Theory of heavy muonic atoms and access to the nuclear properties

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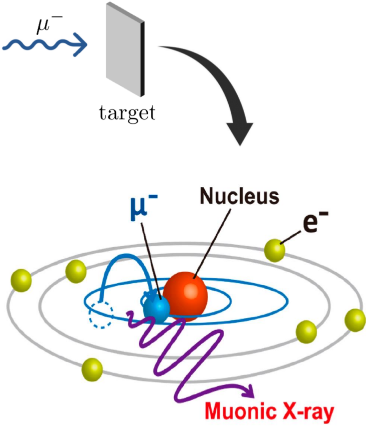
Max Planck Institute for Nuclear Physics (Heidelberg)

Muonic Atoms at PSI'2022
15 October, 2022

MAX-PLANCK-GESELLSCHAFT

Access to muonic atoms



http://www.mdpi.com/2412-382-X/1/1/11/htm

- capture and cascade: $10^{-12} - 10^{-9}$ s

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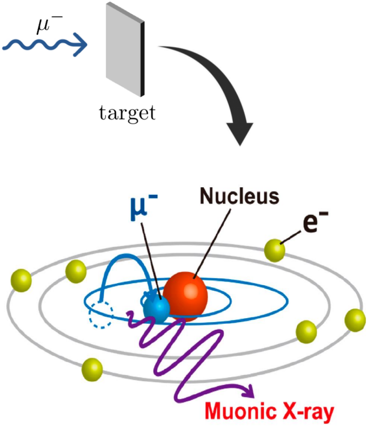

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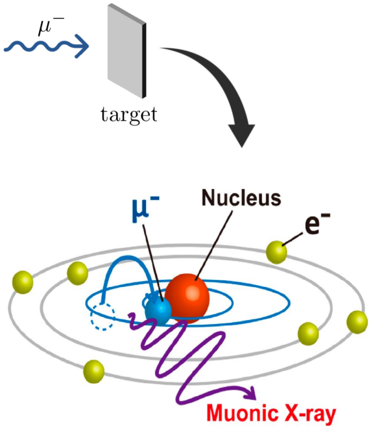

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$$\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$$

$$\mu^- + p \rightarrow n + \nu_\mu$$

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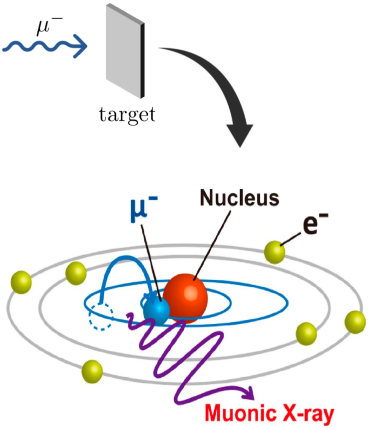

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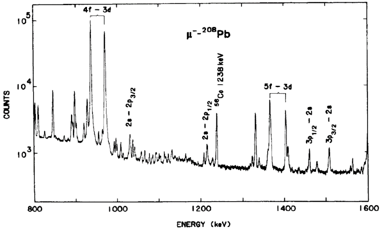

Motivation for muon physics: Theory

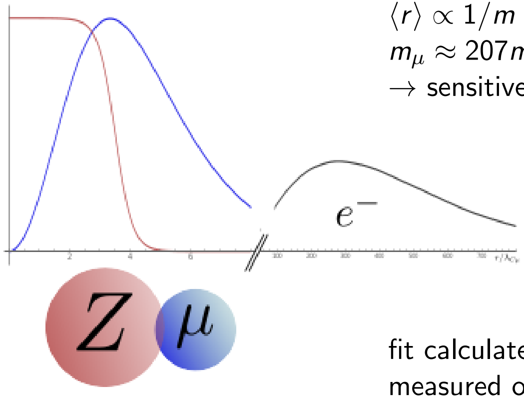

Fig: Niklas Michel

$\langle r \rangle \propto 1/m$
$m_\mu \approx 207 m_e$
$\rightarrow$ sensitive to nuclear structure

fit calculated spectra to measured ones
$\rightarrow$ determine nuclear parameters

Motivation for muon physics: A fine-structure anomaly

muonic ^{90}Zr, $^{112-124}$Sn, ^{208}Pb: very poor fit, $\chi^2/\mathrm{DF} = 187$

P. Bergem *et al.*, Phys. Rev. C **37** 2821 (1988)

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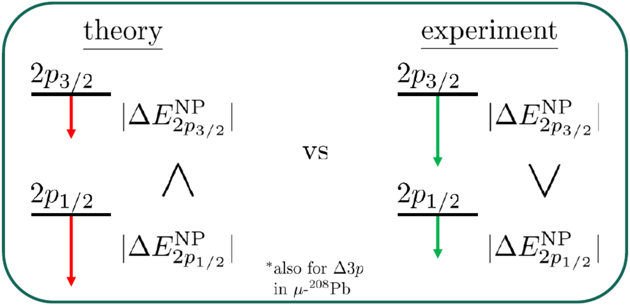

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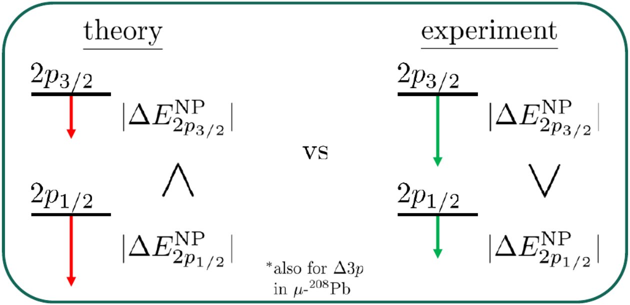

$2p_{1/2}$ is closer to a nucleus and should be affected more strongly

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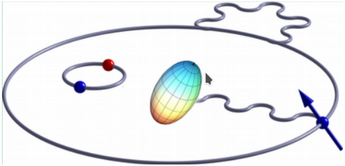
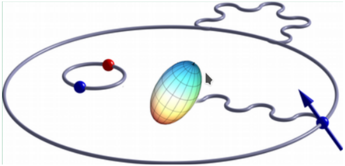
Outline





Basic and brief theory

- Muons are close to the nucleus, relativistic $\rightarrow$ Dirac equation

A. S. M. Patoary and NSO, EPJD **72**, 54 (2018)
N. Michel, NSO, and C. H. Keitel, PRA **96**, 032510 (2017)
N. Michel and NSO, PRA **99**, 042501 (2019)

Basic and brief theory

- Muons are close to the nucleus, relativistic → Dirac equation
- Extended nucleus: sphere,

$$V_{\mathrm{Sph}}(r) = \begin{cases} a + br^2; & r \leq R \\ -\frac{Z\alpha}{r}; & r \geq R \end{cases}$$

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- Extended nucleus: sphere, Fermi,

$$\rho^F_{a,c}(r_\mu) = \frac{N}{1 + e^{(r-c)/a}}$$

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Basic and brief theory

- Muons are close to the nucleus, relativistic $\rightarrow$ Dirac equation
- Extended nucleus: sphere, Fermi, deformed Fermi distribution

$$\rho_{a,c,\beta}(r_\mu,\vartheta_\mu) = \frac{N}{1 + e^{[r - c(1+\beta \mathrm{Y}_{20}(\vartheta_\mu))]/a}}$$

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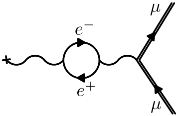

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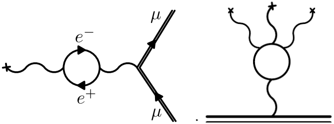

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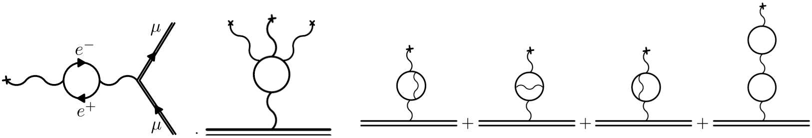

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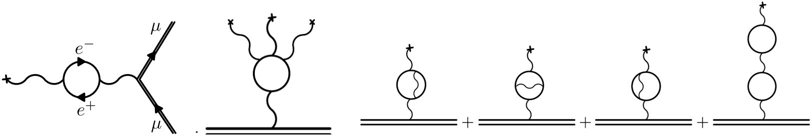

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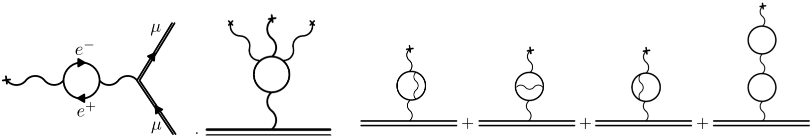

- Electron screening effect

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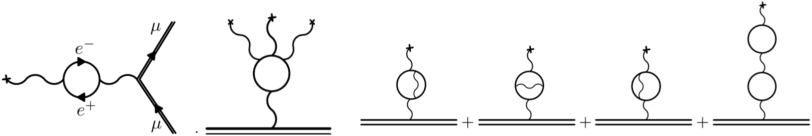

- Electron screening effect
- HFS: electric quadrupole and magnetic dipole

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Dynamic hyperfine structure in muonic atoms

Muonic FS ond/or HFS $\approx$ nuclear rotational states energies
$\rightarrow$ Muonic transitions can excite nucleus!

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Toy model: ^{185}Re with $I \in \{\frac{5}{2}, \frac{7}{2}, \frac{9}{2}\}$ + muonic $(2p_{1/2}, 2p_{3/2})$

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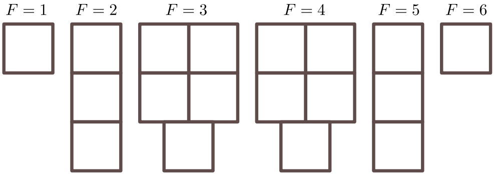
for given F, M_F: 6 unperturbed states: $|FM_F(n\kappa)(IK)\rangle$



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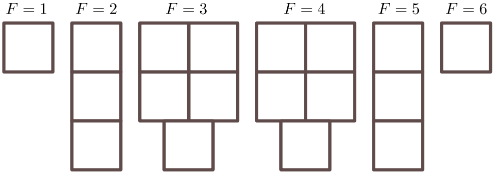

diagonalise quadrupole interaction in every F-block

Dynamic hyperfine structure in muonic atoms for Re

$$E_{5/2} = 0\,\text{keV}$$
$$E_{7/2} = 125\,\text{keV}$$
$$E_{9/2} = 284\,\text{keV}$$

$$E_{2p_{1/2}} = -4059\,\text{keV}$$
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$$\Delta E_{2p} = 149\,\text{keV}$$

Figure courtesy: N. Michel

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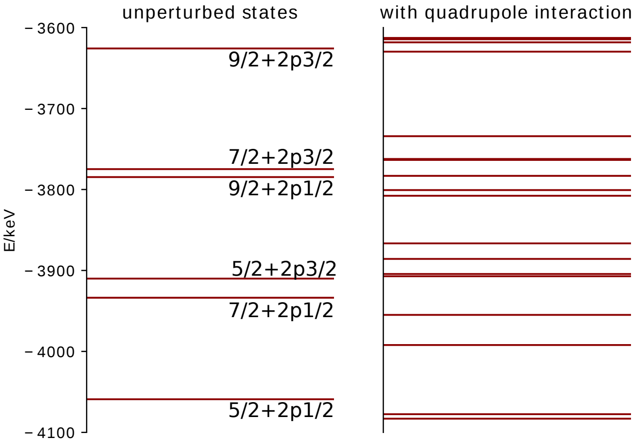

Figure courtesy: N. Michel

PHYSICAL REVIEW LETTERS **128,** 203001 (2022)

Evidence Against Nuclear Polarization as Source of Fine-Structure Anomalies in Muonic Atoms

Igor A. Valuev[1,*] Gianluca Colò[2,3] Xavier Roca-Maza[2,3] Christoph H. Keitel[1] and Natalia S. Oreshkina[1,†]

[1]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*

[2]*Dipartimento di Fisica, Università degli Studi di Milano, via Celoria 16, I-20133 Milano, Italy*

[3]*INFN, Sezione di Milano, via Celoria 16, I-20133 Milano, Italy*

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Self-energy correction to the energy levels of heavy muonic atoms

Natalia S. Oreshkina[1,*]

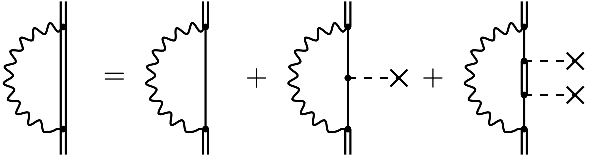
[1]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*

(Dated: June 14, 2022)

Self-energy correction

$$\langle a|\Sigma(E)|b\rangle = \frac{i}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \sum_n \frac{\langle an|I(\omega)|nb\rangle}{E-\omega-\varepsilon_n(1-i0)},$$

$$I(\omega, \mathbf{x}_1, \mathbf{x}_2) = \frac{(1-\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2)\exp\left(i\sqrt{\omega^2+i0}x_{12}\right)}{4\pi x_{12}}.$$



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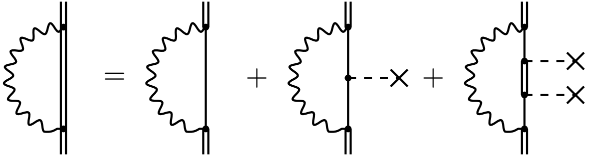
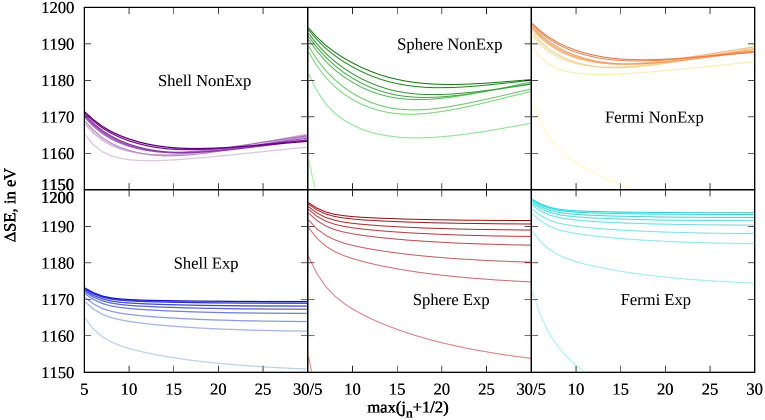


- Order-of-magnitude improvement for $1s_{1/2}$ state
- Rigorous for $2p_{1/2}$, $2p_{3/2}$
- Three nuclear models: shell, sphere, Fermi
- Two integration grids
- Contains infinite summation over intermediate κ

Models and grid compared: $\mu - {}^{90}_{40}\mathrm{Zr}$



ΔE_{SE} contribution to the $1s_{1/2}$ state of the muonic zirconium in units of eV as a function of maximal intermediate angular momentum j_n for different nuclear models and numerical grids. The colors of the lines on every panel change depending on the number of used DKB basis functions from light for $n_{\mathrm{DKB}} = 50$ to dark for $n_{\mathrm{DKB}} = 150$

SE: results

Ion	State	Final	Previous
$\mu - {}^{90}$Zr	$1s_{1/2}$	1191(4)	1218
	$2p_{1/2}$	6.99(5)	1
	$2p_{3/2}$	46.52(6)	41
	$\Delta 2p$	39.53(8)	40
$\mu - {}^{208}$Pb	$1s_{1/2}$	3225(15)	3373
			3270(160)*
	$2p_{1/2}$	453(5)	413
	$2p_{3/2}$	745(5)	707
	$\Delta 2p$	292(7)	294

The previous results are from [Haga *et al.*, PRC **75**, 044315 (2007)], * is from [Cheng *et al.*, PRA **17**, 489 (1978).]

Nuclear polarization effect

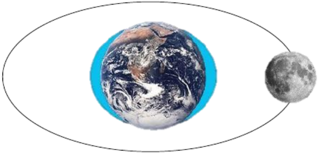


Image source: www.universetoday.com

$$V_{\mathrm{Coul}}(r) = -\frac{\alpha Z}{r}$$

$$V_{\mathrm{ext}}(r) = -\alpha \int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

$$V_{\mathrm{NP}}(r) = -\alpha \sum_{Z} \frac{1}{|\mathbf{r}-\mathbf{r}_{\mathbf{N_i}}|}$$

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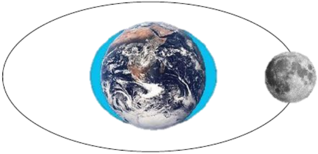

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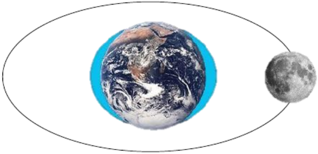
$$H = H_N + \boldsymbol{\alpha}\mathbf{p} + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

$$\Delta E_I = {\sum_N}' \frac{\langle I|\Delta V|N\rangle\langle N|\Delta V|I\rangle}{E_I - E_N}$$

Nuclear polarization effect



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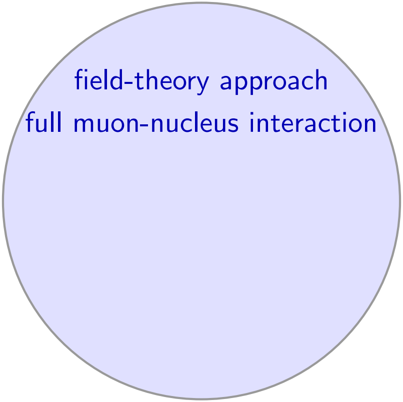
Only longitudinal (Coulomb) part

Also: transverse part, only via field-theory approach

Our goal



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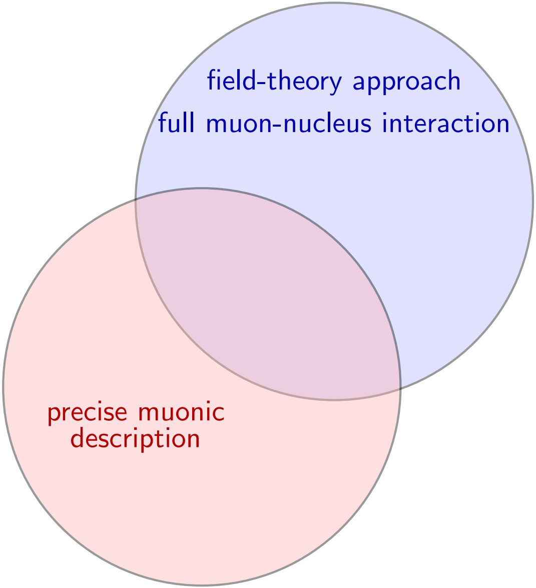
field-theory approach
full muon-nucleus interaction
precise muonic
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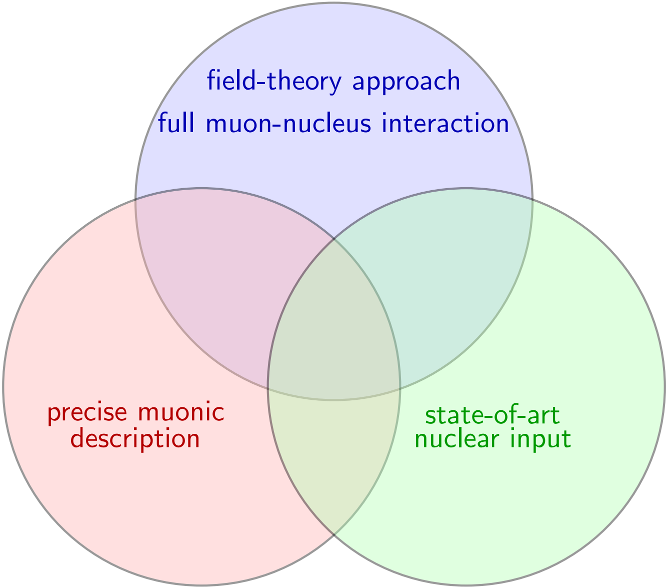
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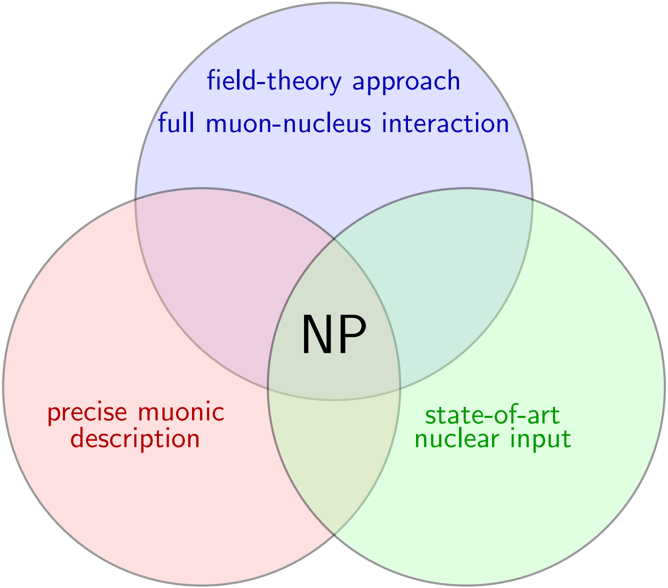
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state-of-art nuclear input

Transverse part of muon-nucleus interaction

$$H = H_N + \boldsymbol{\alpha}\mathbf{p} + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

$$\Downarrow$$

$$H = H_N + \boldsymbol{\alpha}(\mathbf{p} - e\mathbf{A}(\mathbf{r}, \mathbf{r}_{N_i})) + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

- Longitudinal (or Coulomb) interaction $V(\mathbf{r}, \mathbf{r}_{N_i})$ always $\Delta E^{\mathrm{NP}}_{2p_1/2}| > \Delta E^{\mathrm{NP}}_{2p_3/2}|$

Tanaka and Horikawa, Nucl. Phys. **A580**, 291 (1994)

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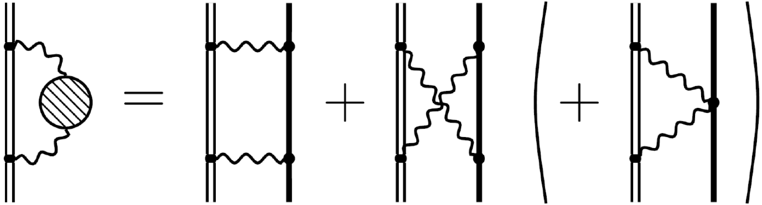
- However, the anomalies still persisted (for more than 40 years)

Tanaka and Horikawa, Nucl. Phys. **A580**, 291 (1994)

Total leading-order nuclear polarization



$$\Delta E^{\mathrm{L}}_{\mathrm{NP}} = -i(4\pi\alpha)^2 \sum_{i'I'} \iint \frac{d\boldsymbol{q}\, d\boldsymbol{q'}}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega,\boldsymbol{q}) D_{\zeta\nu}(\omega,\boldsymbol{q'}) \langle iI|j^{\mu}_{m}(-\boldsymbol{q}) J^{\xi}_{N}(\boldsymbol{q})|i'I'\rangle \; \langle i'I'|J^{\zeta}_{N}(-\boldsymbol{q'}) j^{\nu}_{m}(\boldsymbol{q'})|iI\rangle}{(\omega+\omega_m - iE_{i'}\epsilon)(\omega-\omega_N+i\epsilon)},$$

$$\Delta E^{\mathrm{X}}_{\mathrm{NP}} = +i(4\pi\alpha)^2 \sum_{i'I'} \iint \frac{d\boldsymbol{q}\, d\boldsymbol{q'}}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega,\boldsymbol{q}) D_{\zeta\nu}(\omega,\boldsymbol{q'}) \langle iI'|j^{\mu}_{m}(-\boldsymbol{q}) J^{\xi}_{N}(\boldsymbol{q})|i'I\rangle \; \langle i'I|J^{\zeta}_{N}(-\boldsymbol{q'}) j^{\nu}_{m}(\boldsymbol{q'})|iI'\rangle}{(\omega+\omega_m - iE_{i'}\epsilon)(\omega+\omega_N-i\epsilon)},$$

$$\Delta E^{\mathrm{SG}}_{\mathrm{NP}} = -i(4\pi\alpha)^2 \sum_{i'} \iint \frac{d\boldsymbol{q}\, d\boldsymbol{q'}}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega,\boldsymbol{q}) \delta^{\xi\zeta} D_{\zeta\nu}(\omega,\boldsymbol{q'}) \langle i|j^{\mu}_{m}(-\boldsymbol{q})|i'\rangle \; \langle i'|j^{\nu}_{m}(\boldsymbol{q'})|i\rangle}{(\omega+\omega_m - iE_{i'}\epsilon)} \frac{\langle I|\rho_N(\boldsymbol{q}-\boldsymbol{q'})|I\rangle}{m_p}.$$

summations over entite muonic (i') and nuclear (I') spectra

Muonic spectrum

Dirac equation:

$$[\boldsymbol{\alpha}\mathbf{p} + \beta m_\mu + V_0(\mathbf{r})]\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

V_0 from Fermi nuclear charge distribution

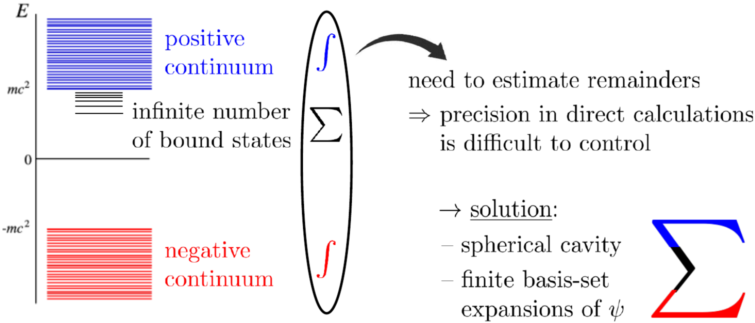


Fig: Igor Valuev

Nuclear spectrum

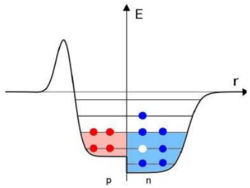
effective nuclear force
$\rightarrow$ Skyrme interaction

⬇

Hartree-Fock calculations

⬇

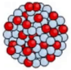
single-nucleon states
(occupied and unoccupied)



basis of particle-hole configurations

Image source: arXiv:1410.7167

nuclear excited states $|\nu\rangle$
$\rightarrow$ random phase approximation



GIF source: gfycat.com

⬇

matrix elements
$\langle \nu | \hat{J}^{\xi}_{N}(\mathbf{r}) | \mathcal{O} \rangle$

Fig: Igor Valuev

Skyrme-type nuclear interaction

Tony Skyrme in 1946

Fig:

https://en.wikipedia.org/wiki/Tony_Skyrme

$$
\begin{aligned}
V(\mathbf{r}_1,\mathbf{r}_2) = & \; t_0(1+\chi_0 P_\sigma)\delta(\mathbf{r}) \\
& + \frac{1}{2} t_1 (1+\chi_1 P_\sigma)[\mathbf{P}^{\dagger 2}\delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{P}^2] \\
& + t_2(1+\chi_2 P_\sigma)\mathbf{P}^\dagger \cdot \delta(\mathbf{r})\mathbf{P} \\
& + \frac{1}{6} t_3 (1+\chi_3 P_\sigma)\rho^\lambda(\mathbf{R})\delta(\mathbf{r}) \\
& + iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[\mathbf{P}^\dagger \times \delta(\mathbf{r})\mathbf{P}]
\end{aligned}
$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \mathbf{R} = \frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$$

$$\mathbf{P} = \frac{1}{2i}(\nabla_1 - \nabla_2), P_\sigma = \frac{1}{2}(1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$

10 parameters

Nuclear wave functions dependence

Calculations details

- complete muonic Dirac spectrum

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- 9 different parametrizations of the Skyrme interaction
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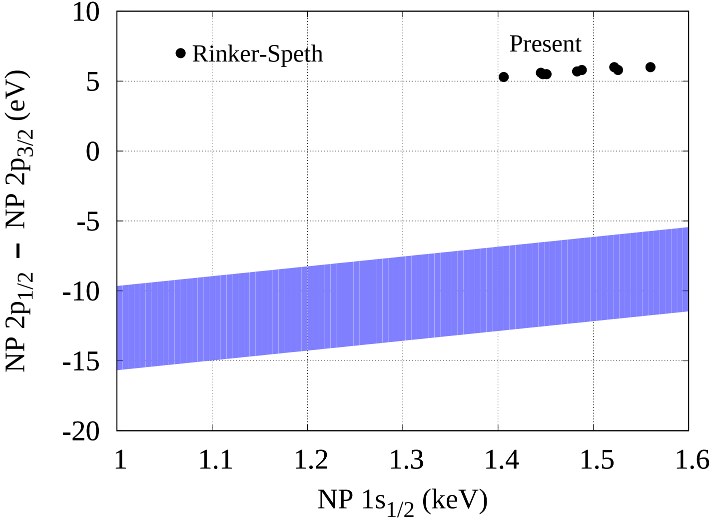
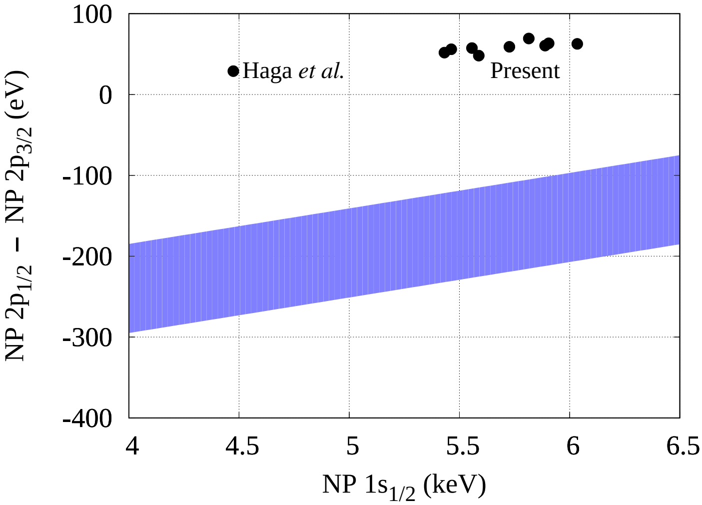
- RMS value changes the NP predictions
- Comparison between theory and free-parameter fit of the experimental data

Nuclear polarization correction ^{90}Zr



around 15 eV (5 σ) gap remains practically constant

Nuclear polarization correction ^{208}Pb



around 150 eV, or 4 σ standard deviations gap

RMS values: why should we care?

Image: Homer Simpson

- Muonic spectroscopy provides RMS for majority of stable nuclei
- High importance for QED, PNC, VFC, ...
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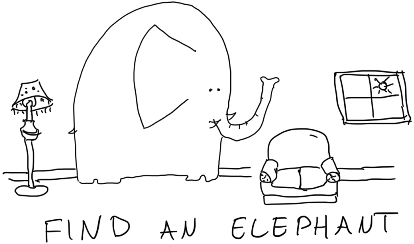
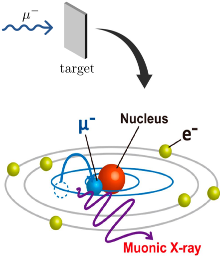
However!

- Fine-structure anomaly
- Poor fit $\chi^2/\mathrm{DF} = 187$
- Estimation for theory
- How much can we trust it?



Summary and Outlook

- Muonic atom: QED for "heavy electron"
- Probe of nuclear parameters and new approaches



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I. A. Valuev, G. Colò, X. Roca-Maza, C. H. Keitel, and N. S. Oreshkina, Phys. Rev. Lett. **128**, 203001 (2022)
A. Antognini *et al.*, Phys. Rev. C **101**, 054313 (2020)
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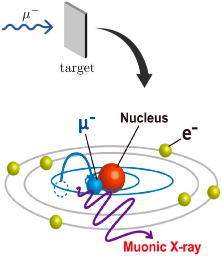

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Summary and Outlook

- Muonic atom: QED for "heavy electron"
- Probe of nuclear parameters and new approaches
- New rigorous values for SE and NP
- State-of-the-art theory predictions
- No resolution of a fine-structure puzzle
- Re-evaluation of RMS values could be addressed

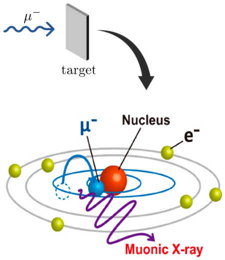

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Image: Science Cartoons by Tom Gauld

Thank you for your attention