

WP1 : Leptonic Processes at NNLO

From Amplitudes to Cross Sections

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Overview

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goal $d\Gamma^{(2)} = \int d\Phi_n 2 \operatorname{Re} \Delta_n^{(2)} \Delta_n^{(0)*} + |\Delta_n^{(1)}|^2$

$$+ \int d\Phi_{n+1} 2 \operatorname{Re} \Delta_{n+1}^{(1)} \Delta_{n+1}^{(0)*}$$

$$+ \int d\Phi_{n+2} |\Delta_{n+2}^{(0)}|^2$$

assume $\Delta_{n,n+1,n+2}^{(0), (1)}$ known, $\Delta_n^{(2)}$ known with $m_e = 0$

steps

- ① massification
- ② HVP corrections
- ③ IR divergences
- ④ numerical stability

QED vs. QCD

- ⊖ legme physical
- ⊕ only soft div.
- ⊖ $E_f \ll m_e \ll Q$

④ Massification [Mitov, Moch 07; Becher, Neubert 11; Memeke 18] 3/7

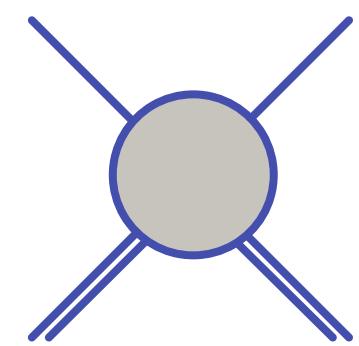
hierarchy

$$p_j^2 = m_j^2 \ll Q^2$$

↑@2-loop

↑ heavy particle

factorisation



process indep. → extract from simple process with MoR

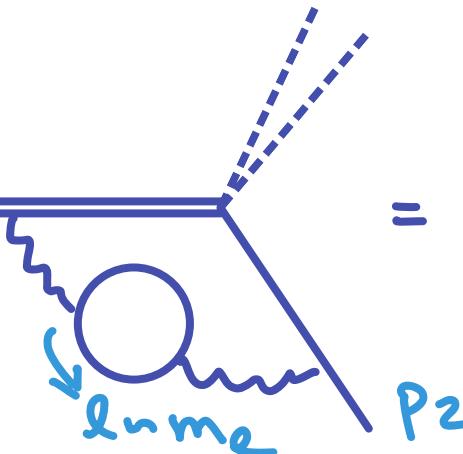
$$= \text{Feynman diagram with } m=0 \text{ and } s \text{ loop} + \mathcal{O}(m) \quad \text{→ more details in Wf2}$$

↓
legs and const.

anomaly

[Becher, Neubert 11]

$$S_{\mu \rightarrow \pi\pi}^{(2)}$$



$$= \int [dl] \frac{\pi(\ell^2)}{[\ell^2]^2 [\ell \cdot p_1]^{4+\eta} [\ell \cdot p_2]} \sim \frac{1}{\eta}$$

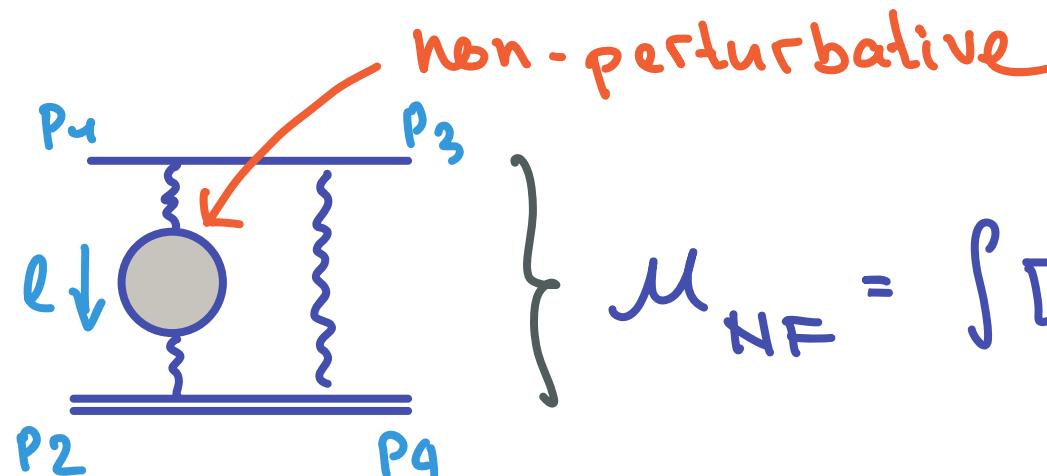
cancels $1/\eta$ in $\pi^{(2)}$

→ breaks fact. & larger massification error

→ different method for fermion loops

② HVP corrections

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dispersive
↑
time-like

hyperspherical
↑
space-like
[Fael 19]

$$\left\{ \begin{array}{l} M_{NF} = \int [dl] \frac{\pi(l^2)}{l^2} \tilde{M}_{NF}(l, p_i) \\ M_{NF} \sim \int_0^\infty \frac{dz}{z} R(z) \end{array} \right.$$

↑ semi-num. approach
for HVP and leg. VP

$$\int_0^\infty \frac{dz}{z} \frac{R(z)}{q^2 - z + i0^+}$$

$$[dl] \frac{\tilde{M}_{NF}}{l^2 - z + i0^+} \hat{=} \boxed{\text{---}}$$

- ⊕ one-loop tools can be used → COLLIER
- ⊖ hadronic resonances in $R(z)$

[Dennert, Dittmaris,
Hafes 17]

$$M_{NF} \sim \int_0^\infty dQ^2 \pi(-Q^2) \int dl \tilde{M}_{NF} \quad \left. \begin{array}{l} \text{non-trivial Wick-rot.} \\ \text{→ } l^0 \rightarrow iQ^0, p_i^0 \rightarrow i\vec{p}_i^0 \end{array} \right\}$$

- ⊕ $\pi(-Q^2)$ smooth for $Q^2 \leq 0$
- ⊖ non-trivial analytic cont. in s-chann
↳ don't know how to do this...

③ IR divergences $\rightarrow m_\ell \neq 0$ w/ soft divergences only

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eikonal

$$\text{Diagram with } E_\ell \text{ outgoing} = \varepsilon \cdot \text{Diagram} + \mathcal{O}(E_\ell^0) \quad [\text{Yennie, Frautschi, Suura 61}]$$

$$= \varepsilon \mu_n + \mathcal{O}(\Delta^{-1})$$

slicing

$$\int d\Phi_\ell \mu_{n+1} = \underbrace{\int_{E_\ell > \Delta} d\Phi_\ell \mu_{n+1}}_{\text{analytic}} + \underbrace{\int_{E_\ell < \Delta} d\Phi_\ell \mu_{n+1}}_{\text{analytic}}$$

⊕ all-order generalisation trivial

⊖ $\Delta \ll Q, m_\ell$ for accurate result

subtraction

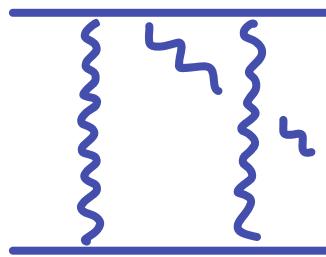
$$\int d\Phi_\ell \mu_{n+1} = \int d\Phi_\ell (\mu_{n+1} - \mu_{CT}) + \underbrace{\int d\Phi_\ell \mu_{CT}}_{\text{analytic}} \stackrel{?}{=} \varepsilon \cdot \mu_n$$

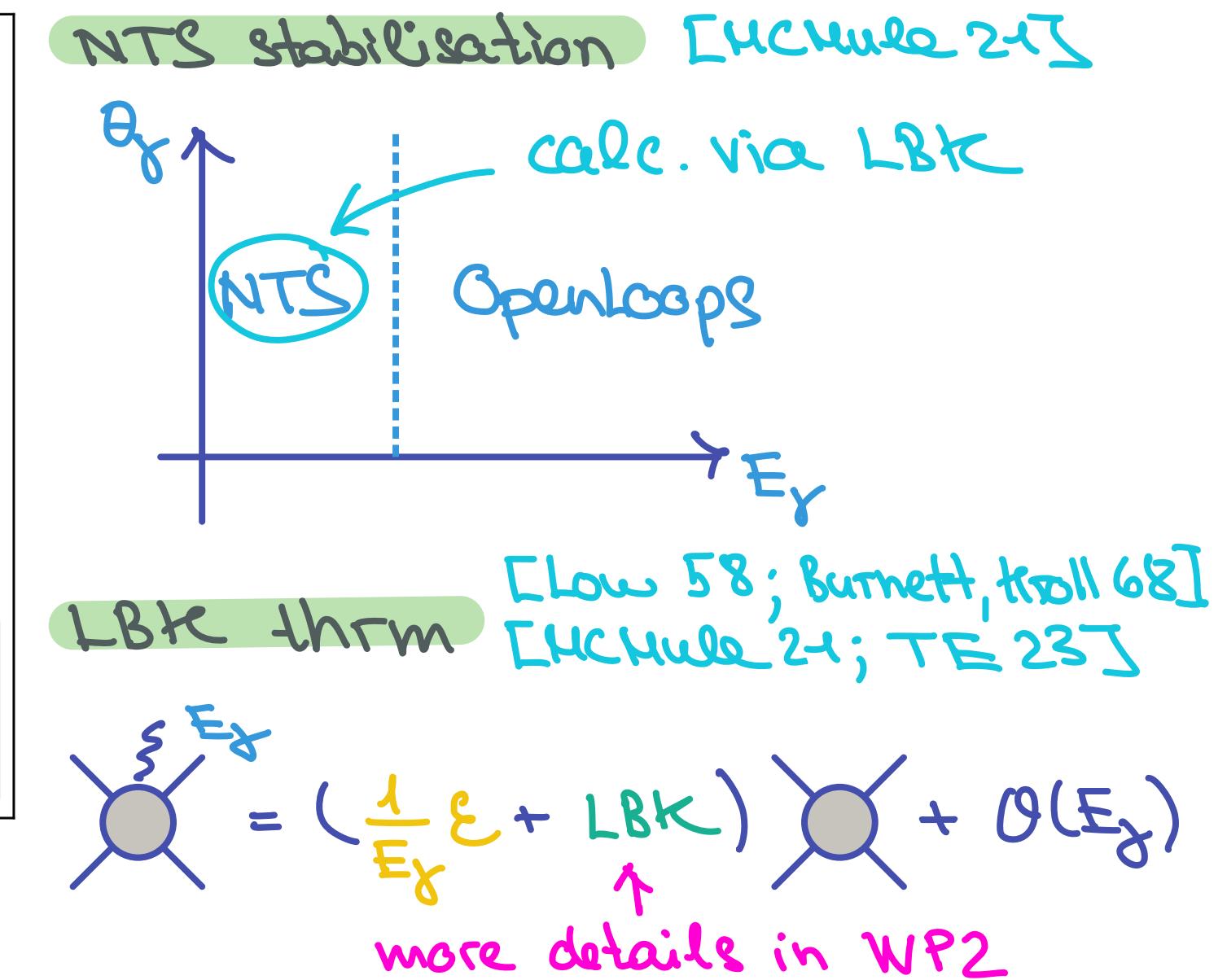
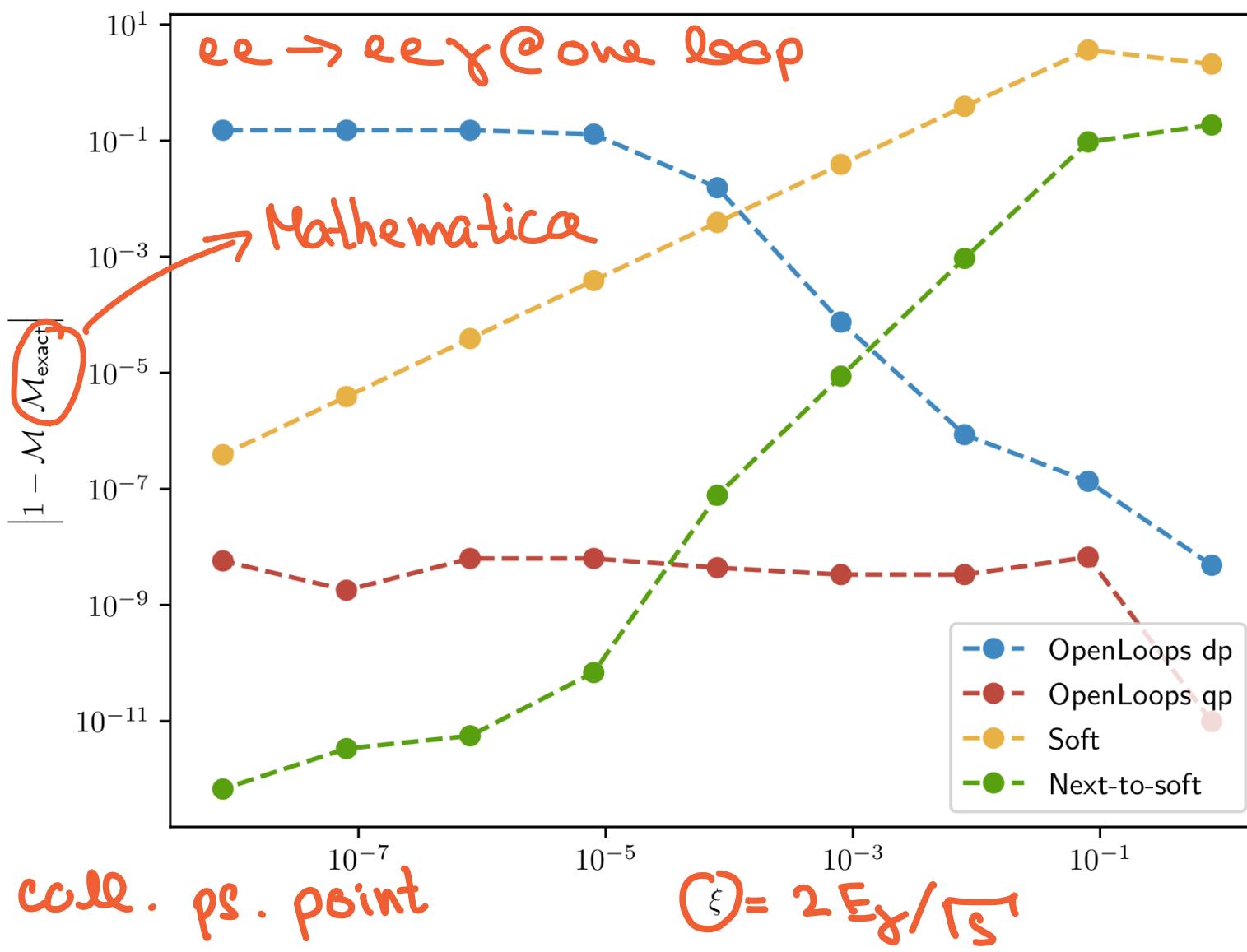
⊕ known @ all orders (FKS^e) [NCMule 19]

⊖ implementation more involved

④ Numerical stability

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 numerically delicate if $E_\gamma \ll m_e \ll Q$ (esp. soft-cell)
 ↳ COLLIER [Denner, Dittmaier, Hofer 17]
 OpenLoops [Buccioni, Lang, Linckert, Maierhöfer, Pozzorini, Zhang, Zoller 19]
 ... ↳ not optimised for $m_f \neq 0$



Conclusions

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amplitudes \rightarrow cross sections @ NNLO understood :

- massification
- IR slicing / subtraction
- NTS stabilisation

nr general methods, i.e. can be applied to

$\mu e \rightarrow \mu e$, $e e \rightarrow \mu \mu$, $e e \rightarrow \gamma^* \gamma^*$



[Calame, Chiesa, Hasan, Montagna, Niccosini, Piccinini 20]

[Banerjee, TE, Signer, Ulrich 20]

[Broggio et al. 22]