WP3: processes with hadrons

Introduction and comments on FsQED

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5th Workstop / Thinkstart: RC and MC tools for Strong 2020

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Outline

WP3 session

- Introduction and FsQED
- Two-pion channel: Gilberto Colangelo
- Three-pion channel: Bastian Kubis

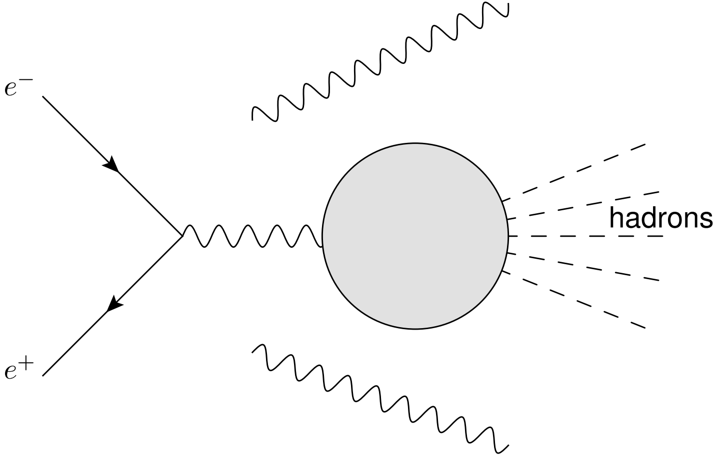
- Hadrons in Phokhara: Henryk Czyz (remote)
- Comparison of different MCs: Fedor Ignatov

- Perspectives on CMD-3 result: Fedor Ignatov

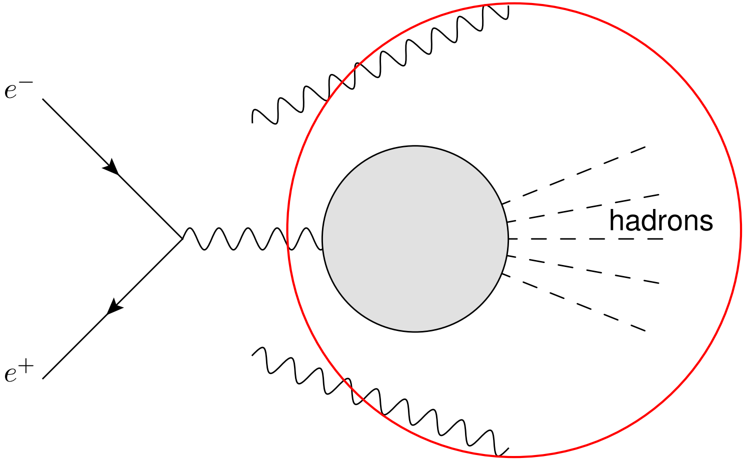
WP3 session

- input talks $\sim$ 10 min. + $\sim$ 15 min. open discussion
- additional discussion slots scheduled
- “satellite talk” on CMD-3 towards the end of session: some people will join via zoom

Processes with hadrons



Processes with hadrons



Hadronic vacuum polarization in muon $g - 2$

importance of different channels: $\rightarrow$ White Paper (2020)

- 2π: 73% of total HVP
- 3π: 7% of total HVP
- $2K$: 5% of total HVP
- > 1.8 GeV (without $\bar{c}c$): 7% of total HVP

FsQED

Adrian's question:

What is "scalar QED with form factors"? Can this be defined systematically in an order-by-order expansion in α*? Is it possible to write down a Lagrangian for this? Would it even make sense to go beyond LO in such a setup?*

Processes with hadrons

Low-energy hadronic interactions in principle described by **QCD+QED** (with tiny corrections due to weak interactions):

$$\mathcal{L}_{\mathrm{QCD+QED}} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_\psi \bar{\psi}(i\not{D} - m_\psi)\psi + \mathcal{L}_{\mathrm{gf}}\,,$$

$$D_\mu = \partial_\mu + igT^A G^A_\mu + ieQA_\mu\,,$$

but QCD is **non-perturbative** at low energies

Non-perturbative methods and their limitations

lattice QCD:

- limited access to timelike region via finite-volume effects

  $\rightarrow$ Lüscher (1991), Lellouch, Lüscher (2001)

- computationally expensive

Non-perturbative methods and their limitations

chiral perturbation theory (χPT): systematic EFT, describes interaction of lightest mesons (π, K, η) and photons → Gasser, Leutwyler (1984, 1985), Urech (1995)

$$\mathcal{L}^{\gamma}_{\chi\text{PT}} = \mathcal{L}^{\gamma}_{p^2} + \mathcal{L}^{\gamma}_{p^4} + \dots \,,$$

$$\mathcal{L}^{\gamma}_{p^2} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\text{gf}} + e^2 F_0^4 Z \langle QUQU^\dagger \rangle + \frac{F_0^2}{4}\langle (D_\mu U)(D^\mu U)^\dagger \rangle + \frac{F_0^2}{4}\langle \chi U^\dagger + U\chi^\dagger \rangle \,,$$

$$U(x) = \exp\left(i\frac{\pi(x)}{F_0} \right) \,, \quad \pi(x) = \sum_{a=1}^{8} \lambda^a \pi^a(x) \,,$$

$$D_\mu U = \partial_\mu U - iUeQA_\mu + ieQA_\mu U \,.$$

Non-perturbative methods and their limitations

chiral perturbation theory (χPT):

- limited to the region of **very low energies** $\ll 4\pi F_\pi \sim 1$ GeV
- no (explicit) hadronic resonances, extension of validity range requires unitarization
- higher-order corrections: **low-energy constants** sometimes poorly known, especially in EM sector

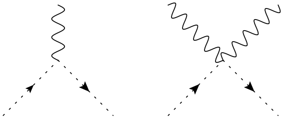
Non-perturbative methods and their limitations

chiral perturbation theory (χPT):

- (renormalizable part of) leading chiral Lagrangian contains **scalar QED** interactions:

$$\mathcal{L}^{\gamma}_{p^2} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\pi^+)(D^\mu\pi^-) + (D_\mu K^+)(D^\mu K^-)\,,$$

$$D_\mu\phi = \partial_\mu\phi + ieQ_\phi A_\mu\phi$$



Non-perturbative methods and their limitations

hadronic models:

- e.g., RChT: inclusion of resonances in χPT as explicit DoF

  $\rightarrow$ Ecker, Gasser, Pich, de Rafael (1989), + Leutwyler (1989)
- not a full-fledged EFT with mass gap and power counting
- multiplying sQED result by form factors (FsQED) *a priori* an ad hoc prescription
- models often phenomenologically successful, but **no systematic improvement** possible

Non-perturbative methods and their limitations

dispersion relations:

- use unitarity, analyticity, crossing, gauge invariance
- step 1: gauge-invariant **tensor decomposition** of photon amplitude $\rightarrow$ Bardeen, Tung (1968), Tarrach (1975)
- step 2: determine single-particle pole residues, multi-particle discontinuities from **unitarity relation** $\Rightarrow$ fixes imaginary parts
- step 3: reconstruct real parts via **dispersion integrals**, using analyticity

Non-perturbative methods and their limitations

dispersion relations:

- approximations: need to **truncate infinite sum** in unitarity relation
- limited knowledge about **sub-processes** needed as input

Dispersion relations

example: $\gamma^*\gamma^* \to \pi^+\pi^-$

- step 1: BTT tensor decomposition

$\rightarrow$ Colangelo, Hoferichter, Procura, Stoffer (2015)

$$W_{\mu\nu} = \sum_{i=1}^{5} T^i_{\mu\nu} A_i \,, \quad q_1^\mu T^i_{\mu\nu} = q_2^\nu T^i_{\mu\nu} = 0$$

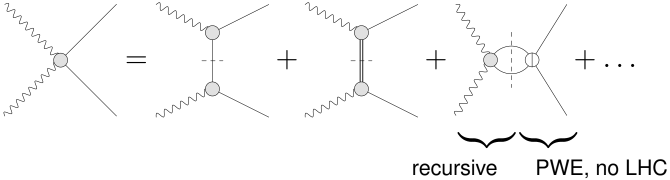
e.g., $T_1^{\mu\nu} = q_1 \cdot q_2 g^{\mu\nu} - q_2^\mu q_1^\nu$

A_i free of kinematic singularities and zeros

Dispersion relations

example: $\gamma^*\gamma^* \to \pi^+\pi^-$

- step 2: intermediate states in unitarity relation(s)



Dispersion relations

example: $\gamma^*\gamma^* \to \pi^+\pi^-$

- step 3: solve dispersion relations

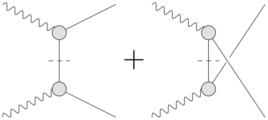
$$h_i(s) = \Delta_i(s) + \frac{\Omega(s)}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{K_{ij}(s,s') \sin\delta(s') \Delta_j(s')}{|\Omega(s')|}$$

  - $\Delta_i(s)$: inhomogeneity due to left-hand cut
  - $\Omega(s)$: Omnès function with $\pi\pi$ phase shift $\delta(s)$ as input
  - $K_{ij}(s,s')$: integration kernels

Dispersion relations: coming back to FsQED

example: $\gamma^*\gamma^* \to \pi^+\pi^-$

- pole term in fixed-s DR:



$$A_1^\pi = -F_\pi^V(q_1^2)F_\pi^V(q_2^2)\left(\frac{1}{t-M_\pi^2}+\frac{1}{u-M_\pi^2}\right),$$

$$A_4^\pi = -F_\pi^V(q_1^2)F_\pi^V(q_2^2)\frac{2}{s-q_1^2-q_2^2}\left(\frac{1}{t-M_\pi^2}+\frac{1}{u-M_\pi^2}\right),$$

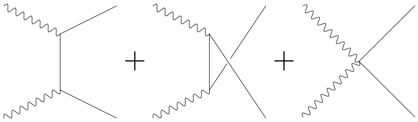
$$A_2^\pi = A_3^\pi = A_5^\pi = 0\,,$$

where $\langle\pi^+(k)|j_{\rm em}^\mu(0)|\pi^+(p)\rangle = (k+p)^\mu\, F_\pi^V\big((k-p)^2\big)$.

Dispersion relations: coming back to FsQED

example: $\gamma^*\gamma^* \to \pi^+\pi^-$

- sQED:

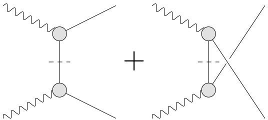


$$A_1^{\rm Born} = -\left(\frac{1}{t-M_\pi^2} + \frac{1}{u-M_\pi^2}\right),$$
$$A_4^{\rm Born} = -\frac{2}{s-q_1^2-q_2^2}\left(\frac{1}{t-M_\pi^2} + \frac{1}{u-M_\pi^2}\right),$$
$$A_2^{\rm Born} = A_3^{\rm Born} = A_5^{\rm Born} = 0\,.$$

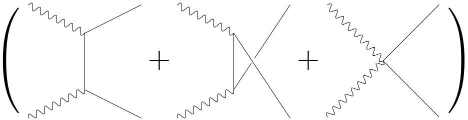
Dispersion relations: coming back to FsQED

example: $\gamma^*\gamma^* \to \pi^+\pi^-$

- $\Rightarrow$ pole terms $=$ FsQED:



$$= \quad F_\pi^V(q_1^2)F_\pi^V(q_2^2) \times$$



Summary and discussion items

- on hadronic side: limiting factor **non-perturbative dynamics**
- resonances in intermediate energy range ($1 \ldots 2$ GeV): no systematic field-theoretic approach
- most promising: **dispersion relations**, but require process-specific analyses and usually leave residual model dependence
- how to feed into MCs?