Dispersive analysis of proton form factors & recoil correction to hyperfine splitting

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Muonic Atoms @ PSI 2022
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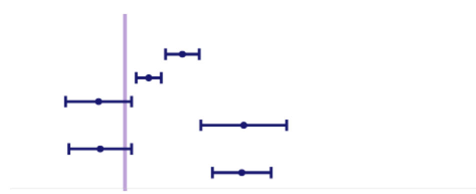
Proton radius measurements

Spectroscopy of hydrogen-like atom

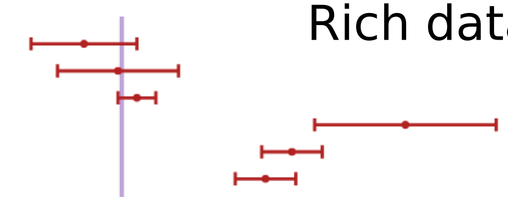
Elastic scattering of proton-lepton

$e\mathrm{H}$ Lamb shift



$e^- p \to e^- p$

Rich data



0.84 fm

Nucleon form factors

$\mu\mathrm{H}$ Lamb shift

$\mu^- p \to \mu^- p$

no data available

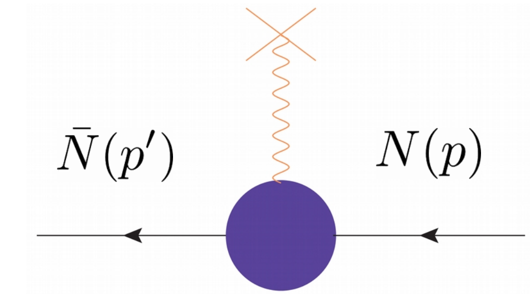
Radius data taken from

A. Antognini et al. Annual Review of Nuclear and Particle Science 72 389(2022)

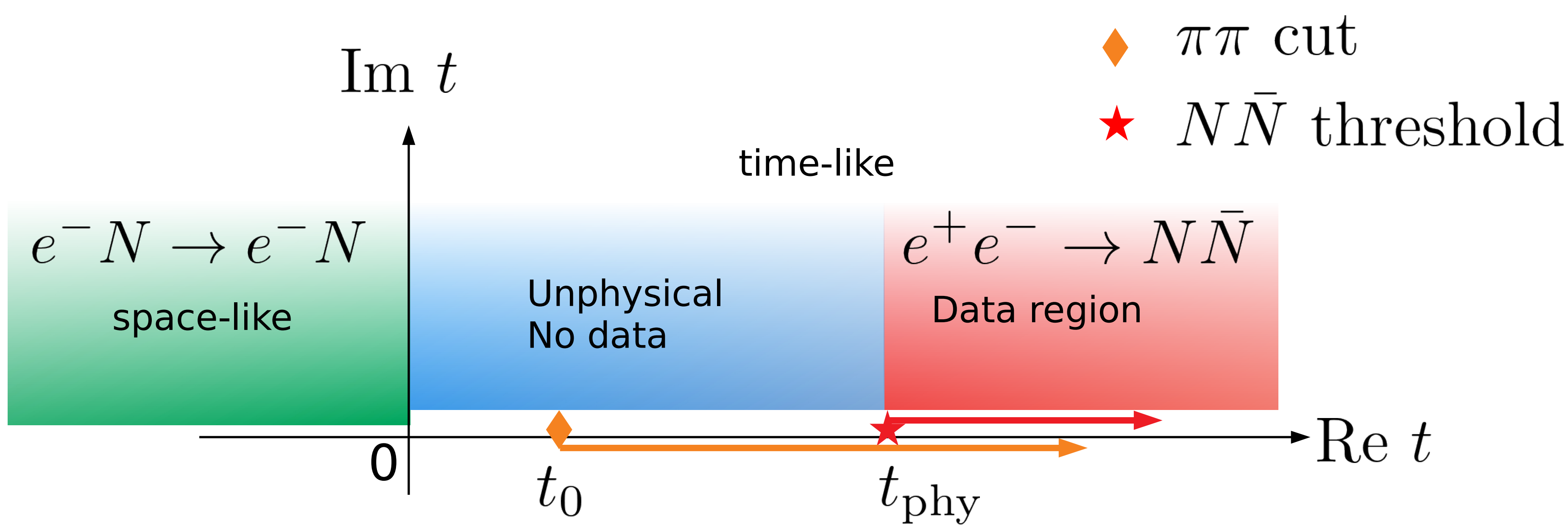
Nucleon electromagnetic form factors

- Definition

$$\langle p' | j_\mu^{\text{em}} | p \rangle = \bar{u}(p') \left[F_1(t)\gamma_\mu + i\frac{F_2(t)}{2m}\sigma_{\mu\nu}q^\nu \right] u(p) \,,$$



- Kinematics



FFs are real below t_0 and complex above it.

Dispersive parameterization of NFFs

- Dispersion relation

$$F(t) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\, F(t')}{t' - t - i\epsilon}\, dt'$$

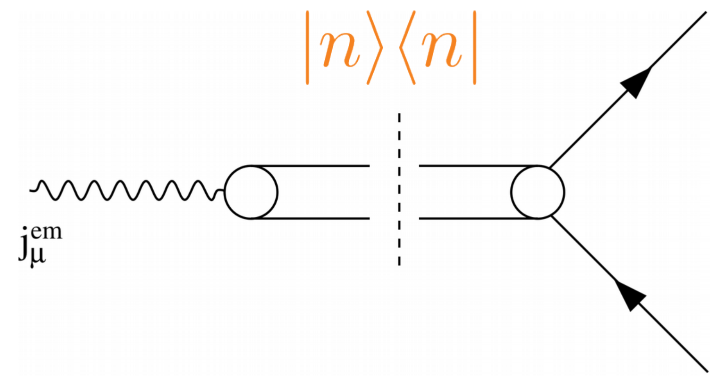
- Many advantages
  - ☞ Unitarity and Analyticity guaranteed,
  - ☞ Works in the whole t-region,
  - ☞ pQCD constraints on asymptotic behavior of NFFs can be added consistently,
  - ☞ Connects to data from various processes (e.g. πN scattering).

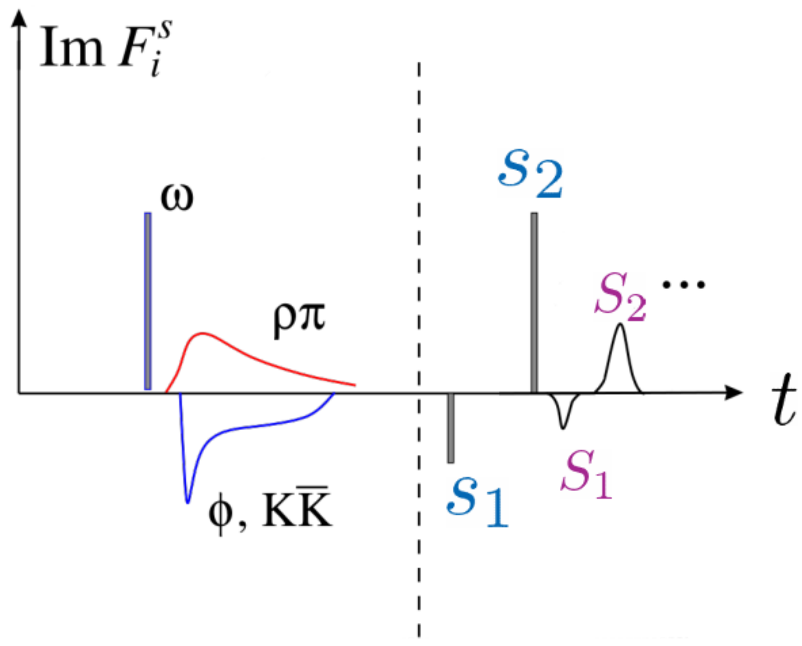
Spectral Function of DR NFFs

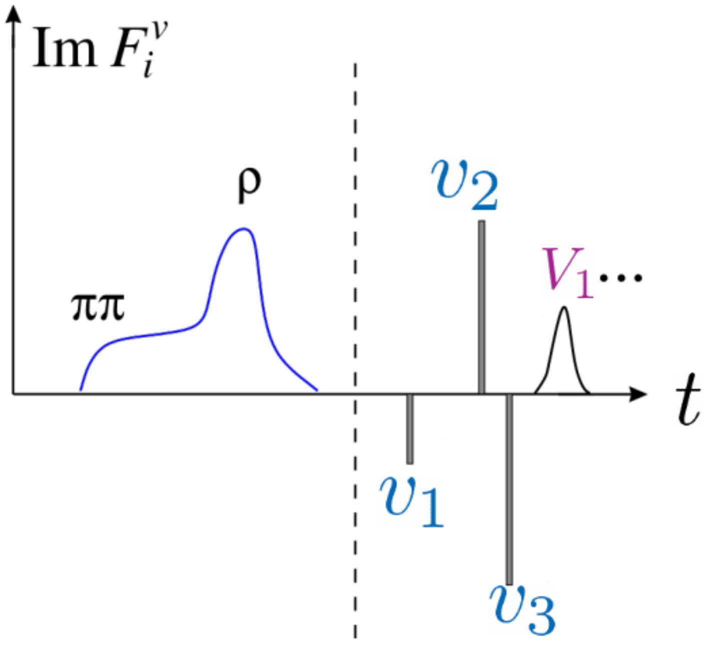
- Spectral Decomposition

$$\mathrm{Im}\langle N(p)\bar{N}(\bar{p})|j_\mu^{\mathrm{em}}|0\rangle$$

$$\sim \sum_n \langle N(p)\bar{N}(\bar{p})|n\rangle\langle n|j_\mu^{\mathrm{em}}|0\rangle$$







Theoretical constraints

- Normalizations--4

$$F_1^p(0) = 1\,,\, F_1^n(0) = 0\,,\; F_2^p(0) = \kappa_p\,,\, F_2^n(0) = \kappa_n\,.$$

- Neutron charge radius--1 A. A. Filin, *et al.* PhysRevLett124, 082501(2020)

$$\langle r_n^2 \rangle = -0.105^{+0.005}_{-0.006}\ \mathrm{fm}^2$$

- Superconvergence relations from pQCD--6

$$\int_{t_0}^{\infty} \mathrm{Im}\, F_i(t)\, t^n dt = 0\,, \quad i = 1, 2$$

with $n = 0$ for F_1, $n = 0, 1$ for F_2

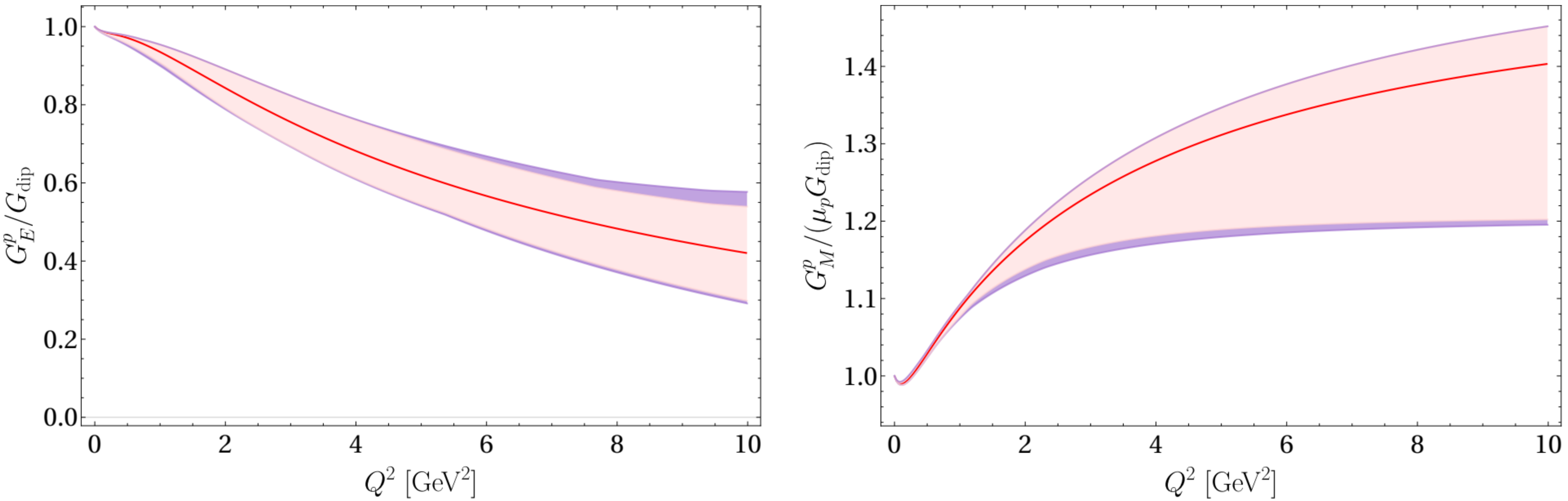
Data basis

Region	Observables	Source	$\lvert t\rvert$ GeV^2	number
Spacelike($t < 0$)	$d\sigma/d\Omega$	MAMI	0.00384–0.977	1422
		PRad	0.000215–0.058	71
	$\mu_p G_E^p/G_M^p$	JLab	1.18–8.49	16
	$\mu_n G_E^n/G_M^n$	world	1.58–3.41	4
	G_E^n	world	0.14–3.41	29
	G_M^n	world	0.071–10.0	49
Timelike($t > 0$)	$\lvert G_{\rm eff}^p\rvert$	world	3.52–20.25	153
	$\lvert G_{\rm eff}^n\rvert$	world	3.53–9.49	32
	$\lvert G_E^p/G_M^p\rvert$	BaBar	3.52–9.0	6
	$d\sigma/d\Omega$	BESIII	3.52–3.80	10

Number of data points: 1792

NFFs from best fits

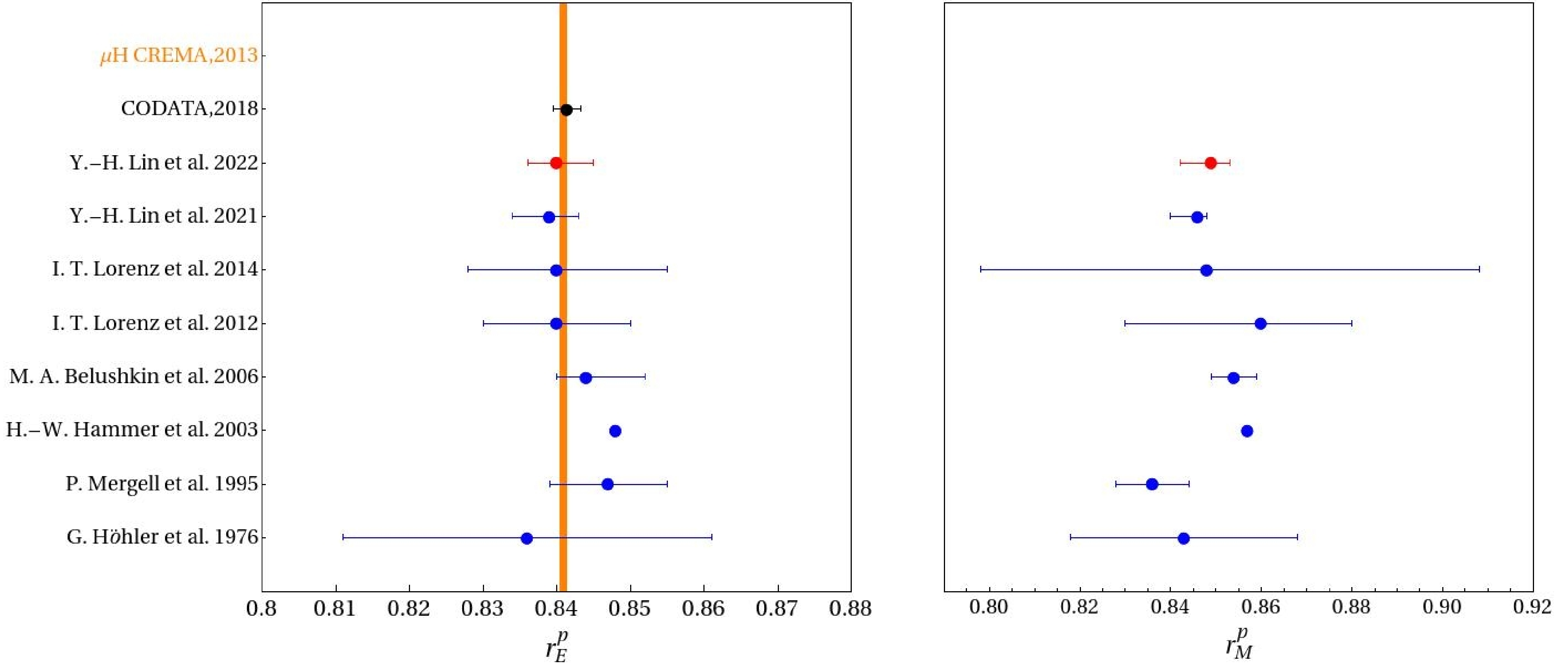
- Spectral ingredients in the best fit $\chi^2/\mathrm{dof} = 1.238$
  - ☞isoscalar $\omega, \phi, s_1, s_2, s_3, S_1, S_2, S_3 + K\bar{K} + \rho\pi$
  - ☞isovector $v_1, v_2, v_3, v_4, v_5, V_1, V_2, V_3 + \pi\pi$



Proton radius

- Our determination

$$r_E^p = 0.840^{+0.003+0.002}_{-0.002-0.002}\ \text{fm},\ r_M^p = 0.849^{+0.003+0.001}_{-0.003-0.004}\ \text{fm}$$

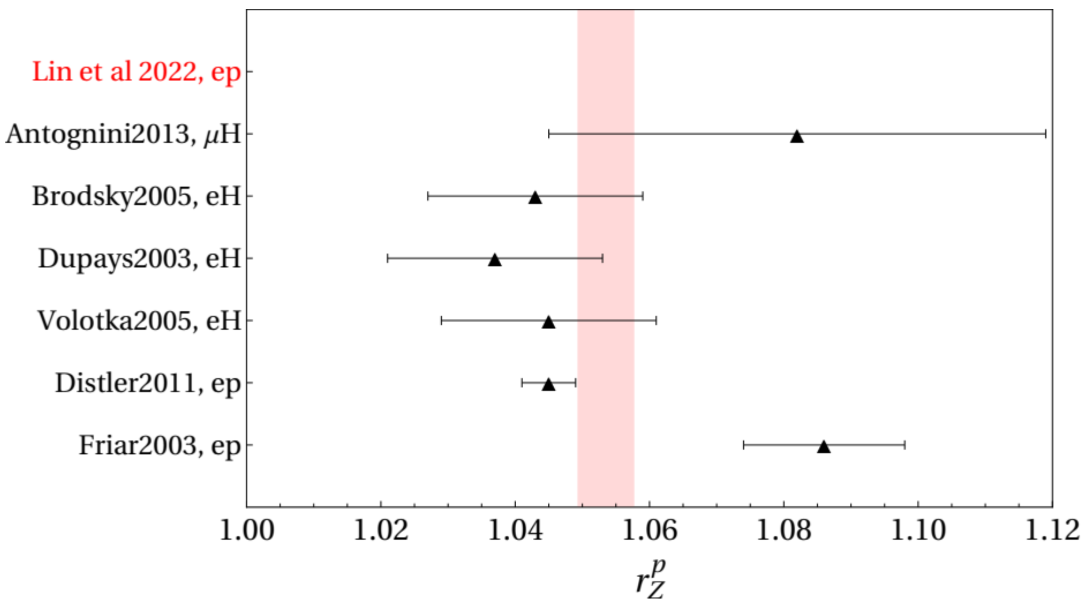


Zemach radius and third Zemach moment

- Zemach radius

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left(\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa_N} - 1\right)$$

$$r_z = 1.054^{+0.003+0.000}_{-0.002-0.001}\ \text{fm},$$



- third Zemach moment

$$\langle r^3\rangle_{(2)} = \frac{48}{\pi}\int_0^\infty \frac{dQ}{Q^4}\left(G_E(Q^2)^2 + \frac{Q^2}{3}\langle r^2\rangle - 1\right)$$

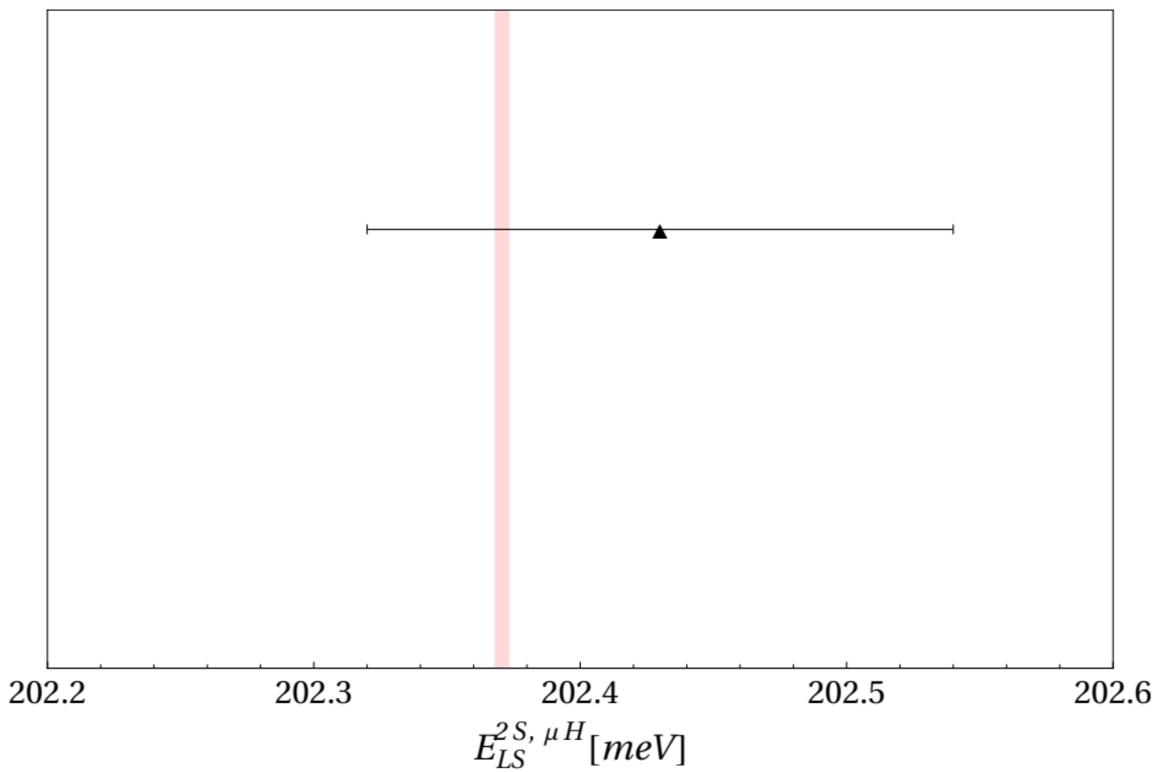
$$\langle r^3\rangle_{(2)} = 2.310^{+0.022+0.014}_{-0.018-0.015}\ \text{fm}^3.$$

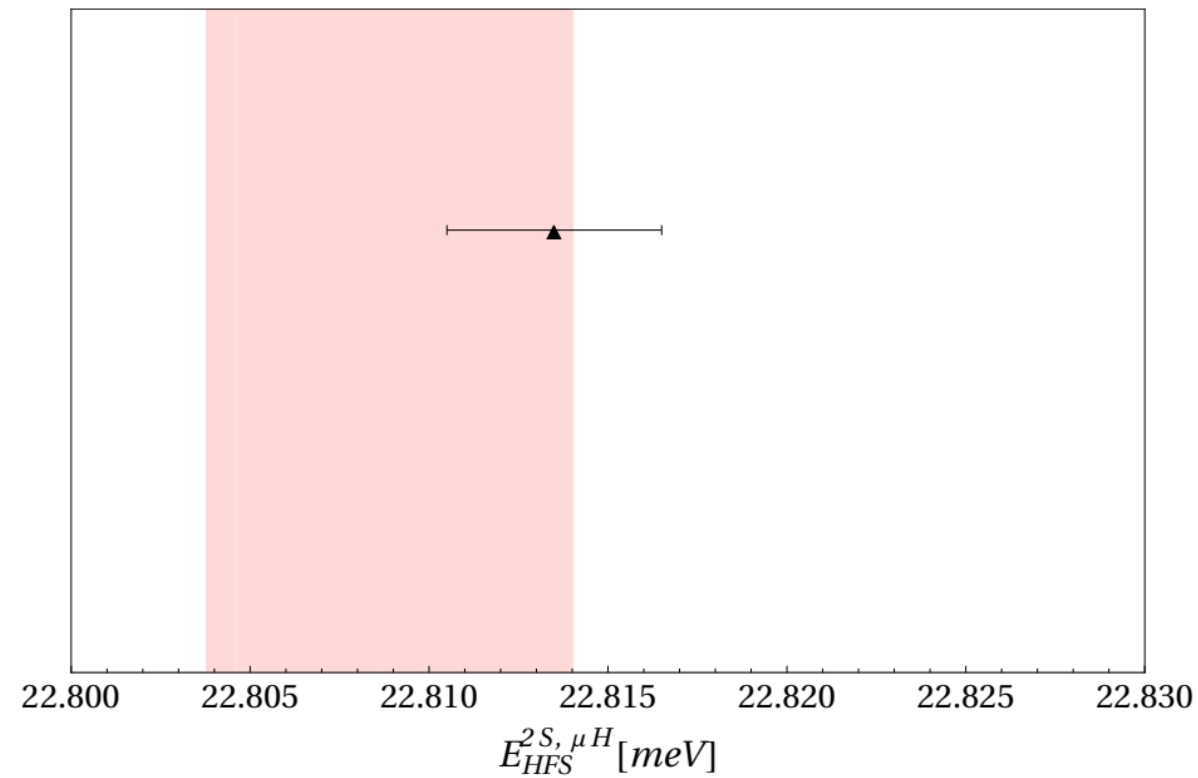
μH spectroscopy observables

$$\Delta E_{\text{LS}} = 206.0336(15) - 5.2275(10)\langle r^2\rangle_p + 0.0347\langle r^3\rangle_{(2)}$$

$$\Delta E_{\text{HFS}} = 22.9843(30) - 0.1621(10) r_z$$

A. Antognini, *et al.* AnnalsPhys, 331 127(2013)





Red bands taken from A. Antognini, *et al.* Science, 339 417(2013)

Recoil–finite-size correction to 1S-HFS

$$E^{\text{th}}_{\text{HFS}} = E_F + \Delta E_{\text{QED}} + \Delta E^{2\gamma}$$

$$E^{\text{th}}_{\text{HFS}}(\text{H}) = 1418840.082(9) + 1613.024(3)$$
$$+ E^{\text{H}}_F \Big(1.01558(13)\Delta^{\text{H}}_{\text{Z}} + 0.99807(13)\Delta^{\text{H}}_{\text{recoil}} + 1.00002\Delta^{\text{H}}_{\text{pol}}\Big) \quad [\text{kHz}]$$

$$E^{\text{th}}_{\text{HFS}}(\mu\text{H}) = 182.443 + 1.354(7)$$
$$+ E^{\mu\text{H}}_F \Big(1.01958(13)\Delta^{\mu\text{H}}_{\text{Z}} + 1.01656(4)\Delta^{\mu\text{H}}_{\text{recoil}} + 1.00402\Delta^{\mu\text{H}}_{\text{pol}}\Big) \quad [\text{meV}]$$

$$\Delta_Z = -2Z\alpha m_r r_{\text{Z}}$$

$$\Delta_{\text{recoil}} = \frac{Z\alpha}{\pi(1+\kappa)} \int_0^\infty \frac{\mathrm{d}Q}{Q} \left\{ \frac{G_M(Q^2)}{Q^2} \frac{8mM}{v_l + v} \left(2F_1(Q^2) + \frac{F_1(Q^2) + 3F_2(Q^2)}{(v_l+1)(v+1)} \right) - \frac{8m_r G_M(Q^2) G_E(Q^2)}{Q} - \frac{m F_2^2(Q^2)}{M} \frac{5+4v_l}{(1+v_l)^2} \right\}$$

$v = \sqrt{1+4M^2/Q^2} (v_l = \sqrt{1+4m^2/Q^2})$

Recoil–finite-size correction to 1S-HFS

$$\Delta_{\text{recoil}} = \frac{Z\alpha}{\pi(1+\kappa)} \int_0^\infty \frac{\mathrm{d}Q}{Q} \left\{ \frac{G_M(Q^2)}{Q^2} \frac{8mM}{v_l + v} \left(2F_1(Q^2) + \frac{F_1(Q^2) + 3F_2(Q^2)}{(v_l+1)(v+1)} \right) - \frac{8m_r G_M(Q^2) G_E(Q^2)}{Q} - \frac{m F_2^2(Q^2)}{M} \frac{5+4v_l}{(1+v_l)^2} \right\}$$

$$\Delta^{\mu\mathrm{H}}_{\text{recoil}} = (837.6^{+1.7+2.2}_{-1.0-0.1}) \times 10^{-6} = (837.6^{+2.8}_{-1.0}) \times 10^{-6} = (837.6^{+2.8}_{-1.0})\ \text{ppm}$$

$$\Delta^{\mathrm{H}}_{\text{recoil}} = (526.9^{+1.1+1.3}_{-0.3-0.2}) \times 10^{-8} = (526.9^{+1.7}_{-0.4}) \times 10^{-8}$$

Summary

- NFFs is extracted from the latest experimental data over the full range of momentum transfer by using dispersion theory for the first time.
  - ☞ Spacelike data 0.000215-0.977 GeV^2
  - ☞ Timelike data 3.52-20.25 GeV^2
- DR analysis on NFFs data provide robust and consistent proton radius over decades, agrees with the small one.
- The obtained NFFs lead to a significant reduction on the theoretical uncertainties of recoil–finite-size correction to HFS.

$$\Delta^{\mu\mathrm{H}}_{\mathrm{recoil}} = (837.6^{+2.8}_{-1.0})\ \mathrm{ppm}$$

$$\Delta^{\mathrm{H}}_{\mathrm{recoil}} = (526.9^{+1.7}_{-0.4}) \times 10^{-8}$$

Recent determinations