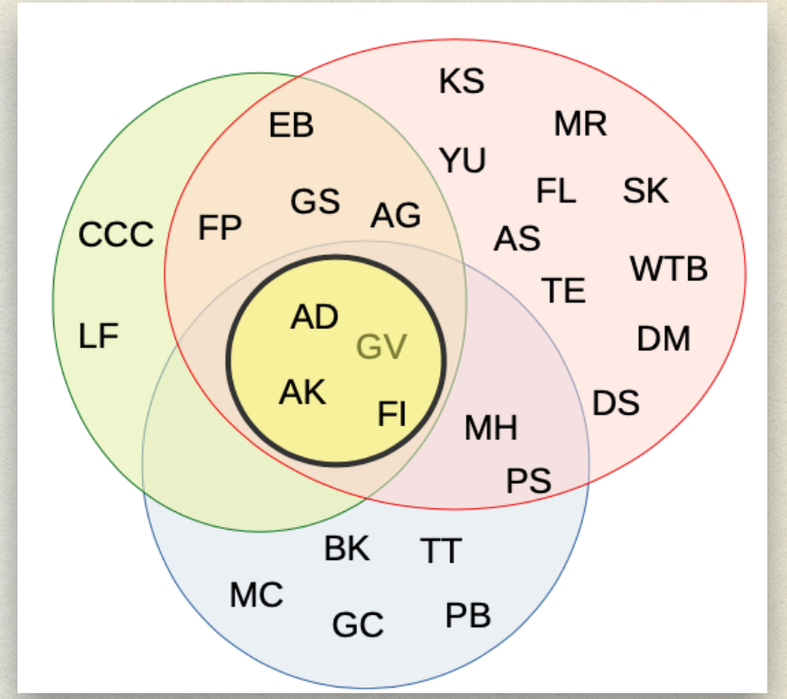
WP1



- From Diagrams to Amplitudes :: WTB
- From Amplitudes to Cross sections :: TE
- From Cross sections to Event :: YU

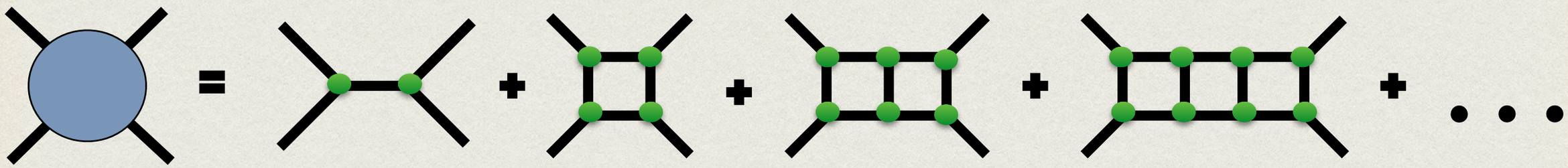
- Dispersive Approach to Massive two-loop Amplitudes :: AG

From Feynman diagrams to Amplitudes

William J. Torres Bobadilla
Max-Planck-Institut Für Physik

5th Workstop / Thinkstart:
Radiative corrections and Monte Carlo tools for Strong 2020
April 5 — 9, 2029
Zurich, Switzerland

Standard approach @multi-loop level



Draw all Feynman diagrams [Hann] [Nogueira]

Generate integrands

Profit of DimReg

[Chetyrkin, Tchakov] [Laporta]

Use Integration-By-Parts identities

Sector Decomposition [Heinrich et al] [Smirnov]

Tropical geometry [Borinsky] [Arkani-Hamed et al]

Numerically

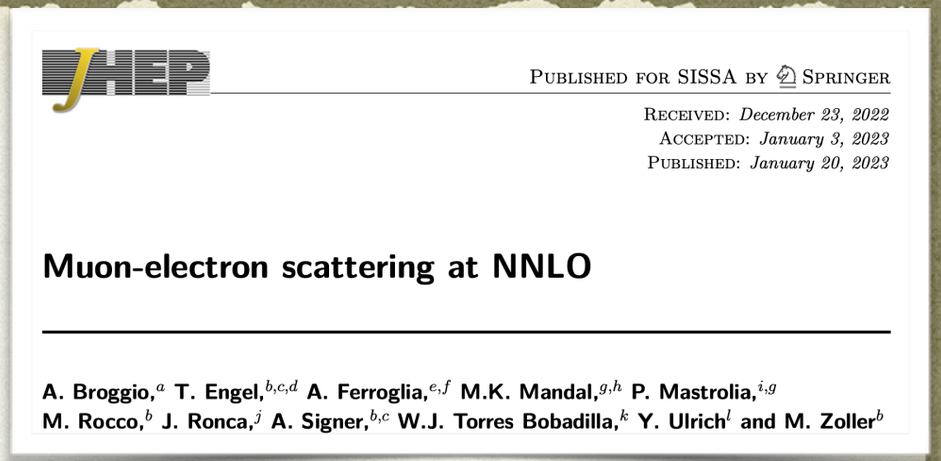
LTD approach [Rodrigo, W.J.T. et al] [Capatti et al] [Runkel et al]

Evaluate integrals

Analytically

Diff. Eqs. [Kotikov] [Remiddi, Gehrmann] [Henn]

$e\mu$-scattering @ NNLO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Muon-electron scattering at NNLO

A. Broggio,[a] T. Engel,[b,c,d] A. Ferroglia,[e,f] M.K. Mandal,[g,h] P. Mastrolia,[i,g]
M. Rocco,[b] J. Ronca,[j] A. Signer,[b,c] W.J. Torres Bobadilla,[k] Y. Ulrich[l] and M. Zoller[b]

$$\mu^+(p_1) + e^-(p_2) \to \mu^+(p_3) + e^-(p_4)$$

Anatomy

- Real-Real contribution
  Tree-level (n+2)-particles

[OpenLoops framework]

- Real-Virtual Contribution
  one-loop (n+1)-particles

- Virtual-Virtual Contribution
  two-loop n-particles

[Mandal, Mastrolia, Ronca, WJT *et al* (2021)]

$$\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

+ Subtractions & MC integrations

[McMule framework]

Outline

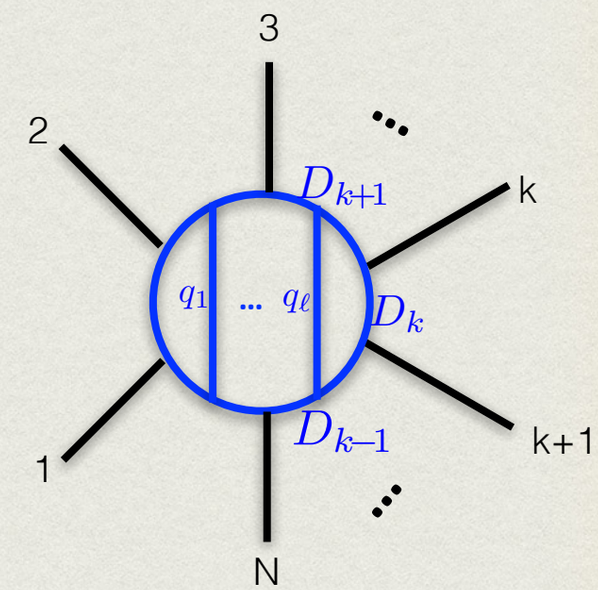
- Feynman integrals
- electron-muon scattering w/ $m_e^2 = 0$
- Perspectives on electron-muon scattering w/ $m_e^2 \neq 0$
- Summary & Future directions

Multi-loop Feynman integrals

- In loop calculations, one finds

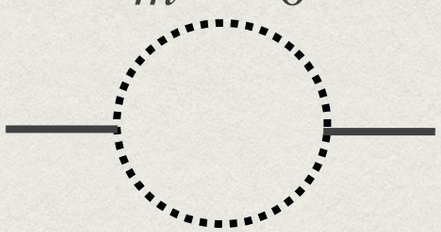
$$J_N^{(L),D}(1,\ldots,n;n+1,\ldots,m) = \int \prod_{i=1}^{L} \frac{d^D \ell_i}{\imath\, \pi^{D/2}} \frac{\prod_{k=n+1}^{m} D_k^{-\nu_k}}{\prod_{j=1}^{n} D_j^{\nu_j}}$$

$$D_i = q_i^2 - m_i^2 + \imath 0$$



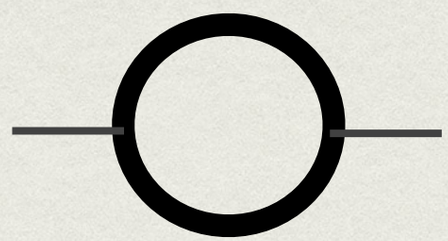
- Complexity easily increases:

$m^2 = 0$



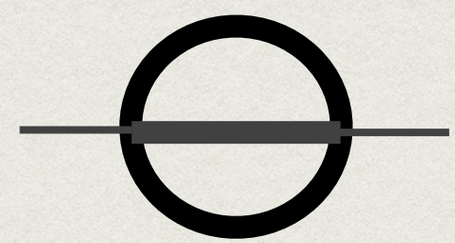
known at all orders

$m^2 \neq 0$



square roots

$m^2 \neq 0$



elliptic integrals

- DEQ :: Feynman integrals are not independent (IBP relations)

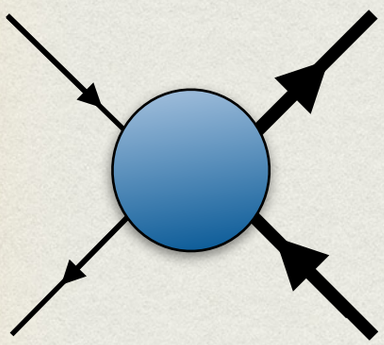
$$\partial_x \vec{J}(x) = A_i(x,\epsilon)\, \vec{J}(x)$$

Several methods (and tools) to analytically or numerically integrate them

Analytic evaluations

[Mandal, Mastrolia, Ronca, WJT *et al* (2021)]
[Mandal, Mastrolia, Ronca, WJT (2022)]

☑ $e^+e^- \to \mu^+\mu^-$ & $q\bar{q} \to t\bar{t}$ at two loops



$$\mathcal{A}(\alpha) = 4\pi\alpha\left[\mathcal{A}^{(0)} + \left(\frac{\alpha}{\pi}\right)\mathcal{A}^{(1)} + \left(\frac{\alpha}{\pi}\right)^2\mathcal{A}^{(2)} + \mathcal{O}\left(\alpha^3\right)\right]$$

- In the massless electron limit ($m_e^2 = 0$) 4-point process depending on 3 scales

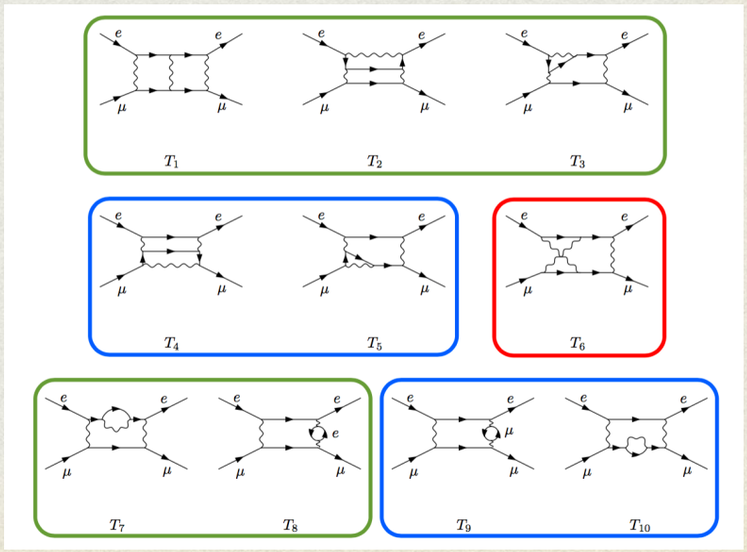
$$s = (p_1 + p_2)^2 \,, \quad t = (p_2 - p_3)^2 \,,$$
$$u = (p_1 - p_3)^2 \,, \quad s + t + u = 2M^2 \,.$$

- Integrand/integral reductions

$$\mathcal{M}^{(2)}\left(e^+e^- \to \mu^+\mu^-\right) = \sum_k c_k\left(s, t, m^2, \epsilon\right) I_k^{(2)}\left(s, t, m^2, \epsilon\right)$$

$O(10000)$ monomials

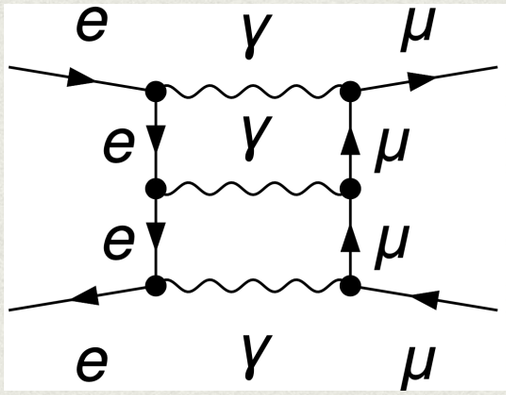
$O(100)$ MIs

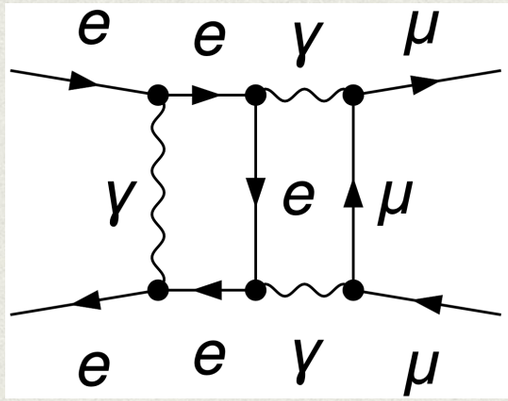


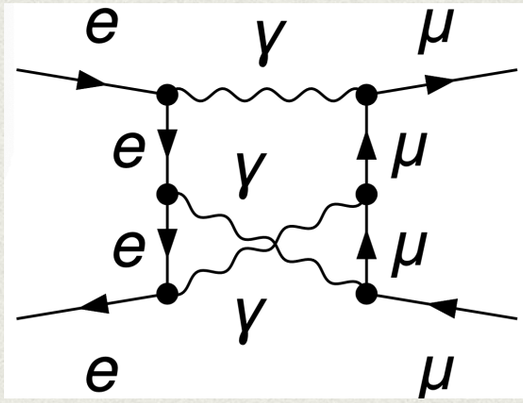
electron-muon scattering w/ $m_e^2 \neq 0$

What is new & problematic?

- Follow diagrammatic approach







[Heller 2021]

#MIs	37	~51	~95
#MIs top sector	2	4	4

- Integration-by-parts identities

`FiniteFlow` [Peraro]

`Kira & FireFly`

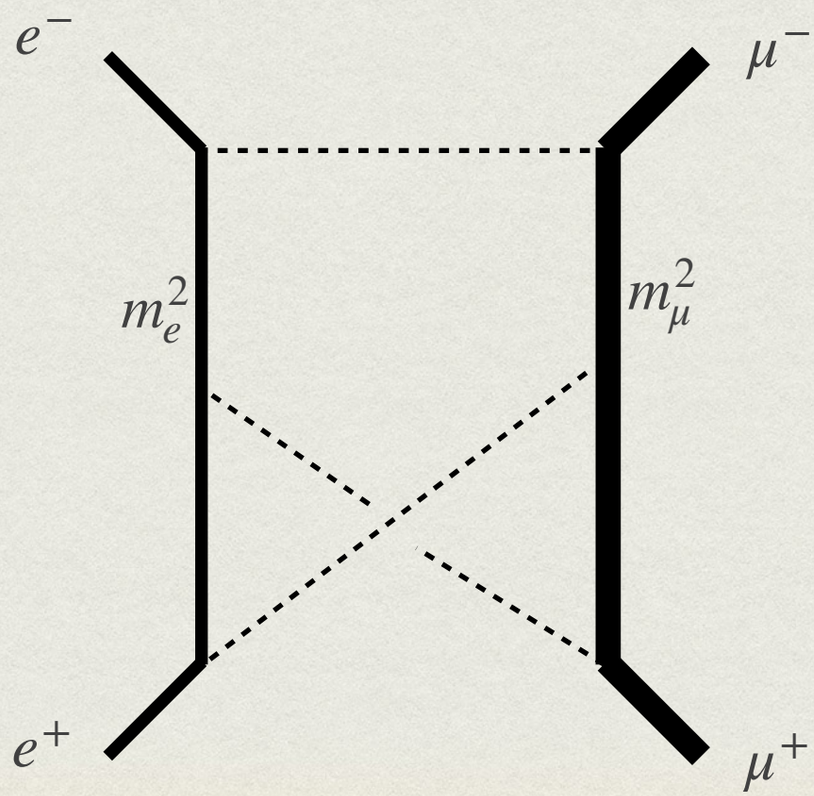
[Klappert, Lange, Maierhöfer, Usovitsch]

☑ All IBP reductions easily handed by reconstruction over finite fields

Numerical evaluation of Feynman integrals

- We are used to
  — Sector Decomposition —> `PySecDec, Fiesta`
  — Auxiliary Mass Flow :: DEQ in $x \sim \imath 0$
  — Series expansions :: solve DEQ along path —> `DiffExp`, `SeaSyde`

- Insights from tropical geometry —> `FeynTrop`



?

Numerical evaluation of Feynman integrals

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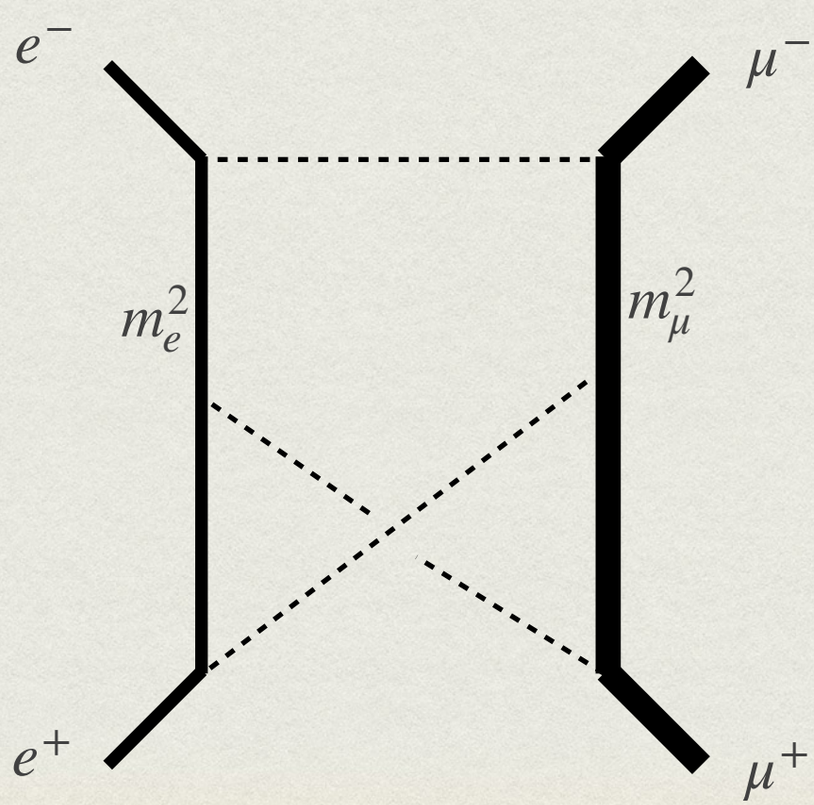



```
Started integrating using 16 threads and N = 1e+07 points.
Finished in 8.98043 seconds = 0.00249456 hours.

-- eps^0: [1.000  +/- 0.019 ]  +  i * [ 0.974  +/- 0.016 ]
-- eps^1: [ 3.98  +/-  0.29 ]  +  i * [ 6.57   +/-  0.28 ]
-- eps^2: [ 8.70  +/-  2.41 ]  +  i * [ 20.04  +/-  2.54 ]
-- eps^3: [21.519 +/- 14.125]  +  i * [30.395  +/- 15.982]
-- eps^4: [89.751 +/- 66.827]  +  i * [-16.737 +/- 75.607]
```

```
Started integrating using 16 threads and N = 1e+08 points.
Finished in 89.3534 seconds = 0.0248204 hours.

-- eps^0: [0.9879  +/- 0.0089]  +  i * [1.0054  +/- 0.0094]
-- eps^1: [ 3.78   +/-  0.17 ]  +  i * [ 7.17   +/-  0.17 ]
-- eps^2: [ 6.49   +/-  1.68 ]  +  i * [ 25.54  +/-  1.49 ]
-- eps^3: [ 3.858  +/- 11.402]  +  i * [ 63.38  +/-  8.82 ]
-- eps^4: [-14.775 +/- 58.255]  +  i * [127.640 +/- 38.927]
```

Recap

- ⭐ Straightforward generation of integrand from Feynman diagrams

  `Aida`

- ⭐ Integration-by-parts identities :: out of the box

  `FiniteFlow`

  `Kira & FireFly`

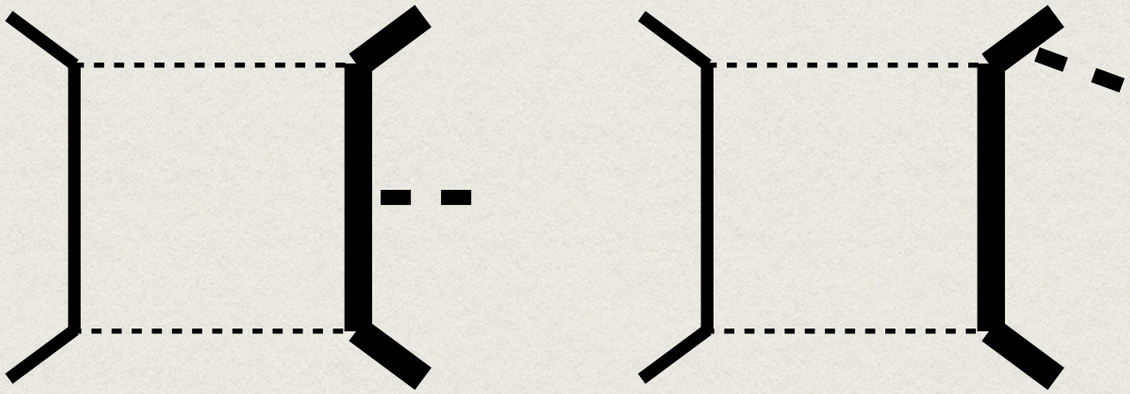
- ⭐ Explore use of new tools to evaluate Feynman integrals

  `FeynTrop?`

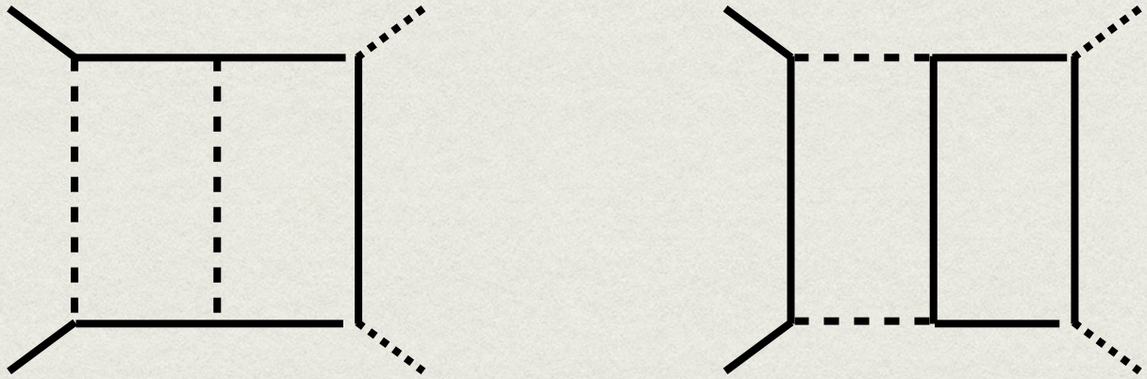
Open questions/directions

Numerically or analytically?

- Provide me with loop integrands to play around



- Keep in mind e+e- —> y* y*



+ non-planar

Result for massless electrons

[Henn, Melnikov, Smirnov (2014)]
[Caola, Henn, Melnikov, Smirnov (2014)]