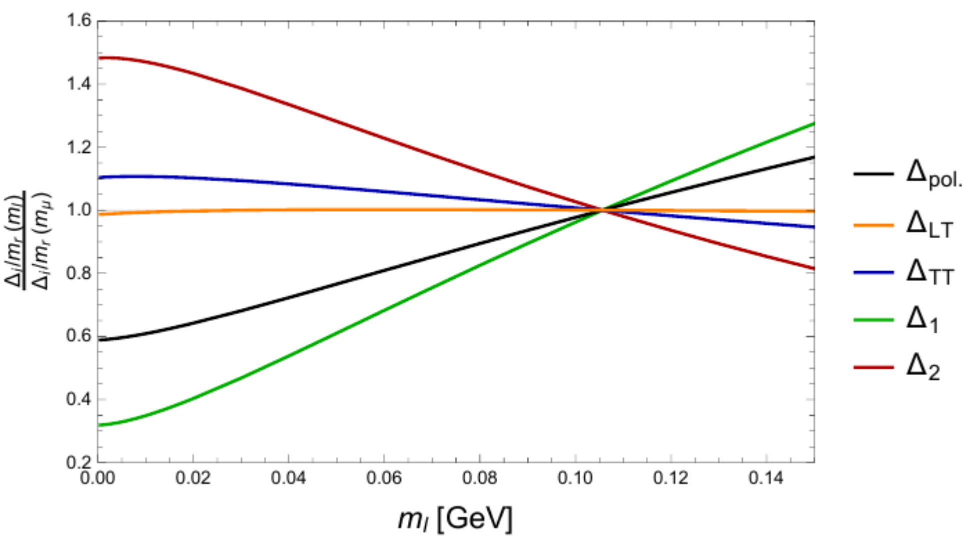



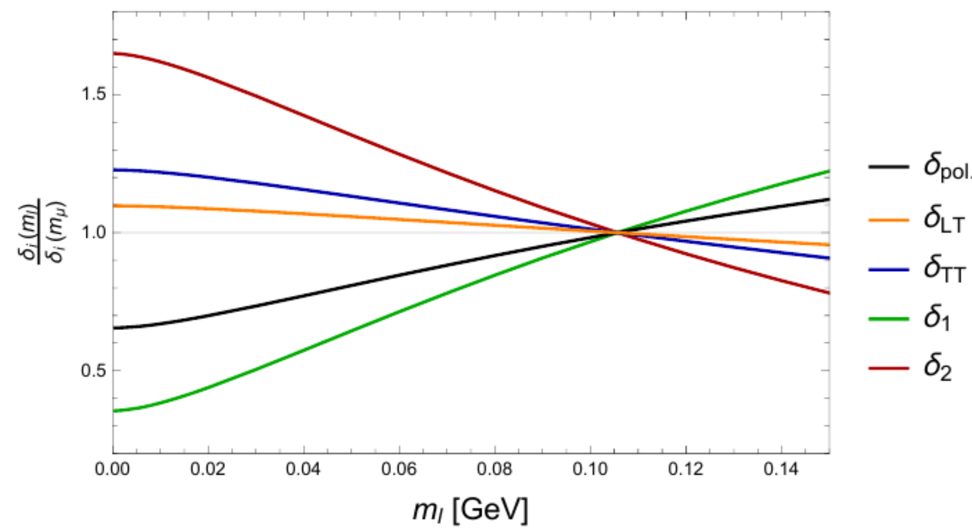

Hagelstein, VL, Pascalutsa (2022)

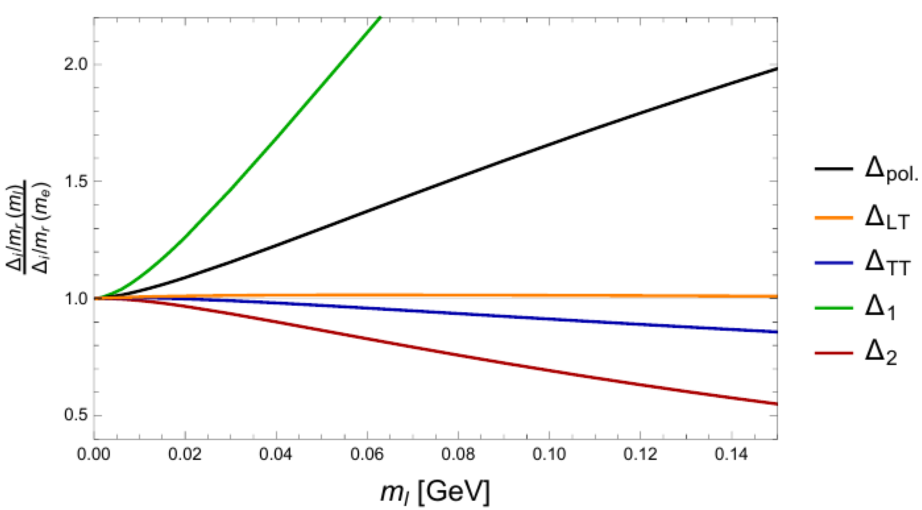


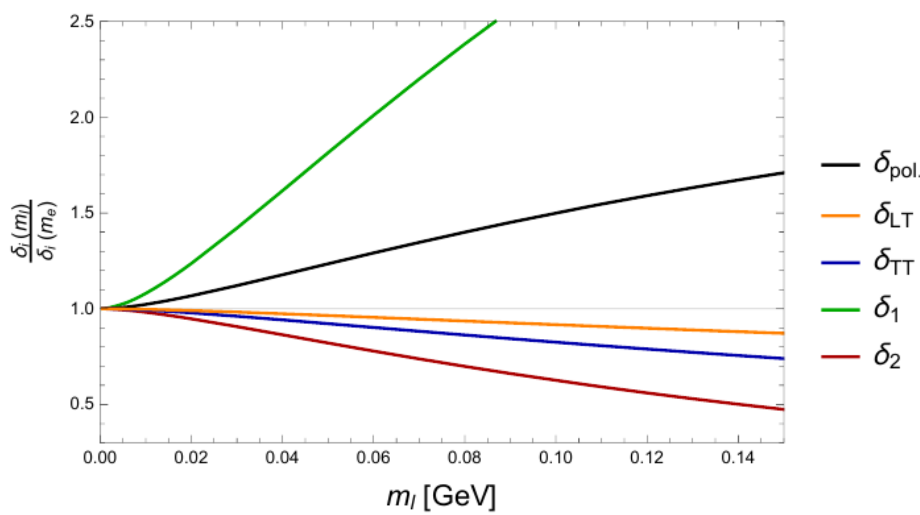

- The LT and TT contributions are large and almost cancel each other
- The LO BχPT result is nearly zero
- Sizeable uncertainty

# HFS of μH in Covariant BχPT: Scaling w. Lepton Mass




- Cancellations seem to affect the scaling
- Can the data-driven evaluations be affected?

# HFS of μH in Covariant BχPT: More Cancellations

$$E_{\text{hfs}}(nS) = \frac{8}{3}\frac{Z\alpha}{(an)^3}\frac{1+\kappa}{mM}\left(1+\Delta_{\text{QED}}+\Delta_{\text{weak}}+\Delta_{\text{strong}}\right)$$

$$\Delta_{\text{strong}} = \Delta_Z + \Delta_{\text{recoil}} + \Delta_{\text{pol}}$$

$$\Delta_{\text{pol.}} = \Delta_1 + \Delta_2 = \frac{Zm}{2\pi(1+\kappa)M}(\delta_1+\delta_2),$$

$$\delta_1 = 2\int_0^\infty \frac{dQ}{Q}\left\{\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)/Z^2 + F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0} dx\, x^2 g_1(x,Q^2)\right.$$
$$\left.\times \frac{1}{(v_l+v_x)(1+v_x)(1+v_l)}\left(4+\frac{1}{1+v_x}+\frac{1}{v_l+1}\right)\right\},$$

$$\delta_2 = 96M^2\int_0^\infty \frac{dQ}{Q^3}\int_0^{x_0} dx\, g_2(x,Q^2)\left(\frac{1}{v_l+v_x}-\frac{1}{v_l+1}\right)$$

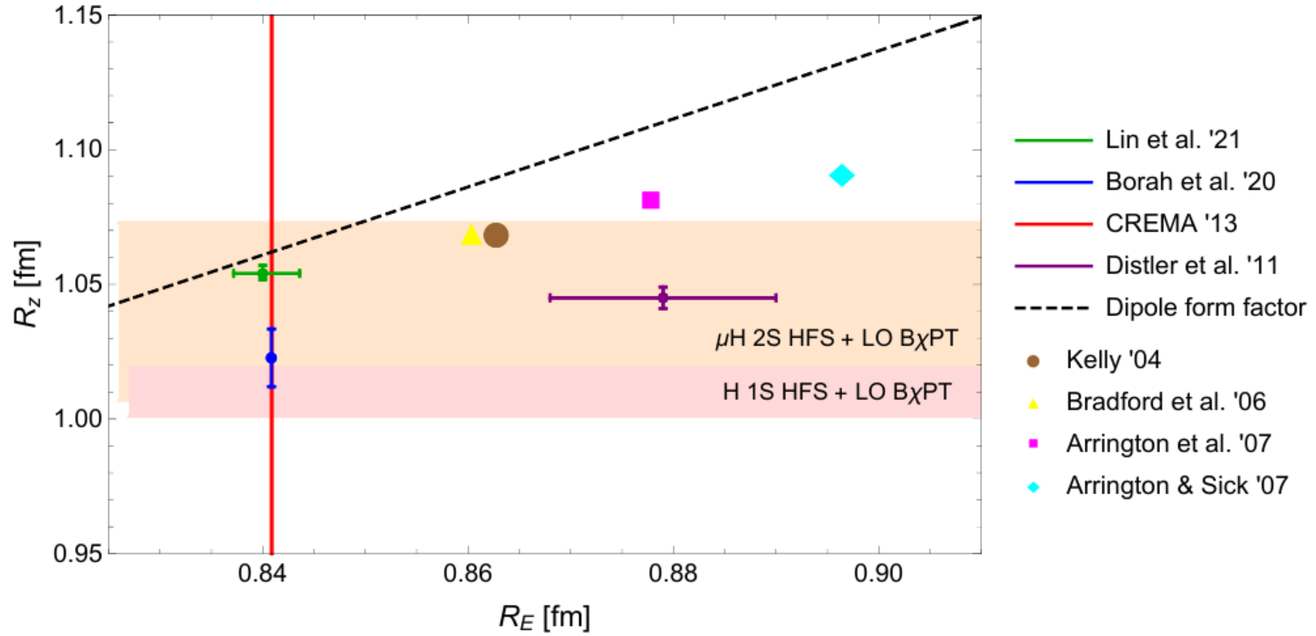
$$I_1(0) = -\frac{\kappa^2}{4} \qquad F_2(0) = \kappa$$

- Cancellation between the Pauli form factor and the intelastic contributions
- Enhanced at low $Q$!

# HFS of μH in Covariant BχPT: Zemach Radius

- Results:

$$R_Z(\text{H}) = 1.010(9)\,\text{fm}$$
$$R_Z(\mu\text{H}) = 1.040(33)\,\text{fm}$$

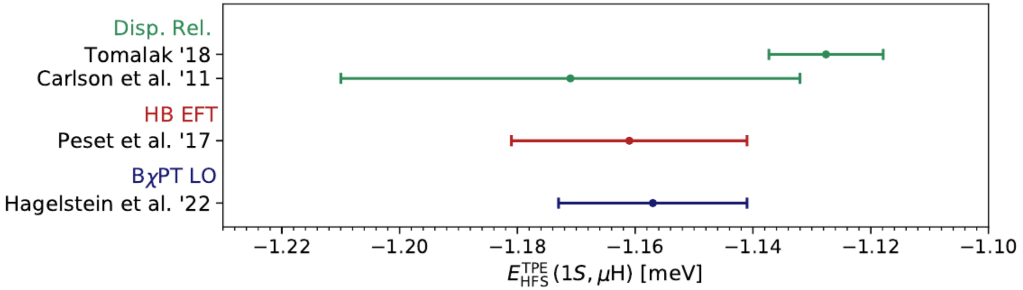



- Zemach radius extracted is smaller than in most of other

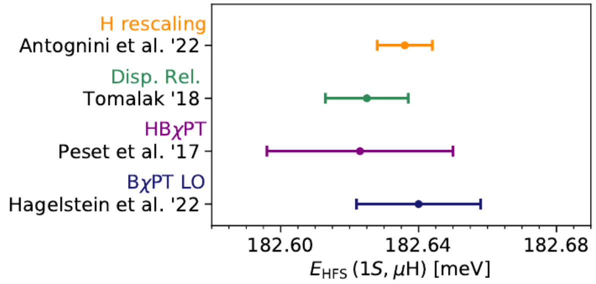
- The smallness of the Zemach radius compensates the smallness of the polarisability contribution:

$$\Delta_{\text{strong}} = \Delta_Z + \Delta_{\text{recoil}} + \Delta_{\text{pol}} \qquad \Delta_Z = -2Z\alpha m_r R_Z$$

2γ

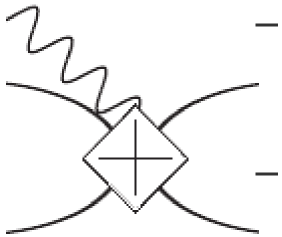


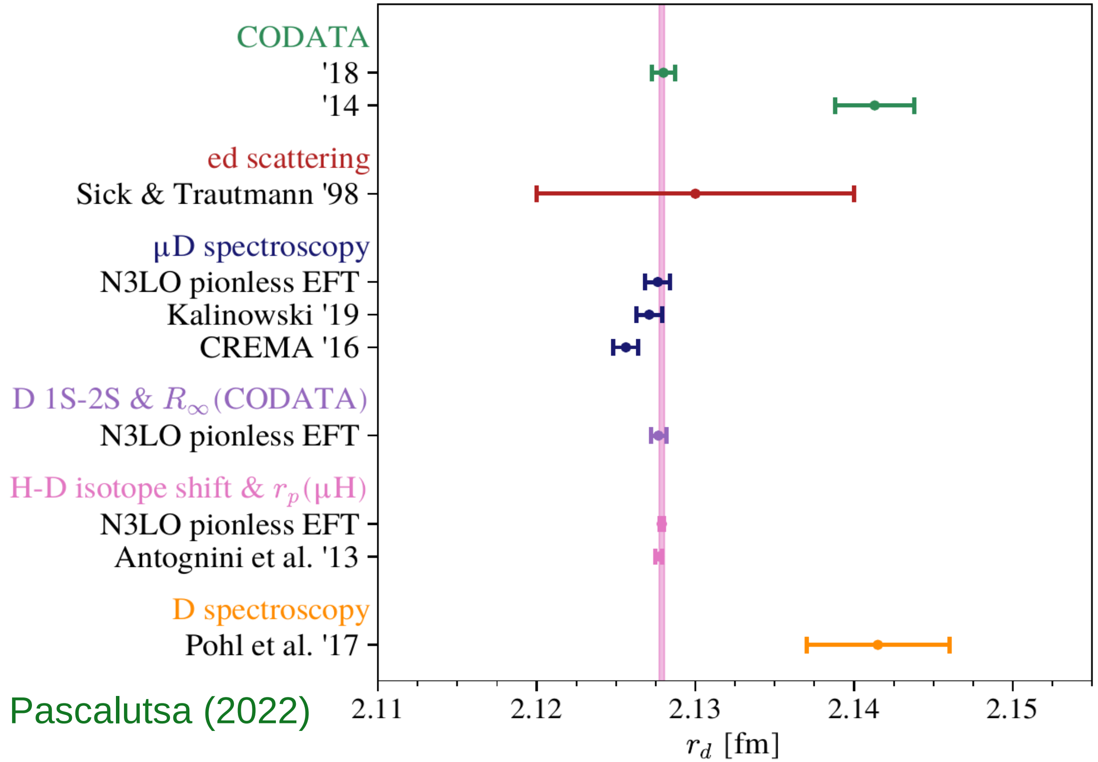

Total HFS



# Deuteron Charge Radius and 2γ Exchange in μD

- Reassessed with pionless EFT
- μD, D, and H-D isotope shift all consistent with one another
- One unknown LEC
  - extracted from H-D isotope shift
  - important for the D charge form factor





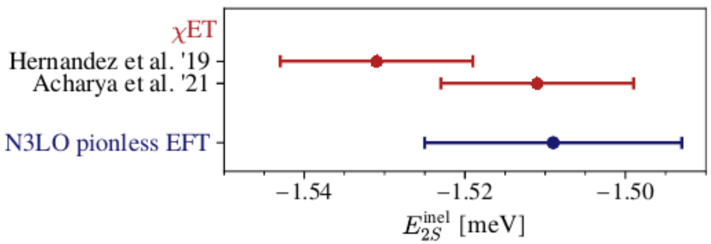

VL, Hagelstein, Pascalutsa (2022)

- Agreement also with the precise empirical value of 2γ exchange correction in μD
- Higher-order in $\alpha$ terms are important in D
  - Coulomb $\left[\mathcal{O}(\alpha^6 \log \alpha)\right]$
  - eVP and 3γ $\left[\mathcal{O}(\alpha^6)\right]$ Kalinowski (2019)
- Higher-order in pionless EFT single-nucleon terms are taken into account (pols+subtraction)

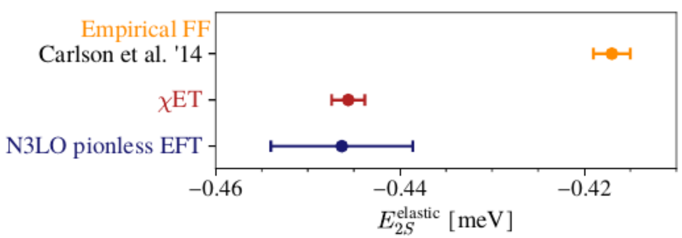
| | $E_{2S}^{2\gamma}$ [meV] |
|---|---|
| Theory prediction | |
| Krauth et al. '16 [5] | $-1.7096(200)$ |
| Kalinowski '19 [6, Eq. (6) + (19)] | $-1.740(21)$ |
| $\not\pi$EFT (this work) | $-1.752(20)$ |
| Empirical ($\mu$H + iso) | |
| Pohl et al. '16 [3] | $-1.7638(68)$ |
| This work | $-1.7585(56)$ |

# Deuteron Charge Radius and 2γ Exchange in μD

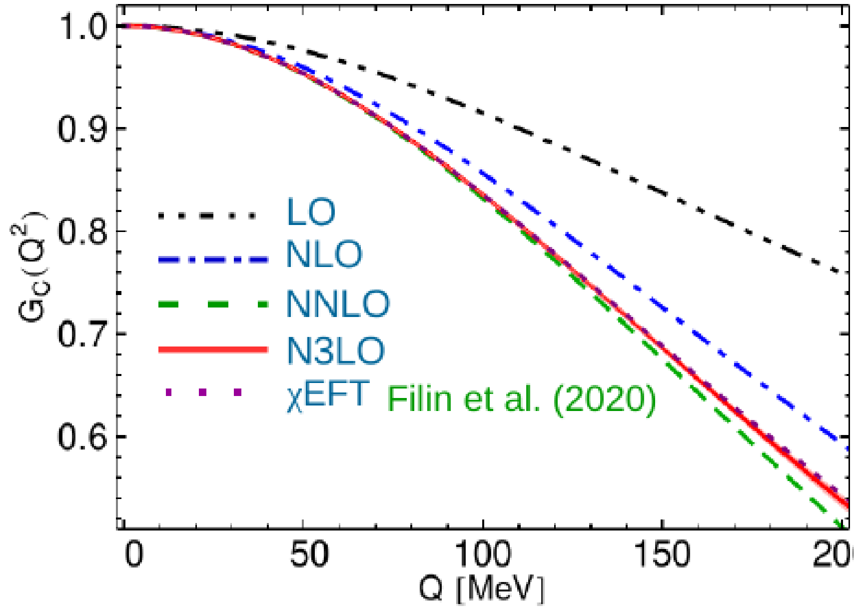
- The deuteron charge form factor is consistent with χEFT
- Inelastic 2γ agrees with other calculations



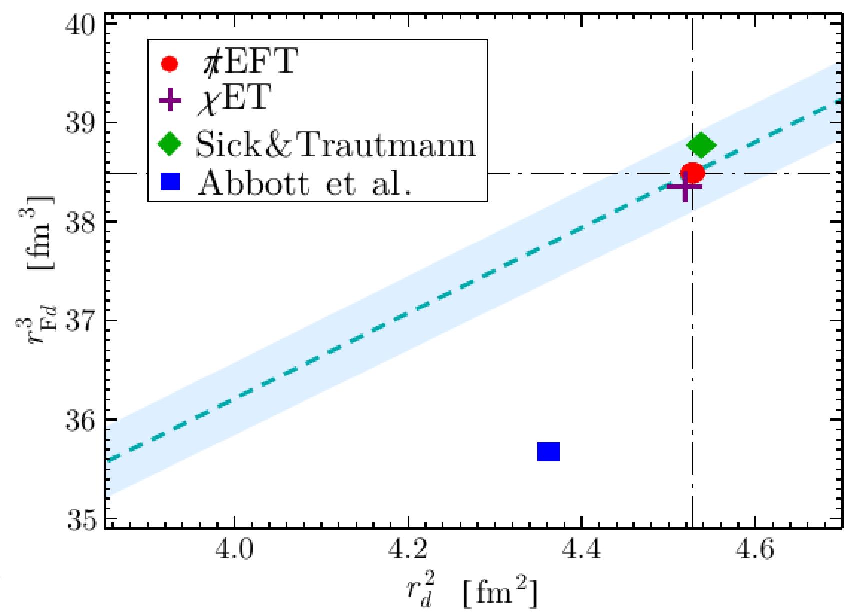

- Elastic 2γ is several std. deviations larger than with Abbott et al. form factor




- Correlation between the charge and Friar radii; can be used to test ff parametrisation
- Abbott et al. parametrisation appears to not give good results for low Q properties
- Agreement with χEFT vindicates both EFTs




VL, Hiller Blin, Pascalutsa (2021)



# Problems to Address

Proton

- Subtraction function: can it be constrained with a better precision?
  - LEX constraints from VVCS, Euclidean Subtraction, LQCD?
- HFS:
  - better theory accuracy possible?
  - can data-driven calculations be affected by cancellations?
  - are there potential pitfalls with lepton mass scaling?

Deuteron and heavier muonic atoms:

- Nuclear part
  - seems to be under control (pionless/chiral EFT)
  - smaller uncertainty should be possible
  - empirical data/parametrisations may sometimes fail
- Nucleon part
  - becoming increasingly important with bigger Z and A
  - neutron?

# Thank You for Your Attention!