MC generators
$e^+e^- \to \pi^+\pi^-(\gamma)$

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University of Liverpool

5th Workstop / Thinkstart:
Radiative corrections and Monte Carlo tools
Zurich, 5 June 2023

Introduction

- Current level of analysis requires knowledge of the integrated cross-section of main channels $e^+e^- \rightarrow e^+e^-, \pi\pi, \mu\mu$ with precision <= 0.1%

- Some of the analysis are based on predicted differential distributions, knowledge of the differential cross-section is also crucial

MC generators e+e- → π+π-

First generation of MC tools were developed for initial studies at LO or of some specific amplitudes. they have ~ O(%) precision

AFKQED (BaBar)
ISR at LO +Structure Function

depends on the event selection (can be as good as phokhara)

EVA (KLOE)
Tagged photon, ISR at LO + Structure Function, FSR: point-like pions

O(%)

BaBaYaga 3.5 (KLOE)
ISR via Parton Shower approach, no FSR

1%

FASTERD (KLOE)
ISR at LO, dedicated for different FSR models study (and e+e- → φ/ρ → ..)

O(%)

KKMC
YFS exponentiation for soft photons + hard part and subleading terms in some approximation

High accuracy only for muon pairs

Carlomat
A general purpose program for automatic computation of LO cross sections

Probably many others….. But when we speak about precision, only very few are available

MC generators e+e- → π+π- (γ)

Most precise 2π MC generators:

PHOKHARA

quoted accuracy 0.5% for differential cross section

developed for ISR process with 1 real photon + addition

Complete set of NLO to e+e- → π+π-γ:

most recent 10.0 version includes NNLO FSR,

and 1real + two virtual photon box diagram in sQED approx.

FSR from the pointlike pion (some models with intermediate f0,σ are possible)

No logarithmically enhanced corrections, no 0-photon soft part

has limited precision for scanned mode (w/o γ)

MCGPJ

accuracy 0.2% for total cross section

exact NLO (to e+e- → π+π-) + logarithmically enhanced correction

using ISR jets along beam with collinear structure functions

box diagram with above sQED approach (GVMD or dispersive)

FSR from the pointlike pion

No some of virtual, soft corrections for e+e- → π+π-γ

Not designed to be used for ISR studies

ISR

Scan mode

Both generators has different region of applicability

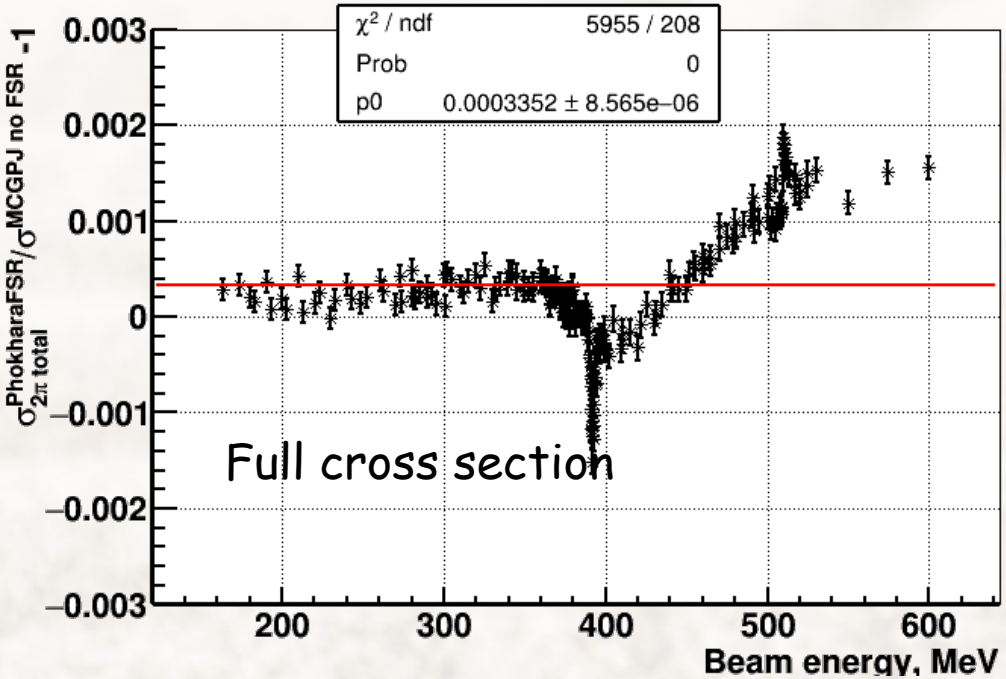
MCGPJ/Phokara

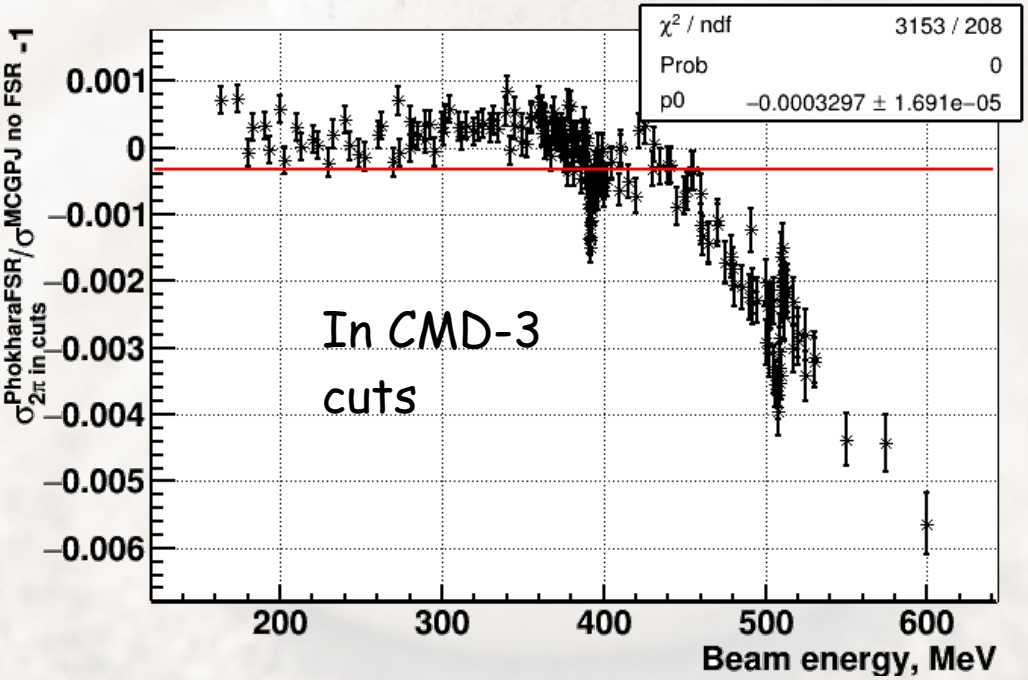
Phokara 10.0: For scan mode doesn't have FSR

CMD-3 cuts:
$|\Delta\varphi|<0.15$ rad, $|\Delta\theta|<0.25$ rad
$1<(\theta^+ + \pi - \theta^-)/2<\pi-1$ rad
$P^{+-} > 0.45\ E_{beam}$

ISR and Fπ cross check

MCGPJ with FSR off,
Phokara 10 with same Fπ as in MCGPJ





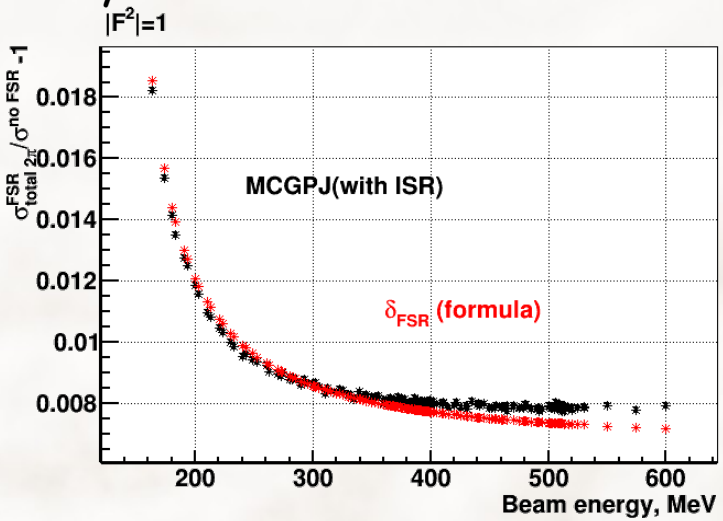
Cross section is consistent at ~0.05% at ρ-peak
(at phi ~ 0.25%)

MCGPJ FSR contribution

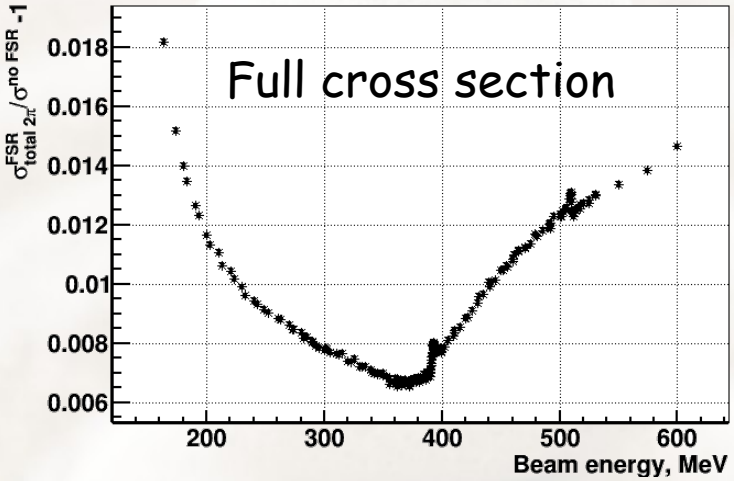
With Fpi=1 FSR is consistent with analytical formula at < 0.05%

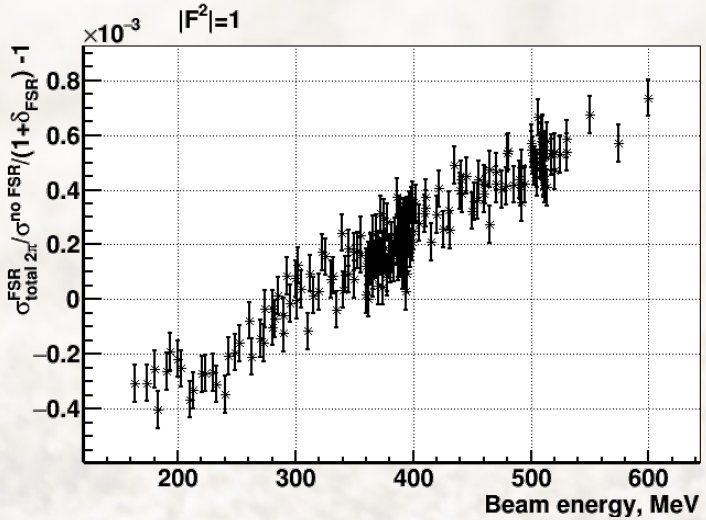
With full formfactor behaviour is different because of ISR return.

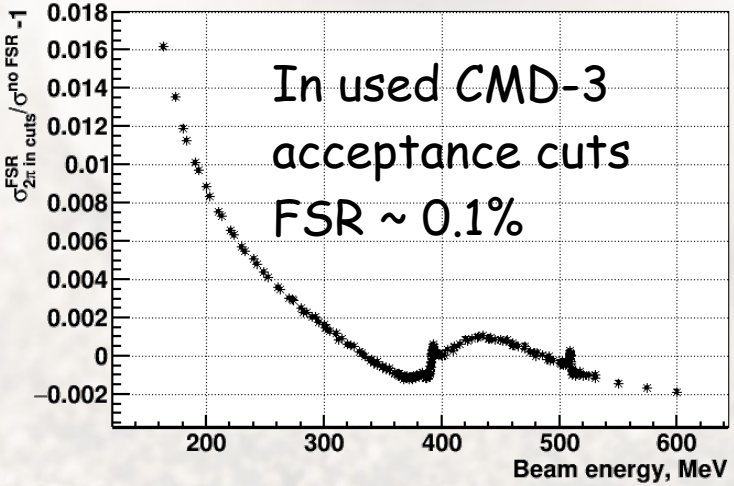
But looks reasonable



$$\frac{\sigma^{\mathrm{FSR}}}{\sigma^{\mathrm{no\,FSR}}} - 1$$





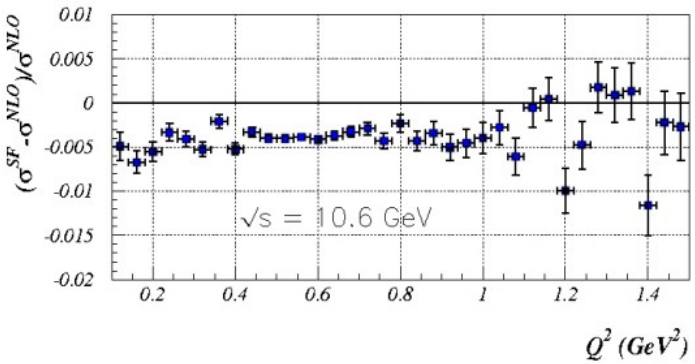


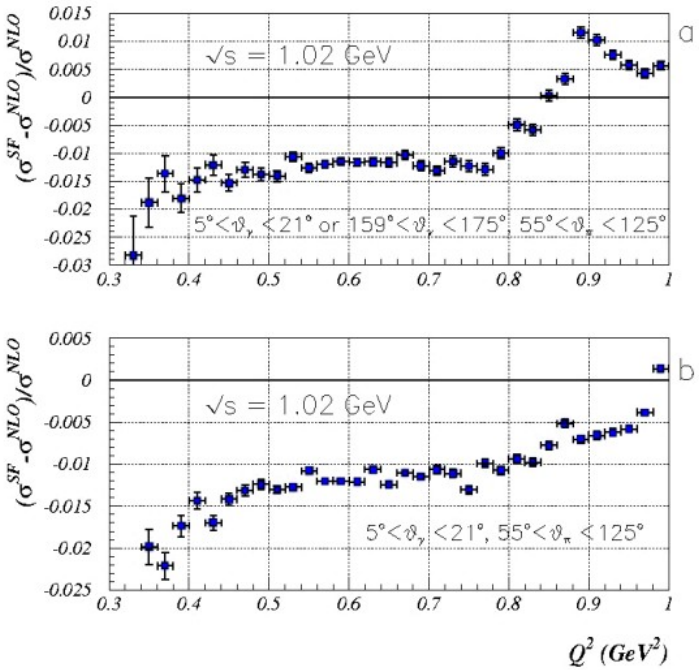
Phokhara vs EVA structure functions

Many studies with previous generators were performed during development of Phokhara

German Rodrigo, Henryk Czyz et al., Eur.Phys.J.C 24 (2002) 71-82 https://inspirehep.net/literature/568265

NLO vs SF





LL: EVA [Binner, Melnikov,Kühn]

EVA4π [Czyz,Kühn]

- resums big logs $L=\mathrm{Log}(s/m_e^2)$ to all orders
- Extra collinear emission integrated out: no momentum conservation
- Untagged photon: double counting

NLO: PHOKHARA

- LL+subleading terms (1%)
- Full angular dependence
- Momentum conservation
- Tagged or untagged photons
- ISR accuracy 0.5%

Phokhara vs KKMC

Phokhara NLO vs KKMC with photons exponential resumption
Comparison for μ+μ- channel (KKMC has only limited terms for ππ)

S. Jadach, Acta Phys.Polon.B 36 (2005) 2387
https://inspirehep.net/literature/685361

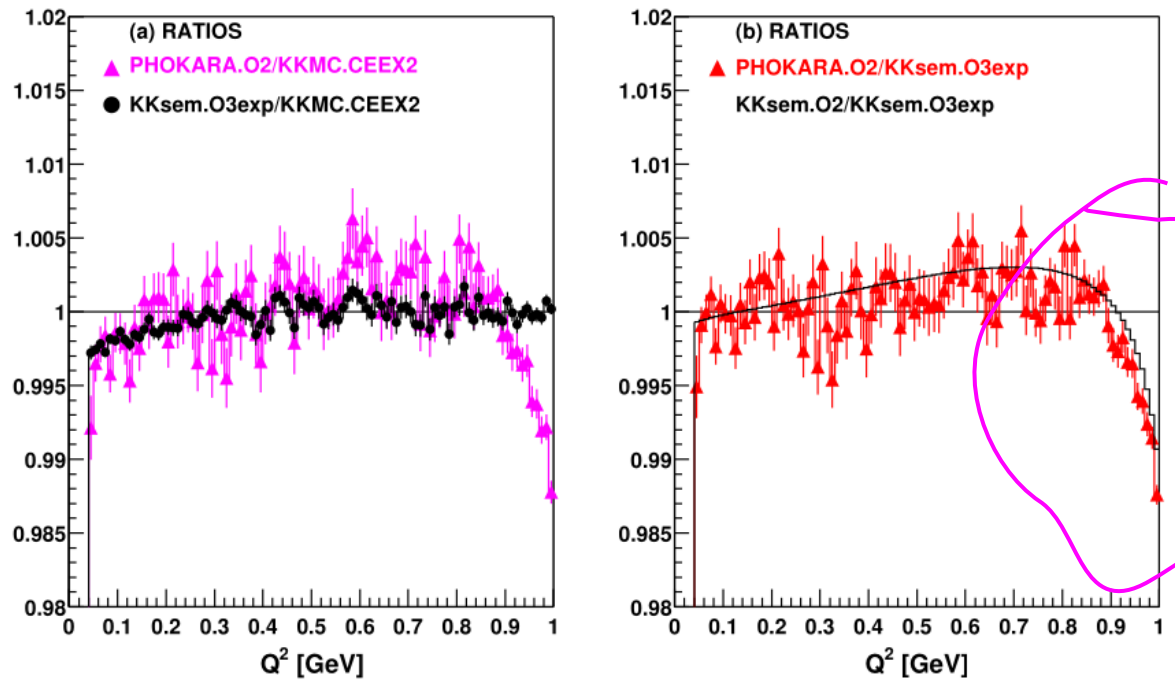


Figure 1: Mu-pair mass (square) spectrum in case of ISR only. $\sqrt{s} = 1.01942$GeV.

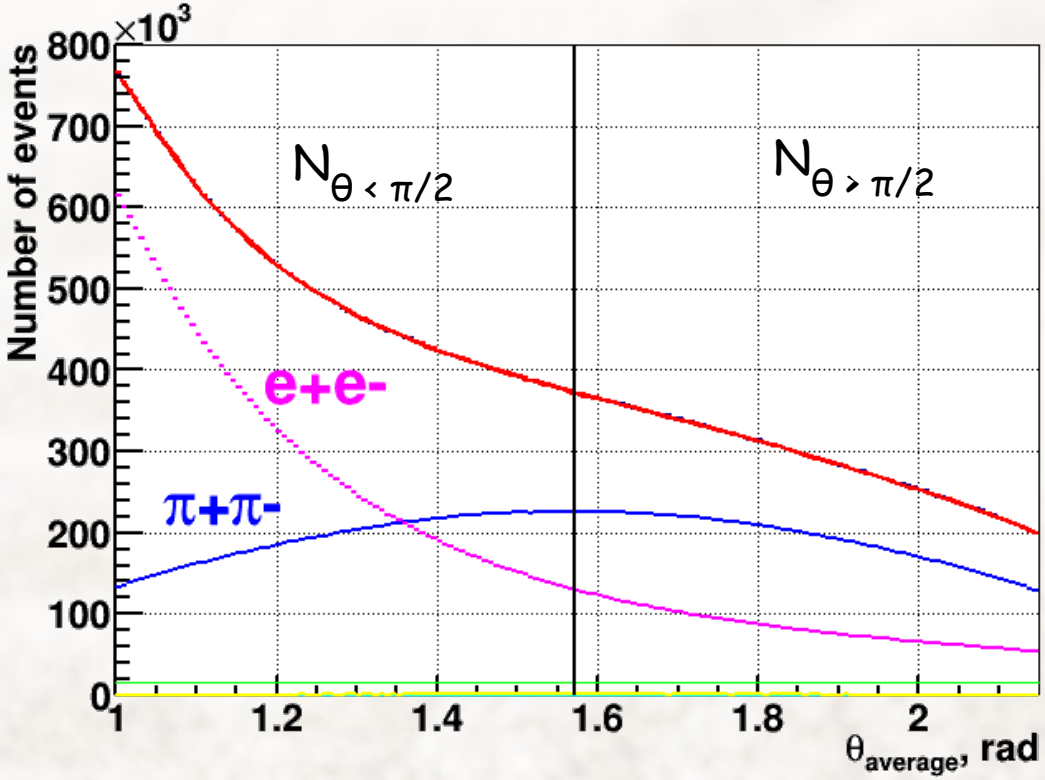
Lack of soft photon resumption in Phokhara (logarithmically enhanced corrections)

First step to implement NNLO logarithmic corrections were performed in Phokhara (not in official version) with stated effect of ~0.5%:

Szymon Tracz, Acta Phys.Polon.B 50 (2019) 1955-1964
https://inspirehep.net/literature/1772370

Forward backward charge asymmetry

dσ/dθ spectra



Asymmetry definition:

$A = (N_{\theta < \pi/2} - N_{\theta > \pi/2})/N$

Sensitive to:

- ✗ angle-related systematics
- ✗ used model of γ-π interaction

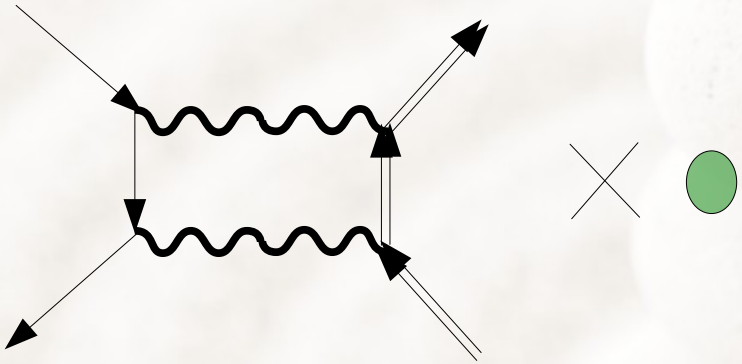
At first try:

1% inconsistency for π+π- was observed between data and MC prediction

sQED assumptions for radiative corrections

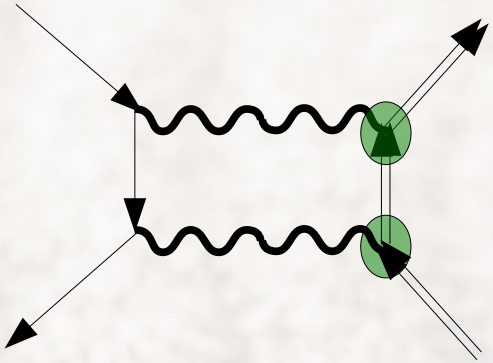
The radiative correction calculations is commonly done in the sQED approach,
It's mean that the calculations are performed without form factor,
then final Amplitude is scaled by $F(q^2)$

Scalar QED approach



$A = sQED*F(s)$

Proper way



$A \sim \int F(q_1)F(q_2)$

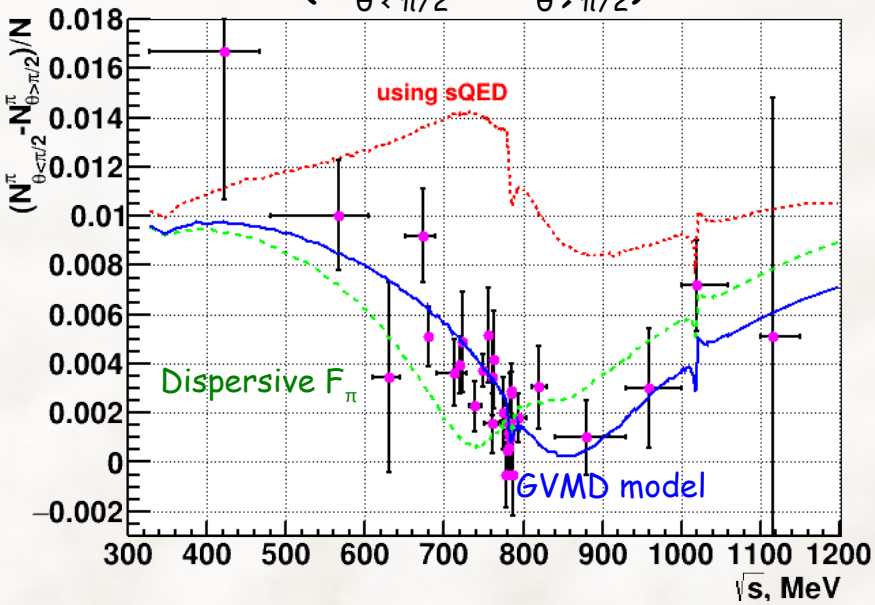
Proper way will be to put $F(q^2)$ to each vertex

N.B. It will be important to re-calculate radiative corrections
with above sQED for ISR measurement

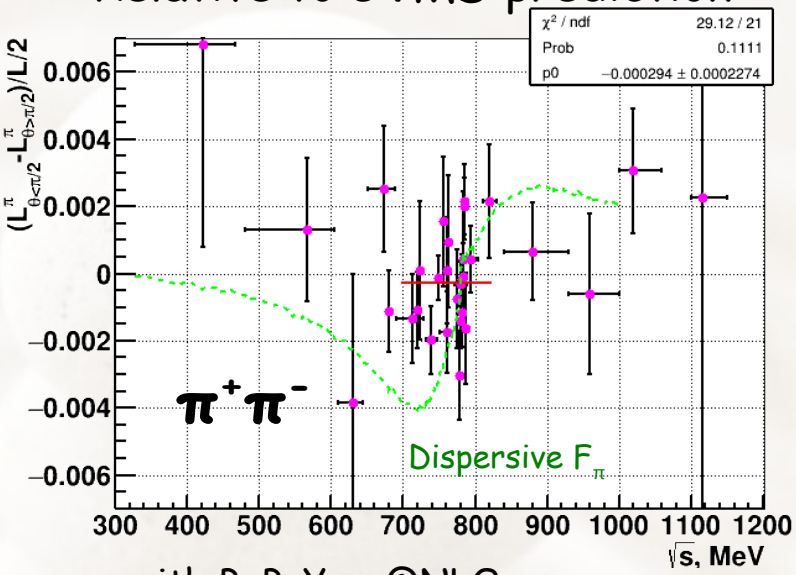
Charge asymmetry in e+e- -> π+π-

Precalculated amplitude of box diagram above sQED was added to MCGPJ

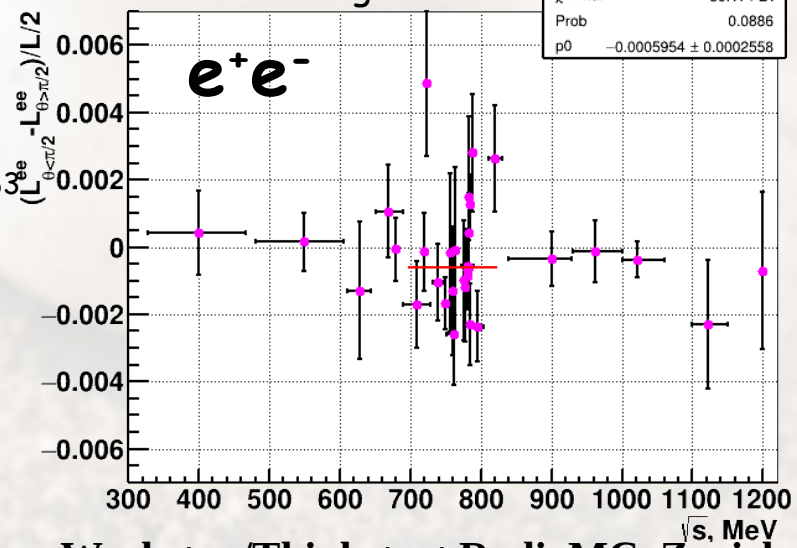
$A = (N_{\theta < \pi/2} - N_{\theta > \pi/2})/N$



Relative to GVMD prediction



with BaBaYaga@NLO



Conventional sQED approach gives ~ 1% inconsistency

The theoretical model within GVMD was introduced, describes well the CMD-3 data R.Lee et al., Phys.Lett.B 833 (2022) 137283

was confirmed by calculation in dispersive formalism

M.Hoferichter et al., JHEP 08 (2022) 295

Average at $\sqrt{s}$ = 0.7-0.82 GeV:

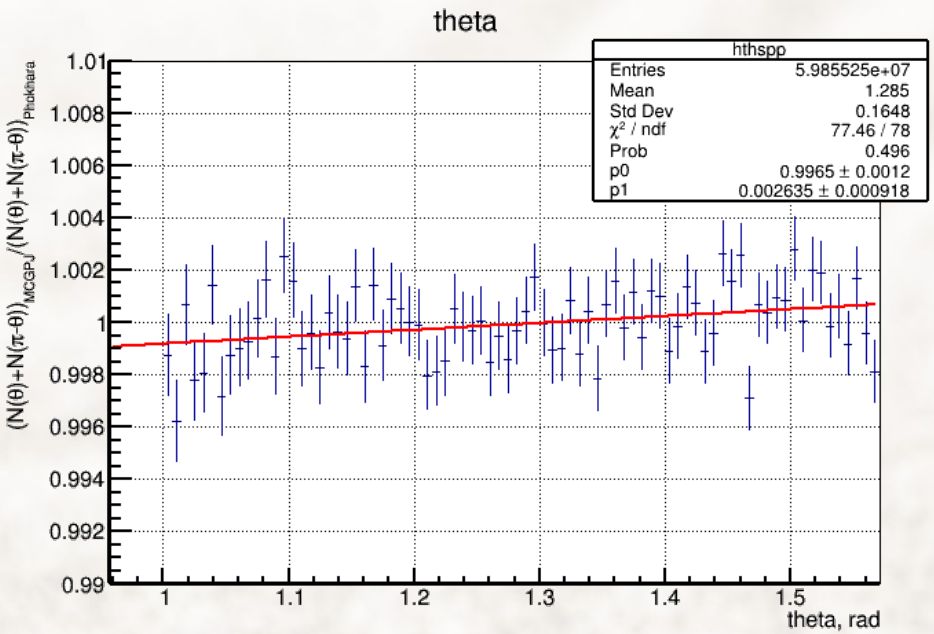
$\pi^+\pi^-$: $\langle\delta A\rangle$ = -0.029 ± 0.023 %

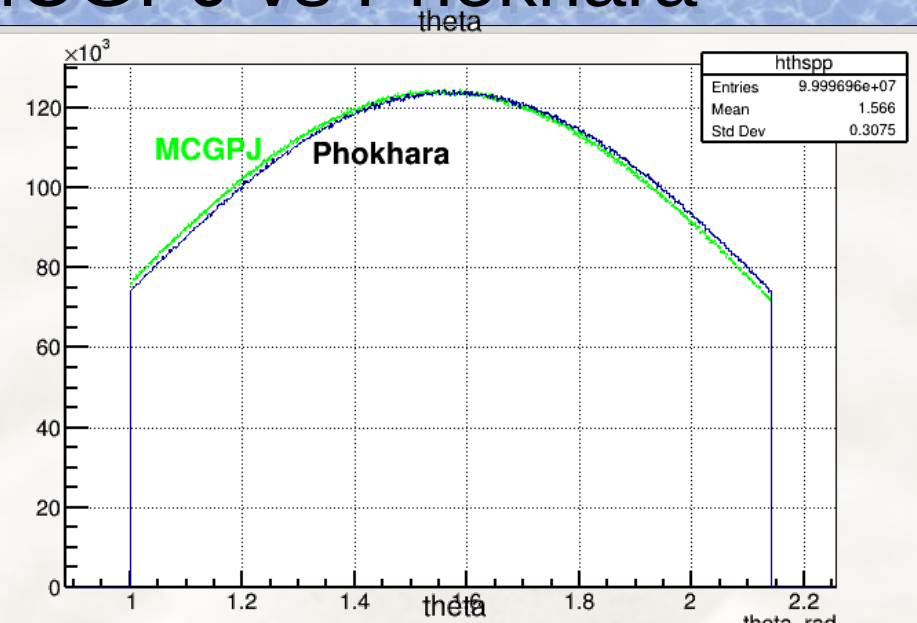
e^+e^-: $\langle\delta A\rangle$ = -0.060 ± 0.026 %

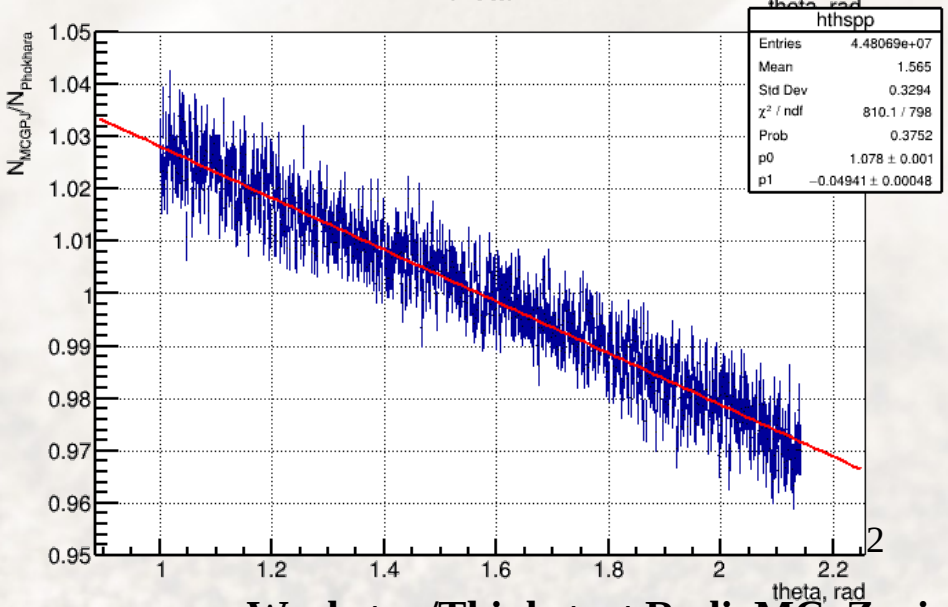
ππ Angle distribution, MCGPJ vs Phokhara

Phokara 10.0: For scan mode without FSR

Don't have any charge asymmetry.





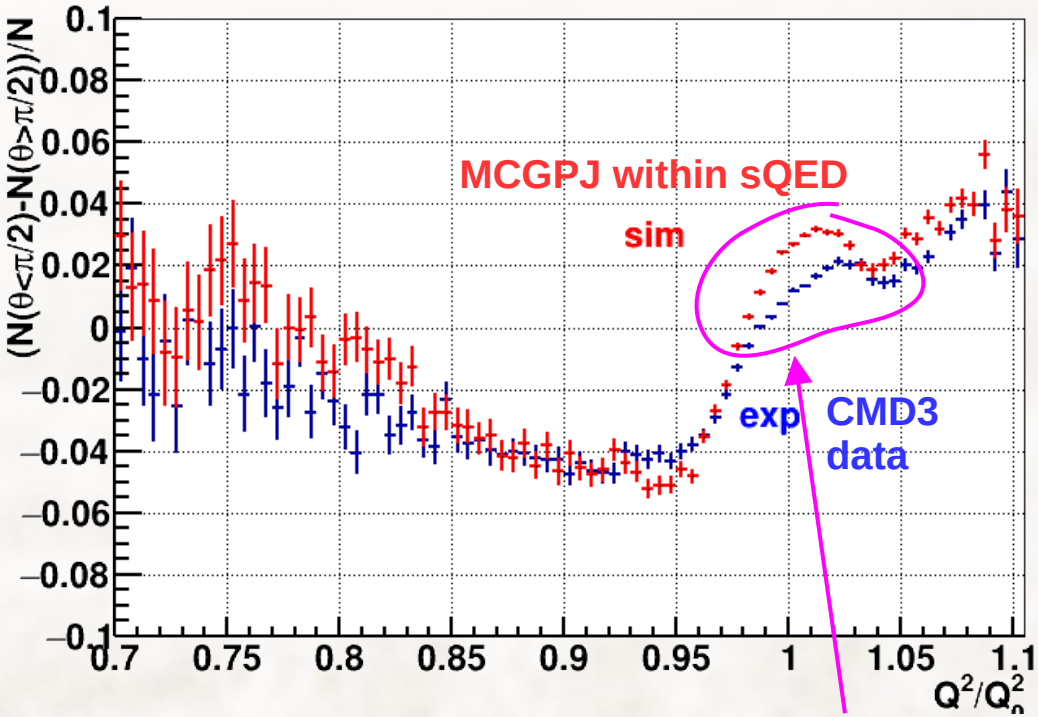


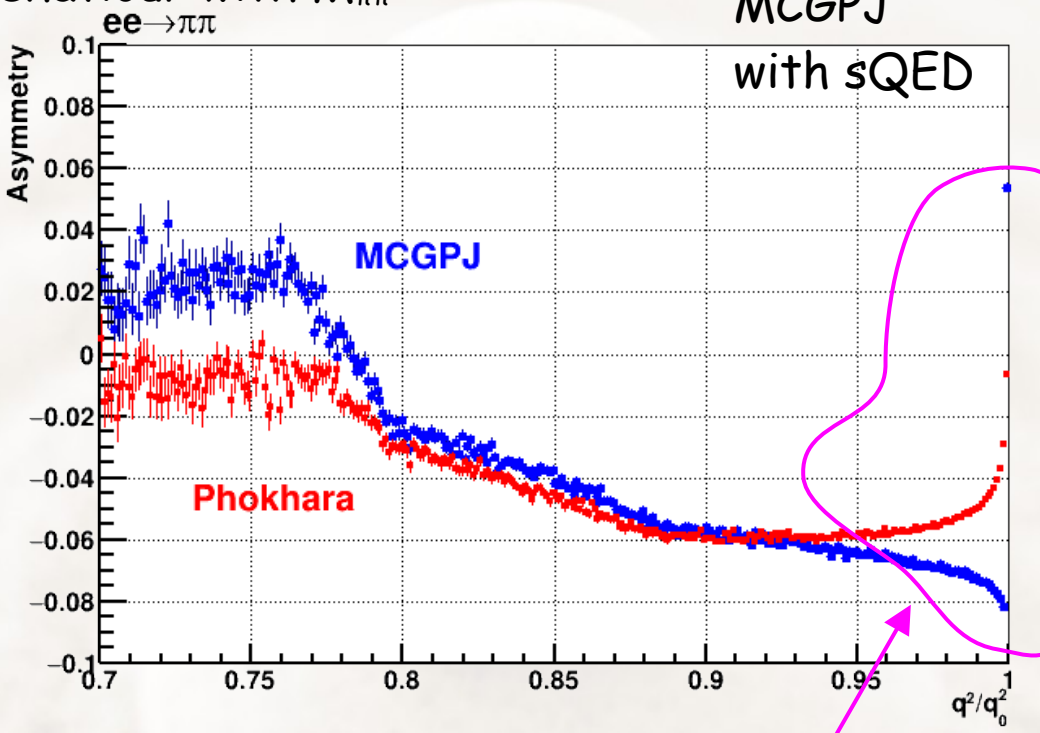
Symmetrized over θ=π/2 differential cross section consistent at ~ <0.15%

At level of experimental sensitivity

Asymmetry for MCGPJ/Phokhara

Asymmetry behaviour with $M_{\pi\pi}$





Difference from sQED limitation
for virtual correction (in box diagram)

MCGPJ have only one photon on large angle or collinear jets along e+e- (but integrally over range gives proper number)
Phokhara don't have point at $M_{\pi\pi}^2 \sim s_0$

KLOE new analysis of x5 statistics have plans to look on the asymmetry with $M_{\pi\pi}$ calculation with above sQED for $e^+e^- \to \pi^+\pi^- + \gamma$ will be required

Summary

e+e- → π+π-(γ) generator

For scan experiment: Unfortunately only MCGPJ available with declared 0.2% precision
For ISR: only Phokhara with 0.5% precision

It is quite desirable to have precise e+e- →π+π-(γ) generator with ~0.1%:
- NNLO + logarithmically enhanced corrections
- with above sQED calculation
- cover both ISR and scan modes (with radiator function up $M_{\pi\pi}^2 = s_0$)

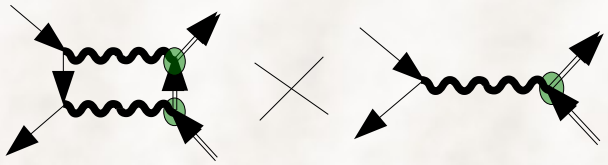
KLOE new analysis will stress and may be limited in some parts by radiative correction
- probably first most important question: 1 real with box diagram above sQED
- next question: logarithmically enhanced corrections for 0.1% precision

CMD-3 analysis can be clarified in some parts by more precise generators

How it can affect pion form factor measurements?

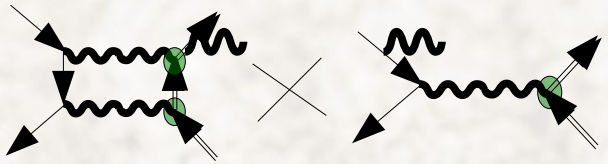
Usually event selections in analyses are charge/angle symmetric

Main effect at lowest order comes from:
Interference of box vs born diagrams



=> only charge-odd contribution
effect is integrated out
in full cross-section

Interference of ISR & box vs FSR (or v.v.)



=> charge-even
can affect integrated cross-section

Backup

ISR measurements

The team:
F. Campanario, G. Rodrigo, Sz. Tracz (Valencia)
H.C., J. Gluza, (Katowice)
T. Jeliński, D. Zhuridov (left physics)

Henryk Czyz
the Muon g-2 Theory Initiative Worksop 2019

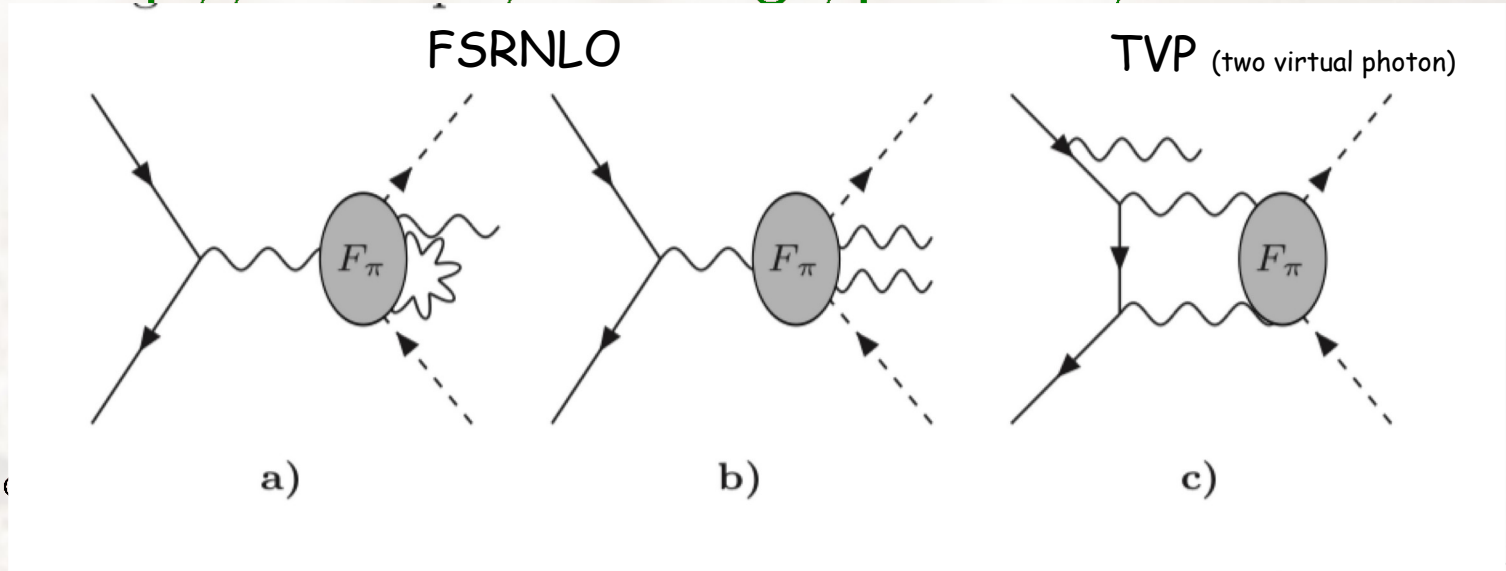
NLO $e^+e^- \to \pi^+\pi^-\gamma$

Status - finished: arXiv:1903.10197

F. Campanario et al.
Phys.Rev.D 100 (2019) 7, 076004

⇒ sQED + form factors:
FSR at NLO and pentaboxes ready and fully tested

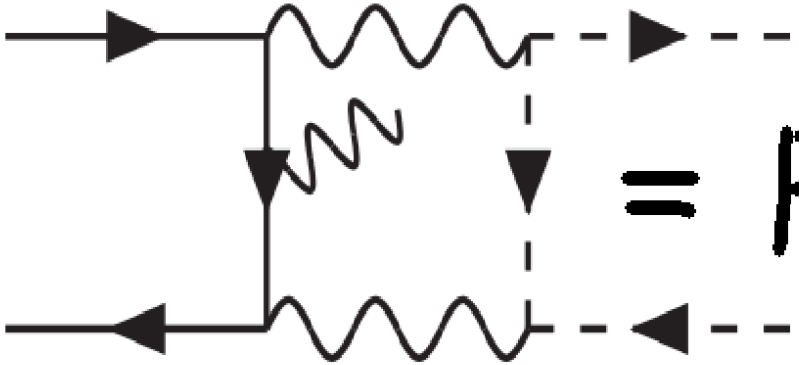
⇒ Phokhara10.0
http://ific.uv.es/∼ rodrigo/phokhara/



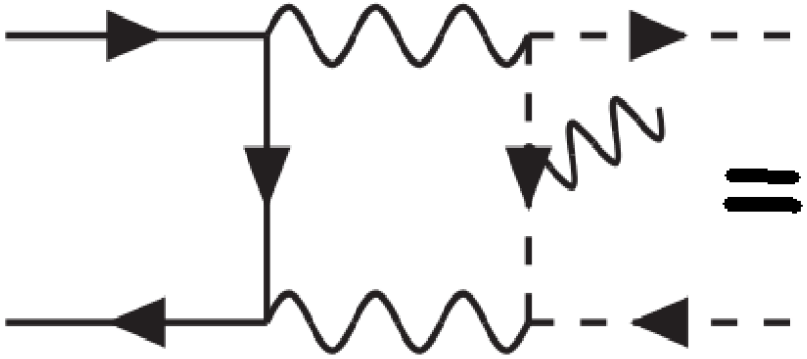
sQED assumption

Henryk Czyz
the Muon g-2 Theory Initiative Worksop 2019

Model assumptions



$= F(q^2) \cdot sQED$

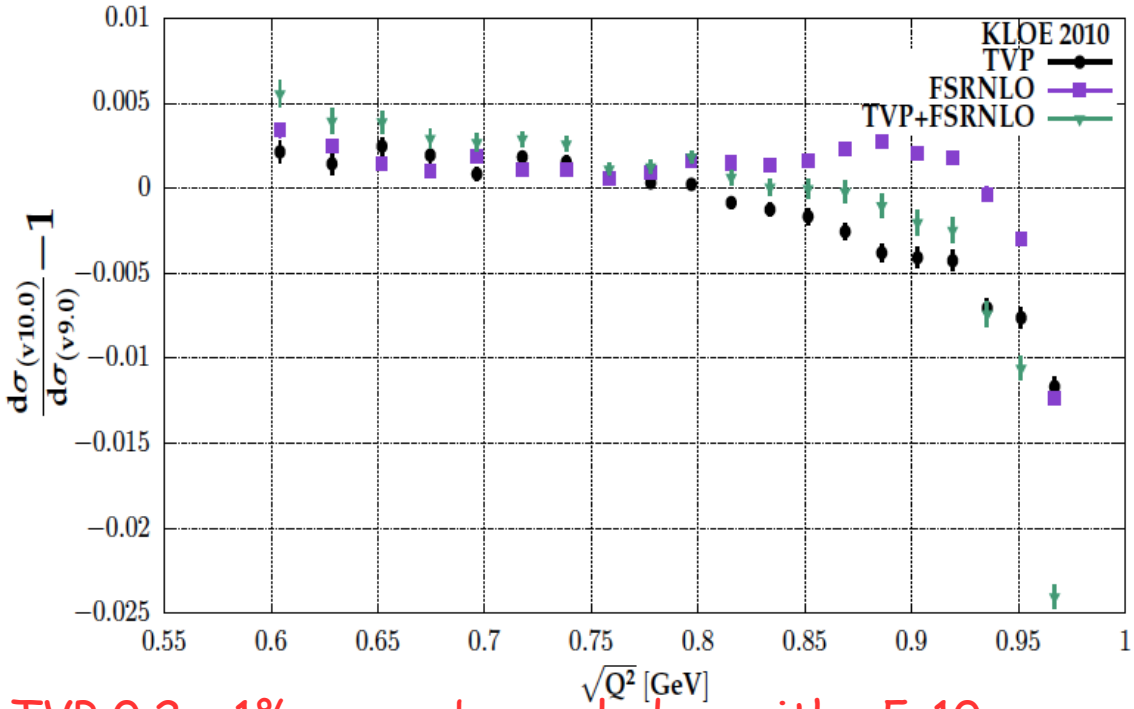


$= F(s) \cdot sQED$

H. Czyż Radiative corrections in PHOKHARA and EKHARA MC generators, 12

ISR measurements

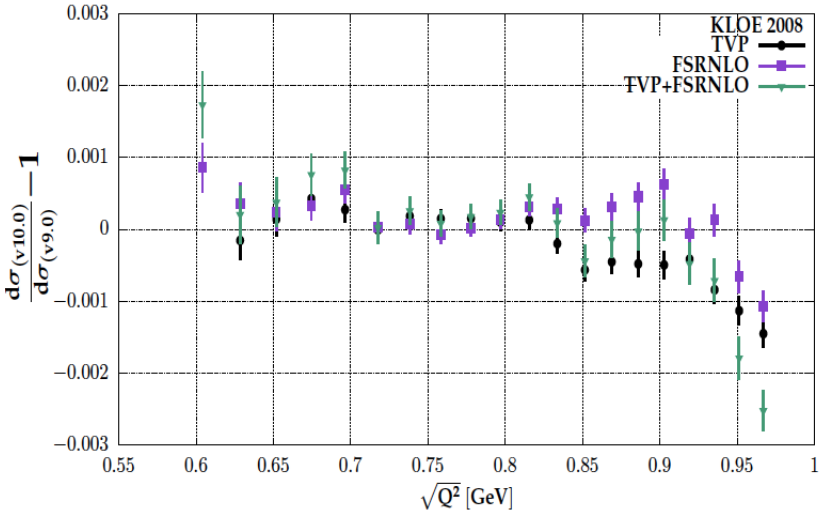
Complete NLO: KLOE-large



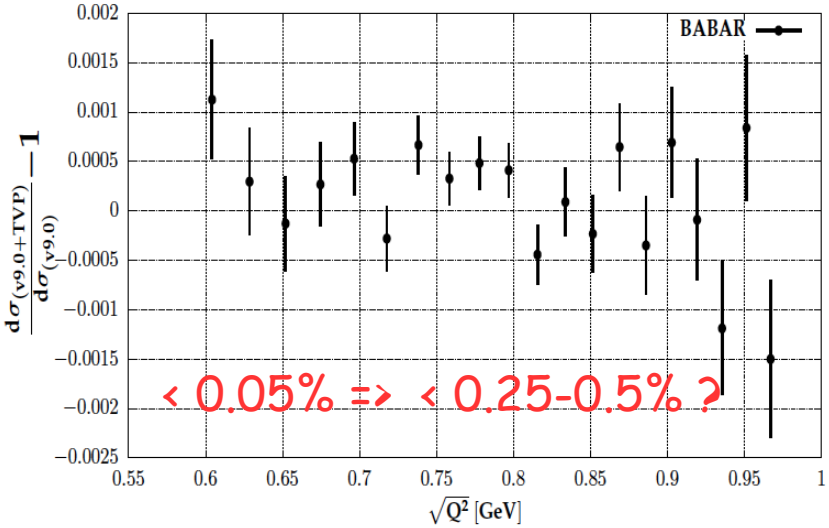
TVP 0.2 – 1% => can be scaled up with x5-10 to 1-5% correction?

KLOE-2010 with tag photon measurement can be affected

Complete NLO: KLOE-small



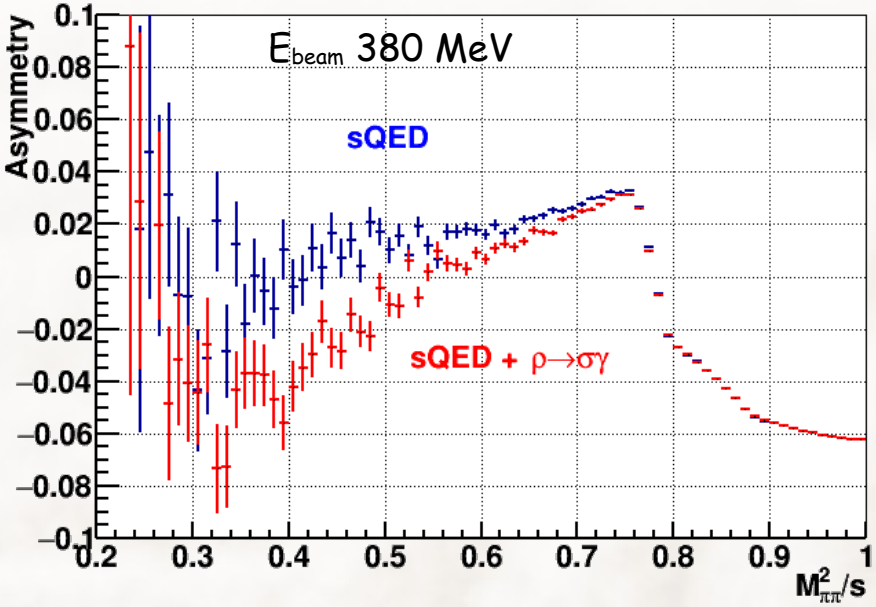
Complete NLO: BaBar

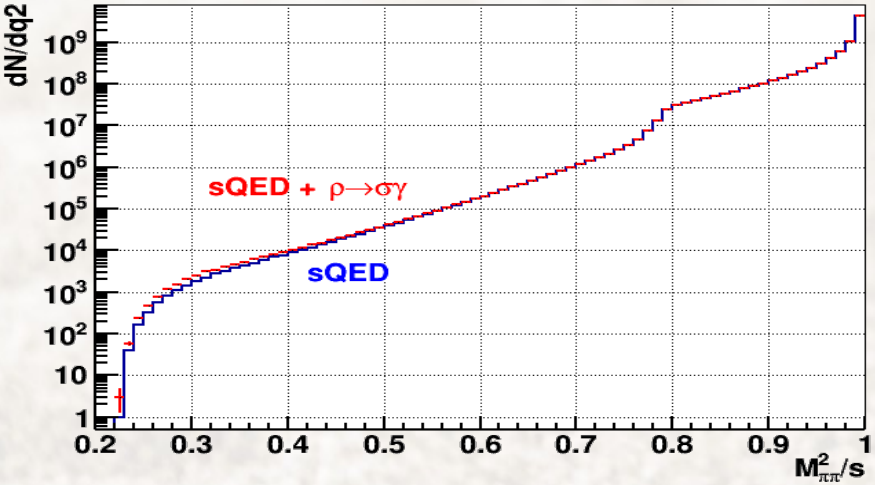


< 0.05% => < 0.25-0.5% ?

Scalar production

Could it be: $e^+e^- \to \rho \to \sigma\gamma$ or $a_1^{\pm}\pi^{\pm}$?





With help of FASTERD generator

O. Shekhovtsova, G. Venanzoni, G. Panccheri, Comp.Phys.C. 180 (2009) 1206-1218

Mixed in $\rho \to \sigma\gamma$ instead of $\varphi \to (f_0+\sigma)\gamma$
in non structure model
with some rough σ production parameters

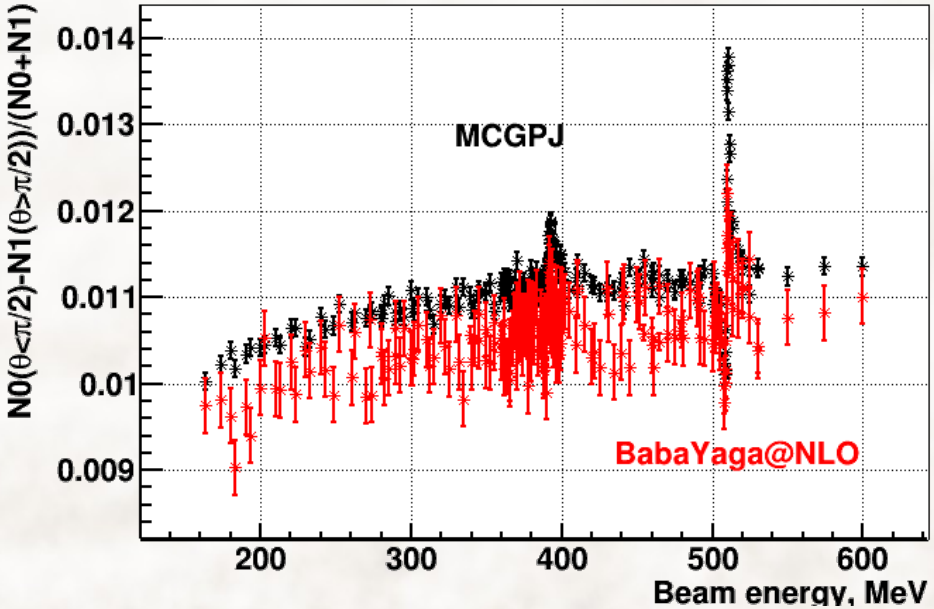
$|\delta A|$ ~< 2×10^{-5} effect only in far tails

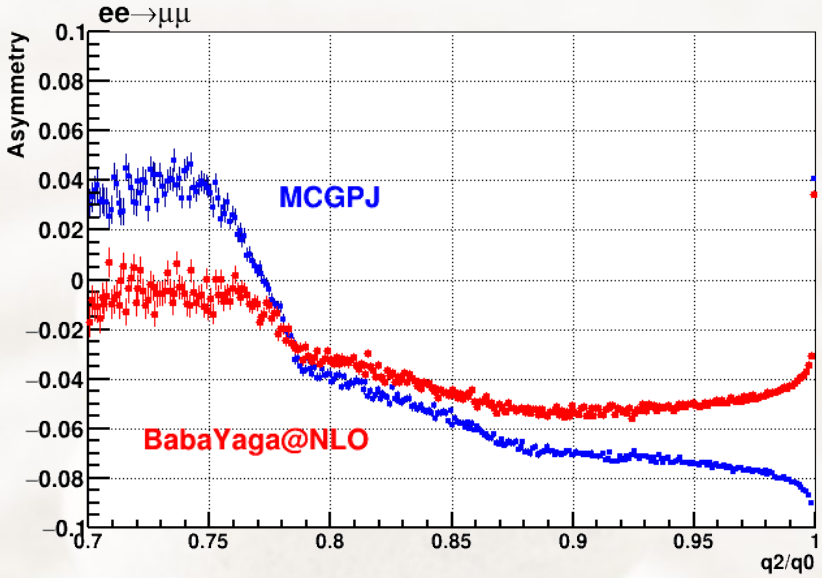
Br ($\rho \to \sigma\gamma$) ~ 1×10^{-4} [x2 Br($\rho \to \pi^0\pi^0\gamma$)]
Interference with sQED $e^+e^- \to \pi^+\pi^-\gamma$: => ~ 1×10^{-3}
x Collinearity selection cuts 1×10^{-2}
Total rate ~ 10^{-5} too small to affect something

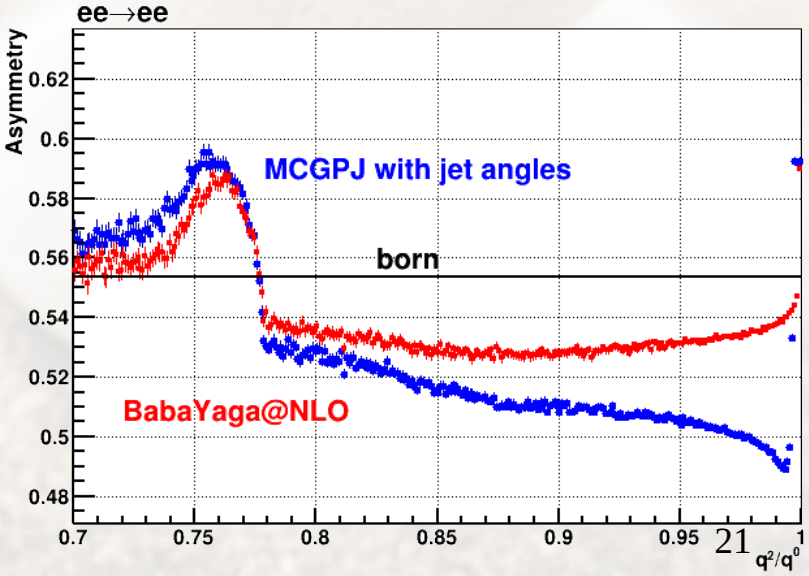
$\rho \to a_1^{\pm}\pi^{\pm}$ effect should be same or less:
Phys.Rev.D 76 (2007) 033001

Generators MCGPJ/BabaYaga@NLO

Для μ+μ- интегральная асимметрия совпадает
между MCGPJ/BabaYaga@NLO
с абс. точностью ~0.05% (5% относительная точность)

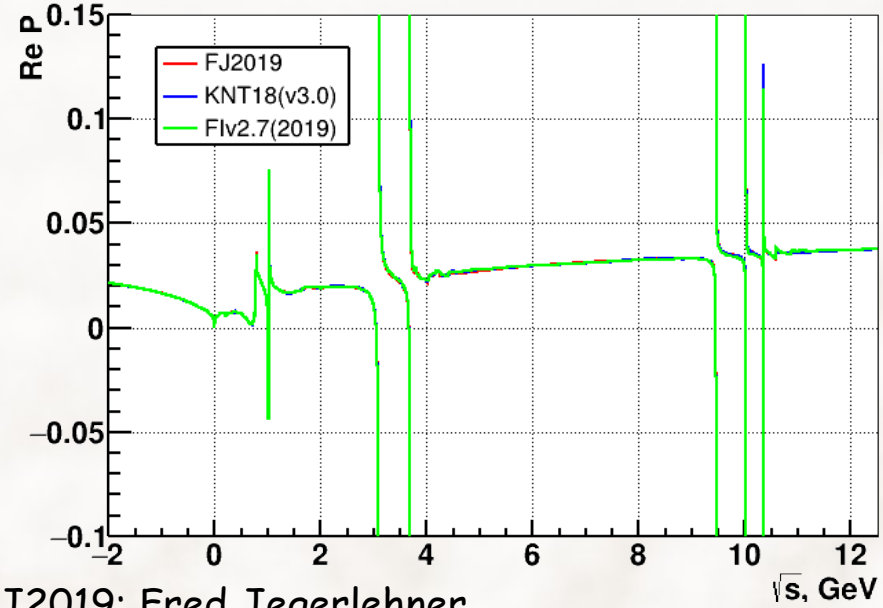


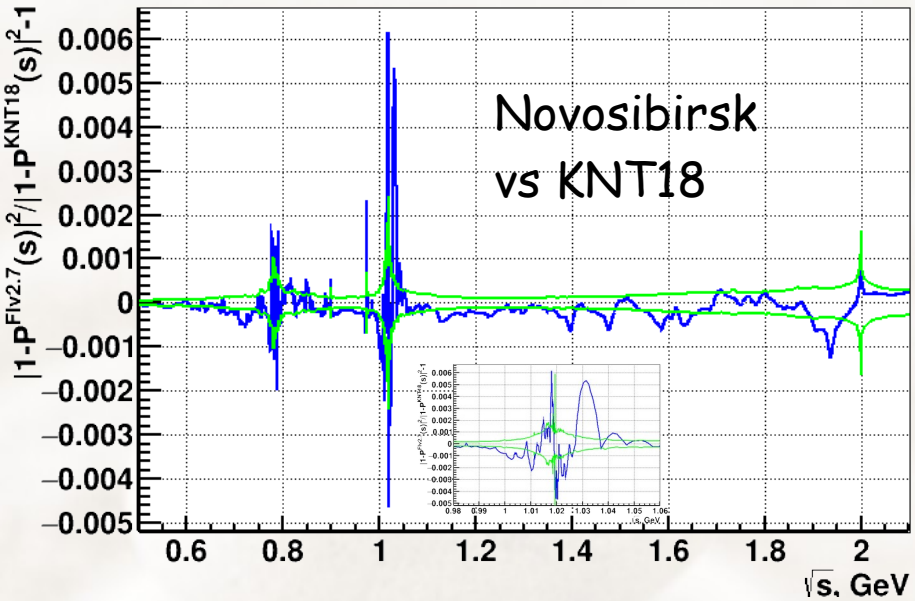


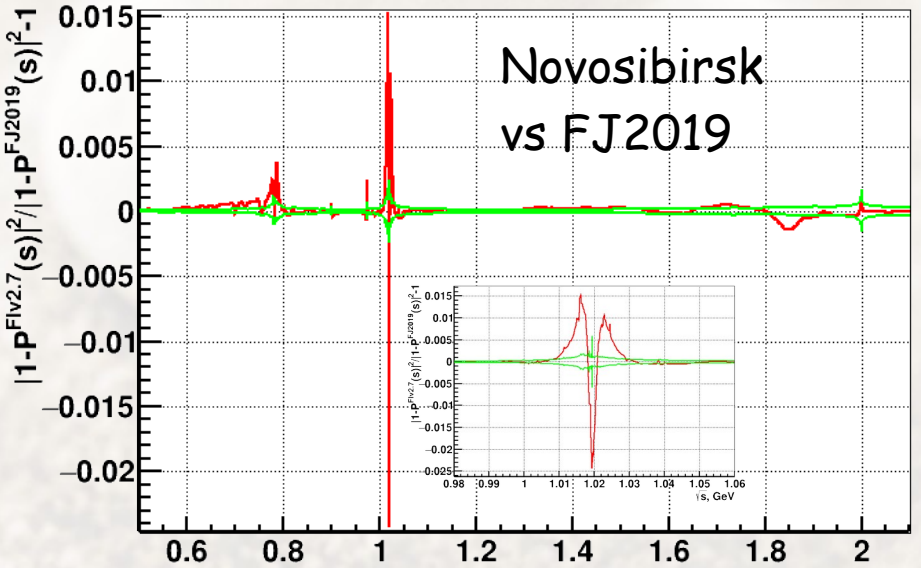


BabaYaga@NLO моделирует фотоны рекурсивно
У нас только один фотон на большой угол
Поведение BabaYaga около q2~ 1 более физично
Скорее всего это отличие дает эффект в систематику
разделения по P из-за разницы генераторов

Vacuum polarization





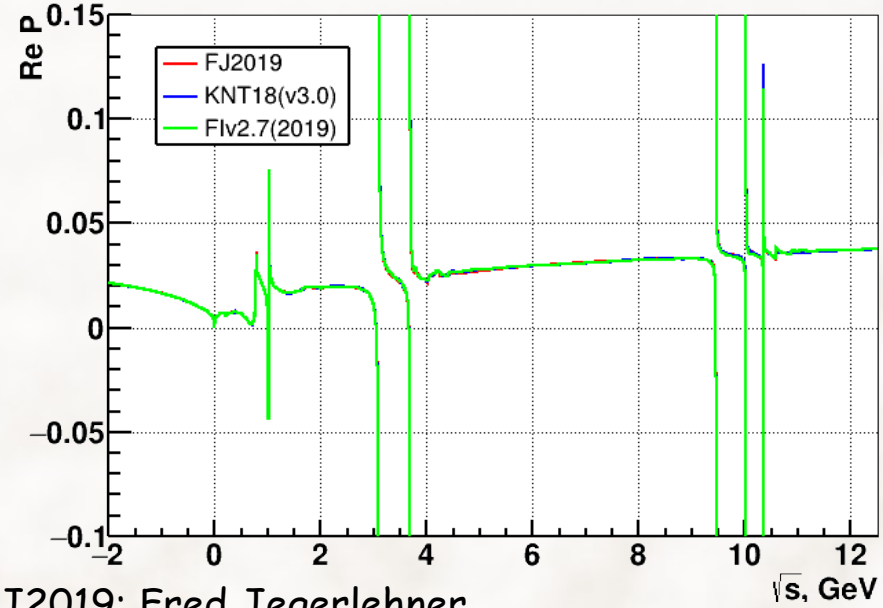


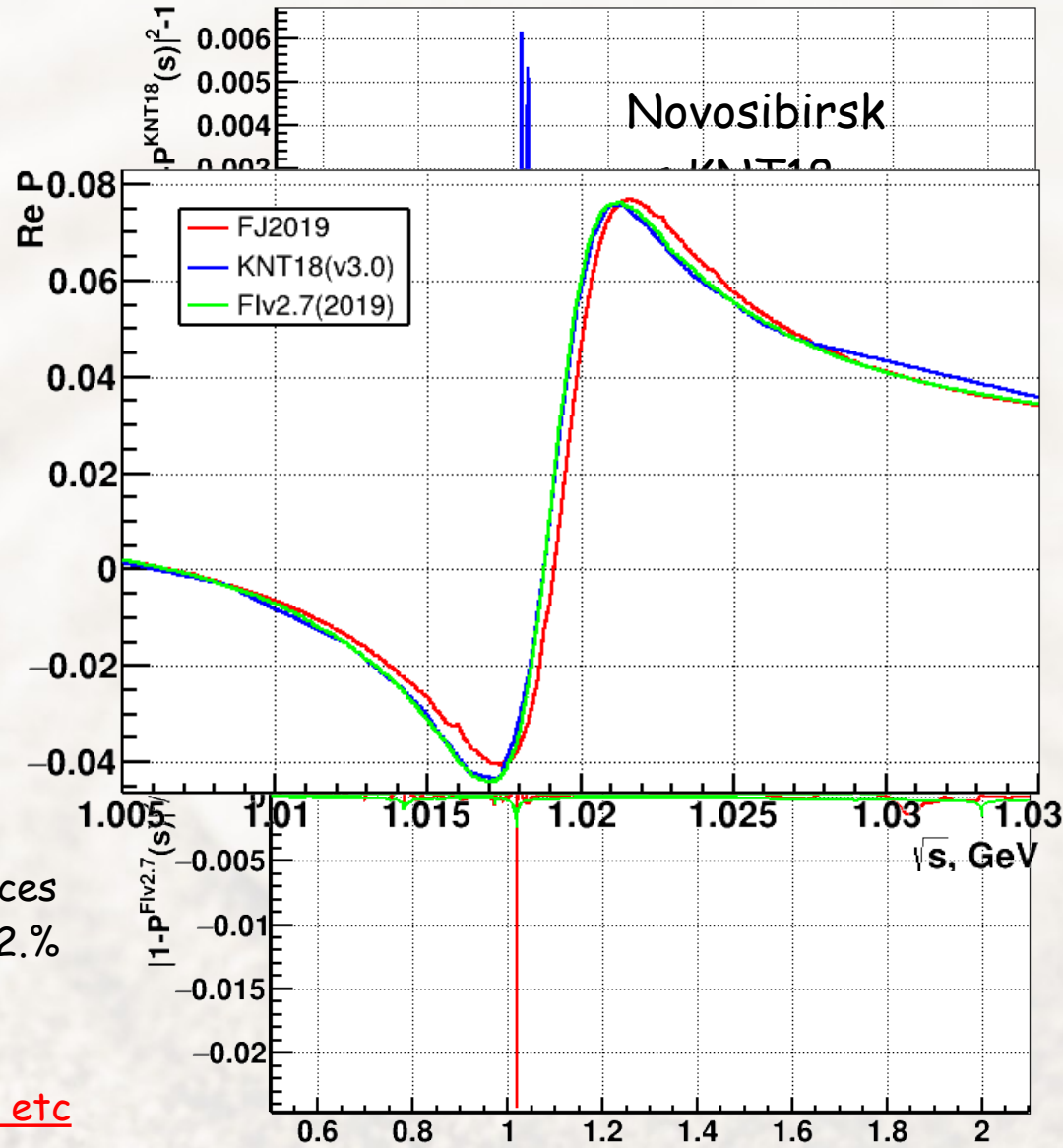
- FJ2019: Fred Jegerlehner
  http://www-com.physik.hu-berlin.de/~fjeger/software.html
- KNT18(v3.0): A Keshavarzi, D Nomura, T Teubner
- FIv2.7(2019): Novosibirsk VP
  https://cmd.inp.nsk.su/~ignatov/vpl/

VP consistent at 0.05-0.1% outside of narrow resonances
At phi – statistical inconsistency ~0.5%, FJ up to 1.5-2.%

Fred is using dressed phi with PDG parameters
(should be bare Mφ, which shifted by 254 keV)

Vacuum polarization





- FJ2019: Fred Jegerlehner
  http://www-com.physik.hu-berlin.de/~fjeger/software.html
- KNT18(v3.0): A Keshavarzi, D Nomura, T Teubner
- FIv2.7(2019): Novosibirsk VP
  https://cmd.inp.nsk.su/~ignatov/vpl/

VP consistent at 0.05-0.1% outside of narrow resonances
At phi – statistical inconsistency ~0.5%, FJ up to 1.5-2.%
Fred is using dressed phi PDG parameters
(should be bare Mφ, which shifted by 254 keV)
Be careful with VP using at narrow resonances φ, J/ψ, etc

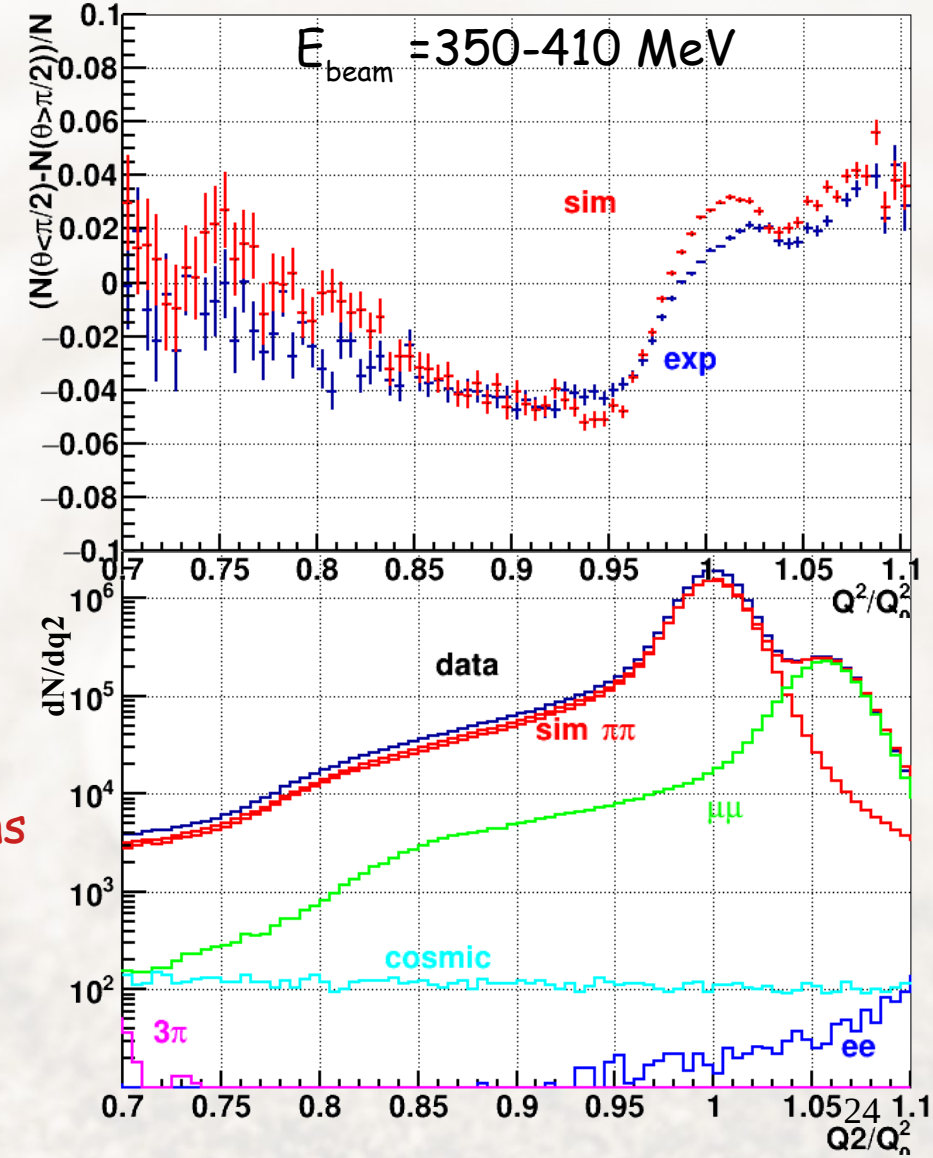
Asymmetry with Mπ2

<u>Asymmetry vs $M_{\pi\pi}^2$</u>

Sample of 2π can be selected by energy deposition as MIP with E_{LXe}^{+-}<100 MeV (with some admixture of 2μ)

Comparison with full mixed simulation

Main difference comes from $M_{\pi\pi}^2/s \sim 1$: correspond to virtual/soft radiative corrections



Phokhara vs EVA

Phokhara NLO with Eva collinear Structure Function

German Rodrigo, Henryk Czyz et al., Eur.Phys.J.C 24 (2002) 71-82 https://inspirehep.net/literature/568265

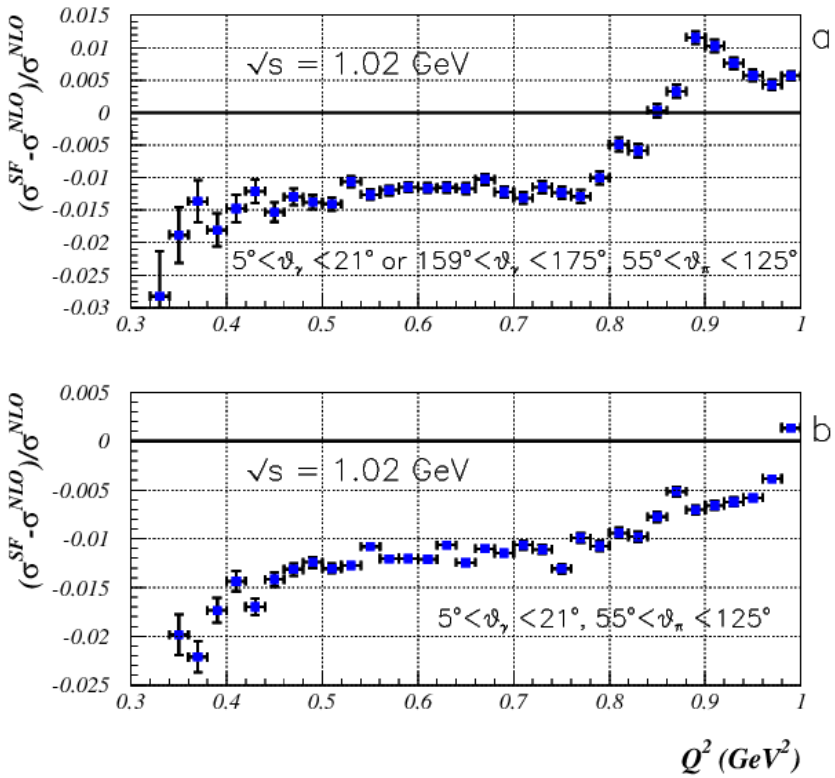


Fig. 7. Comparison between the collinear approximation by structure functions and the fixed order NLO result. Cuts from Table 1 for the lower figure. In the upper figure, same cuts as below, with the addition of the symmetric photon configuration $159° < \theta_\gamma < 175°$.

Depend on used cuts

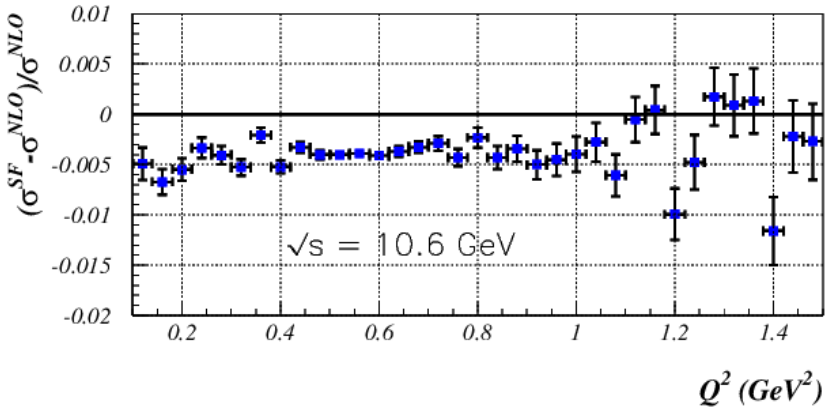


Fig. 9. Comparison between the collinear approximation by

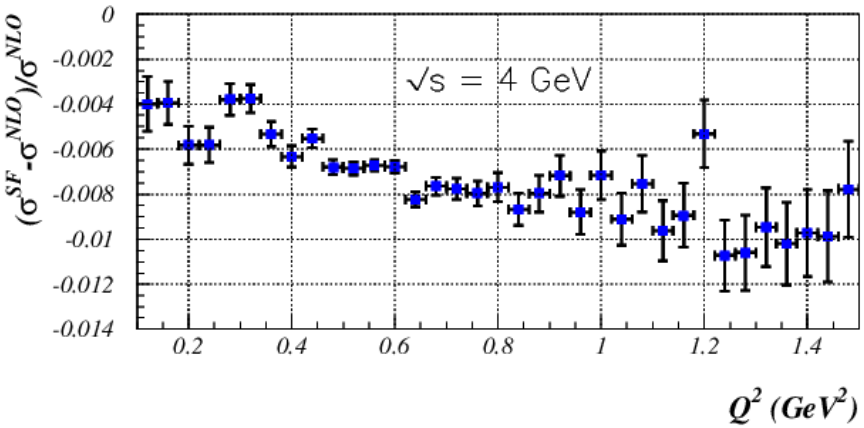


Fig. 11. Comparison between the collinear approximation by structure functions and the fixed order NLO result. Cuts from

MC generators e+e- → e+e-

Several MC generators available with 0.1-0.5% precision.
Most recent e+e- -> e+e- (gamma) generators
include exact NLO + Higher Order terms in some approximation:

0.1%
e+e-, μ+μ-

BabaYaga@NLO (KLOE,BaBar, BESIII)
Parton shower approach: n photons with angle distribution,
interference for 1 photon radiation

Accuracy 0.2%
e+e-, μ+μ-, π+π-,
etc

MCGPJ (VEPP-2000)
1 real photon (from any particle)
+ photon jets along all particles (collinear Structure function)
v2: + jets angle distributions

0.5% (~0.1%?)
e+e-

BHWIDE (LEP)
n real photons by Yennie-Frautschi-Suura (YFS) exponentiation method
interference on O(α) level

<0.1%
e+e-, etc

McMule
Fixed order NNLO

under development
e+e−, μ+μ- , ZH, ...

ReneSANCe (from Dubna)
NLO + leading log corrections for ISR

And there are other generators for μ+μ-:
PHOKHARA (KLOE) μ+μ-, π+π- etc , KKMC (μ+μ-), etc

BabaYaga@NLO vs MCGPJ generators

Only two e+e- → e+e- generators available with claimed precision ~ 0.1%
BabaYaga@NLO & MCGPJ

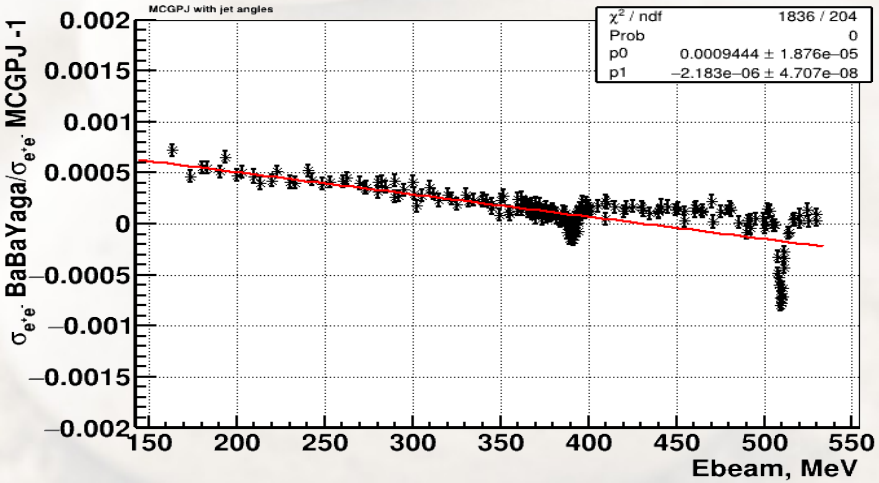
Integrated cross-section is consistent at the level <0.1%
(0.06-0.% for 2E = 0.3-1.0 GeV)

<u>In CMD3 Selection cuts:</u>

$|\Delta\varphi|<0.15$, $|\Delta\theta|<0.25$, $1< \theta_{average}<\pi -1$, $P^{+-} >0.45\ E_{beam}$

<u>Calculated cross-section at E beam=391.48 MeV</u>

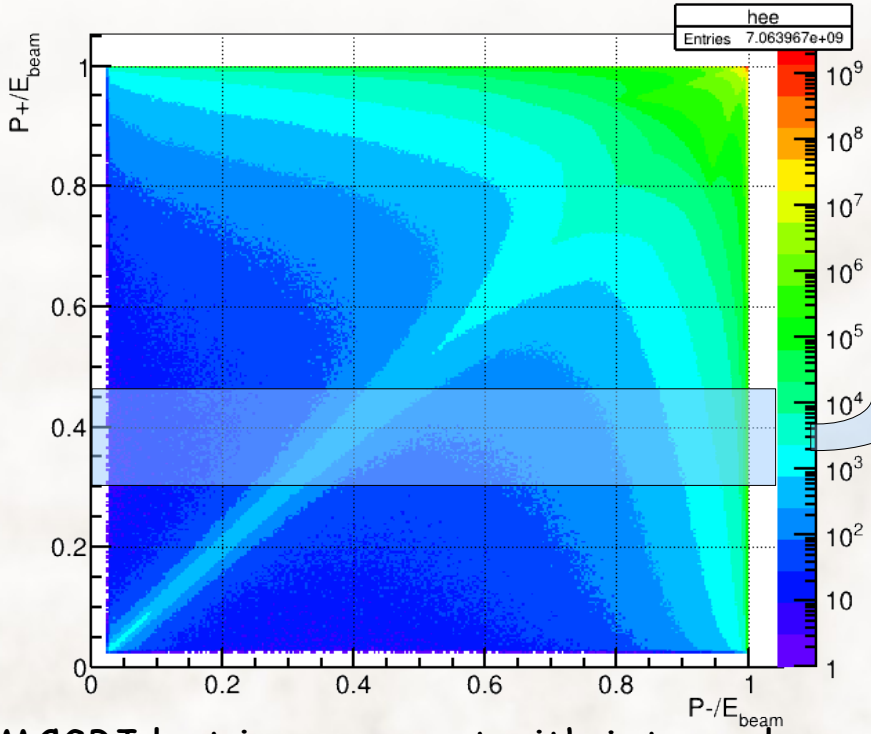
MCGPJ : 751.269 +- 0.007 nb
BabaYaga@NLO : 751.223 +- 0.009 nb
Δ ~ 0.01%

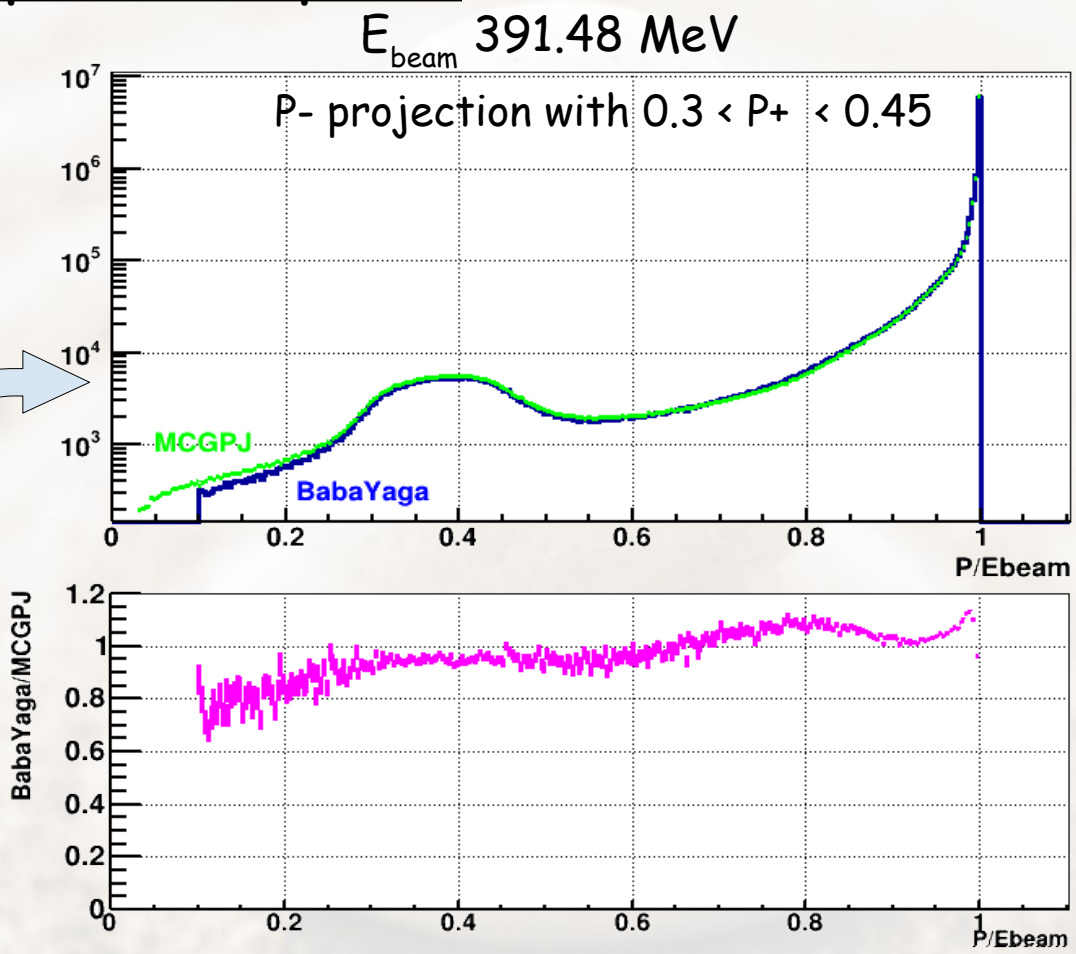


N.B. MCGPJ last improvement with introduction of jet angle distribution greatly improved differential distribution, but gives only modest change of total cross-section: -0.06%

MCGPJ vs BabaYaga bhabha P+ vs P- spectrum

Differential over momentum spectrum comparison





MCGPJ last improvement with jets angles reduce discrepancy from x1.6-3 to x1.1

Momentum spectrum still disagree at level ~ 10%

Tails comes from e+e- → e+e- γγ , NNLO order

Very desirable to have more precise generators

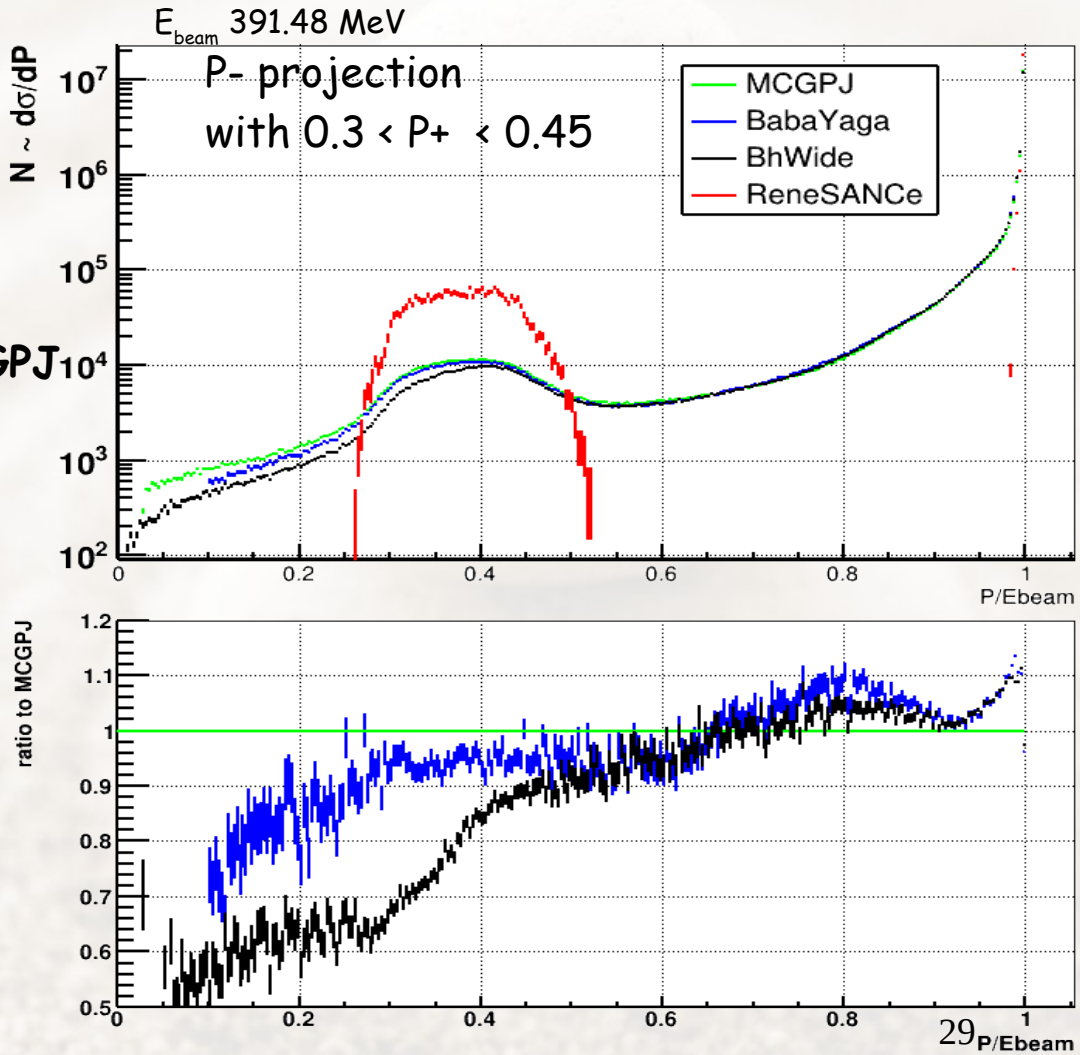
Such discrepancy gives 0.3% systematic for π+π- at ρ-peak using momentum analysis at CMD3

Other e+e- generators

Differential momentum spectrum comparison

Integrated cross-section (in CMD3 selection cuts) at E beam=391.48 MeV:

		Δ to MCGPJ
MCGPJ vs jetangles	: 751.269 +- 0.007 nb	
BabaYaga@NLO	: 751.223 +- 0.009 nb	<0.01%
BHWIDE v1.05	: 751.428 +- 0.006 nb	0.02%
SANCe v1.2.0	: 754.75 +- 0.05 nb	0.5%



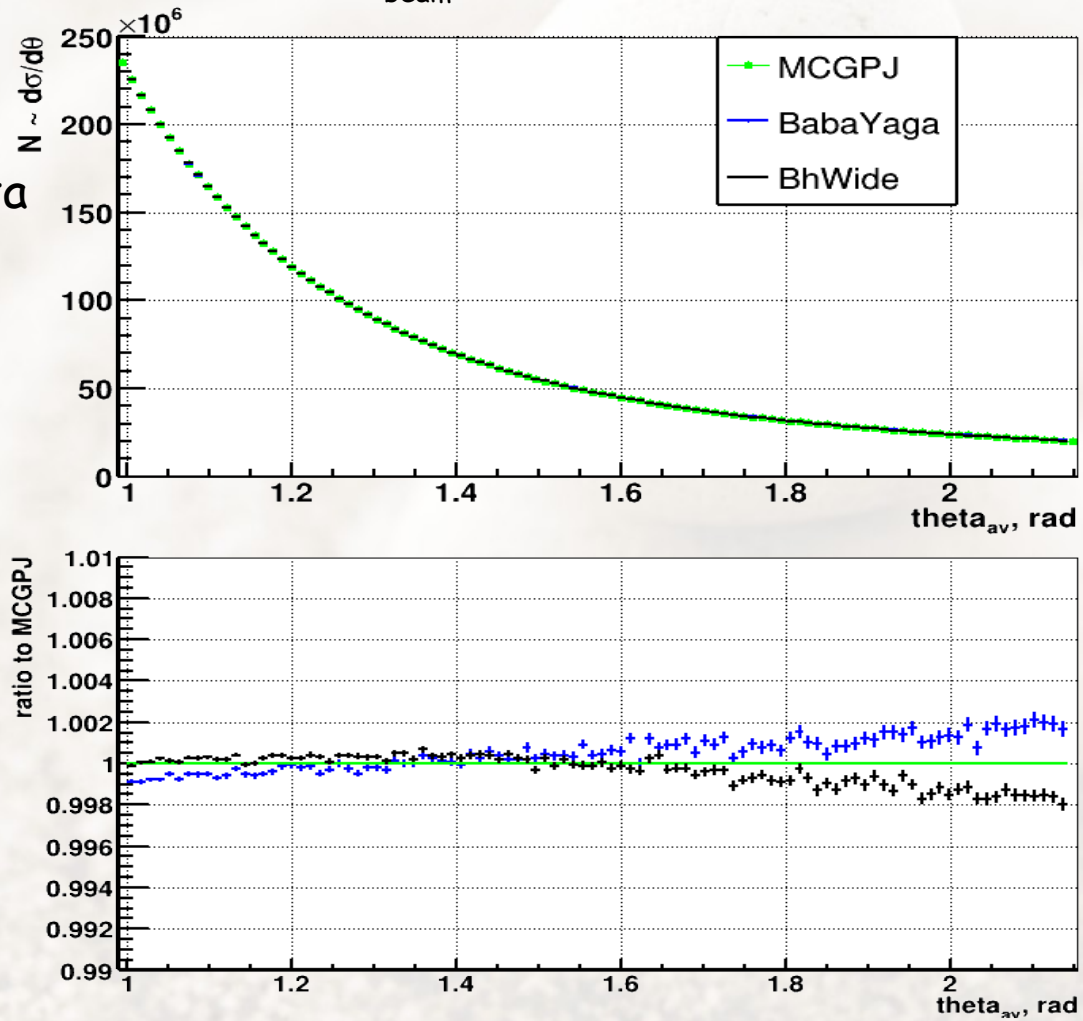
Other e+e- generators

Differential over angle spectrum comparison

E_{beam} 391.48 MeV

Differential cross section over theta consistent/or inconsistent at level ~0.1-0.2%

But we are already sensitive to it in the asymmetry study with CMD3 as shown in presentation yesterday

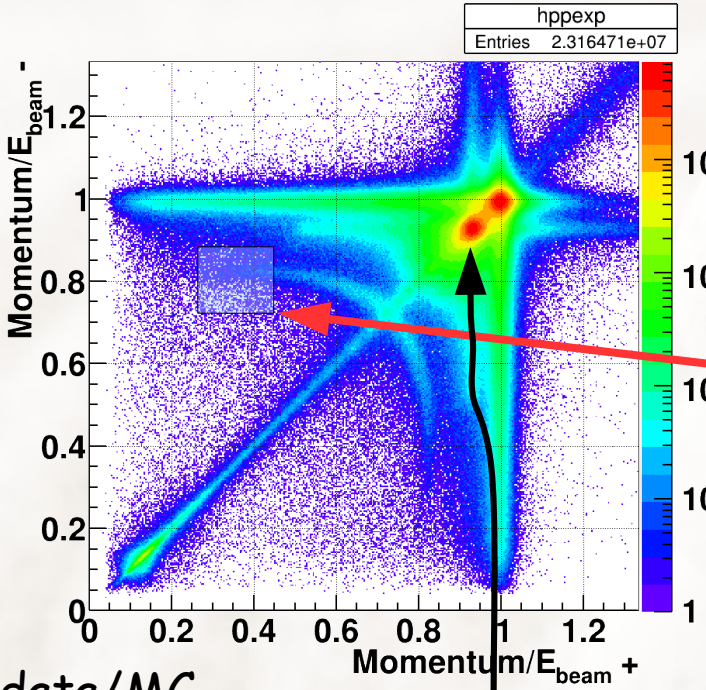


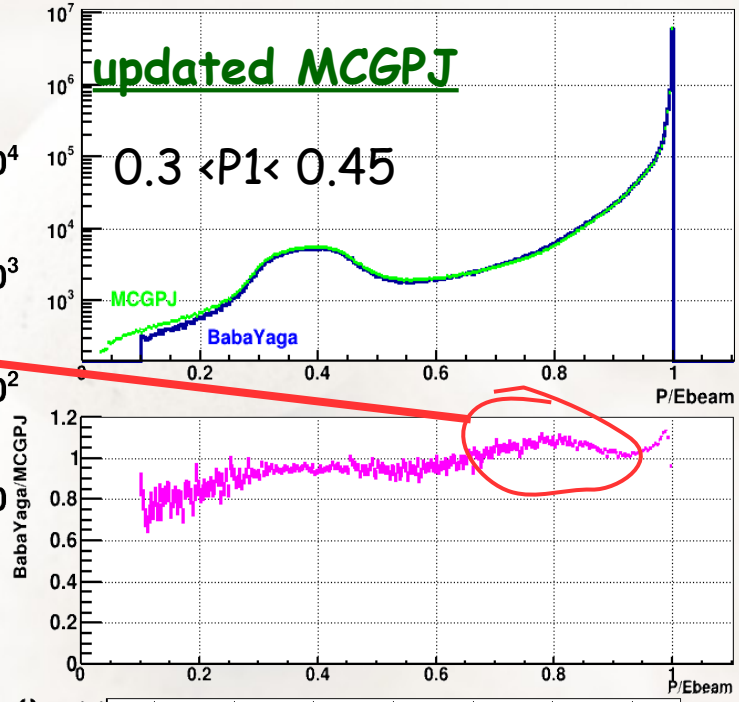
MCGPJ vs BabaYaga spectrums

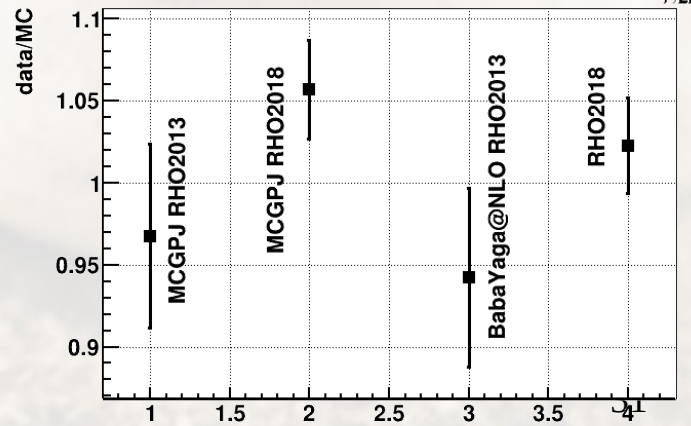
2013+2018 data

Can be looked region where no 2π events:
0.3 <P1 < 0.4 &&
0.75 < P2 <0.85
+ box vise-versa

Ebeam < 375 MeV to suppress 3π







	data/MC
MCGPJ	1.038 +- 0.026
BabaYaga@NLO	1.006 +- 0.026

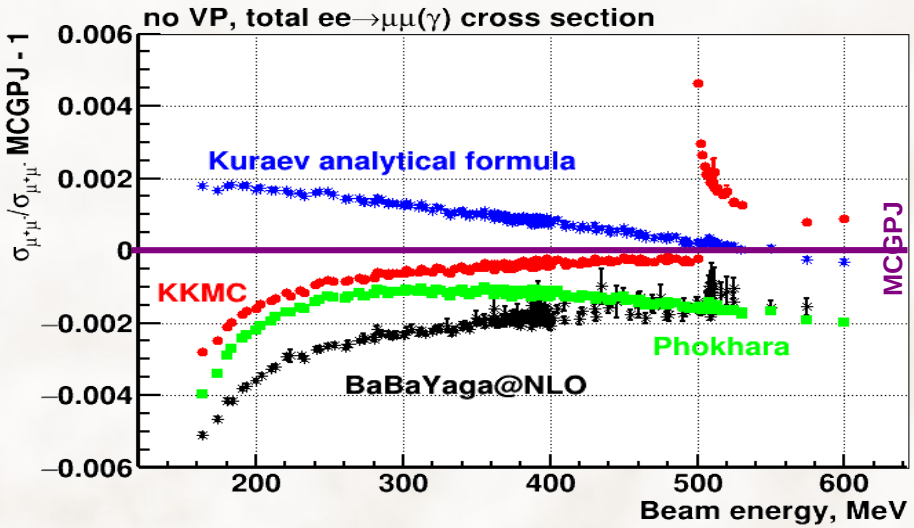
It is necessary to have statistic ~ x10 more (or somehow to suppress 3π events)

And for 2π analysis more crucial spectrum in another part, where pion peaks: P1,P2 ~ 0.9 E_{beam}

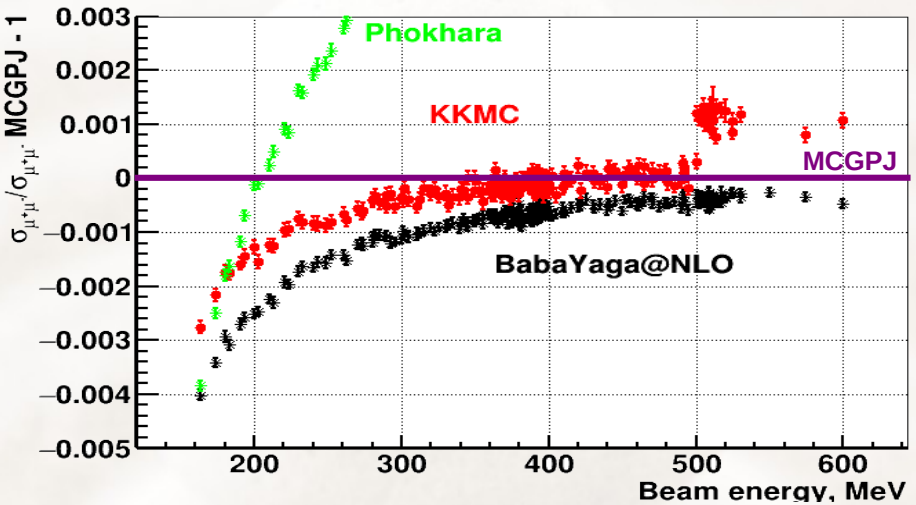
e+e- → μ+μ-(γ) cross-section

Comparison relative to MCGPJ, VP off

Total cross section



in CMD3 selection cuts



KKMCe v 4.32, Phokhara v10.0, BabaYaga@NLO, MCGPJ

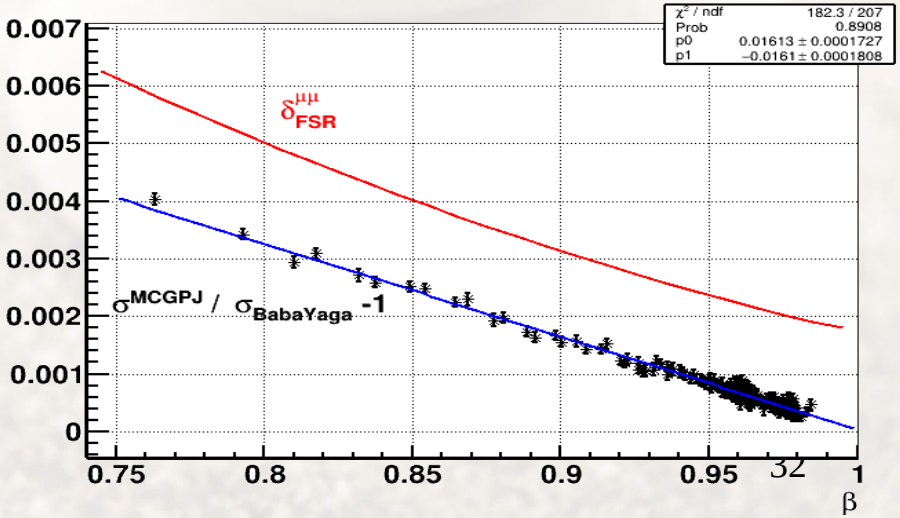
KURAEV analytical formula for e+e-→μ+μ-(γ)

total cross-section: Phys.Rev.D72:114019,2005(arXiv:hep-ph/0505236)

KKMC was design for LEP energies

MCGPJ for μ+μ- is still without jets angular distribution

Phokhara has limited precision for scanned mode (w/o ISR γ)



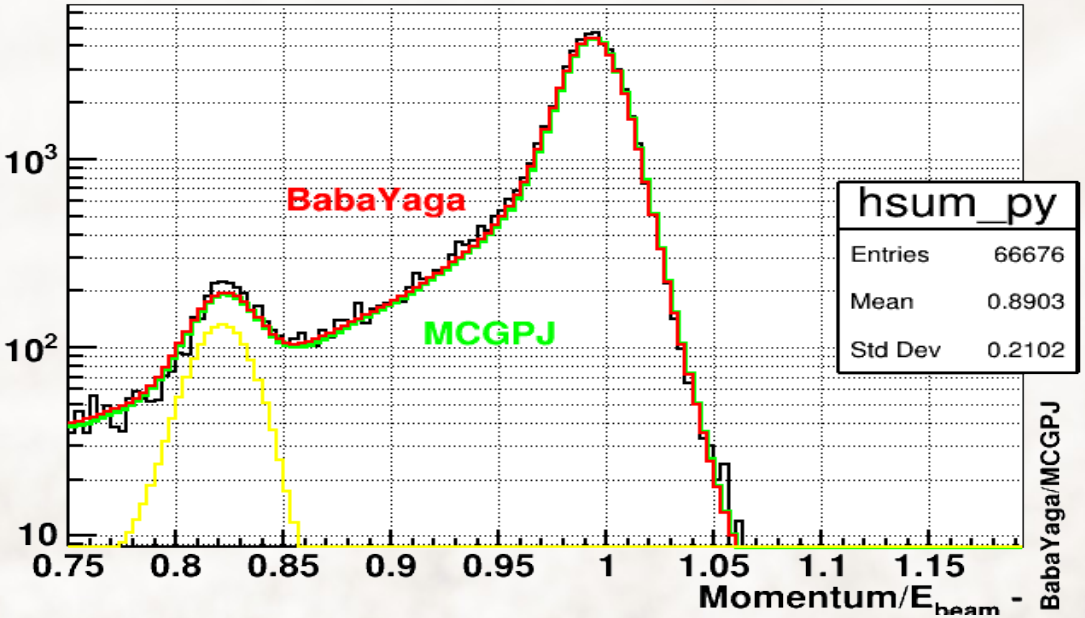
It is commonly used FSR correction in approx. with E>>Mμ:

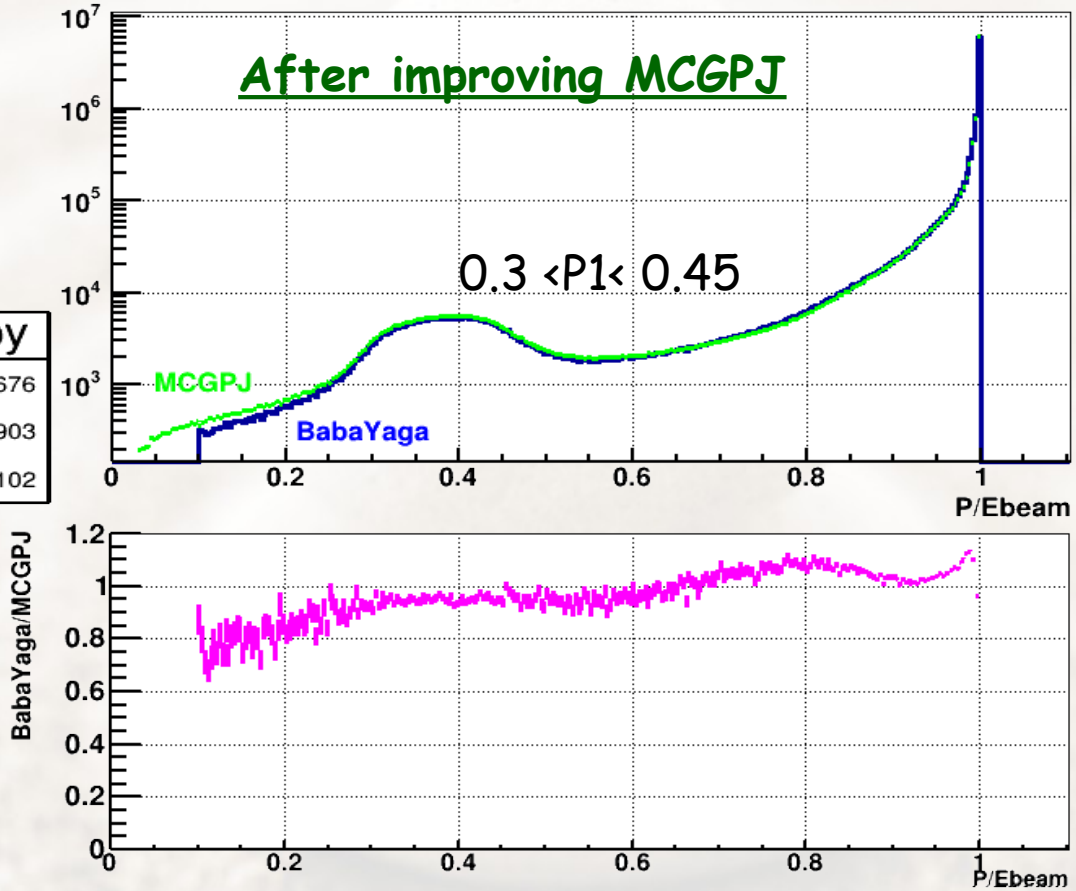
missed dependency δ_{FSR} virtual ~ $2\alpha\pi/\beta_\mu$ with $\beta_\mu \to 0$

MCGPJ vs BabaYaga spectrums

After adding angle distribution for jets, etc ...

Momentum spectrum still disagree at level ~ 10%
Need more experimental data for cross-check





Momentum spectrum disagree at level ~ 10%
Need more experimental data for cross-check
We need more theoretical input for MC

Result in systematic of π+π- measurement → 0.0 – 0.4%

MCGPJ modifications

Several steps for upgrading MCGPJ were done:

photon jets angular distribution with proper kinematic:

$$f(c=\cos(\theta), x=\omega/E) \sim \frac{1}{pk} - \frac{x(1-x)}{1+(1-x)^2}\frac{m^2}{(pk)^2}$$

$$\sim \frac{1}{1-\beta c} - \frac{1-x}{1+(1-x)^2} * \frac{1-\beta^2}{(1-\beta c)^2}$$

Born cross-section boost shift rewritten with virtuality of lepton

? how well factorization is working now(|ISR|*|BornShift|*|FSR|)

In case jets along lepton → leptons was near real, but now it is not

Structure function for FSR: To be consistent with single photon behavior, it started to be used relative to energy of particle after radiation:

$$D(z,s) \sim \frac{1}{2} b (1-z)^{\frac{b}{2}-1} \ldots, b = \frac{2\alpha}{\pi}(L-1), L = \log\left(\frac{s}{m^2}\right), s \rightarrow s(1-x)^2$$

rebalance of jet compensator:

not necessary to keep minimal cone θ from which exact 1 photon Berends is used

some question still under inspection: (some effects of my(not theorist) not understanding at level ~ 0.05%)

1)? is it consistent definition of Berneds soft part versus Jets soft part….

2) problem to construct generator…, now can be used in weighting mode

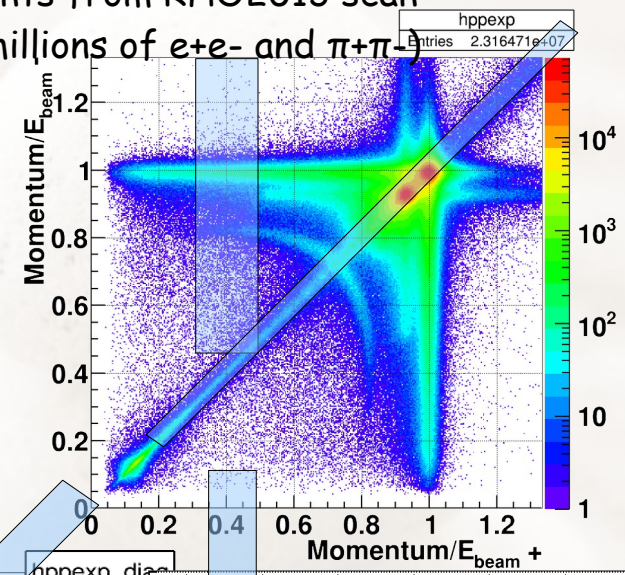
No positive balance of Matrix element between exact Berends 1 photon vs always 4 jet configuration:

how to subtract only 1 photon from always 4 jet event…

MC generator, MCGPJ

High experimental precision relies on high theoretical precision of MC tools:

All events from RHO2013 scan
(~ 10 millions of e+e- and π+π-)



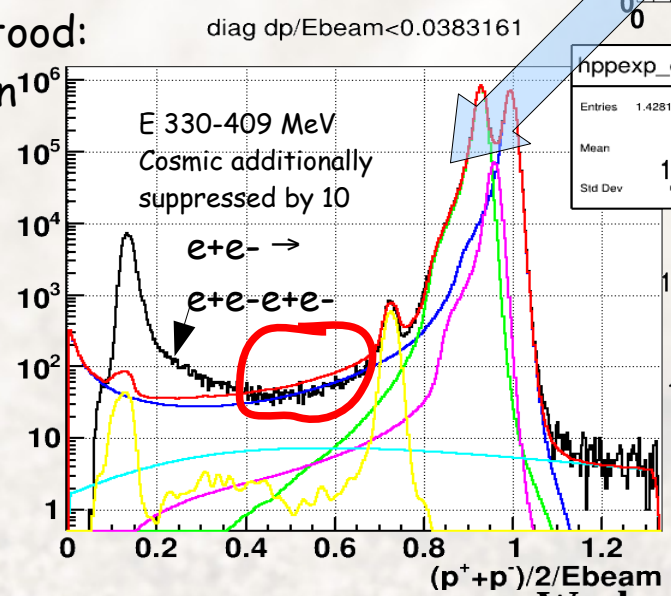
Several MC generators available with 0.1-0.5% precision.

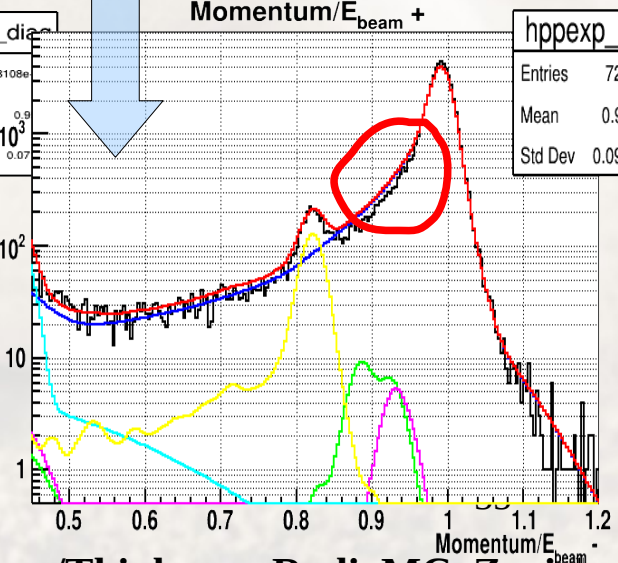
<u>MCGPJ generator (0.2%)</u> is used by Novosibirsk group:

1 real γ + γ jets along all particles (with collinear Structures function)

High statistics allowed us to observe
a discrepancy in momentum distribution
of experimental data vs theoretical spectra from MCGPJ

The source of the discrepancy is understood:
also important γ jets angular distribution

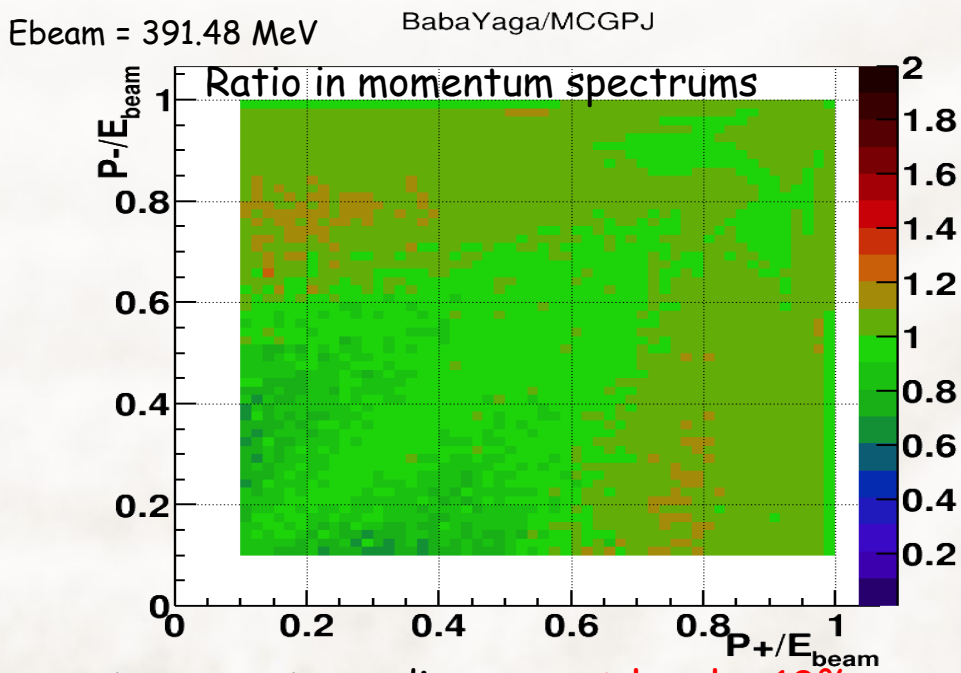


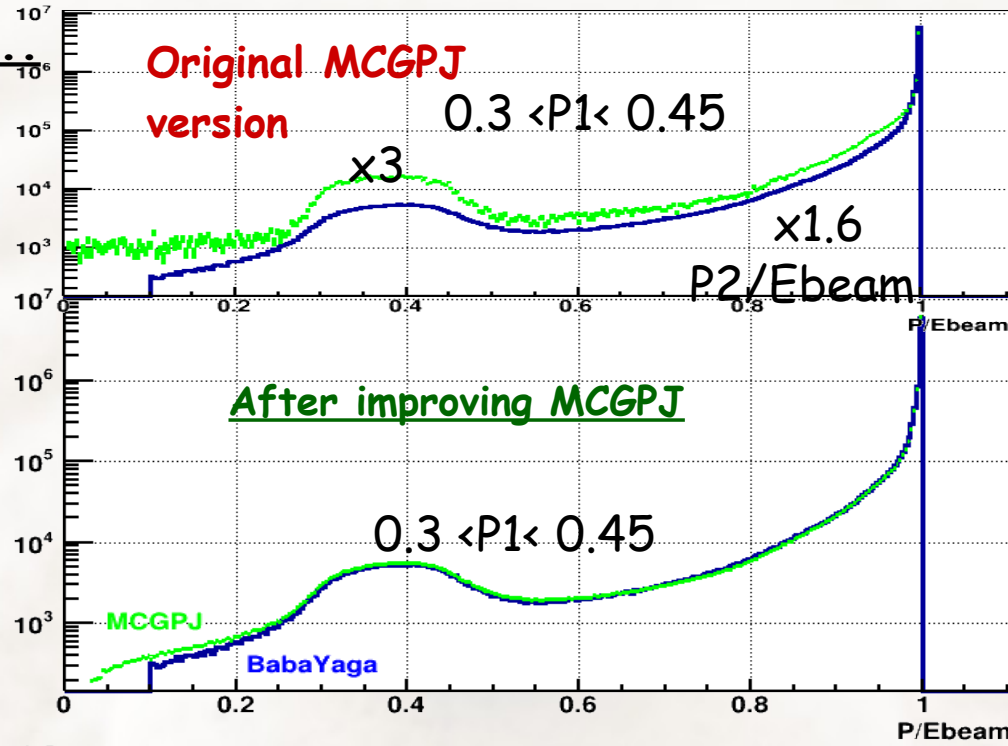


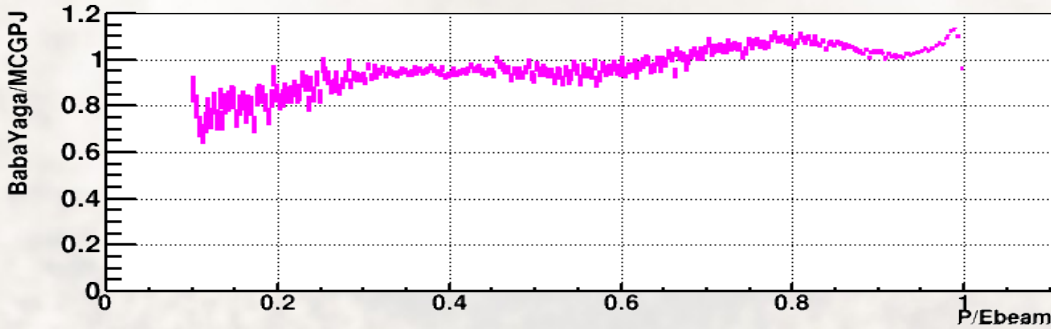
Several steps for upgrading MCGPJ
were done.
But still some question under inspection

MCGPJ vs BabaYaga spectrums

After adding angle distribution for jets, etc ...







Momentum spectrum disagree at level ~ 10%

Need more experimental data for cross-check

We need more theoretical input for MCGPJ

Result in systematic of π+π- measurement

→ 0.0 – 0.4%

For precision ~<0.1% necessary to have exact e+e-→e+e-(γγ) NNLO generator