Parton Showers (WP4)
a.k.a. QED resummation in Monte Carlos

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5$^{\text{th}}$ Workstop/Thinkstart:
Radiative Corrections and Monte Carlo Tools for Strong 2020

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QED resummation

- Instead of focussing on calculating order-by-order exact corrections in α for a given process, **QED Parton Shower** and **Yennie-Fraustchi-Suura resummation** take a different point of view as starting point:

  they aim at calculating **approximate** and **"universal"** corrections **up to all orders**, by including (the important, leading) contributions arising from soft and/or collinear regions

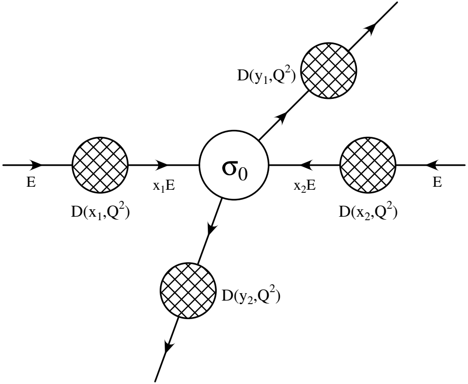
- They rely on the general property of **factorization of soft/collinear divergencies** (enhancements) in QED, which leads to **exponentiation**

⇝ Sometimes, in some phase-space regions, for some observables, for certain experimental cuts, you better have an approximate resummed result than a fixed-order one

$$\alpha < \alpha^2 L^2 \quad \text{somewhere, with } L = \log \frac{s}{m^2}$$

PS: QED collinear Structure Functions

- PS algorithms rely on QCD-inspired Structure Function approach to radiative corrections (it's still called *Parton* Shower although here it describes multiple photon emissions. . .)



- If we are interested only in photon radiation, $D(x, Q^2)$ are the **Leading-Log non-singlet QED SF**

QED DGLAP equation

$D(x, Q^2)$ is the solution of the QED DGLAP equation

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} P_+(y) D(\frac{x}{y}, Q^2)$$

$$P_+(x) = \frac{1+x^2}{1-x} - \delta(1-x) \int_0^1 \frac{1+t^2}{1-t} dt$$

which can be solved analytically (but some “exclusive” information is lost because integrated out) or by a **Monte Carlo iterative solution** (the **Parton Shower**, which is “exclusive”)

$$\begin{aligned} D(x, Q^2) &= \Pi(Q^2, m^2)\delta(1-x) \\ &+ \int_{m^2}^{Q^2} \Pi(Q^2, s') \frac{ds'}{s'} \Pi(s', m^2) \frac{\alpha}{2\pi} \int_0^{x_+} dy P(y) \delta(x-y) \\ &+ \int_{m^2}^{Q^2} \Pi(Q^2, s') \frac{ds'}{s'} \int_{m^2}^{s'} \Pi(s', s'') \frac{ds''}{s''} \Pi(s'', m^2) \times \\ & \left(\frac{\alpha}{2\pi}\right)^2 \int_0^{x_+} dx_1 \int_0^{x_+} dx_2 P(x_1) P(x_2) \delta(x - x_1 x_2) + \cdots \end{aligned}$$

PS: pros and cons

$$\Pi(Q^2, m^2)_\epsilon = e^{-\frac{\alpha}{2\pi}\log\frac{Q^2}{m^2}\int_0^{1-\epsilon} dx P(x)} = e^{-\frac{\alpha}{2\pi}\log\frac{Q^2}{m^2} I_+}$$

is the Sudakov Form Factor, which exponentiates approximate virtual and soft emission up to all orders

✓ Advantages:

- ⇝ the number of emitted photons is not limited (shower)
- ⇝ at each branching, kinematical variables are generated and photons' momenta can be reconstructed $\rightarrow$ **fully exclusive event generation**
- ⇝ it can be truncated at $\mathcal{O}(\alpha^n)$ and consistently compared to fixed-order N^{n}LO calculations.

✗ Disadvantages:

- ⇝ initial-final state radiation interference effects are not naturally included, but they can be recovered by choosing an appropriate photons' angular distribution (eikonal, YFS-inspired)

Carloni Calame, PLB 520 (2001) 16

$$I(k) = \sum_{i,j} \eta_i \eta_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} E_\gamma^2$$

- ⇝ at its LL level, it misses already corrections at $\mathcal{O}(\alpha)$: a matching to NLO is needed

PS: matching (Pavia solution)

- Firstly, the corrected LL cross section can be cast in the form

$$d\sigma_{LL}^{\infty} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n,$$

$$|\mathcal{M}_{1,LL}|^2 = \frac{\alpha}{2\pi}\frac{1+z^2}{1-z}I(k)\ |\mathcal{M}_0|^2\ \frac{8\pi^2}{E^2 z(1-z)}$$

⇝ The multi-differential phase-space is kept **exact**

⇝ Any approximation is shifted on matrix elements

⇝ A mapping of momenta is needed: this is a dirty and ambiguous job.
You hope ambiguities are effects beyond your working accuracy...

PS: matching (Pavia solution)

- A LL PS-corrected differential cross section can be expanded at $\mathcal{O}(\alpha)$

$$
\begin{aligned}
d\sigma^{\alpha}_{LL} &= \left[1 - \frac{\alpha}{2\pi} I_+ \log\frac{Q^2}{m^2}\right] |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \\
&\equiv [1 + C_{\alpha,LL}]\,|\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 ,
\end{aligned}
$$

while the NLO cross section can be always cast as

$$
d\sigma^{\alpha} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1 .
$$

By defining the factors

$$
F_{SV} = 1 + (C_{\alpha} - C_{\alpha,LL}), \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}
$$

the NLO cross section can be re-written (up to terms of $\mathcal{O}(\alpha^2)$) as

$$
d\sigma^{\alpha} = F_{SV}(1 + C_{\alpha,LL})|\mathcal{M}_0|^2 d\Phi_0 + F_H |\mathcal{M}_{1,LL}|^2 d\Phi_1
$$

which brings to the master formula. . .

PS: matching (Pavia solution)

$$d\sigma^{\infty}_{matched} = F_{SV}\,\Pi(Q^2,\varepsilon)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\prod_{i=0}^{n}F_{H,i}\right)|\mathcal{M}_{n,LL}|^2\,d\Phi_n,$$

- ⇝ it's based on LO and NLO building blocks
- ⇝ F_{SV} and F_H are collinear and infrared safe, no double counting of LL terms
- ⇝ the cross-section is still fully differential
- ⇝ its $\mathcal{O}(\alpha)$ expansion coincides with NLO
- ⇝ resummation of LL higher-orders, beyond NLO, is preserved
- ⇝ it can be expanded at $\mathcal{O}(\alpha^2)$ and compared to exact NNLO corrections

✓ Successfully applied to match QED NLO to PS in **`BabaYaga@NLO`**, EWK NLO to PS in **`Horace`** (neutral and charged Drell-Yan) and **`Hto4l`** ($H \to 4\ell$)

✗ generalization to NNLO?

Yennie-Frautschi-Suura soft exponentiation

✓ It all started in this beautiful work, full of insights and clever tricks

D. R. Yennie, S. C. Frautschi and H. Suura
"The infrared divergence phenomena and high-energy processes", Ann. Phys. 13, 379 (1961)

⇝ Many Monte Carlos for LEP (and LHC) developed by S. Jadach and colleagues on this framework (**`Koral[W/Z]`**, **`BH[LUMI/WIDE]`**, **`YFS[WW3/ZZ]`**, **`WINAC`**, **`KKMC`**)

⇝ Nowadays YFS is the basis for QED radiation resummation in **`Sherpa`**.
Applied also to (future) e^+e^- machines

Krauss, Price, Schönherr, SciPost Phys. 13, 026 (2022)

YFS: master formulae

- As usual, the full perturbative series for the emission of an arbitrary number of photons in a given LO process can be written as

$$\mathrm{d}\sigma = \sum_{n_\gamma=0}^{\infty} \frac{1}{n_\gamma!} \mathrm{d}\Phi_Q \left[\prod_{i=1}^{n_\gamma} \mathrm{d}\Phi_i^\gamma\right] (2\pi)^4 \delta^4\left(\sum_{i=1}^{2} p_i - \sum_{j=3}^{N+2} q_j - \sum_{k=1}^{n_\gamma} k_k\right) \left|\sum_{\bar{n}_\gamma=0}^{\infty} \mathcal{M}_{n_\gamma}^{\bar{n}_\gamma+\frac{1}{2}n_\gamma}\right|^2,$$

- After factorizing out all soft virtual and soft real corrections, you end up with something like

$$\mathrm{d}\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} \mathrm{d}\Phi_Q \left[\prod_{i=1}^{n_\gamma} \mathrm{d}\Phi_i^\gamma \tilde{S}(k_i)\, \Theta(k_i,\Omega)\right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{S}(k_j)} + \sum_{\substack{j,k=1\\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j,k_k)}{\tilde{S}(k_j)\tilde{S}(k_k)} + \cdots\right)$$

where

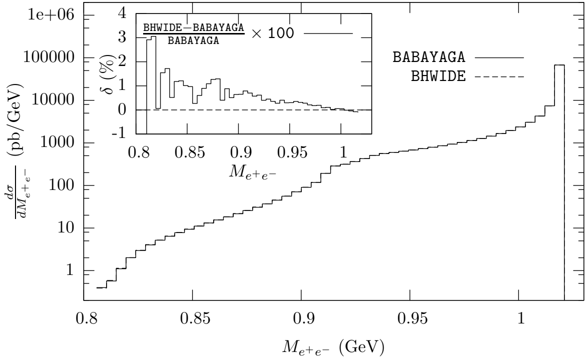
- $e^{Y(\Omega)}$ resums all soft virtual and soft real emissions
- $\tilde{S}(k_i)$ are eikonal factors
- $\tilde{\beta}_n$ are IR-subtracted matrix elements remnants (with n photons)

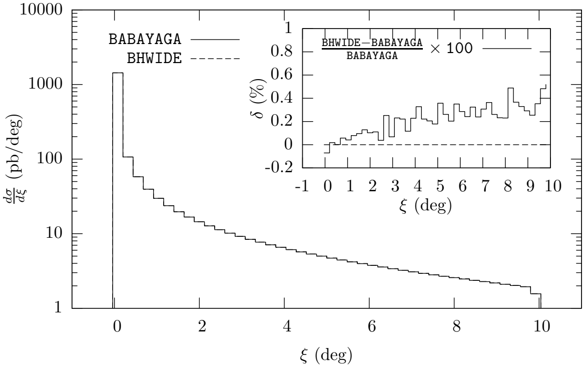
YFS properties

- it relies on factorization of soft virtual and real photon emissions
- fully exclusive event generation
- inclusion of exact higher-order matrix elements more “natural” than in PS
  $\rightarrow$ $\tilde{\beta}_1 \neq 0$ matches to NLO, $\tilde{\beta}_2 \neq 0$ matches to NNLO, ... (I think)
- two flavours:
  - **EEX**
    exclusive exponentiation: based on YFS original paper, works at $|\mathcal{M}|^2$ level
  - **CEEX**
    coherent exclusive exponentiation: works at $\mathcal{M}$ level: only in **KKMC**, drastically more difficult to implement
- a mapping of momenta still necessary

NLO matched PS vs NLO YFS

- distributions: **BabaYaga@NLO** vs. **BHWIDE** (Bhabha $e^+e^- \to e^+e^-(+n\gamma)$, at KLOE)



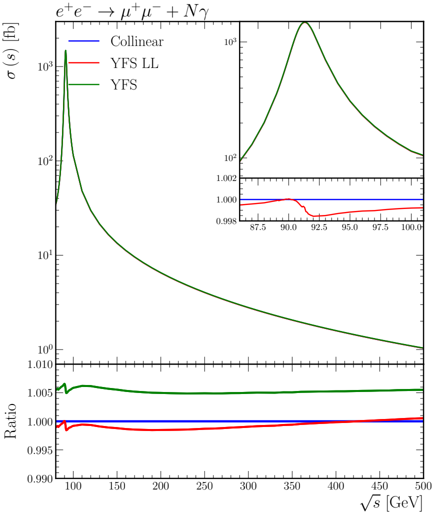


ξ = acollinearity

from Balossini et al., NPB 758 (2006) 227

Collinear SF vs YFS

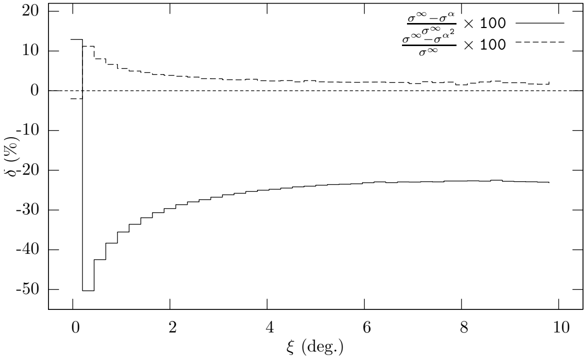
from Krauss, Price, Schönherr, SciPost Phys. 13, 026 (2022)



Total cross-section for $e^+e^- \to \mu^+\mu^-(+n\gamma)$ for $\sqrt{s}$ from 80 to 500 GeV where the ISR is modelled using a collinear (blue) resummation and compared against the soft resummation up to $\mathcal{O}(\alpha^3 L^3)$. The red line represents the strictly LL YFS while the green represent the full YFS calculation

Resummation beyond α^2

- With a complete NNLO generator at hand, can LL resummation beyond α^2 be neglected (again Bhabha at KLOE)?



ξ = acollinearity

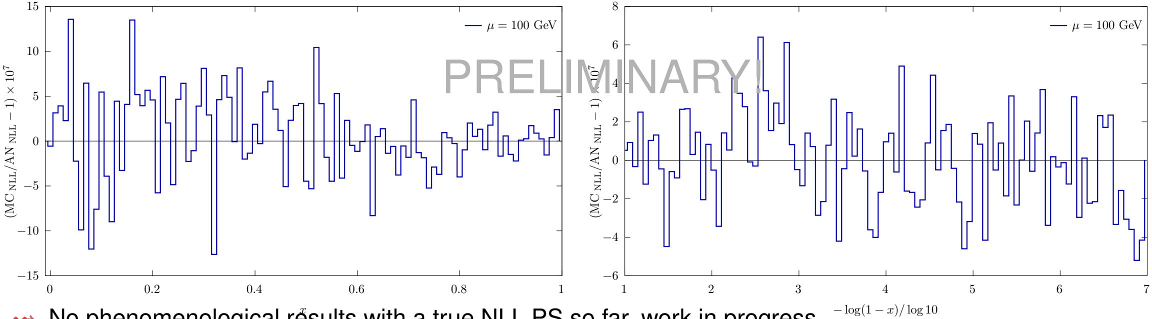
- Resummation beyond α^2 still important (at least for some distributions)!

- Frixione and colleagues recently studied and solved analytically QED DGLAP equations at NLL accuracy, *i.e.* with AP splitting functions at NLO and appropriate NLO initial conditions

Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040

Frixione 2105.06688; Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265

- A PS algorithm can be used to get numerically the solution $D_{NLL}(\mu^2, x)$ (non-singlet)



- No phenomenological results with a true NLL PS so far, work in progress

Q & A

- on Excalidraw
  1. I accidentally erased the first question by Adrian `:-(` ...sorry!
     Was it something like “can a PS describe radiation off pions?” ? If yes,

     For PS: are AP splitting functions the same for fermions and spin-0 particles?
     For YFS: I believe soft photons factorize as eikonal in sQED as well (soft photons are spin-blind), so probably YFS might work out-of-the-box

  2. What is the status of PS (LL, NLL)?

     See previous slide

  3. Can we use NTS to improve Parton Showers? That is doing a LBK PS rather than pure YFS/PS

     I don’t know. Is it there any advantage? Maybe radiation off massive particles?

  4. Does it make sense to include NTS in “hard” corrections factor when full ME is not know?

     Not sure I understand the question...

  5. Use **`Sherpa`** and marry it with pions in the final state, is this doable in a simple way?

     Marek? Lois? Is answer to Q1 enough?

- on Overleaf

NLO+PS Is it feasible (i.e. reasonably easy or already done, e.g. [9]) to produce a code for low-energy $e^+e^- \to \gamma^* + \{\gamma, (\ell^+\ell^-)\}$ with fixed-order NLO matched to (i) YFS parton shower (ii) collinear factorisation parton shower? This could then be truncated at NNLO and compared to fixed-order NNLO. If it is easy, could it be extended to fixed-order NNLO matched to parton shower, for an update of a detailed comparison?

Having an ISR PS is doable in principle, but matching with NLO makes it less a “black box” in my opinion.
Yes, any fixed order truncation can in principle be done.
As of NNLO matching, I had hard times to think it in a PS framework, no good solution so far. In the YFS framework this seems more straighforward (?).

whatLL Is there a QED parton shower that is NLL? (AS gets a different answer every time he asks).

Double counted question. . .