Nuclear structure corrections in light muonic atoms

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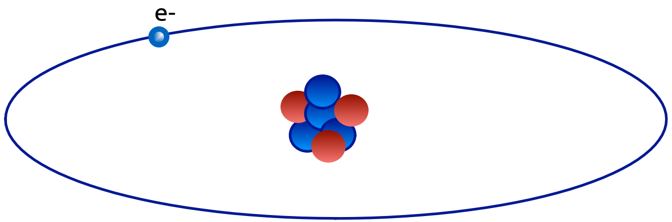
JG|U

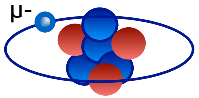
Muonic atoms

Hydrogen-like systems

Ordinary atoms



Muonic atoms



The muon is more sensitive to the nucleus

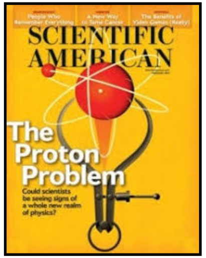
Excellent precision probe for the nucleus

Experimental program at **PSI** of the **CREMA** collaboration

Muonic Hydrogen
- Pohl et al., Nature (2010)
- Antognini et al., Science (2013)

Muonic Deuterium
- Pohl et al., Science (2016)

Muonic Helium-4
- Krauth et al., Nature (2021)



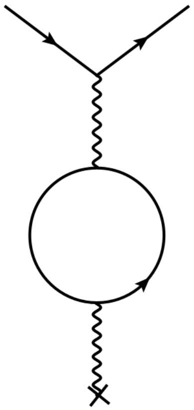
Lamb-shift and charge radius

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta_{\mathrm{NS}}^{(4)} \times r_c^2 + \delta_{\mathrm{NS}}^{(5)} + \delta_{\mathrm{NS}}^{(6)} + \ldots$$

Lamb-shift and charge radius

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta_{\mathrm{NS}}^{(4)} \times r_c^2 + \delta_{\mathrm{NS}}^{(5)} + \delta_{\mathrm{NS}}^{(6)} + \ldots$$

$\delta_{\mathrm{QED}} =$



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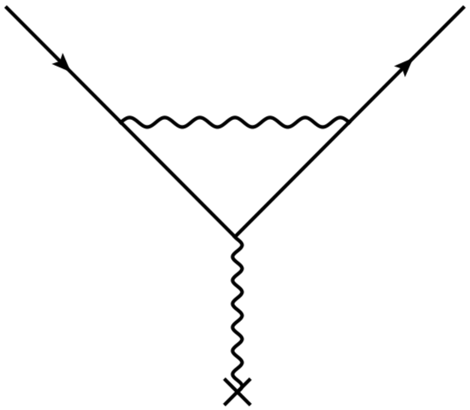
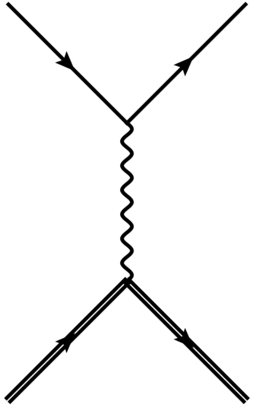
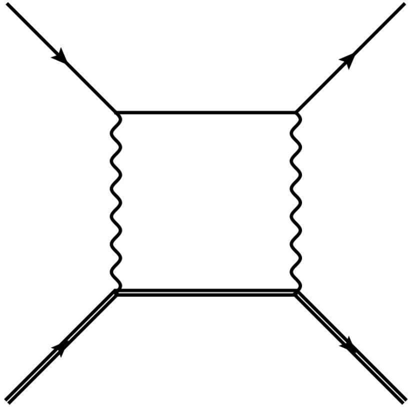


$+ \quad \cdots$

Lamb-shift and charge radius

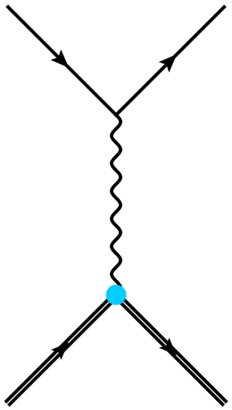
$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta_{\mathrm{NS}}^{(4)} \times r_c^2 + \delta_{\mathrm{NS}}^{(5)} + \delta_{\mathrm{NS}}^{(6)} + \ldots$$

$\delta_{\mathrm{NR}} =$  $+$  $+ \quad \cdots$

Lamb-shift and charge radius

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta_{\mathrm{NS}}^{(4)} \times r_c^2 + \delta_{\mathrm{NS}}^{(5)} + \delta_{\mathrm{NS}}^{(6)} + \ldots$$

$\delta_{\mathrm{NS}} =$



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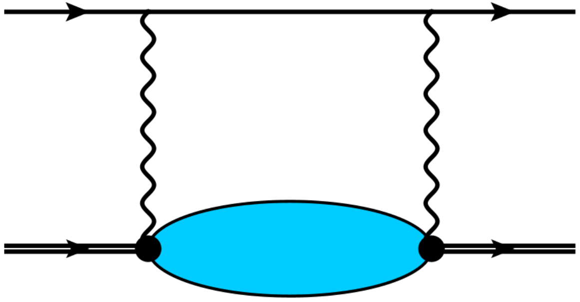


$+ \quad \cdots$

A matter of precision

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta_{\mathrm{NS}}^{(4)} \times r_c^2 + \delta_{\mathrm{NS}}^{(5)} + \delta_{\mathrm{NS}}^{(6)} + \ldots$$

For the muonic Helium-4 ion

$$\delta_{\mathrm{QED+NR}} = +1,668.489(14)\ \mathrm{meV}$$

$$\delta_{\mathrm{NS}}^{(4)} = -106.220(8)\ \mathrm{meV\ fm}^{-2}$$

$$\delta_{\mathrm{NS}}^{(5)} = +9.340(250)\ \mathrm{meV}$$

$$\delta_{\mathrm{NS}}^{(6)} = -0.150(150)\ \mathrm{meV}$$

→

$$r_c = 1.67824(13)_{\mathrm{ex}}(82)_{\mathrm{th}}\ \mathrm{fm}$$

J. J. Krauth et. al. Nature 589,527 (2021)

The Uncertainty is largely dominated by the nuclear structure effects.

Outline

Theory

Evaluation of the NS effects

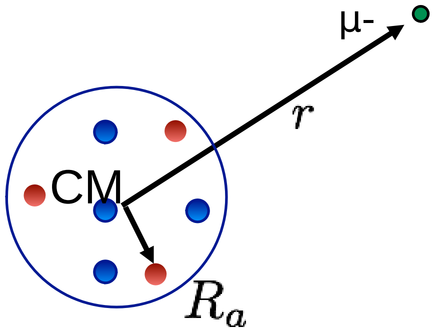
$$\mathrm{H} = \mathrm{H}_N + H_\mu + \Delta V$$

The Nuclear Hamiltonian is included or not depending on whether we look for nuclear sizes or polarizability corrections

$$H_\mu = \frac{\mathbf{p}^2}{2m_r} - \frac{Z\alpha}{r}$$

The corrections to the bulk coulomb interactions are included in perturbation theory

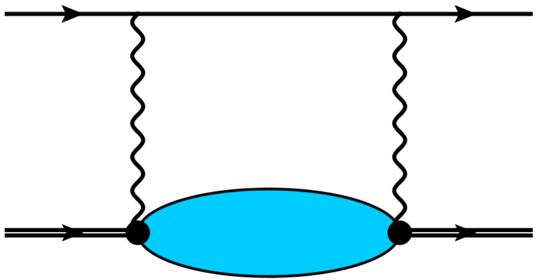
$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\mathbf{r} - \mathbf{R}_a|} \right)$$



Evaluation of the NS effects

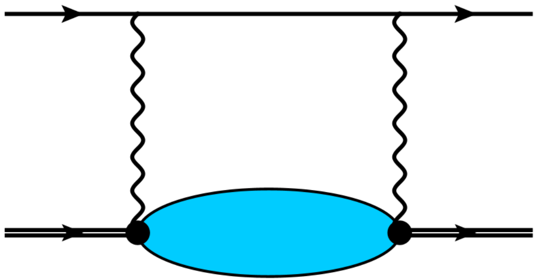
$$\delta_{\mathrm{NS}} = \delta_{\mathrm{FS}} + \delta_{\mathrm{pol}}$$

$$\delta^{(5)}_{\mathrm{pol}} = \langle N_0 \mu | \, \Delta V \; G \; \Delta V \, | N_0 \mu \rangle$$



Evaluation of the NS effects

$$\delta_{\rm NS} = \delta_{\rm FS} + \delta_{\rm pol}$$

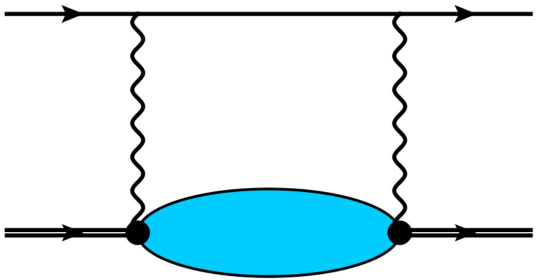


$$\delta^{(5)}_{\rm pol} = \langle N_0 \mu | \Delta V \; G \; \Delta V \, | N_0 \mu \rangle$$

Work in coordinate space

$$= \sum_{N \neq N_0} \int d^3R \; d^3R' \; \rho^p_N(\mathbf{R}) \; W(\mathbf{R}, \mathbf{R}', \omega_N) \; \rho^p_N(\mathbf{R}')$$

Evaluation of the NS effects



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Work in coordinate space

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Taylor expand the lepton matrix element

$$= C \sum_{N \neq N_0} \left(\frac{1}{\omega_N} \right)^{\frac{3}{2}} \int d^3R \; d^3R' \;\; \rho^p_N(\mathbf{R}) \left[\eta^2 - \frac{1}{4}\eta^3 + \frac{1}{20}\eta^4 + \ldots \right] \rho^p_N(\mathbf{R}')$$

Evaluation of the NS effects

$$\delta^{(5)}_{\rm pol} = C \sum_{N\neq N_0} \left(\frac{1}{\omega_N}\right)^{\frac{3}{2}} \int d^3R\, d^3R'\; \rho^p_N(\mathbf{R}) \left[\eta^2 - \frac{1}{4}\eta^3 + \frac{1}{20}\eta^4 + ...\right] \rho^p_N(\mathbf{R}')$$

LO NLO N2LO

$$\eta = \sqrt{2m_r\omega_N}|\mathbf{R}-\mathbf{R}'| \approx \sqrt{\frac{m_r}{m_N}} \approx 0.33$$

. . . Some more algebra . . .

$$\delta^{(5)}_{\rm pol} = \sum_i \left[\; C_i(Z\alpha, m_r)\; \int_0^\infty \mathcal{F}_i(\omega/m_r)\; S_{O_i}(\omega)\; d\omega \;\right]$$

Nuclear response function

Ab-initio Nuclear Theory

Ab-initio methods: Solutions of the time-independent Schrödinger equation for the nuclear states

$$\widehat{H}_{\mathrm{N}}\Psi_{\mathrm{N}} = E_{\mathrm{N}}\Psi_{\mathrm{N}}$$

With controlled approximations.

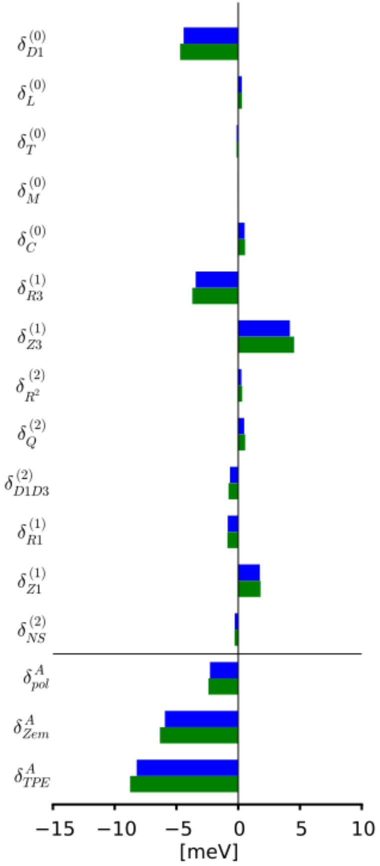
Solutions of Schrödinger Eq. $\longrightarrow$ $S_{O_i}(\omega)$ $\longrightarrow$ $\delta^{(5)}_{\mathrm{pol}}$

The nuclear Hamiltonian is an input and a big source of uncertainty.

Previous work on Helium-4

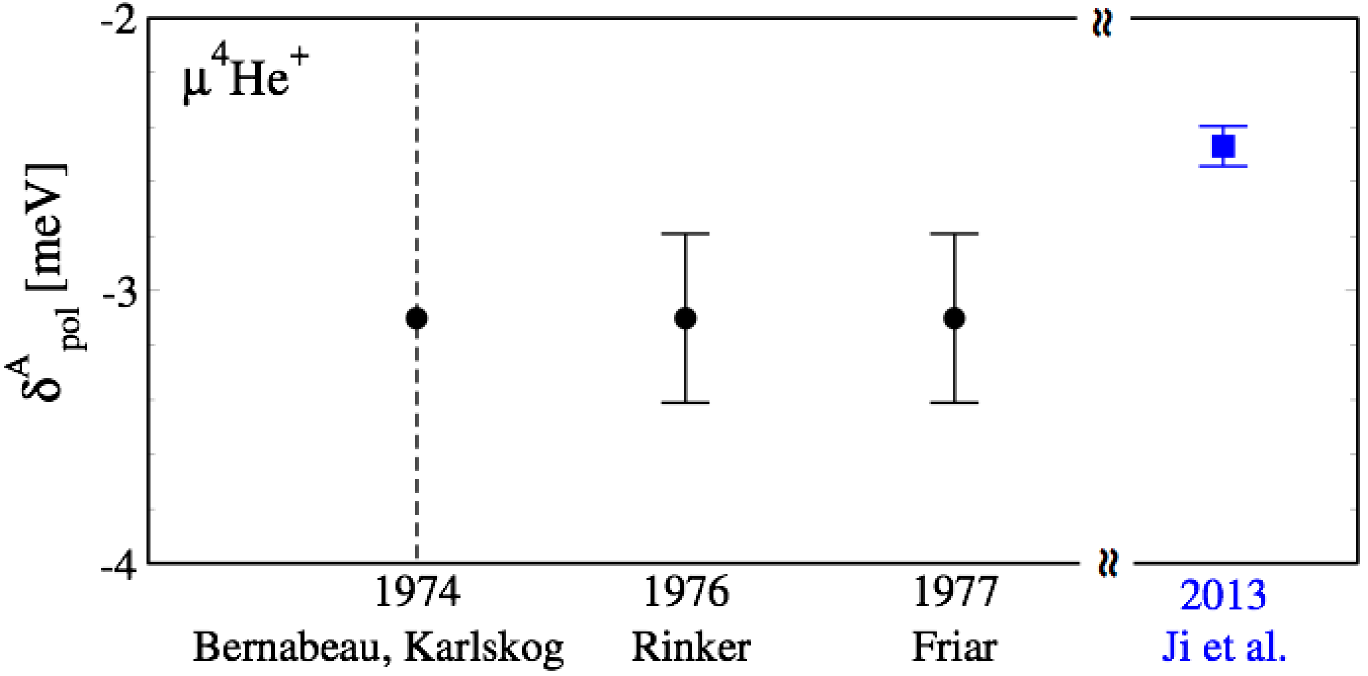
C. Ji, et al. PRL 143402-1 (2013)

C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

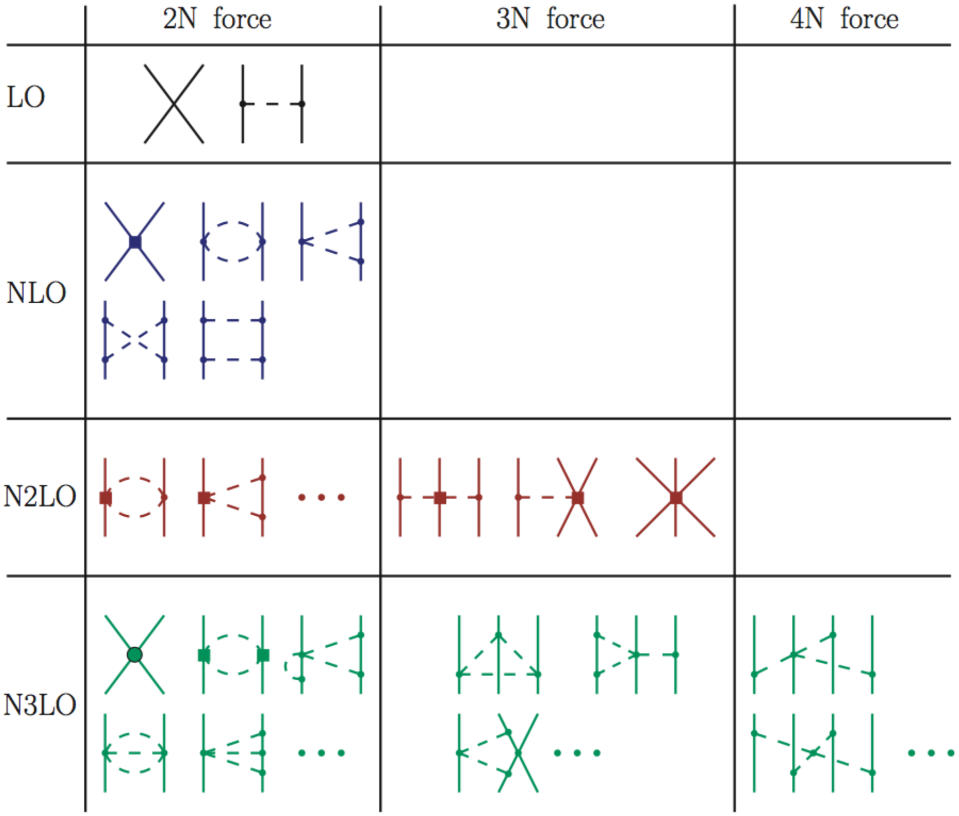


	AV18/UIX	χEFT
$\delta_{D1}^{(0)}$	-4.418	-4.701
$\delta_{L}^{(0)}$	0.289	0.308
$\delta_{T}^{(0)}$	-0.126	-0.134
$\delta_{M}^{(0)}$	0.008	0.006
$\delta_{C}^{(0)}$	0.519	0.553
$\delta_{R3}^{(1)}$	-3.442	-3.717
$\delta_{Z3}^{(1)}$	4.183	4.526
$\delta_{R^2}^{(2)}$	0.259	0.324
$\delta_{Q}^{(2)}$	0.484	0.561
$\delta_{D1D3}^{(2)}$	-0.666	-0.784
$\delta_{R1}^{(1)}$	-0.848	-0.874
$\delta_{Z1}^{(1)}$	1.753	1.811
$\delta_{NS}^{(2)}$	-0.279	-0.293
δ_{pol}^{A}	-2.284	-2.414
δ_{Zem}^{A}	-5.936	-6.337
δ_{TPE}^{A}	-8.220	-8.751

Impact of ab-initio theory



The Hamiltonians



Interactions derived from the **Chiral effective field theory** and written in coordinate space.

Hierarchy among different operators decided after specifying a **power counting scheme**.

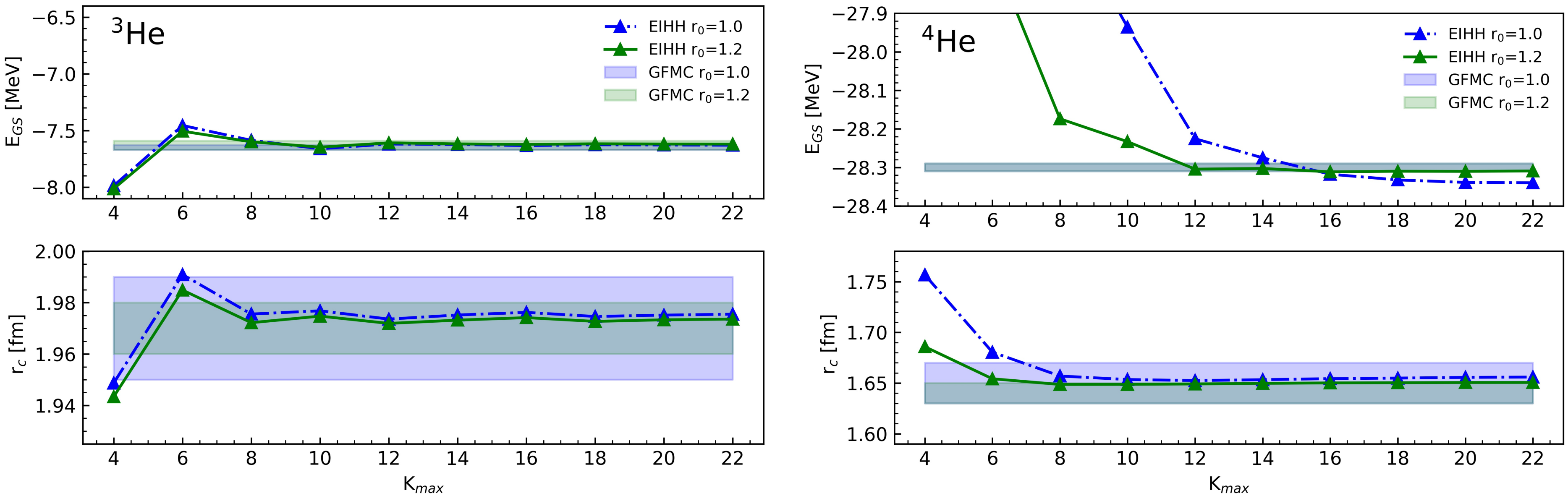
The separation of scales of the operators makes it possible to reliably quantify the uncertainties due to the truncation of the expansion (Ex. With **Bayesian Statistics**).

Outline

Results

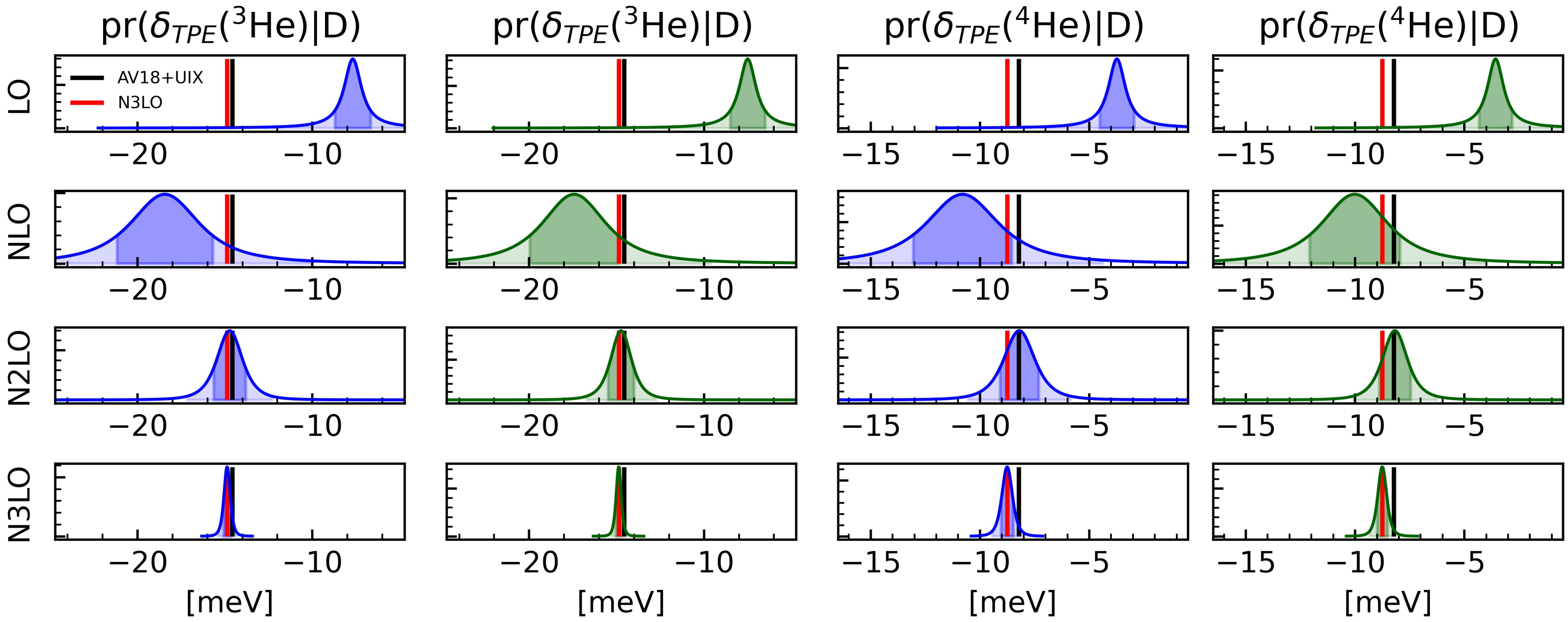
Benchmark tests

S.S. LM, S. Bacca , N. Barnea, Front. Phys. 9, 671869 (2021)



NS corrections in muonic Helium

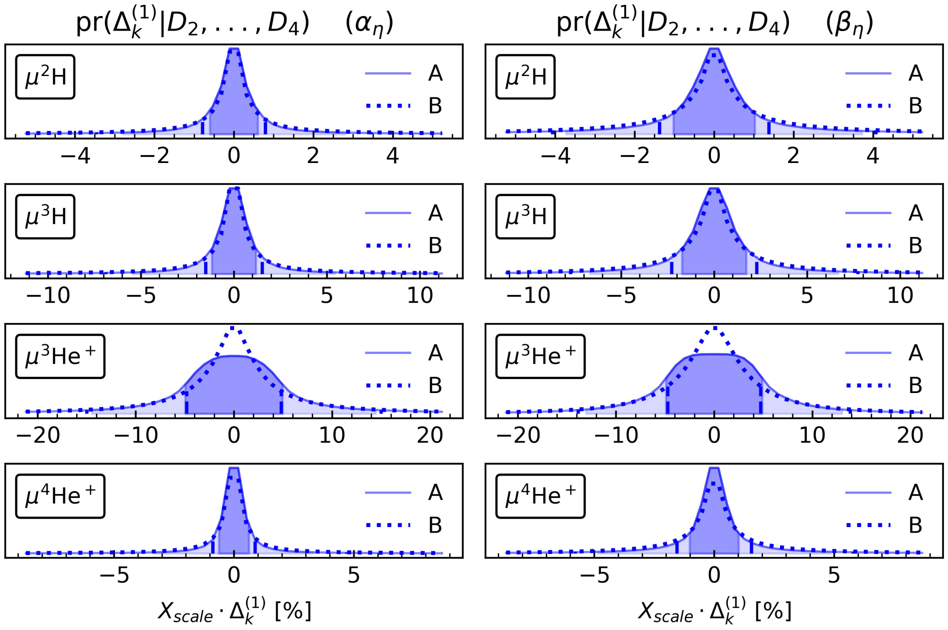
S.S.LM, et al. In preparation for 2022



C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

Eta-expansion uncertainty

$$\delta^{(5)}_{\rm pol} = C \sum_{N \neq N_0} \left(\frac{1}{\omega_N}\right)^{\frac{3}{2}} \int d^3R\, d^3R'\, \rho^p_N(\mathbf{R}) \left[\eta^2 - \frac{1}{4}\eta^3 + \frac{1}{20}\eta^4 + \ldots\right] \rho^p_N(\mathbf{R}')$$



S.S.LM, et al. 2022 J. Phys. G: Nucl. Part. Phys. 49 105101

	μ^2H	μ^3H	μ^3He$^+$	μ^4He$^+$
[1]	0.4%	1.3%	1.1%	0.8%
This work	0.8%	1.5%	4.8%	0.9%

[1] C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

Uncertainties in muonic Helium

S.S.LM, et al. In preparation for 2022

	μ^3He^+			μ^4He^+		
	δ_{pol} [%]	δ_{Zem} [%]	δ_{TPE} [%]	δ_{pol} [%]	δ_{Zem} [%]	δ_{TPE} [%]
Numerical (2019)	0.4	0.1	0.1	0.4	0.3	0.4
Numerical	0.1	0.2	0.1	0.4	0.3	0.2
Nuclear model (2019)	0.7	1.8	1.5	3.9	4.6	4.4
Nuclear model (N2LO)	4.8	6.9	6.2	14.5	9.4	11
Nuclear model (N3LO)	1.6	1.6	1.4	4.1	2.8	3
η-expansion (2019)	1.1		0.3	0.8		0.2
η-expansion	4.8		1.4	0.9		0.2

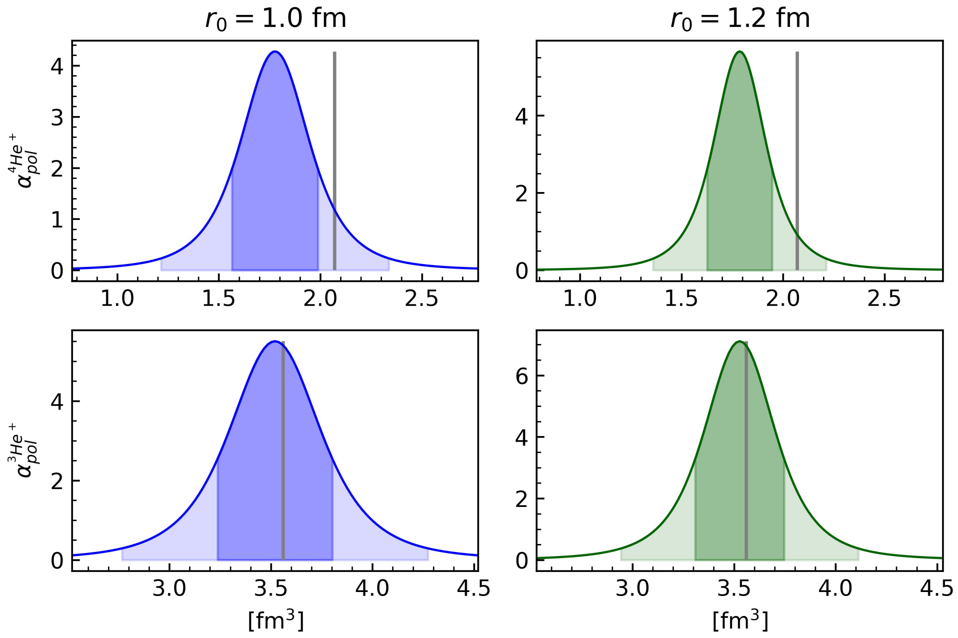
Conclusions

- We re-evaluated numerical, nuclear model and η-expansion uncertainties with chiral perturbation theory and Bayesian techniques.
- Chiral interaction at N2LO can not reduce the uncertainty of $\delta_{\rm NS}^{(5)}$ improvement can be seen from N3LO.
- We need to reduce the η-expansion uncertainties in $\mu^3\mathrm{He}^+$.
- It is important to compute the nuclear structure effects at order $\delta_{\rm NS}^{(6)}$.
- This formalism needs to be extended to calculate nuclear polarizabilities to the HFS.

Backup

NS effects in 3He+ and 4He+

$$\delta^{(5)}_{\rm pol} = -m\alpha^4 \left\langle \sum_a \delta^3(r_a) \right\rangle (m^3 \alpha_{\rm pol})$$



— [1] K. Pachucki, A.M. Moro Phys.Rev.A 75,032521(2007)

S.S.LM, et al. In preparation for 2022

1S-2S	$^3\mathrm{He}^+$	$^4\mathrm{He}^+$
[1]	48(5)kHz	28(3)kHz
This work	48(6)kHz	24(4)kHz

Uncertainty

Uncertainty sources

- **Numerical**
- **Nuclear model**
- **Nucleon model**
- **Truncation of EM multipoles**
- **η-expansion**
- **Expansion in (Zα)**

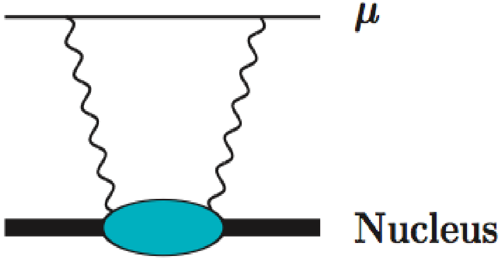
Evaluation of the TPE amplitude

- **Coulomb Distortion**

Contribution coming from intermediate interactions between lepton and nucleus during the TPE process.

We include only the correction of order $(Z\alpha)^6 log(Z\alpha)$

Since we work in the leading dipole approximation, the correction is related to the electric dipole response



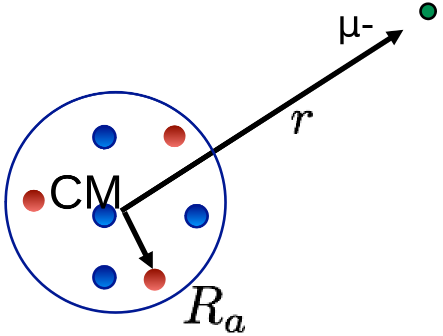
- **Relativistic effects**

They are smaller by factors $\frac{\omega_{th}}{m_r}$

Includes also first effects from electromagnetic currents. Are evaluated in the leading dipole approximation

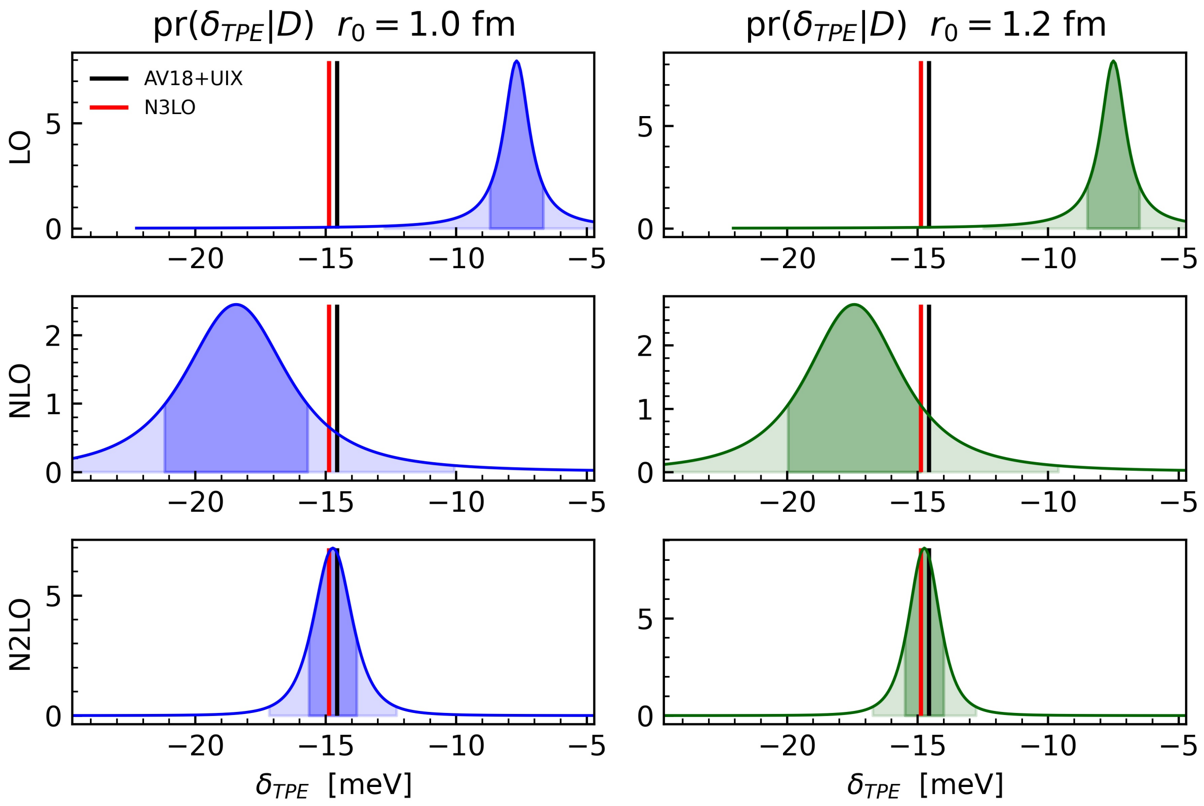
$$\delta^{(0)}_{L(T)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{th}}^{\infty} K_{L(T)}\left(\frac{\omega}{m_r}\right) S_{D1}(\omega)$$

- **Nucleon size effects**



TPE in He-3

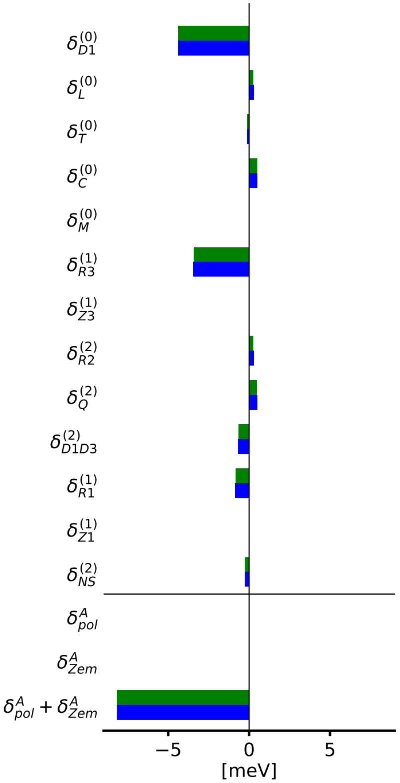
S.S. LM, et al. In preparation



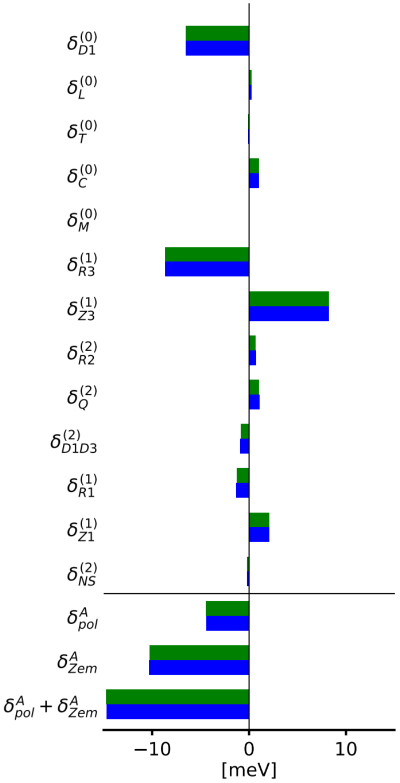
C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

NS corrections in μ4He+ (EKM)

S.S.LM, et al. In preparation for 2022



	$r_0 = 1.2$	$r_0 = 1.0$
$\delta_{D1}^{(0)}$	-4.386	-4.373
$\delta_{L}^{(0)}$	0.272	0.287
$\delta_{T}^{(0)}$	-0.124	-0.125
$\delta_{C}^{(0)}$	0.517	0.514
$\delta_{M}^{(0)}$	0.011	0.011
$\delta_{R3}^{(1)}$	-3.422	-3.477
$\delta_{Z3}^{(1)}$	-	-
$\delta_{R2}^{(2)}$	0.267	0.285
$\delta_{Q}^{(2)}$	0.484	0.505
$\delta_{D1D3}^{(2)}$	-0.668	-0.69
$\delta_{R1}^{(1)}$	-0.846	-0.856
$\delta_{Z1}^{(1)}$	-	-
$\delta_{NS}^{(2)}$	-0.272	-0.277
δ_{pol}^{A}	-	-
δ_{Zem}^{A}	-	-
$\delta_{pol}^{A} + \delta_{Zem}^{A}$	-8.174	-8.196



	$r_0 = 1.2$	$r_0 = 1.0$
$\delta_{D1}^{(0)}$	-6.562	-6.552
$\delta_{L}^{(0)}$	0.232	0.232
$\delta_{T}^{(0)}$	-0.104	-0.104
$\delta_{C}^{(0)}$	1.018	1.015
$\delta_{M}^{(0)}$	0.008	0.008
$\delta_{R3}^{(1)}$	-8.674	-8.691
$\delta_{Z3}^{(1)}$	8.223	8.255
$\delta_{R2}^{(2)}$	0.67	0.696
$\delta_{Q}^{(2)}$	1.043	1.062
$\delta_{D1D3}^{(2)}$	-0.877	-0.899
$\delta_{R1}^{(1)}$	-1.297	-1.302
$\delta_{Z1}^{(1)}$	2.054	2.057
$\delta_{NS}^{(2)}$	-0.191	-0.193
δ_{pol}^{A}	-4.458	-4.411
δ_{Zem}^{A}	-10.277	-10.312
$\delta_{pol}^{A} + \delta_{Zem}^{A}$	-14.735	-14.723