Electroweak nuclear radii constrain the isospin breaking correction to V_{ud}

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We lay out a novel formalism to connect the isospin-symmetry breaking correction to the rates of superallowed nuclear beta decays, δ_{C}, to the isospin-breaking sensitive combinations of electroweak nuclear radii that can be accessed experimentally. We individuate transitions in the superallowed decay chart where a measurement of the neutron skin of a stable daughter even at a moderate precision could already help discriminating between models used to compute δ_{C}. We review the existing experimental situation and make connection to the existing and future experimental programs.

Introduction – Superallowed beta decays of 0^+ nuclei provide currently the best measurement of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ud}, which permits high-precision tests of the Standard Model (SM) prediction through the first-row unitarity constraint $|V_{ud}|^2+|V_{us}|^2+|V_{ud}|^2=1$. Recent improvements in the single-nucleon radiative correction theory [1–6] unveil an apparent violation of the unitarity relation [7] and motivate the renewed interest within the theory and experimental communities. The current most precise extraction of $|V_{ud}|$ is obtained from a global analysis of superallowed nuclear decays following the master formula [8]

$$|V_{ud}|^2\,\mathcal{F}t\,(1+\Delta_R^V)=2984.43\,\mathrm{s}\,. \tag{1}$$

Among the ingredients in the above formula, the r.h.s. combines the very precisely known physical constants with uncertainties far beyond the precision goal relevant to the analysis of beta decays. The l.h.s., along with $|V_{ud}|^2$ and the already mentioned single-nucleon radiative correction Δ_R^V, contains the universal, decay-independent $\mathcal{F}t$-value. The latter is defined by absorbing the experimental, process-specific measurements summarized as ft, with decay-specific nuclear corrections [8],

$$\mathcal{F}t=ft(1+\delta_R')(1+\delta_{\mathrm{NS}})(1-\delta_{\mathrm{C}}). \tag{2}$$

The QED correction δ_R' describes soft-photon effects beyond Coulomb distortion, that bear dependence on bulk nuclear properties. The nucleon structure correction δ_{NS} encodes the nuclear dependence of the γW-box that has to be accounted for upon extracting the single-nucleon radiative correction Δ_R^V in Eq. (1). This separation has recently been addressed in Refs. [2, 9].

The isospin-symmetry breaking (ISB) correction δ_{C} modifies the squared Fermi matrix element M_F from its isospin-limit value M_F^0 as $|M_F|^2=|M_F^0|^2(1-\delta_{\mathrm{C}})$ even in the absence of radiative corrections. It arises from the isospin mixing of the nuclear states arising predominantly from Coulomb repulsion between the protons in the nucleus. Across the nuclear decays relevant for the high-precision extraction of V_{ud}, δ_{C} ranges from $\sim 0.1\%$ for the ^{10}C decay to $\sim 1.5\%$ for the ^{74}Rb decay. Thus, it plays a central role in aligning the experimental, nucleus-dependent ft-values to a nucleus-independent constant $\mathcal{F}t$-value, as required by conservation of vector current (CVC) [8].

At present, this correction is obtained solely from nuclear model calculations; the nuclear shell model calculations with the Woods-Saxon potential [8, 10–13] result in an impressive alignment of the $\mathcal{F}t$-values. However, concerns about possible theory inconsistencies of these calculations [14–16], and significant model dependence [17–23] persist. Modern ab-initio calculations that could help reducing the model dependence are still in the preliminary stage [24, 25]. With this ongoing discussion on δ_{C} in the nuclear theory community, no direct experimental constraints on the ISB correction exist to our knowledge.

In this Letter we devise a new formalism which connects δ_{C} to a set of experimentally accessible quantities that are sensitive to the same ISB nuclear matrix elements. These observables encompass recoil effects in the superallowed decay process, nuclear charge radii across the isotriplet, and the neutron skin of the stable daughter nucleus. The relevant combinations are constructed such that the non-ISB contributions cancel out, and a clean probe of the isospin mixing effects is obtained.

Basic notation – We adopt the "nuclear physics" convention for the isospin projection, $(T_z)_p=-1/2$. We consider β^+ transitions $i\to f$ accross the isotriplet with $T_{z,i}=0$ and $T_{z,f}=+1$ (which we will explain later). The Fermi matrix element is defined as $M_F=\langle f|\hat{\tau}_+|i\rangle$, with $\hat{\tau}_+$ the isospin-raising operator, and the states $|i\rangle$, $|f\rangle$ normalized to 1.

The nuclear states are eigenstates of the full Hamiltonian H which we split as $H=H_0+V$, with H_0 the part that conserves isospin and V the ISB perturbation term. We label the eigenstates of H_0 as $|a;T,T_z\rangle$ where a denotes all quantum numbers unrelated to isospin (we use $a=g$ for the ground state isotriplet that undergoes superallowed beta decay). The corresponding energy eigenvalues are labelled as $E_{a,T}$, which may depend on a and T but not T_z. In the absence of V, the bare Fermi matrix element reads $M_F^0 = \langle g;1,T_{z,f}|\hat{\tau}_+|g;1,T_{z,i}\rangle = \sqrt{2}$.

A key ingredient in our analysis is the isovector monopole operator,

$$\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{\hat{T}}(i) \tag{3}$$

where $\vec{\hat{T}}(i)$ is the isospin operator of the nucleon i, and $\vec{r}_i$ its position. The irreducible tensors of rank 1 in the isospin space with its components are: $M_0^{(1)} = M_z^{(1)}$, $M_{\pm1}^{(1)} = \mp(M_x^{(1)} \pm iM_y^{(1)})/\sqrt{2}$.

Key experimental observables – The charged weak form factors in superallowed decays of spinless nuclei are:

$$\langle f(p_f)|J_W^{\lambda\dagger}(0)|i(p_i)\rangle = f_+(t)(p_i+p_f)^\lambda + f_-(t)(p_i-p_f)^\lambda, \tag{4}$$

where $J_W^{\lambda\dagger}(x) = \bar{d}(x)\gamma^\lambda(1-\gamma_5)u(x)$ is the charged weak current, and $t=(p_i-p_f)^2$. The contribution of $f_-(t)$ to the differential decay rate is suppressed simultaneously by kinematics and by ISB, so we can only probe $f_+(t)$. In the Breit frame ($p_i^0 = p_f^0$), $f_+(0) = M_F$ and we define $f_+(t) = M_F\bar{f}_+(t)$ with $\bar{f}_+(0)=1$. For small t we have,

$$\bar{f}_+(t) = 1 - \frac{t}{6}\frac{\langle f|M_{+1}^{(1)}|i\rangle}{\sqrt{2}M_F} + \mathcal{O}(t^2), \tag{5}$$

and one may safely set $M_F \to \sqrt{2}$ above, given our precision goal. The recoil of the daughter nucleus is a measurable quantity which allows us to determine the nuclear matrix element $\langle f|M_{+1}^{(1)}|i\rangle$. Notice that recoil effects in beta decays have been measured before to extract the $\beta-\nu$ correlation, e.g. at TRIUMF and CERN [26–28].

Further, we define the root mean square (RMS) radii of the proton and neutron distribution in a nucleus ϕ (with the proton number Z_ϕ and the neutron number N_ϕ) as

$$R_{p/n,\phi} = \sqrt{\frac{1}{X}\langle\phi|\sum_{i=1}^{A} r_i^2\left(\frac{1}{2}\mp\hat{T}_z(i)\right)|\phi\rangle}, \tag{6}$$

with $-$ for the proton and $+$ for the neutron and $X = Z_\phi$ or N_ϕ, respectively. These radii naturally connect to the z-component of the isovector monopole operator,

$$\langle\phi|M_0^{(1)}|\phi\rangle = \frac{N_\phi}{2}R_{n,\phi}^2 - \frac{Z_\phi}{2}R_{p,\phi}^2. \tag{7}$$

In absence of ISB, the Wigner-Eckart theorem requires the equality $\langle g;1,1|M_{+1}^{(1)}|g;1,0\rangle = -\langle g;1,1|M_0^{(1)}|g;1,1\rangle$. Hence, the following combined experimental observable

$$\Delta M_A^{(1)} \equiv \langle f|M_{+1}^{(1)}|i\rangle + \langle f|M_0^{(1)}|f\rangle \tag{8}$$

offers a very clean probe of ISB effect. Furthermore, we define another experimentally accessible quantity,

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1R_{p,1}^2 + Z_{-1}R_{p,-1}^2\right) - Z_0R_{p,0}^2 \tag{9}$$

which combines the R_p across the isotripet ($-1,0,1$ denote T_z of the nucleus). Again, $\Delta M_B^{(1)}$ vanishes in the isospin limit, providing another clean probe of isospin mixing effects. $\Delta M_{A,B}^{(1)}$ are the two key experimental observables that we focus on in this Letter.

While the RMS radii $R_{p,n}$ are generally not observable, they are directly related to nuclear charge and weak radii $R_{\mathrm{Ch},\phi}$, $R_{\mathrm{Wk},\phi}$. The former are measurable for both stable and unstable nuclear isotopes, mainly from the atomic spectroscopy [29]. The nuclear RMS charge radii are largely given by R_p, as the corrections due to the charge radii of the proton and the neutron can easily be included. New results for charge radii of unstable isotopes are anticipated, e.g., from the BECOLA facility at FRIB [30].

Nuclear weak radii are accessible with parity-violating electron scattering (PVES) on nuclear targets. The object of interest is the neutron skin $R_n - R_p \propto R_{\mathrm{Wk}} - R_{\mathrm{Ch}}$ which is the subject of a vibrant experimental program at electron scattering facilities [31–35] with the scope of obtaining insights into the properties of the neutron-rich matter with relevance for astrophysics [36]. Since fixed-target PVES is only viable with a stable target nucleus, we concentrate on (observationally) stable superallowed daughter nuclei, most of which are $T_{z,f} = +1$ members of the isotriplet, which motivates the definition of Eq. (8). In addition, RMS charge radii of stable nuclei are known to $0.1-0.01\%$ precision [29], which opens the possibility to extract the respective weak RMS radii with a subpercent precision [37].

The difference in the proton and neutron distributions within a nucleus can generically come from two sources: the neutron excess and ISB effects. In asymmetric nuclei with $N > Z$ the skin is mainly generated by the symmetry energy [38], although even there the ISB effects may be non-negligible [39]. For symmetric nuclei with $N \approx Z$, such as those participating in the superallowed decays, the ISB effect is promoted to the main contributor to the skin. To our knowledge, this is the first time ISB is considered as the main point of interest in the context of a neutron skin measurement.

The connection between $\boldsymbol{\Delta M_{A,B}^{(1)}}$ and $\boldsymbol{\delta_{\mathrm{C}}}$ – To investigate the underlying physics of $\Delta M_{A,B}^{(1)}$, we resort to the perturbation theory formalism outlined in Refs.[14, 15] The only assumption is that the ISB operator V transforms as an isovector ($T=1$, $T_z=0$) [40]. Inserting the full set of intermediate (isospin-symmetric!)

nuclear states, we obtain,

$$\Delta M_A^{(1)} = -\frac{1}{3}\sum_a \frac{\langle a;0||M^{(1)}||g;1\rangle^*\langle a;0||V||g;1\rangle}{E_{a,0}-E_{g,1}} -\frac{1}{2}\sum_{a\neq g} \frac{\langle a;1||M^{(1)}||g;1\rangle^*\langle a;1||V||g;1\rangle}{E_{a,1}-E_{g,1}} -\frac{1}{6}\sum_a \frac{\langle a;2||M^{(1)}||g;1\rangle^*\langle a;2||V||g;1\rangle}{E_{a,2}-E_{g,1}} -\sum_a \frac{\langle a;2||V||g;1\rangle^*\langle a;2||M^{(1)}||g;1\rangle}{E_{a,2}-E_{g,1}} + \mathcal{O}(V^2) \quad (10)$$

and

$$\Delta M_B^{(1)} = \mathfrak{Re}\left\{-\frac{2}{3}\sum_a \frac{\langle a;0||M^{(1)}||g;1\rangle^*\langle a;0||V||g;1\rangle}{E_{a,0}-E_{g,1}} +\sum_{a\neq g} \frac{\langle a;1||M^{(1)}||g;1\rangle^*\langle a;1||V||g;1\rangle}{E_{a,1}-E_{g,1}} -\frac{1}{3}\sum_a \frac{\langle a;2||M^{(1)}||g;1\rangle^*\langle a;2||V||g;1\rangle}{E_{a,2}-E_{g,1}}\right\} + \mathcal{O}(V^2) \quad (11)$$

where the reduced matrix elements are defined via the Wigner-Eckart theorem:

$$\begin{aligned}\langle a;T',T'_z|M^{(1)}_{T''_z}|g;1,T_z\rangle &= C^{11;T'T'_z}_{1T_z;1T''_z}\langle a;T'||M^{(1)}||g;1\rangle \\ \langle a;T',T'_z|V|g;1,T_z\rangle &= C^{11;T'T'_z}_{1T_z;10}\langle a;T'||V||g;1\rangle, \quad (12)\end{aligned}$$

with Cs the Clebsch-Gordan coefficients. Note that our definition of $\Delta M_B^{(1)}$ ensures that the isoscalar operator $\sum_i r_i^2$ in Eq.(6) does not enter the matrix elements at $\mathcal{O}(V)$. Meanwhile, the ISB correction δ_{C} starts at $\mathcal{O}(V^2)$ in accord with the (generalized) Behrends-Sirlin-Ademollo-Gatto theorem [41, 42], and reads

$$\delta_{\mathrm{C}} = \frac{1}{3}\sum_a \frac{|\langle a;0||V||g;1\rangle|^2}{(E_{a,0}-E_{g,1})^2} + \frac{1}{2}\sum_{a\neq g} \frac{|\langle a;1||V||g;1\rangle|^2}{(E_{a,1}-E_{g,1})^2} -\frac{5}{6}\sum_a \frac{|\langle a;2||V||g;1\rangle|^2}{(E_{a,2}-E_{g,1})^2} + \mathcal{O}(V^3). \quad (13)$$

Further insight can be obtained with a more detailed information on V. It is well known that the dominant source of the isospin mixing in the nuclear states is played by Coulomb repulsion between protons [43, 44], with its prevailing part coming from a one-body potential where each proton is subject to a mean field. Furthermore, we take the potential of a uniformly charged sphere of radius R_C, inside which the whole nucleus resides [22]:

$$V_C \approx -\frac{Ze^2}{4\pi R_C^3}\sum_{i=1}^{A}\left(\frac{1}{2}r_i^2 - \frac{3}{2}R_C^2\right)\left(\frac{1}{2} - \hat{T}_z(i)\right). \quad (14)$$

While there is an ambiguity that Z is different across the isotriplet, it is safe to take $Z \approx A/2$, since $|T_z| \ll Z$. Only the isovector component breaks isospin symmetry; taking furthermore into account the fact that the T_z is always a good quantum number as it counts the neutrons and protons in the nucleus, we connect the ISB Coulomb potential with the isovector monopole operator,

$$V_C^{(1)} = (Ze^2/8\pi R_C^3)M_0^{(1)}, \quad (15)$$

and in what follows we will take $V = V_C^{(1)}$. Consequently, we can rewrite Eqs.(10), (11) as:

$$\begin{aligned}\Delta M_A^{(1)} &= \frac{1}{3}\Gamma_0 + \frac{1}{2}\Gamma_1 + \frac{7}{6}\Gamma_2 + \mathcal{O}(V^2) \\ \Delta M_B^{(1)} &= \frac{2}{3}\Gamma_0 - \Gamma_1 + \frac{1}{3}\Gamma_2 + \mathcal{O}(V^2), \;, \quad (16)\end{aligned}$$

where

$$\Gamma_T \equiv -\frac{8\pi R_C^3}{Ze^2}\sum_a \frac{|\langle a;T||V_C^{(1)}||g;1\rangle|^2}{E_{a,T}-E_{g,1}}\;, \quad (17)$$

with $a \neq g$ for $T = 1$. This should be compared to the expression for δ_{C} in Eq.(13) (with $V \to V_C^{(1)}$). We observe that $\Delta M_{A,B}^{(1)}$ and δ_{C} share the same set of reduced matrix elements in the $T = 0, 1, 2$ channels, imposing a strong experimental constraint on δ_{C}. This is one of the central results of this work.

The fact that these quantities essentially probe the same underlying physics means that any nuclear theory approach capable to compute δ_{C} can also be used to compute $\Delta M_{A,B}^{(1)}$, and thus compared to the experiment.

Isovector monopole dominance – An even more straightforward relation between $\Delta M_{A,B}^{(1)}$ and δ_{C} can be established by invoking the concept of isovector monopole dominance [22, 45], which states that the sum over reduced matrix elements of the isovector monopole operator is largely saturated by the contribution from the giant isovector monopole states (IVMS) which we denote as $|M;T,T_z\rangle$, with energies $E_{M,T}$. Furthermore, it is argued that the difference between the reduced matrix elements at different isospin channels of $|M;T\rangle$ are of the order $(N-Z)/A \ll 1$. Hence, in this approximation scheme all matrix elements are equal, $\langle M;T||V_C^{(1)}||g;1\rangle \equiv u$ for $T = 0, 1, 2$. From Eq.(13) it appears that for δ_{C} to be non zero, a splitting between the IVMS energies in different isospin channels $E_{M,0}$, $E_{M,1}$, $E_{M,2}$ must be introduced. This splitting comes about from the symmetry potential with the result from Ref. [22],

$$E_{M,T} - E_{g,1} = \xi\omega[1 + (T^2 + T - 4)\kappa/2], \;\; T = 0, 1, 2 \quad (18)$$

with $\kappa \equiv 2V_1/(\xi\omega A)$, V_1 the strength of the symmetry potential, ω the harmonic oscillator frequency, and ξ a model parameter describing the IVMS strength. With these ingredients we obtain:

$$\delta_{\mathrm{C}} \approx \frac{\kappa(4 - 13\kappa + 12\kappa^2 - \kappa^3)}{(1-2\kappa)^2(1-\kappa^2)^2}\frac{u^2}{\xi^2\omega^2}, \quad (19)$$

Transitions	δ_{C} (%)					$\Delta M_A^{(1)}$ (fm^2)					$\frac{\Delta M_A^{(1)}}{AR^2/4}$ (%)				
	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro
^{26m}Al $\to ^{26}$Mg	0.310	0.329	0.30	0.139	0.08	-2.2	-2.3	-2.1	-1.0	-0.6	3.2	3.3	3.0	1.4	0.8
^{34}Cl $\to ^{34}$S	0.613	0.75	0.57	0.234	0.13	-5.0	-6.1	-4.6	-1.9	-1.0	4.6	5.6	4.3	1.8	1.0
^{38m}K $\to ^{38}$Ar	0.628	1.7	0.59	0.278	0.15	-5.4	-14.6	-5.1	-2.4	-1.3	4.2	11.2	3.9	1.8	1.0
^{42}Sc $\to ^{42}$Ca	0.690	0.77	0.42	0.333	0.18	-6.2	-6.9	-3.8	-3.0	-1.6	4.0	4.5	2.5	2.0	1.1
^{46}V $\to ^{46}$Ti	0.620	0.563	0.38	/	0.21	-5.8	-5.3	-3.6	/	-2.0	3.3	3.0	2.0	/	1.1
^{50}Mn $\to ^{50}$Cr	0.660	0.476	0.35	/	0.24	-6.4	-4.6	-3.4	/	-2.4	3.1	2.3	1.7	/	1.2
^{54}Co $\to ^{54}$Fe	0.770	0.586	0.44	0.319	0.28	-7.8	-5.9	-4.4	-3.2	-2.8	3.3	2.5	1.9	1.4	1.2

Table I: Estimation of $\Delta M_A^{(1)}$ and $|\Delta M_A^{(1)}/(AR^2/4)|$ from different models. A few remarks: $A = 46, 50$ are missing in the RPA calculation, while the DFT calculation gives an unusually large δ_{C} for $A = 38$.

Transitions	$\Delta M_B^{(1)}$ (fm^2)					$\frac{\Delta M_B^{(1)}}{AR^2/2}$ (%)				
	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro
^{26m}Al $\to ^{26}$Mg	-0.12	-0.12	-0.11	-0.05	-0.03	0.08	0.09	0.08	0.04	0.02
^{34}Cl $\to ^{34}$S	-0.17	-0.21	-0.16	-0.06	-0.04	0.08	0.10	0.07	0.03	0.02
^{38m}K $\to ^{38}$Ar	-0.15	-0.42	-0.15	-0.07	-0.04	0.06	0.16	0.06	0.03	0.01
^{42}Sc $\to ^{42}$Ca	-0.15	-0.17	-0.09	-0.07	-0.04	0.05	0.06	0.03	0.02	0.01
^{46}V $\to ^{46}$Ti	-0.12	-0.11	-0.08	/	-0.04	0.03	0.03	0.02	/	0.01
^{50}Mn $\to ^{50}$Cr	-0.12	-0.09	-0.06	/	-0.04	0.03	0.02	0.02	/	0.01
^{54}Co $\to ^{54}$Fe	-0.13	-0.10	-0.07	-0.05	-0.05	0.03	0.02	0.02	0.01	0.01

Table II: Estimation of $\Delta M_B^{(1)}$ and $|\Delta M_B^{(1)}/(AR^2/2)|$ from different models.

we see that it is suppressed by the small energy splitting parameter κ. The same treatment applies to $\Delta M_{A,B}^{(1)}$; they are all proportional to the same unknown reduced matrix element u^2, and could be connected to δ_{C} as:

$$\begin{aligned}\delta_{\mathrm{C}} &\approx -\frac{Ze^2}{8\pi R_C^3}\frac{\kappa(4-13\kappa+12\kappa^2-\kappa^3)}{(\kappa^2-4\kappa+2)(1-2\kappa)(1-\kappa^2)}\frac{1}{\xi\omega}\Delta M_A^{(1)} \\ &\approx -\frac{Ze^2}{8\pi R_C^3}\frac{(4-13\kappa+12\kappa^2-\kappa^3)}{2\kappa(1-2\kappa)(1-\kappa^2)}\frac{1}{\xi\omega}\Delta M_B^{(1)}\ , \qquad (20)\end{aligned}$$

where u^2 now drops out. Hence we have obtained a direct relation between δ_{C} and $\Delta M_{A,B}^{(1)}$, with a proportionality constant bearing a residual model dependence. We notice that $\Delta M_A^{(1)}$ is not suppressed by κ, so its sensitivity to δ_{C} is enhanced by $1/\kappa$; on the other hand $\Delta M_B^{(1)}$ is suppressed by κ^2 so it requires a much higher experimental precision to observe a deviation from zero. Furthermore, the ratio between $\Delta M_{A,B}^{(1)}$ depends only on κ, so a simultaneous measurement of the two may pin down κ, which further solidifies their relation to δ_{C}.

Targeted experimental precision – Following the strategy outlined above, we devise the experimental precision required for the quantities $\Delta M_{A,B}^{(1)}$, which would allow to address the reliability of the estimates of δ_{C} and its uncertainty in a less model-dependent way. First, to fix the proportionality constant, we take:

$$Z \approx A/2,\ \ R_C \approx \sqrt{5/3}\times 1.1\mathrm{fm}\times A^{1/3}, \qquad (21)$$

with the standard expectation for the nuclear RMS radius, $R \approx 1.1\mathrm{fm}\times A^{1/3}$, related to the radius of a nucleus as a uniform sphere by $R^2 = (3/5)R_C^2$. We take further parameters from Ref. [22],

$$V_1 \approx 100\mathrm{MeV},\ \ \omega \approx 41\mathrm{MeV}\times A^{-1/3},\ \ \xi \approx 3. \qquad (22)$$

A more recent discussion of these parameters supporting the above choices can be found in Ref. [46]. Next, we may, e.g., take the estimates of δ_{C} available in the literature and substitute them into the first line of Eq. (20). This returns an estimate of the size of $\Delta M_A^{(1)}$, which informs, how precise the measurement of this quantity should be to discriminate the model dependence of δ_{C}.

Restricting ourselves to superallowed decays with $T_{z,i} = 0$ and $T_{z,f} = +1$ and requiring the daughter nucleus to be (observationally) stable, we study the transitions with $26 \le A \le 54$. We take δ_{C} as calculated in the nuclear shell model with the Woods-Saxon (WS) potential [8], the density functional theory (DFT) [18], the Hartree-Fock (HF) calculation [20], the random phase approximation (RPA) with PKO1 parameterization [21], as well as the "miscroscopic" model of Ref.[22, 47] which gives $\delta_{\mathrm{C}} \approx 2\times 18.0\times 10^{-7}A^{5/3}$. The estimated size of

$\Delta M_A^{(1)}$ indicates the targeted absolute precision in the measurements of $\langle f|M_{+1}^{(1)}|i\rangle$ and $\langle f|M_0^{(1)}|f\rangle$. The latter implies subtracting two large terms, $NR_{n,f}^2/2$ and $ZR_{p,f}^2/2$, each of the typical size $AR^2/4$. Therefore, we may use the ratio $\Delta M_A^{(1)}/(AR^2/4)$ as an estimate of the precision of the RMS radii of the nuclear neutron and proton distributions required to probe the ISB effects.

The results of our numerical analysis are summarized in Table I. We find that most models predict a generic size of $\Delta M_A^{(1)} \sim 1\text{fm}^2$, with a precision level $(1-3)\%$ needed for the $R_{p,f}^2$ and $R_{n,f}^2$ measurements in order to probe the isospin mixing effect, i.e. start seeing a deviation of $\Delta M_A^{(1)}$ from zero. If it turns out that a non-zero $\Delta M_A^{(1)}$ is not observed at this precision, it could indicate that the actual values of δ_C are smaller than most existing model predictions, as suggested in [15, 16]. The model predictions for $\Delta M_A^{(1)}$ span over an order of magnitude for ${}^{38m}\text{K}\to {}^{38}\text{Ar}$, and half that range for ${}^{34}\text{Cl}\to {}^{34}\text{S}$ and ${}^{42}\text{Sc}\to {}^{42}\text{Ca}$ decays, reflecting a similar model dependence in δ_C in these channels. Hence, an experimental study of $\Delta M_A^{(1)}$ for these systems even at a moderate precision will shed light on the model dependence of δ_C.

An analogous analysis for $\Delta M_B^{(1)}$ is summarized in Table II; following Eq.(9), we use $\Delta M_B^{(1)}/(AR^2/2)$ as a measure of the precision goal. We observe that, due to the κ^2-suppression, a much higher precision (0.01-0.1)% is required to probe δ_C experimentally through $\Delta M_B^{(1)}$. However, our outlined theory framework permits an interplay in an opposite way: We know that partial results for charge radii already exist, for instance within $A = 42$ isotriplet, the ^{42}Sc and ^{42}Ca charge radii are measured while the ^{42}Ti radius is missing [29]. The IVMS formalism then allows us to use either model calculations of δ_C or experimental measurements of $\Delta M_A^{(1)}$ to predict the missing radius, which can be directly checked by future experiments [30].

Summary – We for the first time proposed the possibility to constrain the ISB correction δ_C to the rates of superallowed nuclear beta decays by experimental data. Our formalism combines the formalisms proposed by Miller and Schwenk [14, 15, 48] and by Auerbach [22], but enlarges the scope of both. In view of a significant model spread of the ISB correction calculations, we opt for model-independent constraints from two thoroughly devised combinations of measurable quantities, $\Delta M_A^{(1)}$ and $\Delta M_B^{(1)}$. The former is related to nuclear weak radii, while the latter to nuclear charge radii. The information on some nuclear charge radii has been used by Hardy and Towner in the past [13]. Here, we put firm foundations under the connection of δ_C and the charge radii differences across the superallowed isotriplet. The inclusion of the weak nuclear radii, both in the charged current (via the measurement of the nuclear recoil in the superallowed transition) and in the neutral current (via the measurement of the neutron skin of the stable daughter nucleus with PVES) is entirely novel. In a simplified picture with the IVMS dominance, $\Delta M_{A,B}^{(1)}$ and δ_C are all unambiguously interconnected. We individuated transitions in the superallowed decay chart where a measurement $\Delta M_A^{(1)}$ at even a moderate, few percent precision, could already discriminate between models used to compute δ_C; $\Delta M_B^{(1)}$ requires higher precision but partial information already exists. Moreover, this study suggests affinities, never attenuated earlier, between experimental programs and communities in physics of rare isotopes, electron scattering and nuclear astrophysics.

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[1] C.-Y. Seng, M. Gorchtein, H. H. Patel, and M. J. Ramsey-Musolf, Phys. Rev. Lett. **121**, 241804 (2018), 1807.10197.
[2] C. Y. Seng, M. Gorchtein, and M. J. Ramsey-Musolf, Phys. Rev. **D100**, 013001 (2019), 1812.03352.
[3] A. Czarnecki, W. J. Marciano, and A. Sirlin (2019), 1907.06737.
[4] C.-Y. Seng, X. Feng, M. Gorchtein, and L.-C. Jin, Phys. Rev. D **101**, 111301 (2020), 2003.11264.
[5] L. Hayen, Phys. Rev. D **103**, 113001 (2021), 2010.07262.
[6] K. Shiells, P. G. Blunden, and W. Melnitchouk, Phys. Rev. D **104**, 033003 (2021), 2012.01580.
[7] P. Zyla et al. (Particle Data Group), PTEP **2020**, 083C01 (2020).
[8] J. C. Hardy and I. S. Towner, Phys. Rev. C **102**, 045501 (2020).
[9] M. Gorchtein, Phys. Rev. Lett. **123**, 042503 (2019), 1812.04229.
[10] I. S. Towner and J. C. Hardy, Phys. Rev. C **66**, 035501 (2002), nucl-th/0209014.
[11] I. S. Towner and J. C. Hardy, Phys. Rev. C **77**, 025501 (2008), 0710.3181.
[12] J. C. Hardy and I. S. Towner, Phys. Rev. C **79**, 055502 (2009), 0812.1202.
[13] J. C. Hardy and I. S. Towner, Phys. Rev. C **91**, 025501 (2015), 1411.5987.
[14] G. A. Miller and A. Schwenk, Phys. Rev. C **78**, 035501 (2008), 0805.0603.
[15] G. A. Miller and A. Schwenk, Phys. Rev. C **80**, 064319

(2009), 0910.2790.
[16] L. Condren and G. A. Miller (2022), 2201.10651.
[17] W. Satula, J. Dobaczewski, W. Nazarewicz, and M. Rafalski, Phys. Rev. Lett. **106**, 132502 (2011), 1101.0939.
[18] W. Satuła, P. Bączyk, J. Dobaczewski, and M. Konieczka, Phys. Rev. C **94**, 024306 (2016), 1601.03593.
[19] W. E. Ormand and B. A. Brown, Phys. Rev. Lett. **62**, 866 (1989).
[20] W. E. Ormand and B. A. Brown, Phys. Rev. C **52**, 2455 (1995), nucl-th/9504017.
[21] H. Liang, N. Van Giai, and J. Meng, Phys. Rev. C **79**, 064316 (2009), 0904.3673.
[22] N. Auerbach, Phys. Rev. C **79**, 035502 (2009), 0811.4742.
[23] J. Damgaard, Nucl. Phys. A **130**, 233 (1969).
[24] E. Caurier, P. Navratil, W. E. Ormand, and J. P. Vary, Phys. Rev. C **66**, 024314 (2002).
[25] M. S. Martin, S. R. Stroberg, J. D. Holt, and K. G. Leach, Phys. Rev. C **104**, 014324 (2021), 2101.11826.
[26] A. Gorelov et al., Phys. Rev. Lett. **94**, 142501 (2005), nucl-ex/0412032.
[27] S. Van Gorp et al., Phys. Rev. C **90**, 025502 (2014).
[28] V. Araujo-Escalona et al., Phys. Rev. C **101**, 055501 (2020), 1906.05135.
[29] I. Angeli and K. P. Marinova, Atom. Data Nucl. Data Tabl. **99**, 69 (2013).
[30] K. Minamisono, P. Mantica, A. Klose, S. Vinnikova, A. Schneider, B. Johnson, and B. Barquest, Nucl. Instrum. Methods Phys. Res. A **709**, 85 (2013).
[31] K. S. Kumar (PREX, CREX), Annals Phys. **412**, 168012 (2020).
[32] S. Abrahamyan et al., Phys. Rev. Lett. **108**, 112502 (2012), 1201.2568.
[33] D. Adhikari et al. (PREX), Phys. Rev. Lett. **126**, 172502 (2021), 2102.10767.
[34] D. Adhikari et al. (CREX), Phys. Rev. Lett. **129**, 042501 (2022), 2205.11593.
[35] D. Becker et al. (2018), 1802.04759.
[36] B. T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz, Phys. Rev. Lett. **126**, 172503 (2021), 2101.03193.
[37] O. Koshchii, J. Erler, M. Gorchtein, C. J. Horowitz, J. Piekarewicz, X. Roca-Maza, C.-Y. Seng, and H. Spiesberger, Phys. Rev. C **102**, 022501 (2020), 2005.00479.
[38] B. A. Brown, Phys. Rev. Lett. **85**, 5296 (2000).
[39] X. Roca-Maza, G. Colò, and H. Sagawa, Phys. Rev. Lett. **120**, 202501 (2018), 1803.09120.
[40] G. F. Bertsch and A. Mekjian, Ann. Rev. Nucl. Part. Sci. **22**, 25 (1972).
[41] R. E. Behrends and A. Sirlin, Phys. Rev. Lett. **4**, 186 (1960).
[42] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).
[43] L. A. Radicati, Phys. Rev. **87**, 521 (1952).
[44] W. M. MacDonald, Phys. Rev. **100**, 51 (1955).
[45] N. Auerbach, Physics Reports **98**, 273 (1983).
[46] N. Auerbach and M.-L. Bui (2021), 2101.06199.
[47] We found that the expression of $\delta_{\mathbf{C}}$ from Eq.(24) onward of Ref.[22] are all off by a factor 2, so we multiply 2 to their numerical results.
[48] N. Miller et al., Phys. Rev. D **102**, 034507 (2020), 2005.04795.