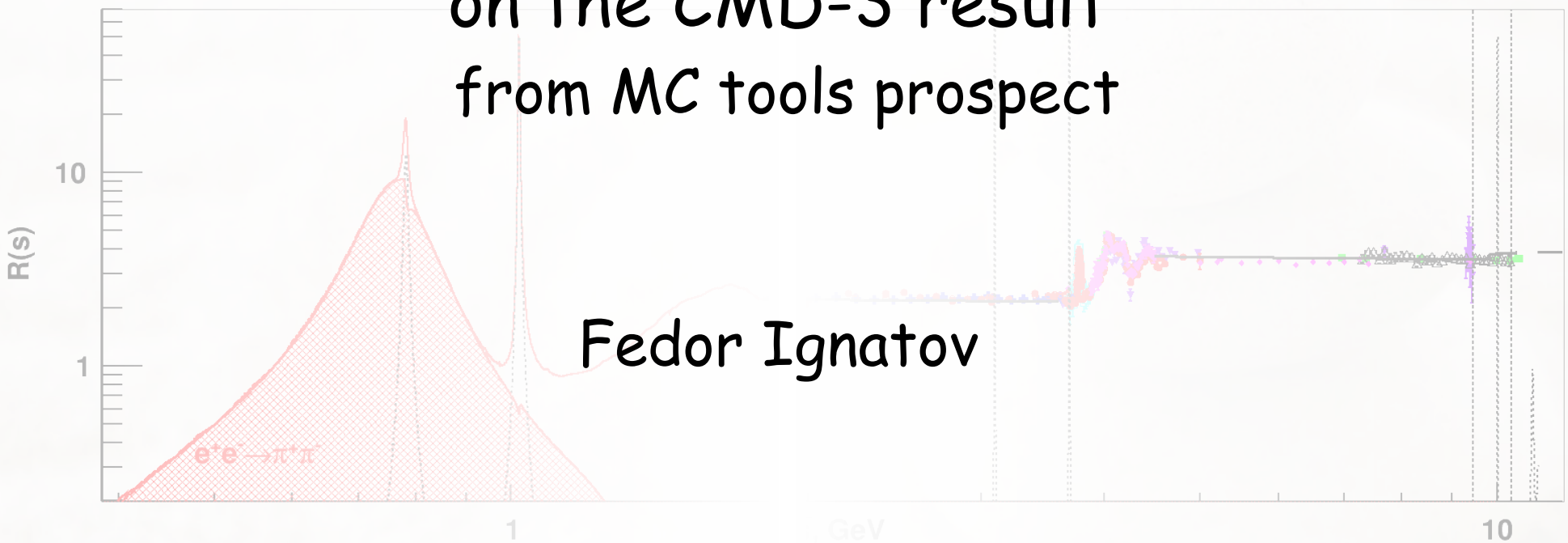
arXiv:2302.08834

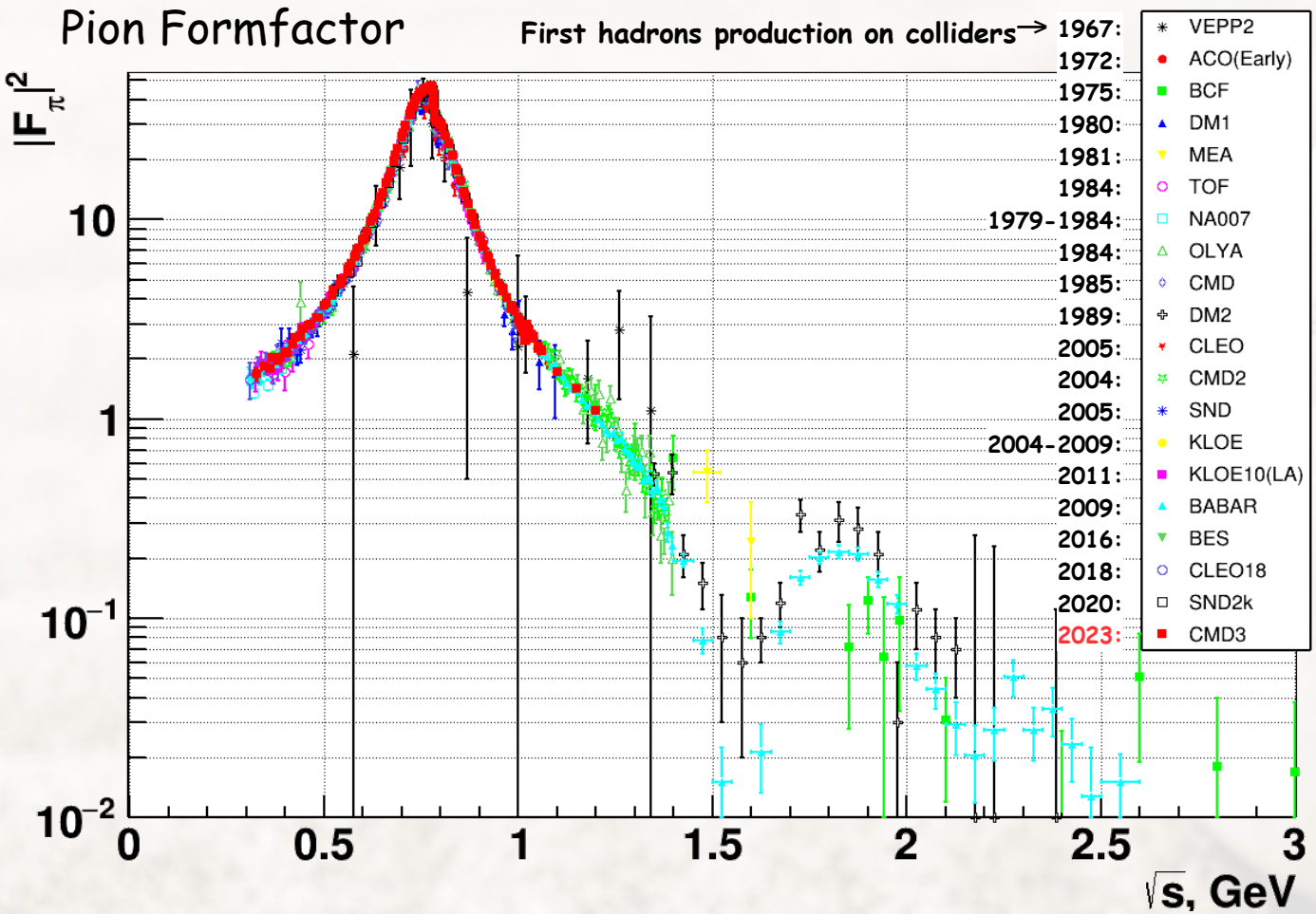
Perspectives on the CMD-3 result from MC tools prospect



Fedor Ignatov

5th Workstop / Thinkstart:
Radiative corrections and Monte Carlo tools
Zurich, 5 June 2023

e+ e− → π+ π− today



Before 1985

Low statistical precision

Systematics >10%

NA7 A few points with >1-5%

1985 - VEPP-2M

with more detailed scan

OLYA systematics 4%

CMD 2%

2004 with CMD2 at VEPP-2M

was boost to systematics: 0.6%

(near same total statistic)

The uncertainty in a_μ(had) was improved by factor 3 as the result of VEPP-2M measurements

New ISR method

e+e- → γ + hadrons (limited only by systematics):

KLOE: 0.8%

BaBar: 0.5%

BES: 0.9%

CLEO: 1.5%

New direct data:

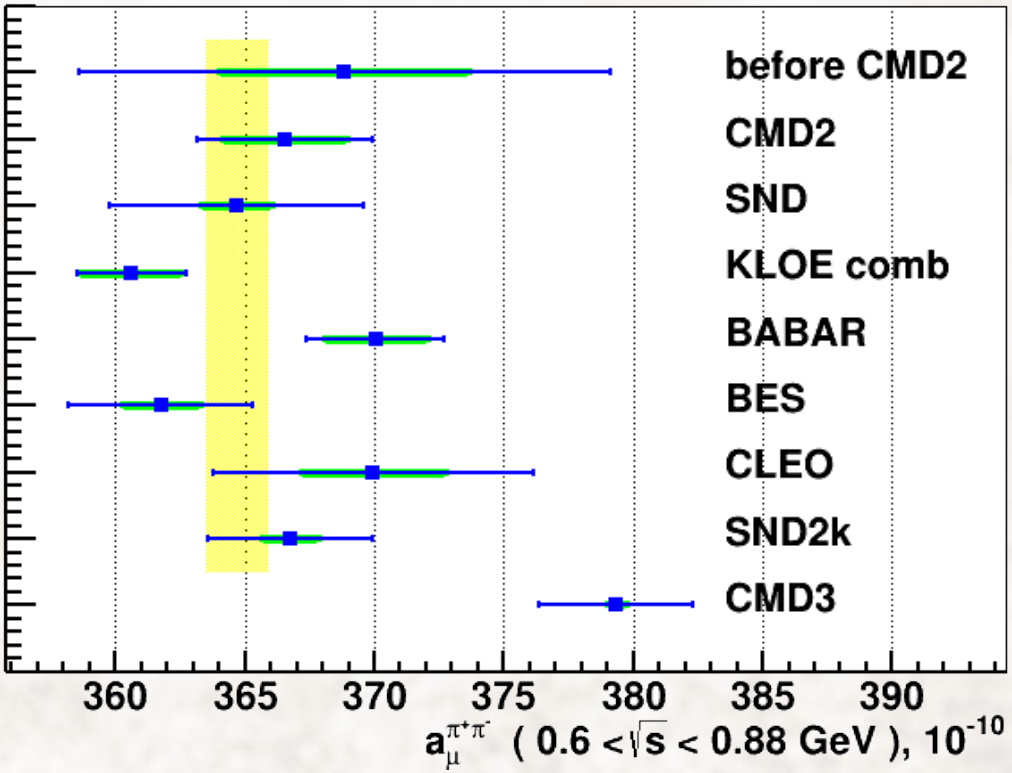
SND2k : 0.8% (with 1./10 of avail. Data)

CMD-3: 0.7%

New g-2 experiments and future e+e- as ILC, FCC-ee require average precision ~0.2%

The π+ π− contribution to a_μ^{had}

$$a_\mu^{had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty}\frac{\sigma_{e^+e^-\to\gamma^*\to hadrons}(s)K(s)}{s}ds$$

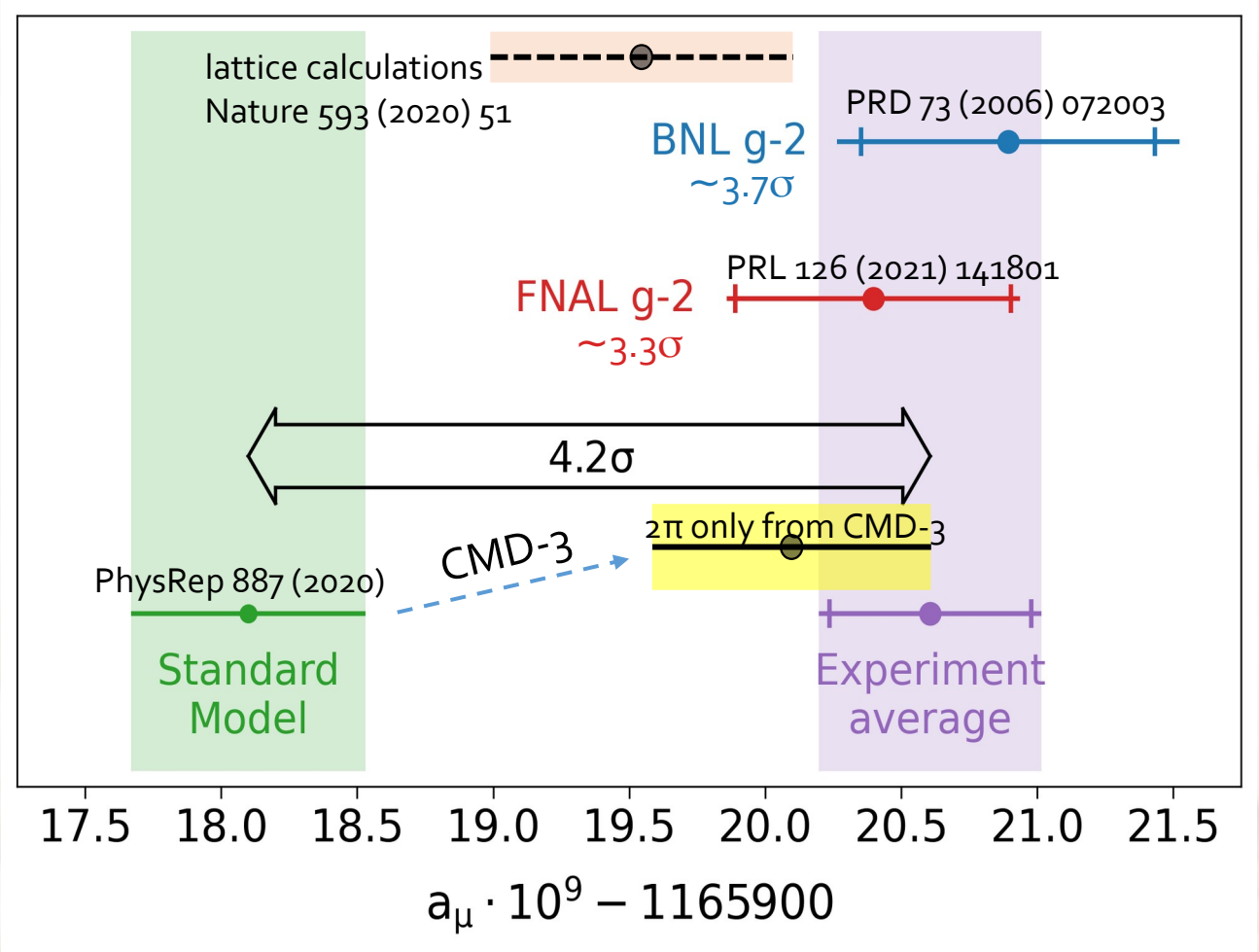


0.6 < √s < 0.88 GeV

	$a_\mu^{\pi\pi,LO}$, 10^{-10}
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

RHO2013	380.06 ± 0.61 ± 3.64
RHO2018	379.30 ± 0.33 ± 2.62 $\times 10^{-10}$
Sum	379.35 ± 0.30 ± 2.95

The impact of CMD-3 on SM prediction of a_μ^{had}



If it will be only CMD-3 than SM will be solved.
But CMD-3 is only one now over many other experiments (BaBar, KLOE, BES, CMD-2, SND, ...)

Unfortunately at the moment, we don't know the reasons of the disagreement between different experiments.

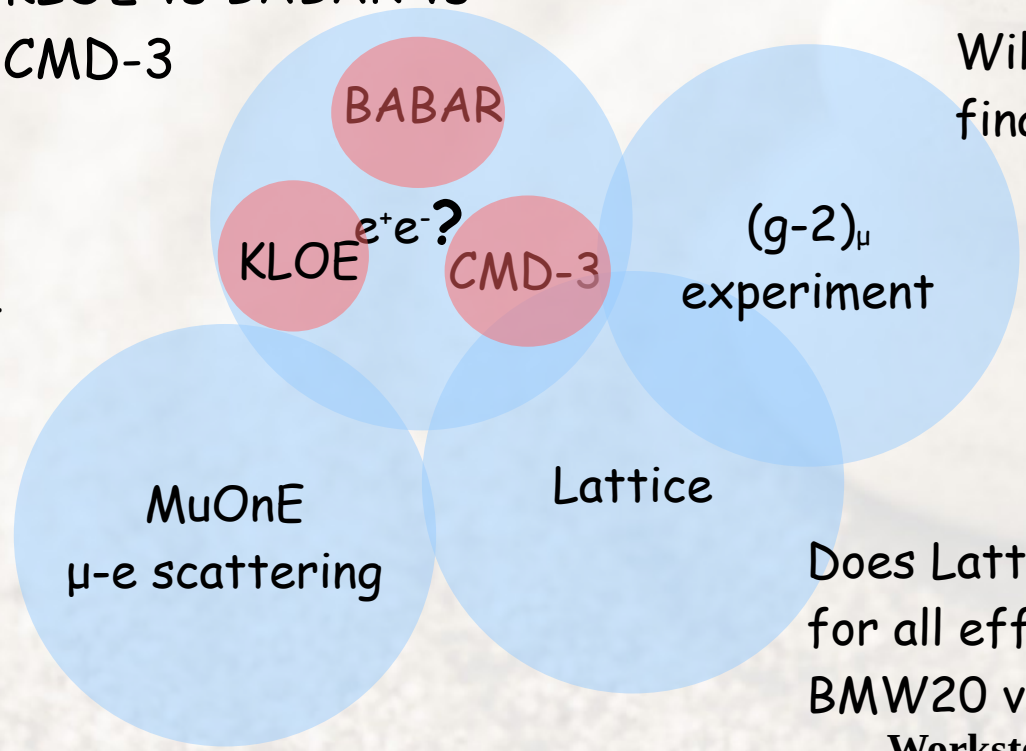
Puzzles in puzzle

Question of comparison:

e+e- vs $(g-2)_\mu$ vs lattice

Where difference comes from:
KLOE vs BABAR vs CMD-3

Will it be confirmed?
final FNAL vs J-PARC



Hard effort against systematics

Does Lattice account for all effects?
BMW20 vs others

On the way

Current plans:

Final editing of the long paper (as in arxiv) has been completed and its is ready for journal submission.

The short paper is still under preparation,

Still we plan to submit both versions to journals at same time

Future plans, it will be another papers:

New ρ scans with improved detector and possibly some specific systematics checks are expected:

more data and it can be done further analysis with some improvement

Analysis at $\sqrt{s} > 1$ GeV is in progress by another person

(exploiting full shower profile information by neural network,

as better separation is required at higher energies)

with same independent steps for efficiency determination, etc for formfactor evaluation

→ cross check between current and new analyses will be required at final stage

Meanwhile

Two long seminars:

KEK seminar, 17 March 2023: https://kds.kek.jp/event/45889/

TI seminar, 27 March 2023: https://indico.fnal.gov/event/59052/

49 questions list was prepared from the panelist nominated by the g-2 Theory Initiative Steering Committee: https://indico.fnal.gov/event/59052/sessions/22020/attachments/165293/219577/Complete_list_of_questions.pdf

Answers had been prepared, some further discussion is expected (shorter list was already given during the TI seminar)

$|F_\pi|^2$ systematic uncertainty

	At $\sqrt{s}$ near ρ peak (except ω peak)
ˣ Radiative corrections	0.2% (2π) ⊕ 0.2% (Fπ) ⊕ 0.1% (e+e-) = **0.3%**
ˣ e/μ/π separation	**0.2%**
ˣ Fiducial volume	**0.5% / 0.8%** (RHO2013)
ˣ Correlated inefficiency	**0.1%**
ˣ Trigger	**0.05%**
ˣ Beam Energy (by Compton σ_E< 50 keV)	**0.1%**
ˣ Bremsstrahlung loss	**0.05%**
ˣ Pion specific loss	**0.2%** nuclear interaction
	0.1% pion decay
	0.7% / 0.9% (RHO2013)

The radiative correction is the next biggest part to the systematic table after quite conservative θ-angle related contribution.
Indirectly theoretical knowledge present in the particle separation and fiducial volume determination as the consistency check

Possible concerns in the analysis related to MC tools:

- Radiative corrections for the $\pi^+\pi^-$ total cross section
- Differential cross section over momentum for the particle separation
- Differential cross section over polar angle for controlling of systematic uncertainty of the fiducial volume determination

Radiative corrections

Measurement of $e^+e^- \to \pi^+\pi^-$ requires high precision calculation of radiative corrections.

Two high precision MC generators were used

MCGPJ(0.2%, e+e-,μ+μ-,π+π-) vs BabaYaga@NLO (0.1%, e+e-,μ+μ-)

They include exact NLO + Higher Order terms in some approximation.

e+e- → e+e-(γ) : great consistency <0.1% in the total cross section

e+e- → μ+μ-(γ) : Mass term in FSR is missed in most of generators (effect 0.4% at √s=0.32 GeV)

e+e- → π+π-(γ) : only MCGPJ available with 0.2% precision (for energy scan experiments)

Achieved precision in current analysis is also sensitive for precision of differential cross sections predictions

e/π separation by momentum requires $d\sigma/dP^+dP^-$ spectra as initial input

Asymmetry study requires $d\sigma/d\theta$ spectra

ππ radiative corrections

Unfortunately only MCGPJ available with declared 0.2% precision (for energy scan experiments)

Phokara and BabaYaga 3.5 are incomplete at NLO level for energy scan mode: there is no FSR

Theoretical estimation of precision for the MCGPJ was estimated as 0.2% and some of main cross checks of the generator were performed.
Also mostly same MCGPJ in π+π- channel was used for CMD-2, SND@VEPP-2000, … this will not give the answer for the difference of measured $|F_\pi|^2$ by CMD-3 with them.

The radiative correction table used in the analysis is part of the arXiv submission, It will be useful for cross-checks them if new generators will be appeared.

Differential cross section from generators

events separation is done either

1) by momentum

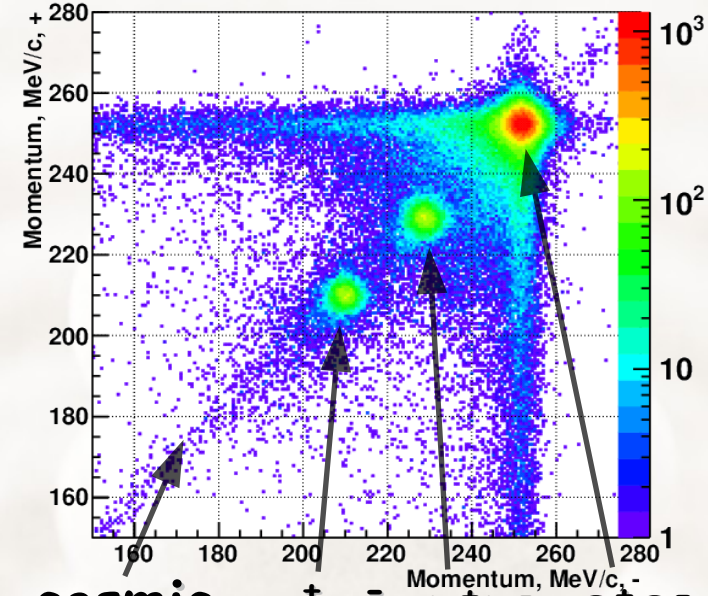
2) or by energy deposition

Separation of $\pi^+\pi^-$, $\mu^+\mu^-$, e^+e^-, …. final states is based on likelihood minimization:

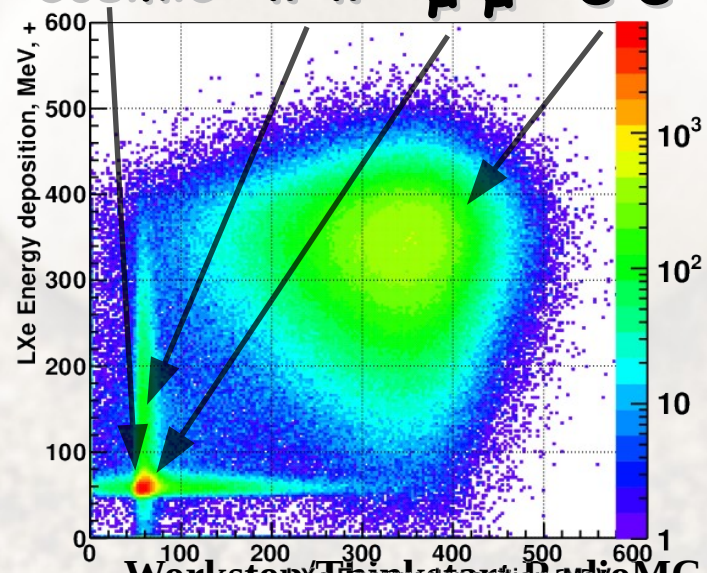
$$-\ln L = -\sum_{events} \ln\left[\sum_i N_i f_i(X^+, X^-)\right] + \sum_i N_i$$

In case of momentum-based separation:
the predicted Momentum spectra from the generators are used as input for PDF construction (+detector effects)

Not the case for energy-deposition based separation – doesn't require knowledge from generators



$P^+ \times P^-$ E_{beam}=250 MeV



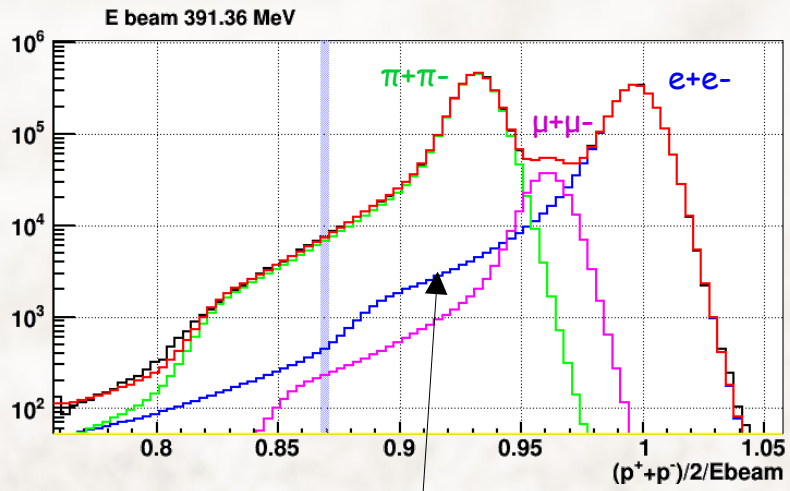
$E^+_{LXe} \times E^-_{LXe}$ E_{beam}=480 MeV

Differential cross section effect on form factor

Differential cross section knowledge is necessary for momentum-based separation (not used in energy deposition separation)

Effect on Nπ+π-/Ne+e- ~ 0.1-0.2% at ρ-peak

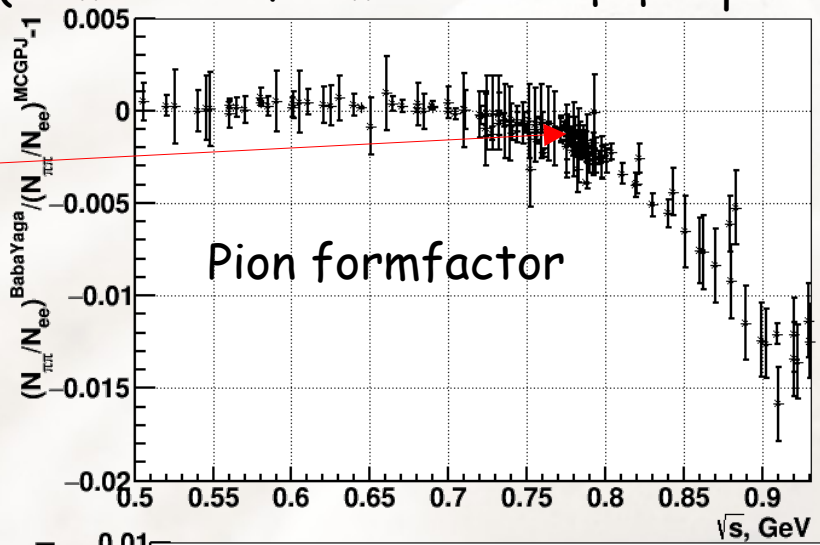
Effect comes when momentum peaks from π+π- and e+e- become close



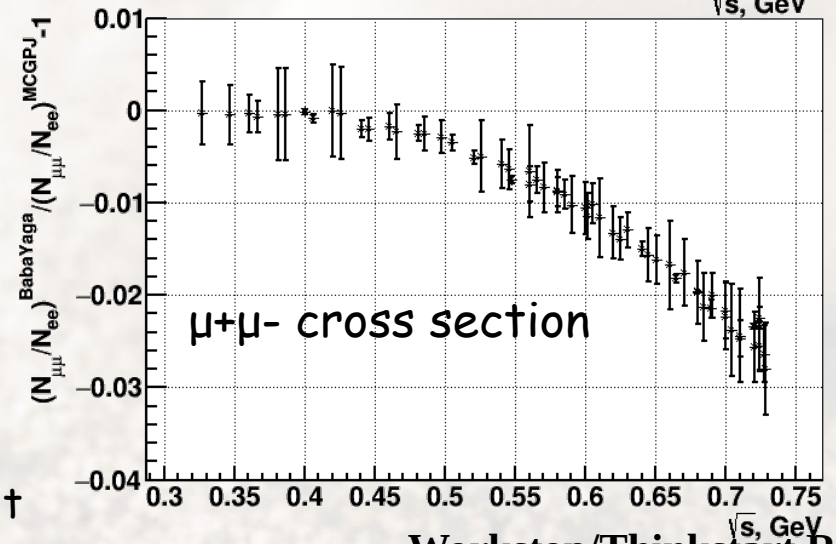
The soft photons radiation distribution is important in this region:

Looks like BaBaYaga@NLO approach with iterative photons generation gives better result

Effect of difference of MCGPJ vs BabaYaga@NLO (cumulative from e+e- and μ+μ- spectra)



μ+μ- effect <0.05-0.1%



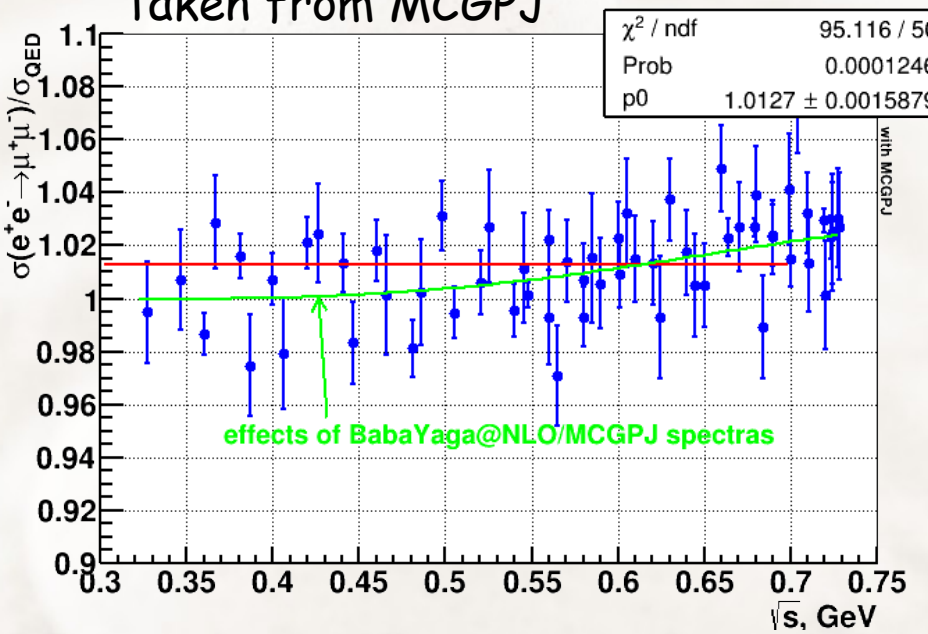
μ+μ- effect ~ 1/4 - 1/3

Radiative corrections

BaBaYaga@NLO shows better agreement with the data:

1) Momentum spectras better describe data:
gives consistent results in $N_{\mu\mu}$/QED
(effect on $|F_\pi|^2$ ~0.2% at $\sqrt{s}$=0.78 GeV, and rising to 1.5% at 0.9 GeV when using momentum-based separation)

effect on $N_{\mu\mu}$/QED
when input $d\sigma/dP^+dP^-$ spectra
taken from MCGPJ



2) Experimental asymmetry in e+e-

Data vs BabaYaga@NLO:

δA = -0.060 ± 0.026 %

Data vs MCGPJ

δA = -0.140 ± 0.026 %

BabaYaga@NLO consistent with NNLO MCMule

δA = +0.006 ± 0.003 % at $\sqrt{s}$=0.76 GeV

We adopted generators usage in this way:

e+e- : BabaYaga@NLO

μ+μ- : BabaYaga@NLO (differential cross section)
MCGPJ (integral)

π+π- : MCGPJ

Better NNLO (+VP + next log terms) generators are quite desirable for higher precision

Differential cross section over momentum

Effect of difference between BabaYaga@NLO/MCGPJ in $d\sigma/dP^+dP^-$ spectra for the momentum-based particle separation is quite moderate on the ρ-peak: 0.1-0.2%

Looks like BabaYaga@NLO quite well describes spectra:

no effect in $\sigma(e+e- \rightarrow \mu+\mu-)/QED$: $\Delta = +0.17 \pm 0.16$ % (at $\sqrt{s} < 0.7$ GeV)

no residual effect in comparison E/P separations at ~0.9 GeV

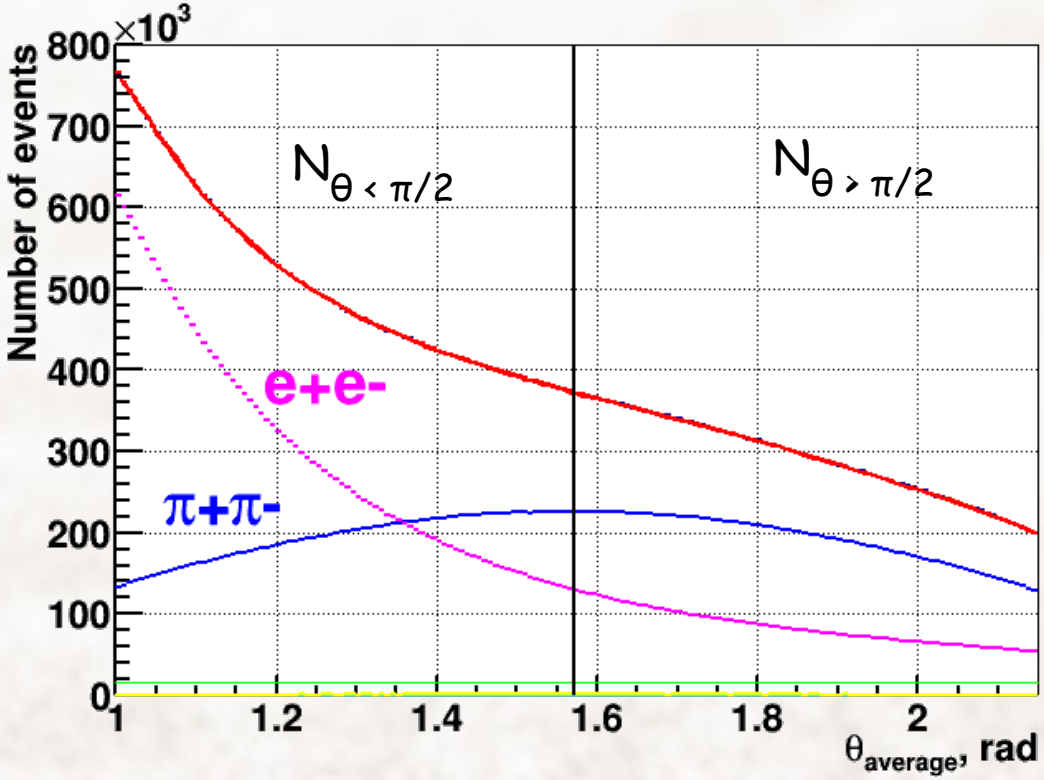
$(N_{\pi\pi}/N_{ee})^E/(N_{\pi\pi}/N_{ee})^P$: $\Delta = -0.22 \pm 0.34$% (at $\sqrt{s} = 0.85$-0.95 GeV, where different generators effect ~ 1.- 1.5%)

Confirming of BabaYaga@NLO momentum spectra for e+e-/μ+μ- and having better than MCGPJ π+π- can give more ensures in the particle separation part.

(N.B. Just fixed order NNLO is not enough for the $d\sigma/dP^+dP^-$ spectra, it should be with logarithmically enhanced corrections via iterative many photons generation)

Forward backward charge asymmetry

dσ/dθ spectra



Asymmetry definition:

$A = (N_{\theta < \pi/2} - N_{\theta > \pi/2})/N$

Sensitive to:

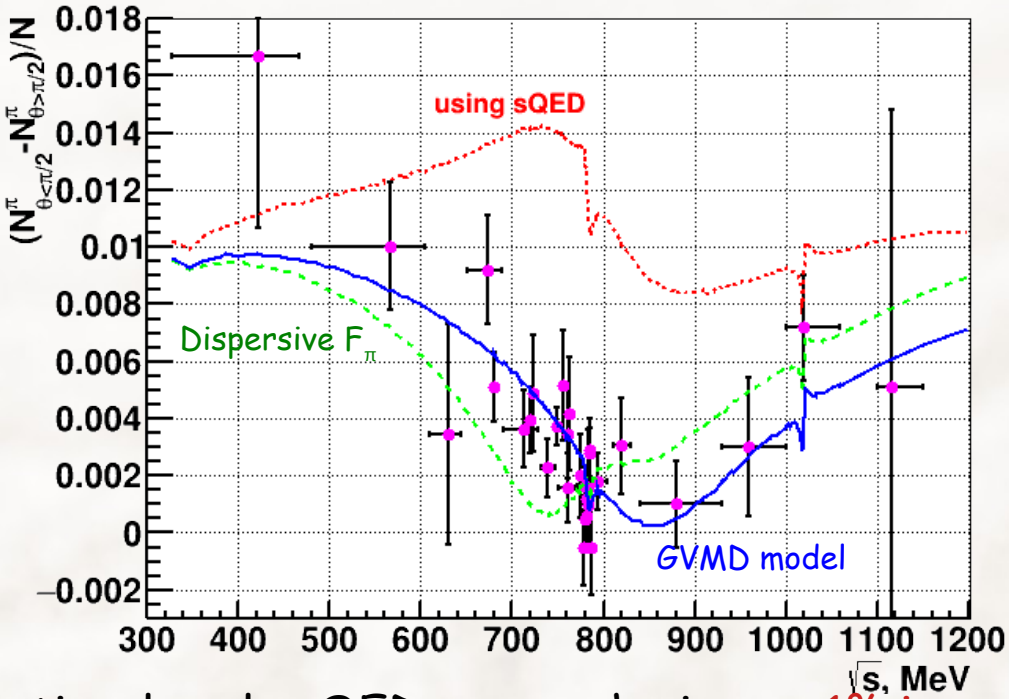
- ✗ angle-related systematics
- ✗ used model of γ-π interaction

At first try:

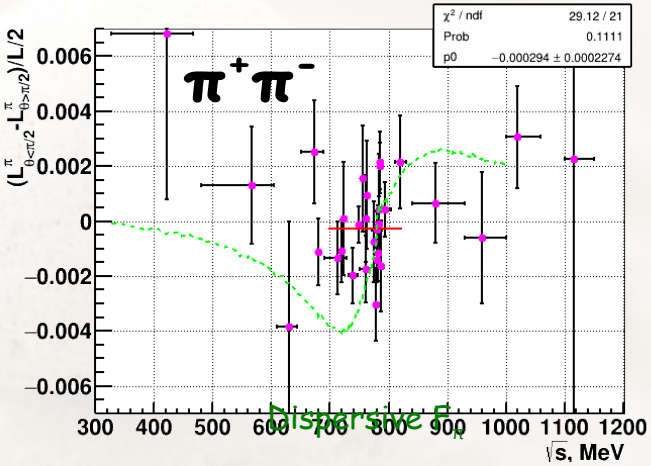
1% inconsistency for π+π- was observed between data and MC prediction

Charge asymmetry in e+e- -> π+π-

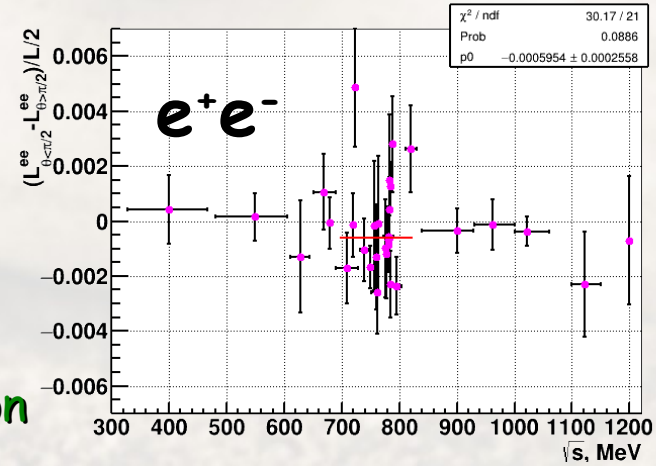
$A = (N_{\theta < \pi/2} - N_{\theta > \pi/2})/N$



Relative to GVMD prediction



to BaBaYaga@NLO
(confirmed by MCMule)



Conventional scalar QED approach gives ~ 1% inconsistency

The theoretical model within GVMD was introduced, describes well the CMD-3 data R.Lee et al., Phys.Lett.B 833 (2022) 137283

was confirmed by calculation in dispersive formalism

M.Hoferichter et al., JHEP 08 (2022) 295

Average at $\sqrt{s}$ = 0.7-0.82 GeV:

$\pi^+\pi^-$: $\langle\delta A\rangle$ = -0.029 ± 0.023 %

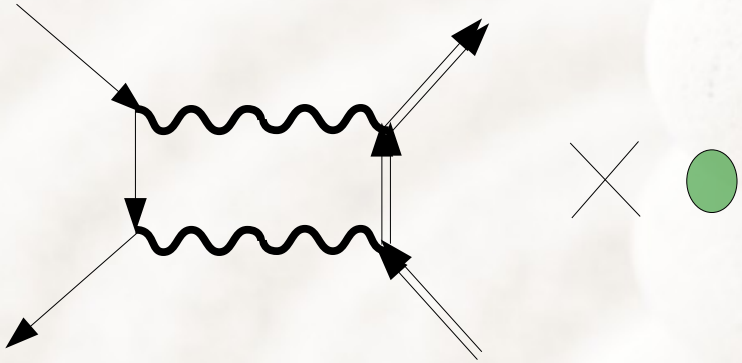
e^+e^-: $\langle\delta A\rangle$ = -0.060 ± 0.026 %

Ensure our θ angle systematics estimation for $|F_\pi|^2$

sQED assumptions for radiative corrections

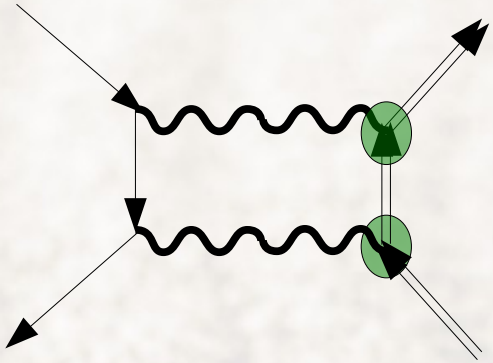
The radiative correction calculations is commonly done in the sQED approach,
It's mean that the calculations are performed without form factor,
then final Amplitude is scaled by $F(q^2)$

Scalar QED approach



$A = sQED*F(s)$

Proper way



$A \sim \int F(q_1)F(q_2)$

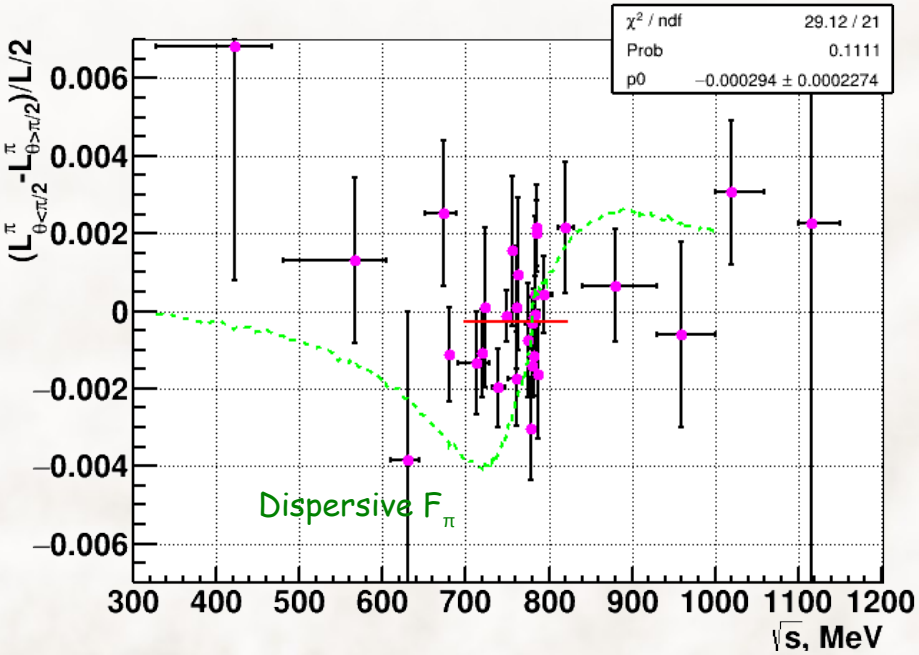
Proper way will be to put $F(q^2)$ to each vertex

N.B. It will be important to re-calculate radiative corrections
with above sQED for ISR measurement

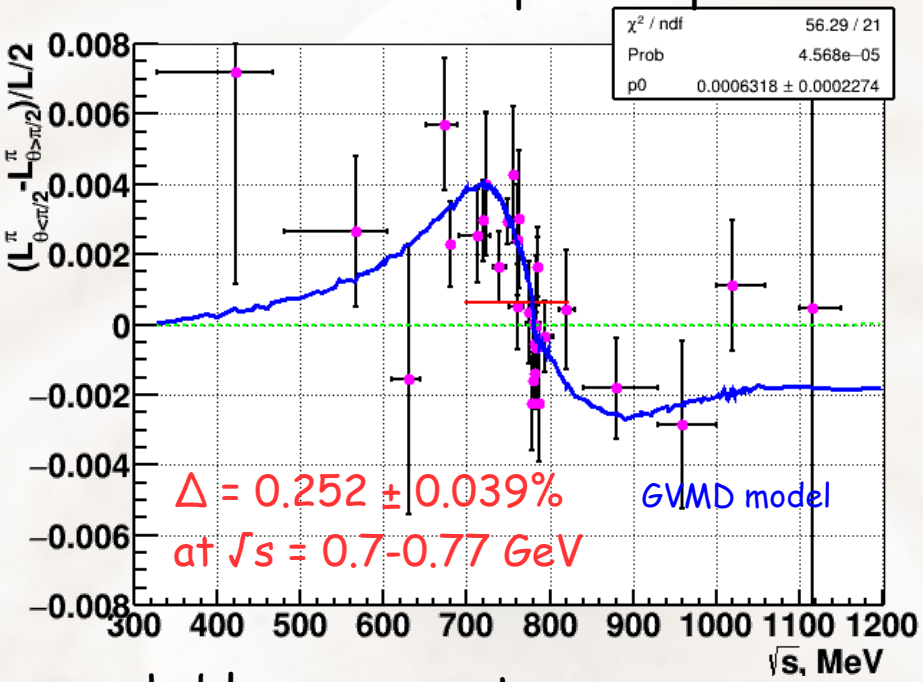
Charge asymmetry in e+e- -> π+π-

π+π- asymmetry

relative to the GVMD prediction



relative to the dispersive prediction



Data vs GVMD prediction has remarkable agreement

But dispersive calculation has better description of F_π as input

Clarification between the GVMD and Dispersive calculations can strengthen confidence on the polar angle determination in the analysis, or can give a sign for a possible detector related effect here (but anyway it is still below of the conservative estimation of θ-related systematic uncertainty for the $|F_\pi|^2$)

Possible progress in MC tools precision or cross checks:

- Radiative corrections for the $\pi^+\pi^-$ total cross section
- Differential cross sections over momentum, angles
  for the $e^+e^- \to \pi^+\pi^-$, e^+e^-, $\mu^+\mu^-$ processes

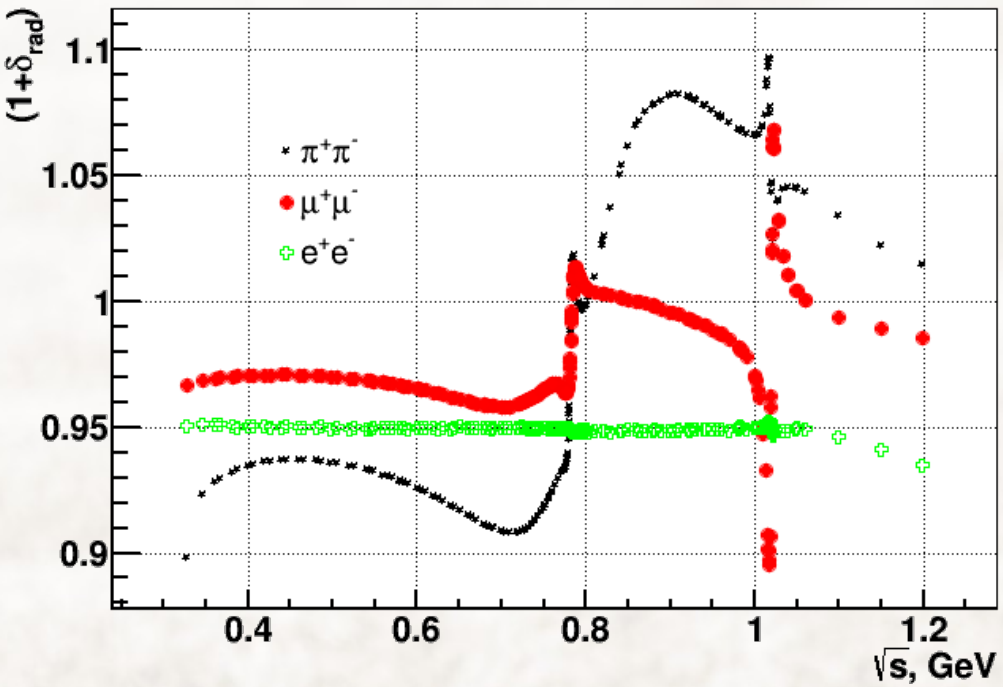
Can help to give more confidence,
or can highlight some detector related effects
in the obtained CMD-3 result

Backup

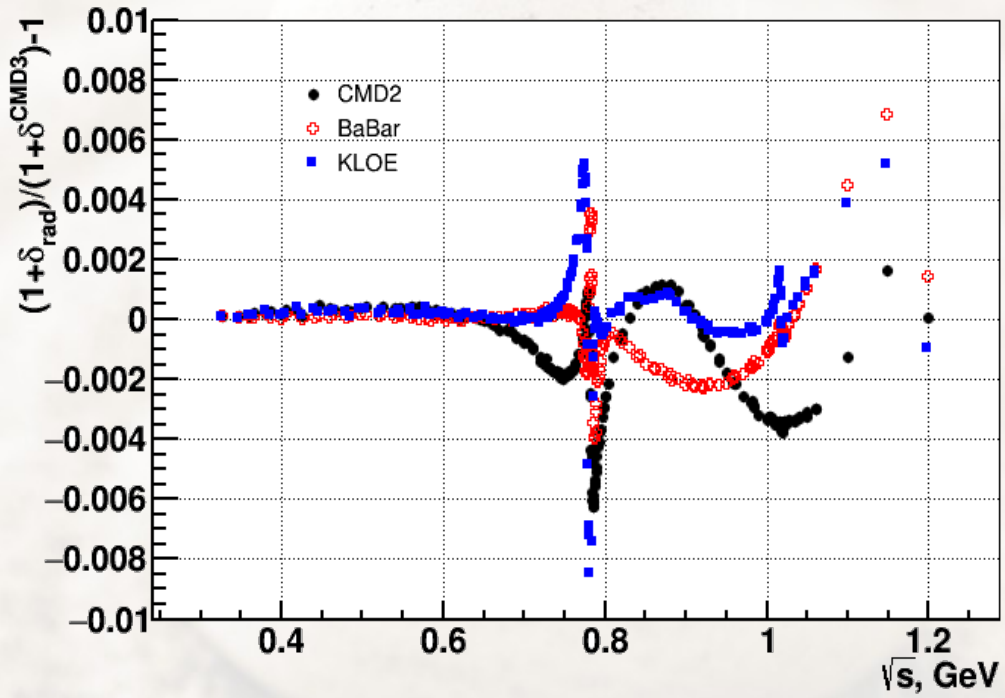
Radiative corrections

Radiative corrections within

$1.<(\pi+\theta^{+}-\theta^{-})/2<\pi-1.\text{rad}$, $|\Delta\phi|<0.15$, $|\Delta\theta|<0.25$



Effect on 2π radiative correction from different $|F|^2_\pi$ parametrizations (over different datasets)



Systematic uncertainty

0.2% (π + π −) ⊕ 0.2% (Fπ , s > 0.74 GeV) ⊕ 0.1% (e+ e−)

N.B. KLOE/BABAR systematic difference in derivative 4%/0.4GeV,
in CMD-3 is also possible up 1%/0.1 GeV → same 0.2% estimation (from Fπ model)

Radiative corrections part of the Long question list

Question 26 (from short list)

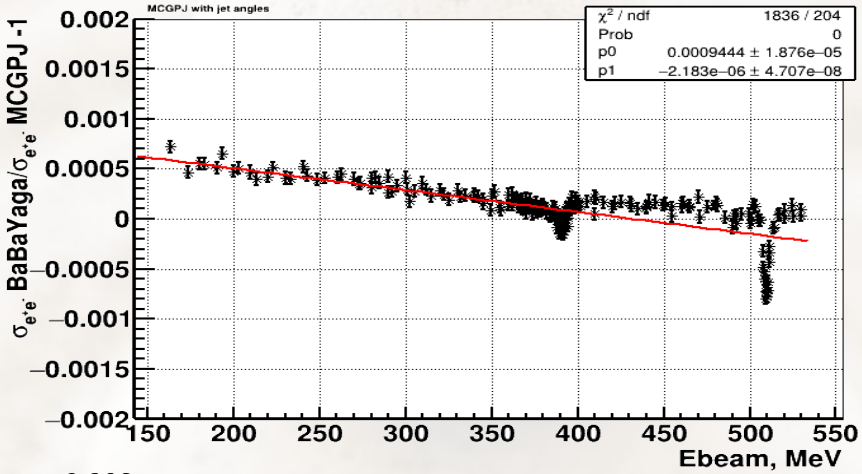
26. Two generators used (MCGPJ, BabaYaga) NLO+NNLO approximative with some differences found for ee: give more information. Does it affect also the μμ and ππ samples?

Please see more details in: https://agenda.infn.it/event/28089/contributions/147298/

Yes, μ+μ- and π+π- differential cross sections have also some uncertainty

<u>e+e-:</u>

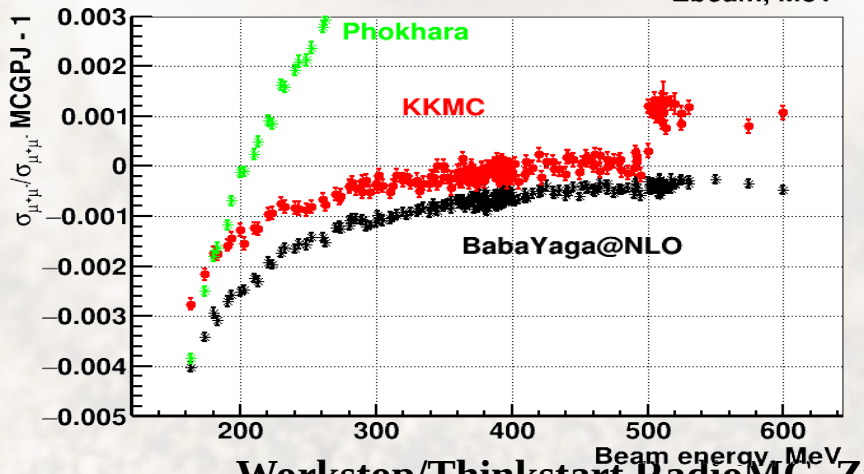
Integrated cross-section is consistent at the level <0.1% between generators



<u>μ+μ-:</u>

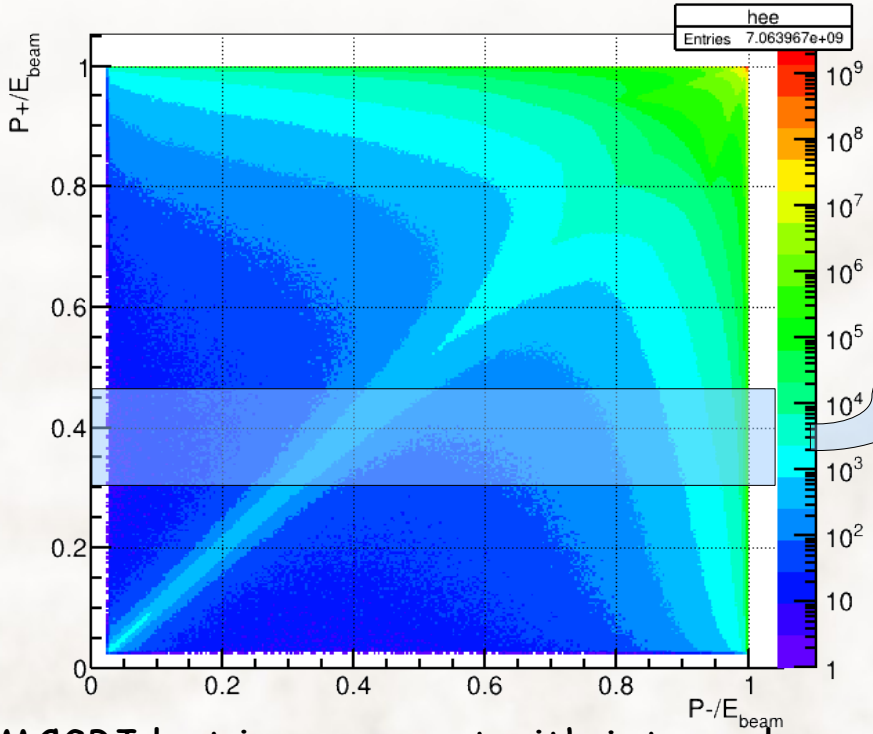
Integrated cross-section is inconsistent up 0.4%

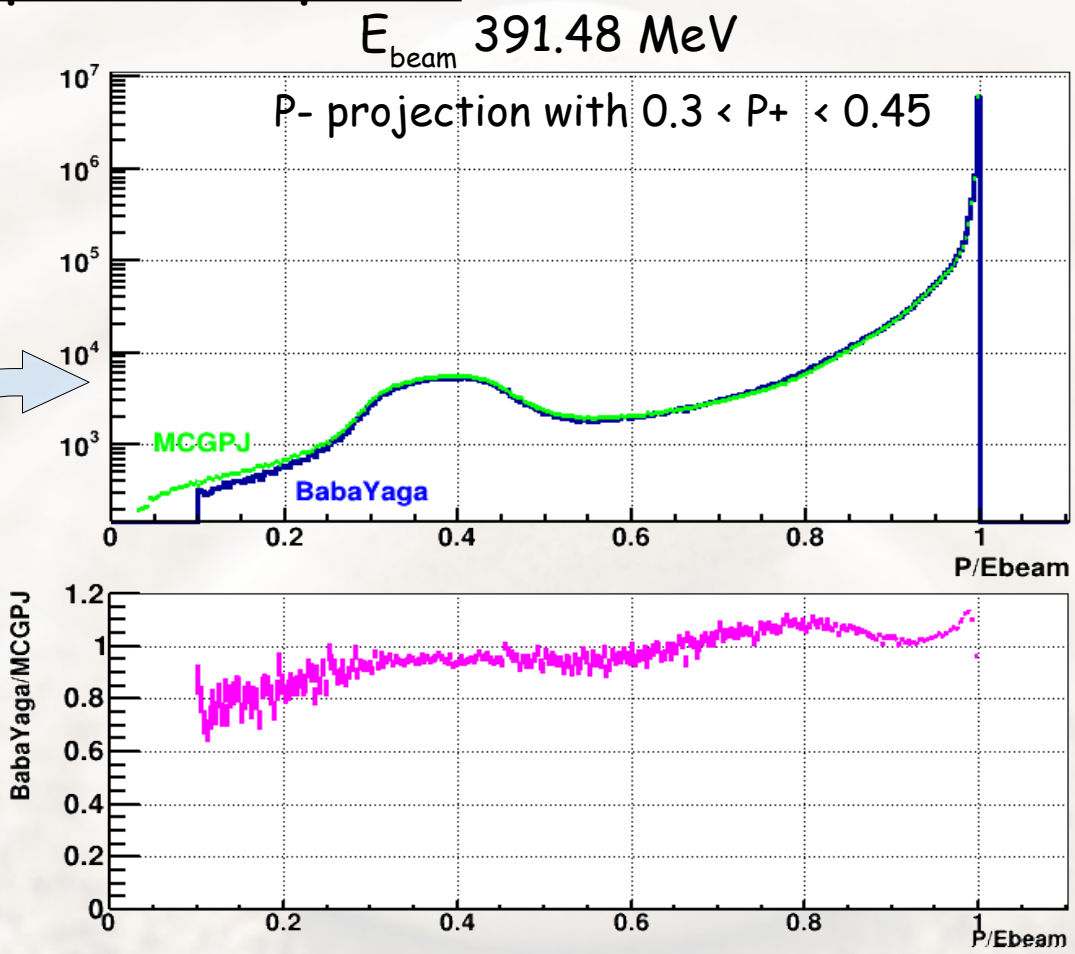
BabaYaga@NLO, KKMC, etc – missed mass term in FSR (arXiv:hep-ph/0505236)



MCGPJ vs BabaYaga bhabha P+ vs P- spectrum

Differential over momentum spectrum comparison





MCGPJ last improvement with jets angles reduce discrepancy from x1.6-3 to x1.1

Momentum spectrum still disagree at level ~ 10%

Tails comes from e+e- → e+e- γγ , NNLO order

Very desirable to have more precise generators

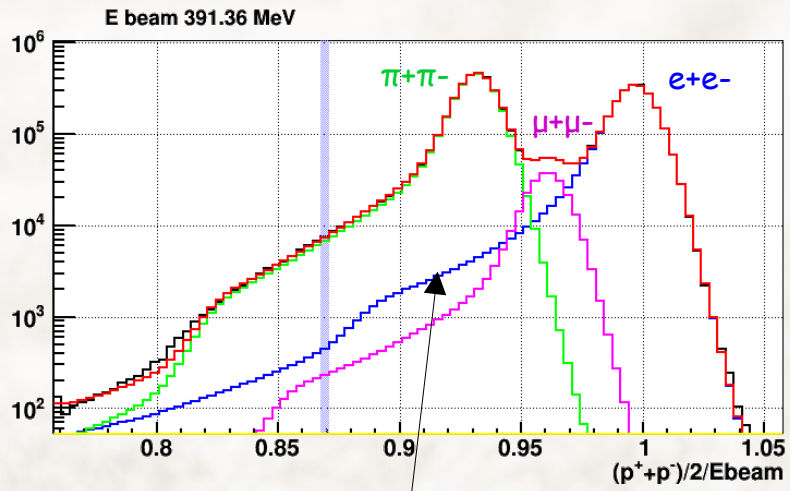
Such discrepancy gives ~0.1-0.2% systematic for π+π- at ρ-peak using momentum analysis at CMD3

Differential cross section effect on form factor

Differential cross section knowledge is necessary for momentum-based separation (not used in energy deposition separation)

Effect ~ 0.1-0.2% at ρ-peak

Effect comes when momentum peaks from π+π- and e+e- become close

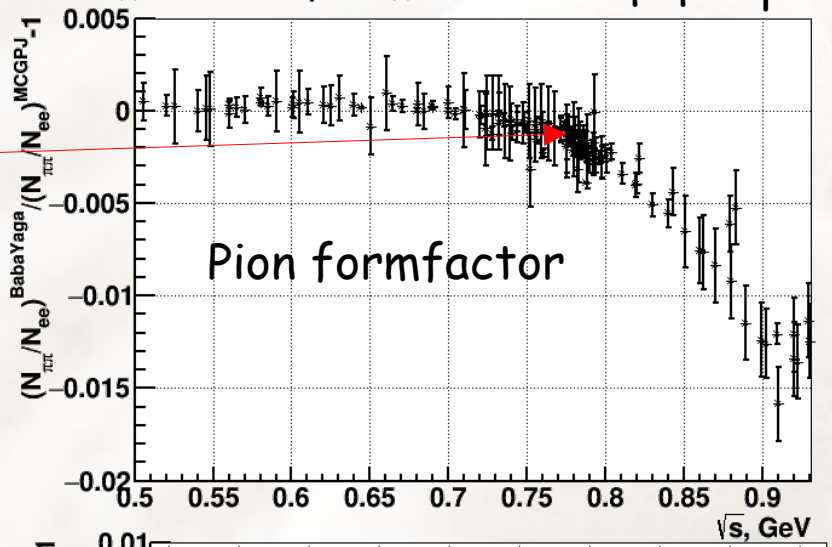


Important here soft photons radiation distribution:

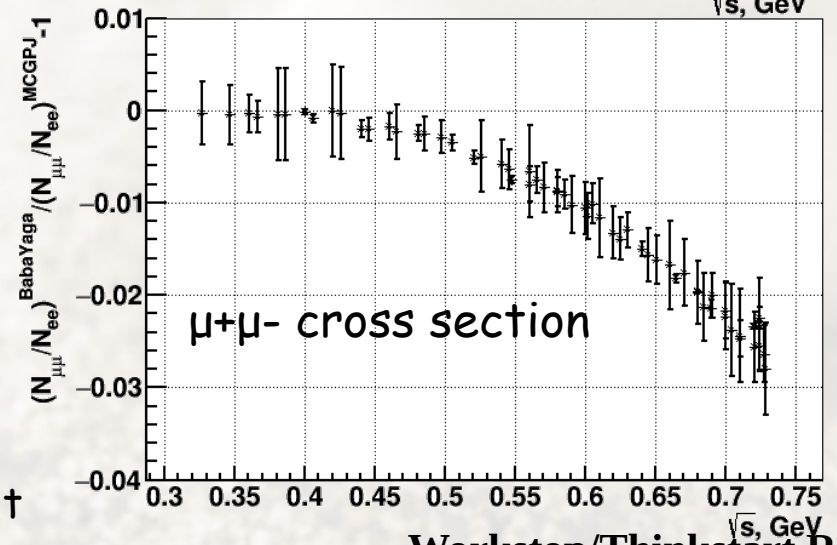
Looks like BaBaYaga@NLO approach with iterative photons generation gives better result

Difference of MCGPJ vs BabaYaga@NLO

Cumulative from e+e- and μ+μ- spectra



μ+μ- effect <0.05-0.1%

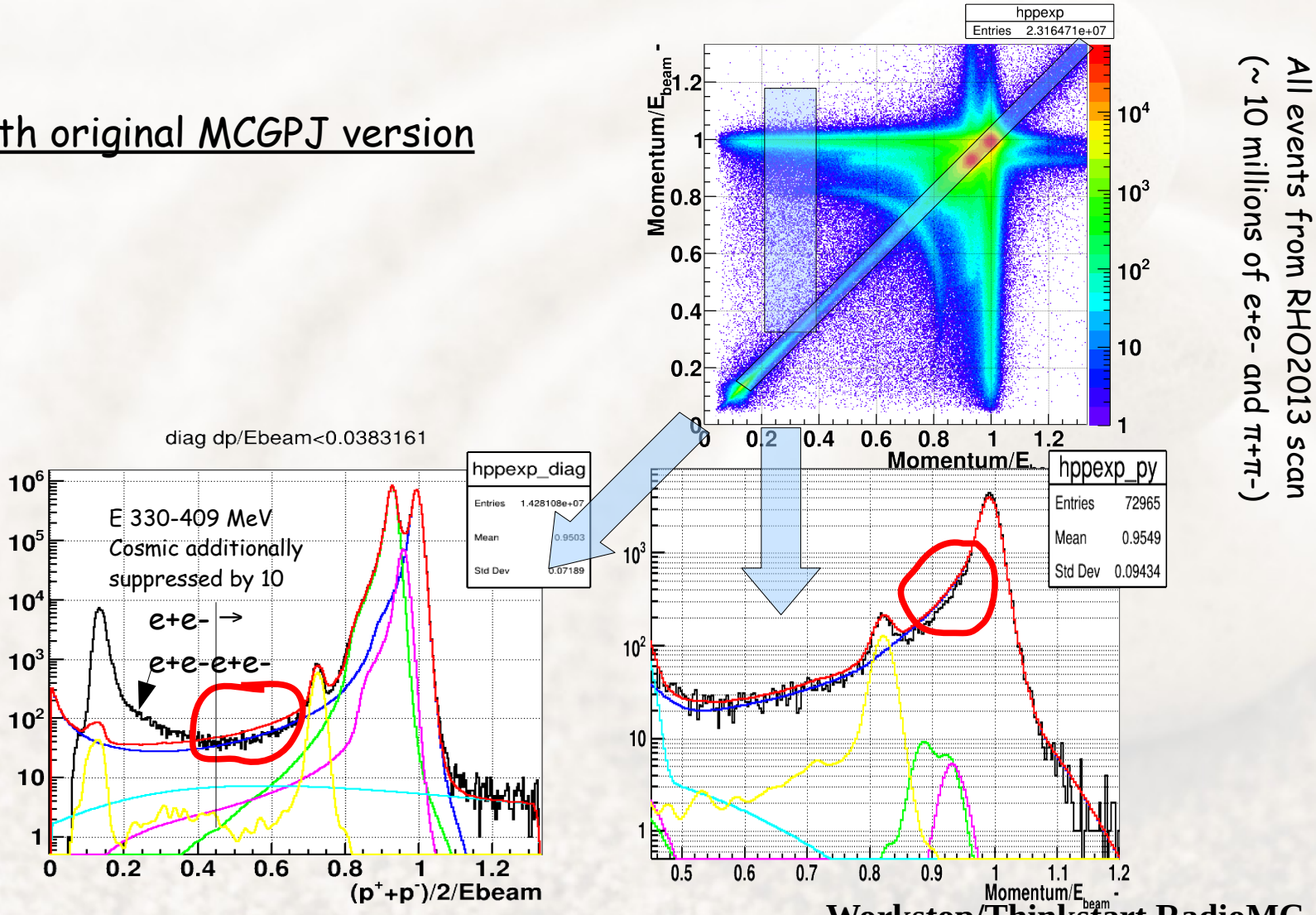


μ+μ- effect ~ 1/4 - 1/3

Question 27

27. A problem is mentioned for the momentum distributions with MCGPJ. Please show Fig. 6-7 using MCGPJ.

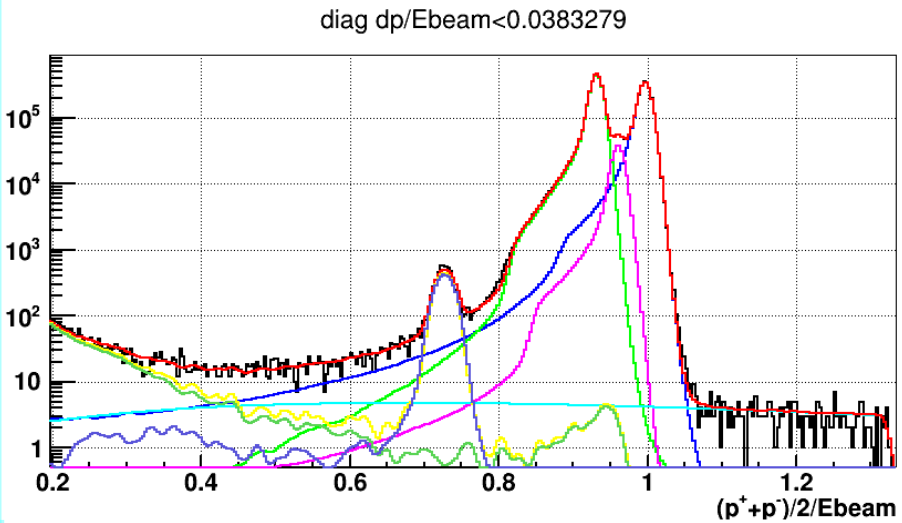
<u>With original MCGPJ version</u>

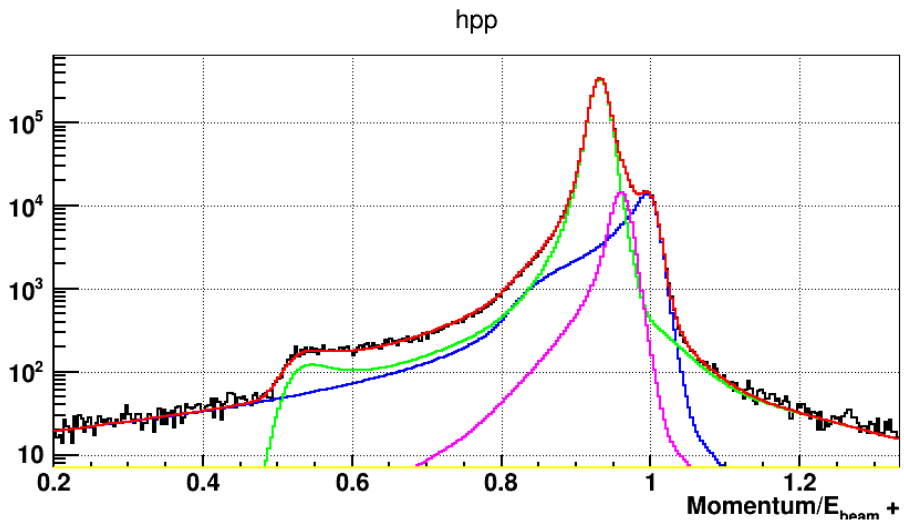


Question 27

BabaYaga@NLO

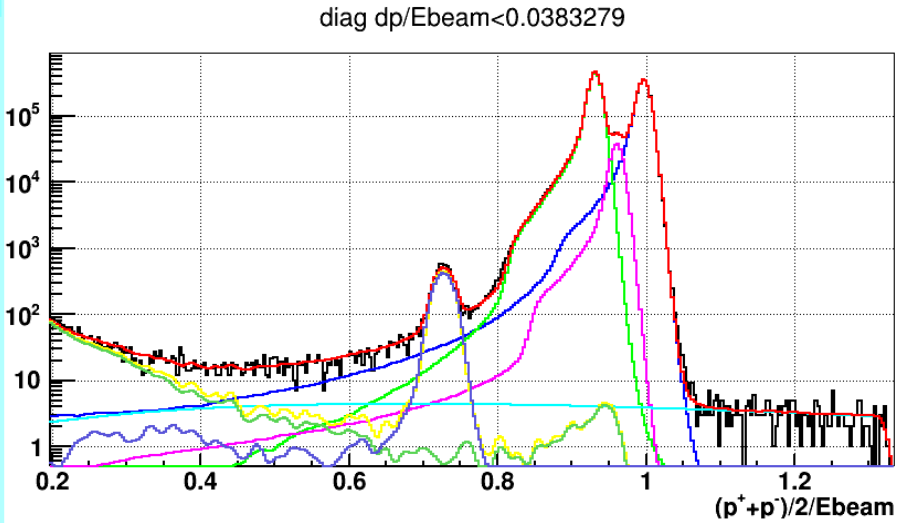
e+e-, μ+μ-

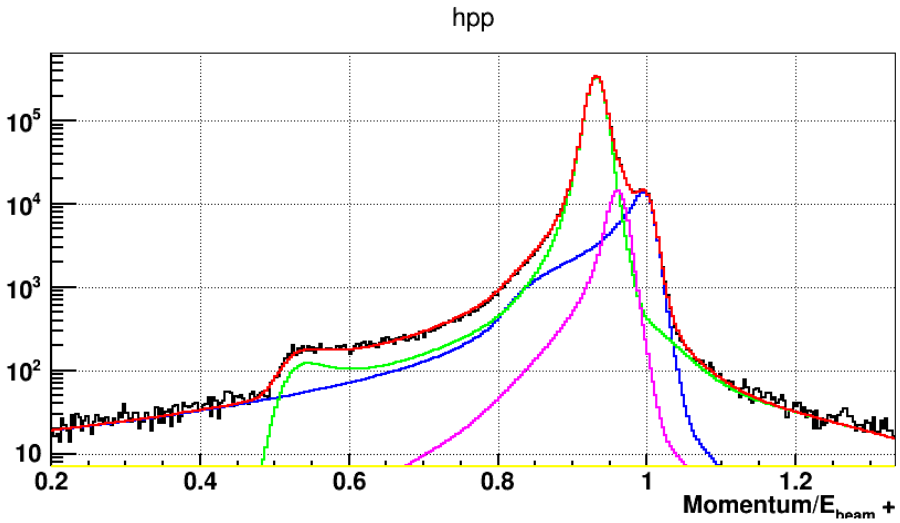




MCGPJ with jet angles

e+e-, μ+μ-





Question 28

28. The problem is claimed to be partly cured by introducing an angular distribution for the photon jets. Is this physical? Wouldn't you expect different angles for each extra photon?

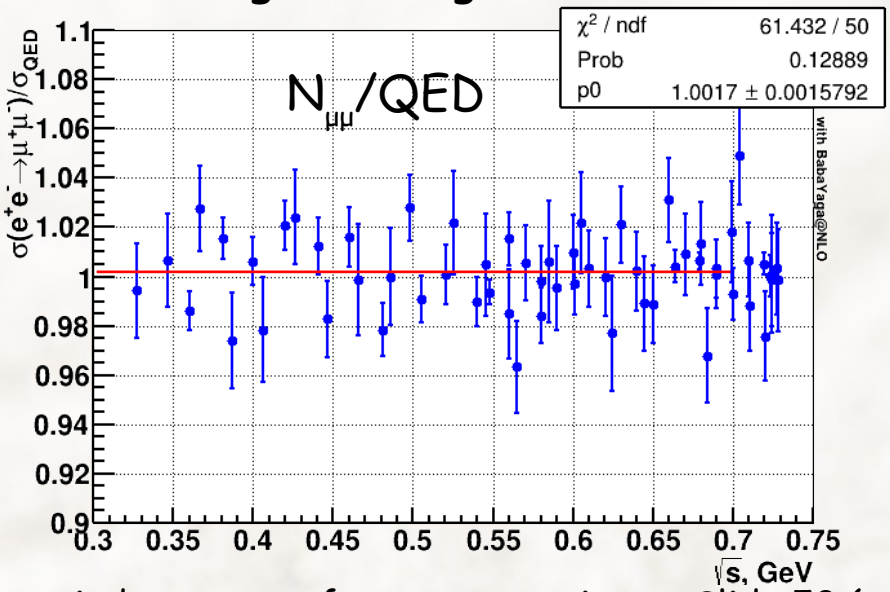
Yes, it is true. It is an approximation.
MCGPJ doesn't have separate extra photons:
There is only jets per lepton with summed energy according to the structure function, or one hard photon on large angle.

Probably this is the reason why BabaYaga@NLO works better:
For momentum-based separation, important difference comes from soft photons radiation region (when momenta of e+e- and π+π- start to be close) – many photon radiation plays role.
Looks like BaBaYaga@NLO approach with iterative photons generation gives better result to describe this soft region.

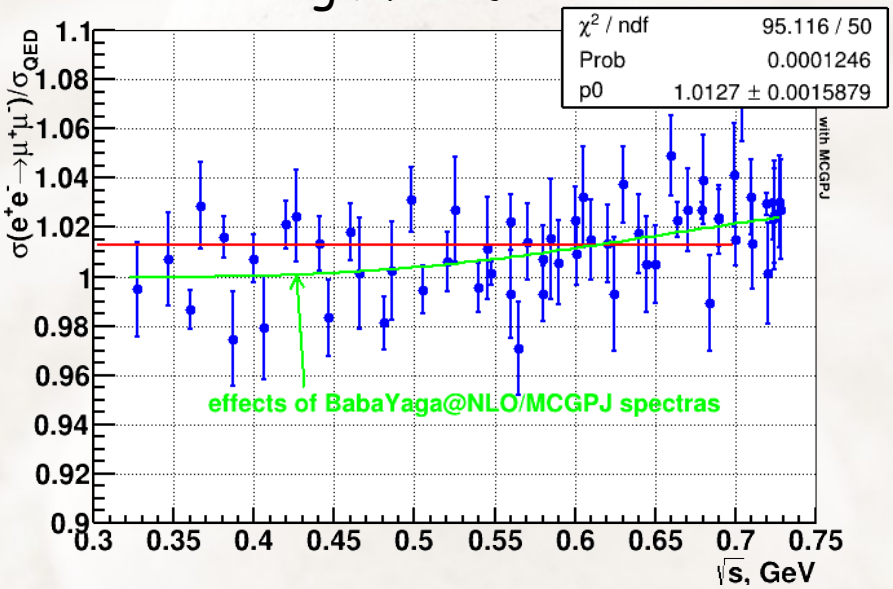
Question 29

29. It seems to affect measurement only above 0.75 GeV for pions, but above 0.4 GeV for muons (Fig. 20, also 1.3% difference quoted p. 36, 10 x larger than the statistical accuracy). In Fig. 30 the agreement is with BabaYaga. Yet MCGPJ is used for pions. Please clarify.

Using BabaYaga@NLO



Using MCGPJ



I was tried to answer for same question on Slide 50 (questioon 26):

The effect in the momentum-based separation comes when peaks of π+π- and e+e- start to be close. Momentum peak of π+π- stay on tail of e+e- momentum distribution and description of this e+e- tail plays role. For example effect on the $N_{\mu\mu}/N_{ee}$ ratio from momentum distribution of μμ itself is 1./4-1./3 less than from e+e-. Same can be expected for π+π-.

Also I tried to use π+π- momentum distribution from Phokhara for PDF construction (next few slides)

→ effect only 0.03% of $|F_\pi|^2$ on Ebeam = 391.36 MeV point.

Questions 30, 32 (from short list)

Question 30: How can you justify a 0.2% error for the ππ mode in MCGPJ given the large uncertainties seen for the Bhabha mode?

Question 32: The RC are large +8% at 0.9 GeV and -9% at 0.7 GeV. What is the uncertainty specific to this analysis, from the used generators. The number 0.2% quoted is for the integrated cross sections ('declared' by MCGPJ authors) , but apparently not listed in Table 2. Also what about NLO+HO differential cross sections? Need to be clarified.

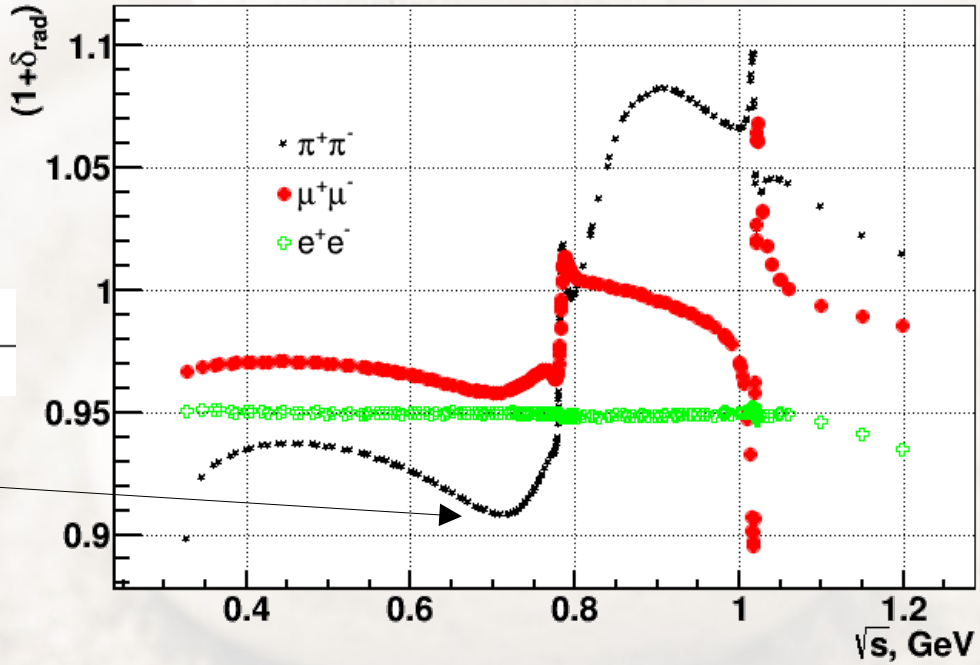
N.B. Integrated cross section in Bhabha mode was always consistent between generators at ~ < 0.1%

0.2% from MCGPJ is listed in Systematics Table 2:

Contribution
$0.2\%\ (\pi^+\pi^-) \oplus 0.2\%\ (F_\pi, \sqrt{s} > 0.74\ \text{GeV}) \oplus 0.1\%\ (e^+e^-)$

+8%/-9% wave comes from F_π and ISR

Uncertainty from different F_π parametrizations is second part in radiative correction uncertainty



Differential cross section doesn't affect energy deposition-based separation.

Looking on Nμμ/Nee in momentum-based separation, the effect from ππ spectra probably is smaller than from e+e- spectra (0.1-0.2% at ρ)

ππ generator

For ππ mode

Unfortunately only MCGPJ available with declared 0.2% precision (for energy scan experiments)

Phokara and BabaYaga 3.5 are incomplete at NLO level for energy scan mode: there is no FSR

Very desirable to have new precise generator with above sQED which will cover ISR up to $E_\gamma=0$

The table with applied radiative corrections in this analysis is part of arXiv submission,
It will be useful for cross-checks if new generators will be appeared.

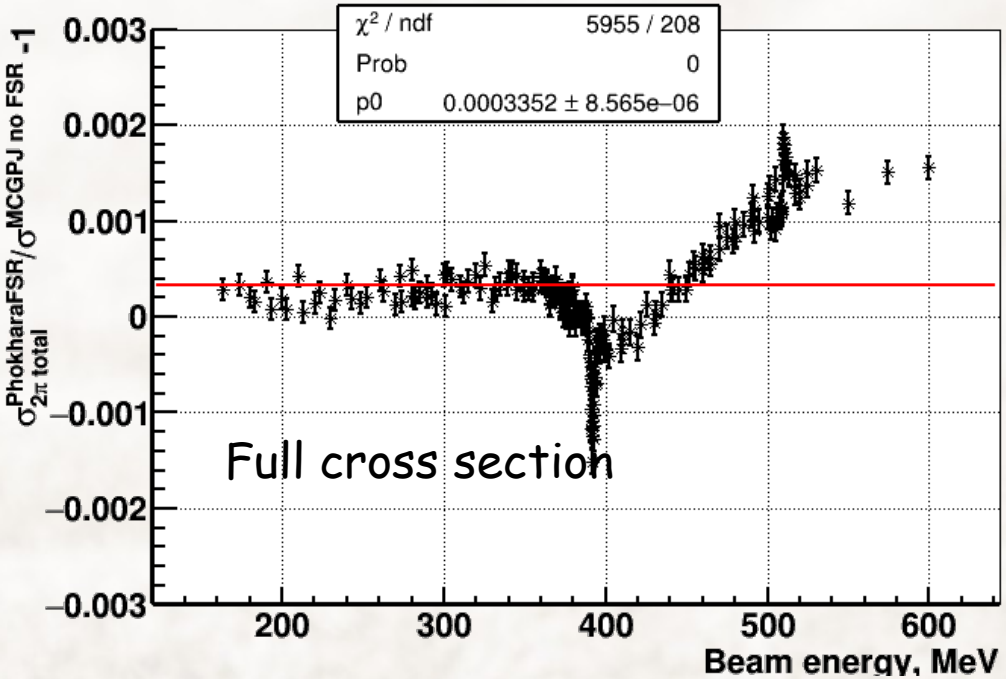
Some cross checks to compare MCGPJ/Phokara were performed

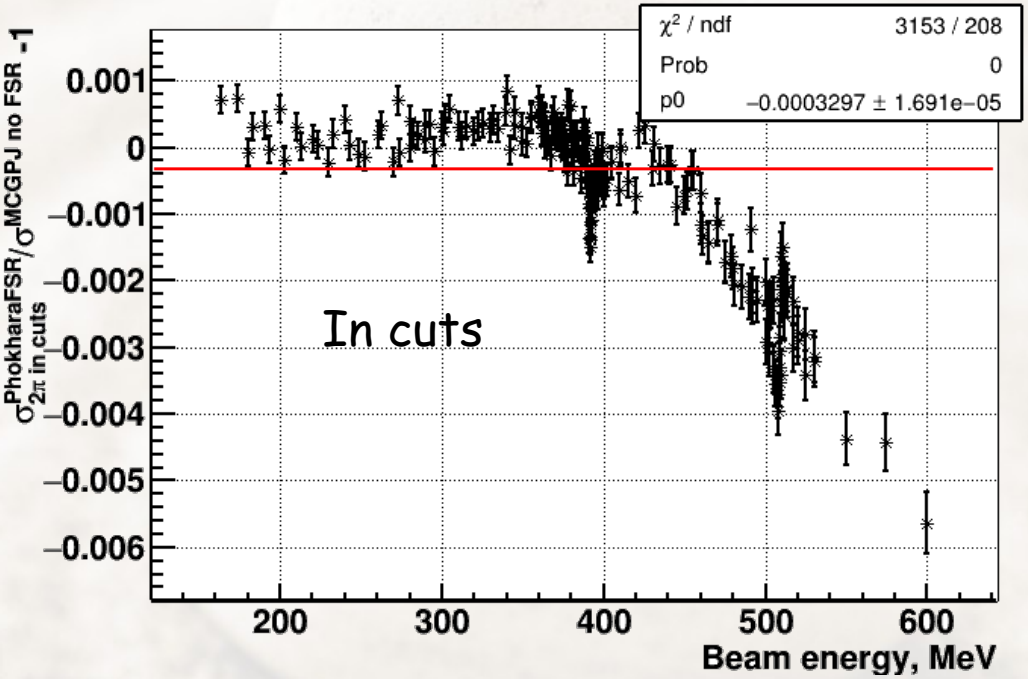
At E_{beam} 391.48 MeV point: If to use Phokara momentum spectra for ππ PDF instead of MCGPJ → 0.03% difference on F_π

MCGPJ/Phokara

ISR and Fπ cross check

MCGPJ with FSR off,
Phokara 10 with same |Fπ| as in MCGPJ, additional VP off





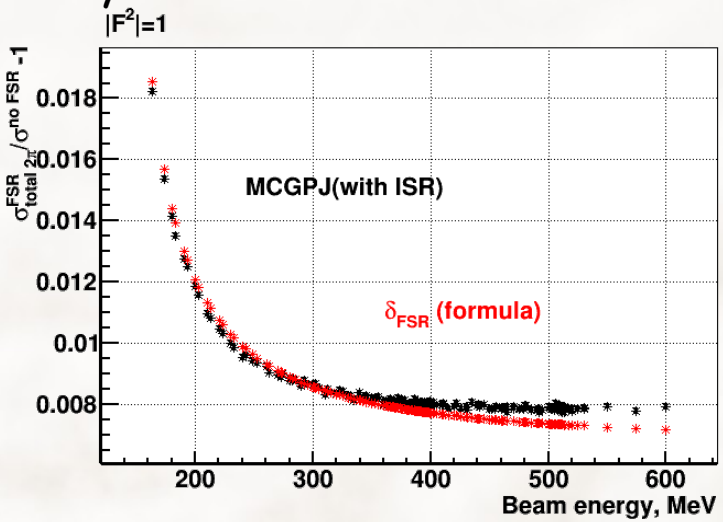
Cross section is consistent at ~0.05% at ρ-peak
(at phi ~ 0.25%)

MCGPJ FSR contribution

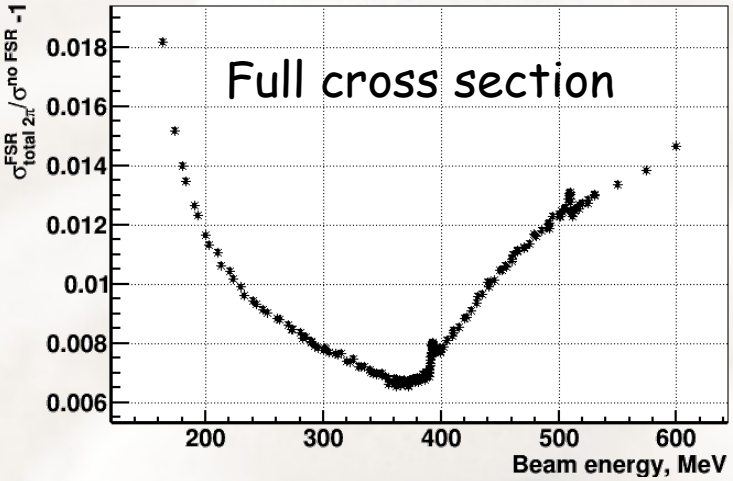
With Fpi=1 FSR is consistent with analytical formula at < 0.05%

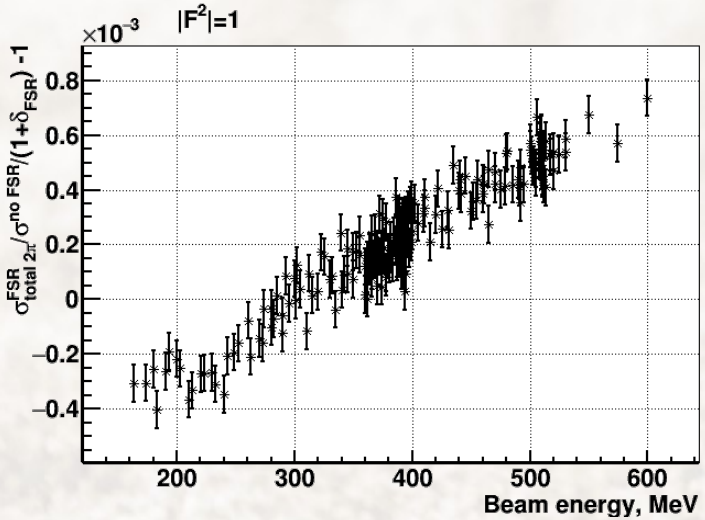
With full formfactor behaviour it is different because of ISR return.

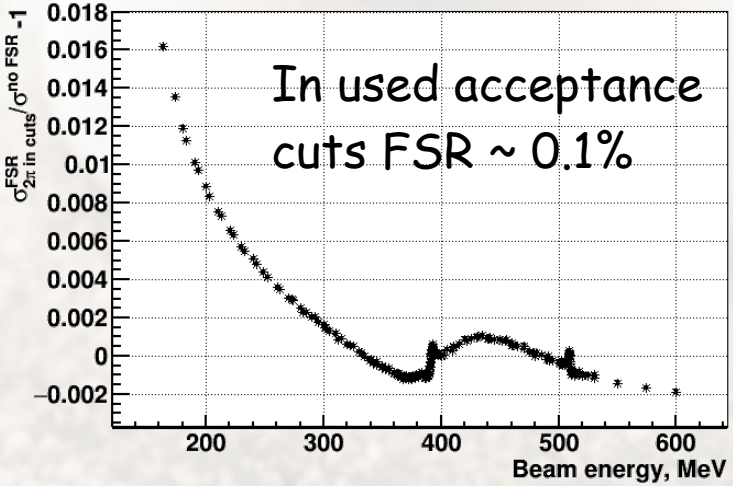
Looks reasonable



$$\frac{\sigma^{\mathrm{FSR}}}{\sigma^{\mathrm{no\,FSR}}} - 1$$







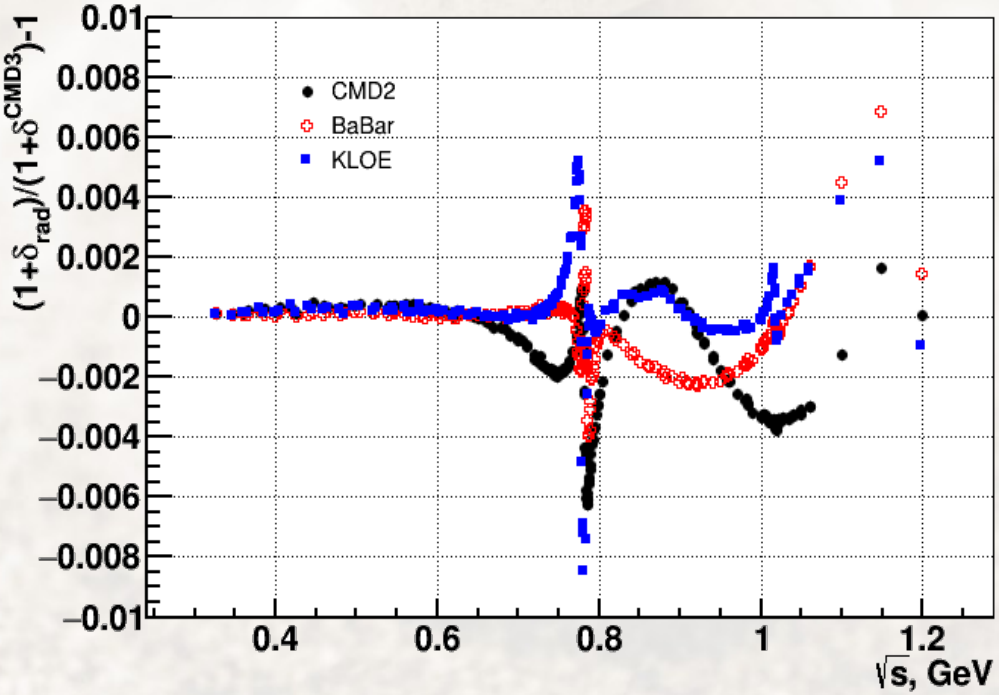
Question 31

31. Why quote a systematic uncertainty on the RC only from form factor parametrizations in other experiments, since the iterative procedure uses the CMD-3 data and so should be self-consistent?

Effect on 2π radiative correction from different $|F|^2_\pi$ parametrizations (over different datasets)

Not only, it is also with CMD3 form factor parametrization. The plot on radiative correction is shown as relative to CMD-3 case.
The radiative correction itself is taken from CMD-3 parametrization – so it is self-consisten.

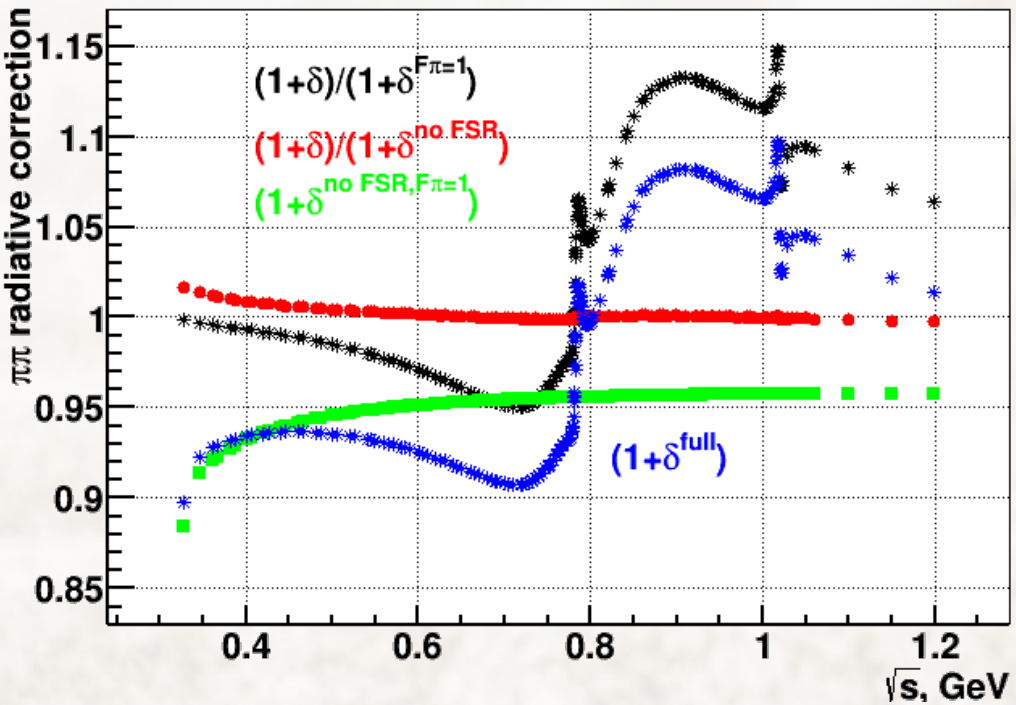
Yes, quoted systematic uncertainty is estimated by looking on different datasets (like theoretical view above different experimental measurement)

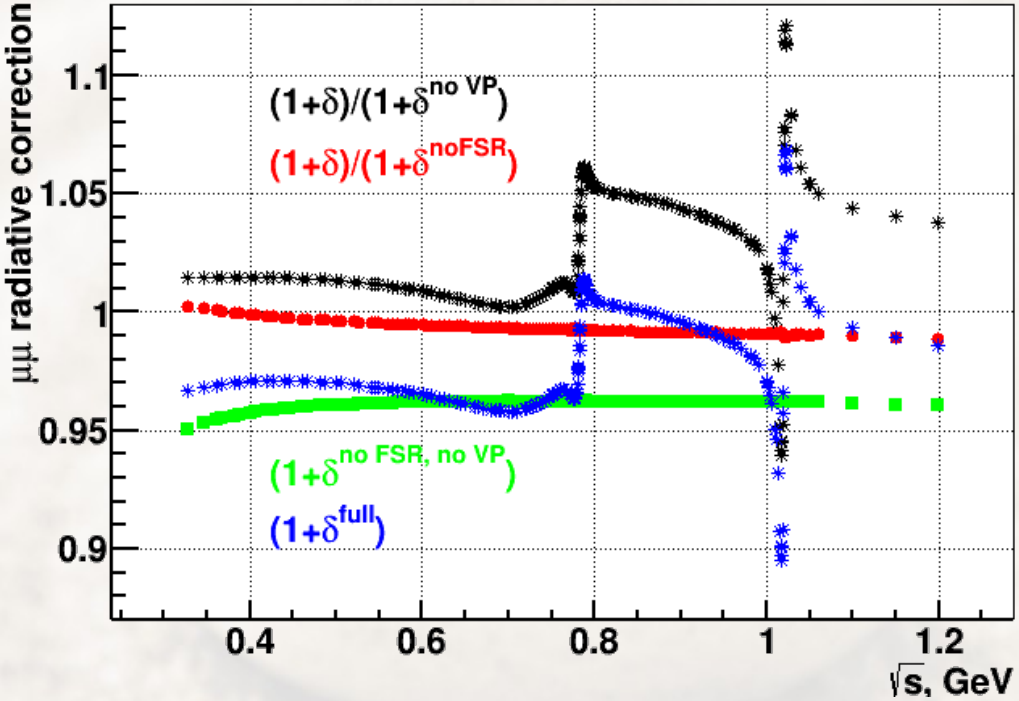


Question 33

33. In Fig 21 would be possible to distinguish the different sources of RC (ISR, FSR and VP) for the three sample (ee, μμ, ππ) also when Babayaga@NLO and MCJPG are used (for ee, μμ)?

Using MCGPJ





For e+e- it is no separate formulas without FSR in the MCGPJ generator.
Effect from the VP is much smaller as t-channel dominated.