`Radiative Corrections and Monte Carlo tools for low-energy hadronic cross sections in e⁺e⁻ collisions'

HVP: Status of the dispersive approach / puzzles



Thomas Teubner



- Introduction & overview
- Data-driven HVP evaluation: basic ingredients, main features
- The most important 2π channel, other channels, total HVP
- Recent new data, one new puzzle
- Outlook, new analyses & pathways to solving the puzzles

SM prediction from Theory Initiative vs. Experiment



SM uncertainty dominated by hadronic contributions, now with δ HVP > δ HLbL

auhadronic: non-perturbative, the limiting factor of the SM prediction



- Q: What's in the hadronic (Vacuum Polarisation & Light-by-Light scattering) blobs?
 A: Anything `hadronic' the virtual photons couple to, i.e. quarks + gluons + photons
 But: low q² photons dominate loop integral(s) and calculate blobs with perturbation theory
- Two very different (model independent) strategies:
 - 1. use wealth of hadronic data, `data-driven dispersive methods': a_{ii}
 - data combination from many experiments, radiative corrections required
 - 2. simulate the strong interaction (+photons) w. discretised Euclidean space-time, `<u>lattice</u> QCD':
 - **I** finite size, finite lattice spacing, artifacts from lattice actions, **QCD** + **QED** needed
 - numerical Monte Carlo methods require large computer resources

a_u^{HVP}: Basic principles of **dispersive** data-driven method



One-loop diagram with hadronic blob = integral over q² of virtual photon, 1 HVP insertion

Causality analyticity dispersion integral: obtain HVP from its imaginary part only

Unitarity → Optical Theorem: imaginary part (`cut diagram') = sum over |cut diagram|², i.e. ∝ sum over all total hadronic cross sections

• Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$ \implies Lower energies more important $\implies \pi^{+}\pi^{-}$ channel: 73% of total $a_{\mu}^{\text{had,LO}}$

- Total hadronic cross section σ_{had} from > 100 data sets for $e^+e^- \rightarrow hadrons$ in > 35 final states
- Uncertainty of $a_{\mu}^{\mu\nu\rho}$ prediction from statistical & systematic uncertainties of input data
- pQCD only at large s, **no modelling** of $\sigma_{had}(s)$, direct data integration

Higher orders & power counting; WP20 values in 10-11







- All hadronic blobs also contain photons, i.e. real + virtual corrections in $\sigma_{had}(s)$
- LO: 6931(40)
- NLO: 98.3(7)

from three classes of graphs: -207.7(7) + 105.9(4) + 3.4(1) [KNT19]

(photonic, extra e-loop, 2 had-loops)

NNLO: 12.4(1) [Kurz et al, PLB 734(2014)144, see also F Jegerlehner]

from five classes of graphs:

8.0 - 4.1 + 9.1 - 0.6 + 0.005

- good convergence, iterations of hadronic blobs _very_ small
- `double-bubbles' very small

HVP: short detour into double-bubbles

• What if the blob in



is a `double-bubble' ?

• Purely leptonic graphs (left diagram below) are part of four-loop QED corrections



- But possibly enhanced contributions from mixed hadronic-leptonic double bubble graphs (right diagram above) are not included in the hadronic NNLO HVP corrections quoted above
- Our recent work has estimated these remaining NNLO contributions to a_μ to be below 1 × 10⁻¹¹ and hence not critical at the level of the experimental accuracy

M Hoferichter + TT, *Phys. Rev. Lett.* 128 (2022) 11, 112002

HVP disp.: cross section (in terms of R-ratio) input



Must build full hadronic cross section/R-ratio...

R(s)

HVP: Recent (oExperiments) talpler puts to phill Pding input σ_{had}(s) data

S. Serednyakov (for SND) @ HVP KEK workshop



 Different methods: `Direct Scan' (tunable e⁺e⁻ beams) & `Radiative Return' (Initial State Radiation scan at fixed cm energy) ✓

• Over last decades detailed studies of radiative corrections & Monte Carlo Generators for σ_{had}(s)

RadioMonteCarLow Working Group report: Eur. Phys. J. C66 (2010) 585-686

► full NLO radiative corrections in ISR MC **Phokhara**: Campanario et al, PRD 100(2019)7,076004

 \sim^{γ}

 Ω^2

ISR

hadrons

e+

e⁻

HVP dispersive: cross section compilation

How to get the most precise σ^{0}_{had} ? Use of $e^+e^- \rightarrow hadrons(+\gamma)$ data:

- Low energies: sum ~35 exclusive channels, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KKππ, ηπ, ..., [now very limited use iso-spin relations for missing channels]
- Above Vs ~1.8 GeV: use of inclusive data or pQCD (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Y)
- Challenge of data combination (locally in Vs, with error inflation if tensions):
 - many experiments, different energy ranges and bins,
 - statistical + systematic errors from many different sources, use of correlations
 - Significant differences between DHMZ and KNT in use of correlated errors:
 KNT allow non-local correlations to influence mean values,
 - DHMZ restrict this but retain correlations for errors, also estimate cross channel corrs.
- σ⁰_{had} means the `bare' cross section, i.e. <u>excluding</u> `running coupling' (VP) effects, but <u>including</u> Final State (γ) Radiation:
 - data need radiative corrections, compilations estimate additional uncertainty,

e.g. in KNT: $\delta a_{\mu}^{had, VP} = 2.1 \times 10^{-11}$, and $\delta a_{\mu}^{had, FSR} = 7.0 \times 10^{-11}$

Rad Corrs: ISR. Scan vs ISR method. Phokara

- ISR is always there, also for `direct scan' measurements, well understood theoretically and routinely taken into account in the experimental analyses (deconvolution of measured hadrons (+γ) cross section to get the cross section w/out ISR)
- In `Radiative Return' analyses, ISR emission defines already the lowest order process, hence higher orders, including FSR, are crucial
- The origin of additional photons can not be determined on an event-by-even basis
- Making use of high luminosities at meson factories, large event numbers can still be achieved with the ISR method, despite the parametric α/π suppression
- Different variants: w. or w/out γ detection (large/small angle), luminosity from Bhabha or $\mu^+\mu^-$
- Crucial Monte Carlo generator: *Phokhara*
 - -- now with complete NLO corrections for $e^+e^- \rightarrow \mu^+\mu^-\gamma$, $\pi^+\pi^-\gamma$
 - -- was not available for the earlier KLOE & BaBar analyses; study of higher orders using the latest version of *Phokhara* indicate that (missing) higher order corrections are not the source of the KLOE vs BaBar discrepancy (see below)

Rad. Corrs.: HVP for running $\alpha(q^2)$. Undressing

• Dyson summation of Real part of one-particle irreducible blobs Π into the effective, real running coupling $\alpha_{\rm QED}$:

$$\Pi = \bigvee_{q}^{q^*} \bigvee_{q} \bigvee_{q}$$

Full photon propagator $\sim 1 + \Pi + \Pi \cdot \Pi + \Pi \cdot \Pi \cdot \Pi + \dots$

$$\rightsquigarrow \qquad \alpha(q^2) = \frac{\alpha}{1 - \operatorname{Re}\Pi(q^2)} = \alpha / \left(1 - \Delta \alpha_{\operatorname{lep}}(q^2) - \Delta \alpha_{\operatorname{had}}(q^2)\right)$$

• The Real part of the VP, $\text{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical* Theorem is directly related to the cross section, $\text{Im}\Pi \sim \sigma(e^+e^- \rightarrow hadrons)$:

$$\Delta \alpha_{\rm had}^{(5)}(q^2) = -\frac{q^2}{4\pi^2 \alpha} \, \mathcal{P} \int_{m_{\pi}^2}^{\infty} \frac{\sigma_{\rm had}^0(s) \, \mathrm{d}s}{s - q^2} \,, \quad \sigma_{\rm had}(s) = \frac{\sigma_{\rm had}^0(s)}{|1 - \Pi|^2}$$

 $[\rightarrow \sigma^0 \text{ requires 'undressing', e.g. via } \cdot (\alpha/\alpha(s))^2 ~ \leadsto ~ \text{iteration needed}]$

• Observable cross sections σ_{had} contain the |full photon propagator|², i.e. |infinite sum|². \rightarrow To include the subleading Imaginary part, use dressing factor $\frac{1}{|1-\Pi|^2}$.

Rad. Corrs.: HVP for running $\alpha(q^2)$. Undressing

 $\Delta lpha(q^2)$ in the time-like: HLMNT compared to Fred Jegerlehner's new routines



 \rightarrow with new version big differences (with 2003 version) gone

- smaller differences remain and reflect different choices, smoothing etc.

Rad. Corrs.: Final State γ Radiation

- Real + virtual , <u>must be included</u> in σ^{0}_{had} as part of the (hadronic) dynamics
- In measured cross sections, virtual and soft/collinear photons are always included,
- but some events with hard real radiation are cut-off by experimental analyses (through event selection/classification, cuts, acceptances):
 - -- limited phase space for hard radiation at low energies in scan mode
 - -- no problem if γ missed but the event counted, but
 - -- possibly important effect in radiative return (ISR) mode, depending on energy

• Experiments account for this and add (back missed) FSR in their data analyses

- using MC generators with corrections based on scalar QED for πs and Ks
 (checked to work ok at low energies when hadronic substructure hardly resolved)
- -- for analyses based on Radiative Return (in particular for the 2π channel), ISR and FSR are an integral part of the MCs used (*EVA*, *Phokhara*)
- -- possible limitations for accuracy discussed at this WorkStop/ThinkStart, work planned for higher order corrections & MC implementation

Rad. Corrs.: inclusive Final State γ Radiation in sQED

- `Schwinger' formula for inclusive (r+v) FSR: $\sigma_{had,(\gamma)}^0(s) = \sigma_{had}^0(s) \left(1 + \eta(s)\frac{\alpha}{\pi}\right)$ [`hard' real radiation (above a cutoff) is finite and easy to calculate as part of $\eta(s)$]
- Example 2π : inclusive correction compared to cross section in the ρ peak region



HVP: Landscape of $\sigma_{had}(s)$ data & most important $\pi^+\pi^-$ channel



[KNT18, PRD97, 114025]

 hadronic channels for energies below 2 GeV

[KNT19, PRD101, 014029]

dominance of 2π

π⁺π⁻ :

- Combination of >30 data sets, >1000 points, contributing >70% of total HVP
- Precise measurements from 6 independent experiments with different systematics and different radiative corrections
- Data sets from Radiative Return dominate, until now...



a_{μ}^{HVP} : $\pi^+\pi^-$ channel KLOE vs. Babar puzzle, enlarged WP error

[Plots from KNT19]



- Tension between different sets, especially between the most precise 4 sets from BaBar and KLOE
- Inflation of error with local χ^2_{min} accounts for tensions, leading to a ~14% error inflation
- Important role of **correlations**; their treatment in the data combination is crucial and can lead to significant differences between different combination methods (KNT vs. DHMZ)
- Differences in data and methods accounted for in WP merging procedure, leading to enlarged error for a_{μ}^{HVP}

HVP: $\pi^+\pi^-$ channel

- **Tension** between data sets from KLOE, BaBar, CMD-2, SND and BESIII in the ρ - ω interference region
- Note that some differences, possibly due to binning effects, are washed out in the dispersion integral for $a_{\mu}^{2\pi}$



Figure from KLOE (+KT) combination paper JHEP 03(2018)173

HVP: $\pi^+\pi^-$ channel

- Combination of same three KLOE data sets by DHMZ (left) and KNT (right), leading to
- different results, depending on use of long-range correlations through systematic errors;
 - -- DHMZ: restricted to error estimate, but not used to determine combination mean values
 - -- KNT: full use of correlated errors in fit, allowing change of mean values within errors



HVP: π⁺π⁻ channel [DHMZ, *Eur. Phys. J.* C 80(2020)3, 241]

- In addition they employ a fit, based on analyticity + unitarity + crossing symmetery, similar to Colangelo et al. and Ananthanarayan+Caprini+Das, leading to stronger constraints/lower errors at low energies
- For 2π , based on difference between result for $a_{\mu}^{\pi\pi}$ w/out KLOE and BaBar, sizeable additional systematic error is applied and mean value adjusted



HVP: Kaon channels [KNT18, PRD97, 114025]





New data:

BaBar: [Phys. Rev. D 89 (2014), 092002.] CMD-3: [Phys. Lett. B 760 (2016) 314.]

 $a_{\mu}^{K_{S}^{0}K_{L}^{0}} = 13.04 \pm 0.19_{\text{tot}}$ HLMNT11: $13.33 \pm 0.16_{\text{tot}}$

Large changes due to new precise measurements on $\phi_{_{19}}$

HVP: Φ in different final states K⁺K⁻, K_s⁰K_L⁰, $\pi^+\pi^-\pi^0$

Direct data integration automatically accounts for all hadronic dynamics, no resonance fits/parametrisations or estimates of mixing effects needed.



For demo. only, does not include latest data

HVP: σ_{had} inclusive region [KNT18]

 \Rightarrow New KEDR inclusive R data [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and BaBar R_b data [Phys. Rev. Lett. 102 (2009) 012001.].



\implies Choose to adopt entirely data driven estimate from threshold to 11.2 GeV

 $a_{\mu}^{\text{Inclusive}} = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$

HVP: White Hadronic vacuum polarization

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
$J/\psi,\psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7,∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_{\mu}^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_{\psi}(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels [CHS 2018, HHKS 2019] 22

HVP: White Paper merging procedure

Conservative merging procedure developed during 2019 Seattle TI workshop:

- Accounts for the different results obtained by different groups based on the same or • similar experimental input
- Includes correlations and their different treatment as much as possible •
- Allows to give one recommended (merged) result, which is conservative w.r.t. • the underlying (and possibly underestimated) systematic uncertainties
- **Note:** Merging leads to a bigger error estimate compared to individual evaluations; • error `corridor' defined by embracing choices goes far beyond χ^2_{min} inflation

 $a_{\mu}^{HVP, LO} = 693.1 (4.0) \times 10^{-10}$ is the result used in the WP `SM2020' value

This result does not include lattice, but in 2020 was compatible with published full results, • apart from the BMW prediction:

 $a_{\mu}^{HVP, LO}$ (BMW) = 707.5 (5.5) × 10⁻¹⁰ [Nature 2021] \rightarrow **1.5/2.1** σ tension w. exp/WP20

Many efforts are ongoing to understand this new puzzle!

a^{HVP}: > 20 years of data based predictions, `pies'



Pie diagrams for KNT compilation:

- error still dominated by the two pion channel ۲
- significant contribution to error from additional ۰ uncertainty from radiative corrections
- further puzzle from most recent CMD-3 data...

- Stability and consolidation over two decades thanks to more and better data input and improved compilation procedures
- Compare with merged DHMZ &

 $a_{\mu}^{had, LO VP}(WP20) = 693.1(4.0) \times 10^{-10}$



HVP: New/updated data sets since KNT19

- **pi+pi-pi0**, BESIII (2019), arXiv:1912.11208
- pi+pi- [covariance matrix erratum], BESIII (2020), Phys.Lett.B 812 (2021) 135982 (erratum)
- K+K-pi0, SND (2020), Eur.Phys.J.C 80 (2020) 12, 1139
- etapi0gamma (res. only), SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- **pi+pi-**, SND (2020), JHEP 01 (2021) 113
- etaomega → pi0gamma, SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- pi+pi-pi0, SND (2020), Eur.Phys.J.C 80 (2020) 10, 993
- pi+pi-pi0, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112003
- pi+pi-2pi0omega, BaBar (2021), Phys. Rev. D 103, 092001
- etaetagamma, SND (2021), Eur.Phys.J.C 82 (2022) 2, 168
- etaomega, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- pi+pi-pi0eta, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- omegaetapi0, BaBar (2021), Phys. Rev. D 103, 092001
- pi+pi-4pi0, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- pi+pi-pi0pi0eta, BaBar (2021), Phys.Rev.D 103 (2021) 9, 092001
- pi+pi-3pi0eta, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- 2pi+2pi-3pi0, BaBar (2021), Phys. Rev. D 103, 092001
- omega3pi0, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- pi+pi-pi+pi-eta, BaBar (2021), Phys. Rev. D 103, 092001
- inclusive, BESIII (2021), Phys.Rev.Lett. 128 (2022) 6, 062004

HVP: New/updated



- No new full KNT update at this stage yet, *preliminary estimates* show no big surprises
- KNT analysis framework blinded in autumn 2022 (see Alex's talk at TI meeting in Edinburgh)
- pi+pi-, inclusion of BESIII (2020 erratum) & SND (2020):



Measurement	$a_\mu(\pi\pi) \times 10^{10}$
This work	$409.79 \pm 1.44 \pm 3.87$
SND06	$406.47 \pm 1.74 \pm 5.28$
BaBar	$413.58 \pm 2.04 \pm 2.29$
KLOE	$403.39 \pm 0.72 \pm 2.50$

(not yet full statistics, systematics?)

 $a_{\mu}^{2\pi}$ [0.305 ... 1.937 GeV] (KNT19) = (503.46 ± 1.91) × 10⁻¹⁰ \rightarrow (503.88 ± 1.79) × 10⁻¹⁰ (prel.)

• inclusive, inclusion of BESIII (2021):

 $a_{\mu}^{\text{incl.}}$ [1.937 ... 11.2 GeV] (KNT19) = (43.55 ± 0.67) × 10⁻¹⁰ → (43.16 ± 0.59) × 10⁻¹⁰ (prel.)



New CMD-3 $\pi^+\pi^-$ data vs. other experiments

Slides from Fedor Ignatov's TI talk 27.3.2023

arXiv:2302.08834



New CMD-3 $\pi^+\pi^-$ puzzle for $a_{\mu}^{\mu\nu\rho}$

Slides from Fedor Ignatov's TI talk 27.3.2023

arXiv:2302.08834



Pathways to solving the (HVP) puzzles

- No easy way out! Signs for Beyond the Standard Model physics?
- BSM at high scales? Many explanations for `4.2σ' puzzle, few seem natural, NP smoking guns in the flavour sector weakened
- BSM `faking' low σ_{had} ? Possible but not probable

[DiLuzio, Masiero, Paradisi, Passera *Phys.Lett.B* 829 (2022) 137037] .. a new Z' [Coyle, Wagner, 2305.02354] ... or even new hadronic states (like sexa-quarks [Farrar, 2206.13460]) ?

- Situation now very complicated due to emerged lattice & CMD-3 puzzles
- More & more precise data are needed (and coming) to solve puzzles
- To avoid any possible bias, **blinded analyses** are now the standard, both for experiments (g-2 and σ_{had}) and lattice
- The third way: MUonE (see talks tomorrow)

KLOE 2π analyses



Large Angle:

2 pion (muon) tracks at 50° < $\vartheta_{\pi,\mu}$ < 130°

Small angle photon selection:

 $\vartheta_{miss} < 15^\circ; \vartheta_{miss} > 165^\circ$

- high statistics for ISR events
- low FSR contribution
- easy to suppress $\varphi \rightarrow \pi^+ \pi^- \pi^0$ background
- photon momentum from kinematics: $\vec{p_{\gamma}} = \vec{p_{miss}} = -(\vec{p_{+}} + \vec{p_{-}})$
- threshold region not accessible

KLOE 2π results



KLOE05

Small Angle analysis of 140 pb⁻¹ @ m_{φ} KLOE Coll. Phys. Lett. B 606 (2005)

KLOE08

Small Angle analysis of 240 pb⁻¹ @ m_{φ} KLOE Coll. Phys. Lett. B 670 (2009)

KLOE10

Large angle analysis of 250 pb⁻¹ @ 1 GeV KLOE Coll. Phys. Lett. B 700 (2011)

KLOE12

KLOE08 with normalisation to $e^+e^- \rightarrow \mu^+\mu^-$ *KLOE Coll. Phys. Lett. B* 720 (2013)

Combination of three sets JHEP 1803 (2018) 173:

 $a\mu^{\pi\pi}$ [0.1 < s < 0.95 GeV²] = (489.8 ± 1.7_{stat} ± 4.8_{sys}) × 10⁻¹⁰

KLOE 2 π uncertainties

We aim to improve:

Ī	Swat $onnong (07)$	$\Lambda \pi \pi_{\alpha}$ aba [4]	$\Lambda \pi \pi \alpha$ motio	
ļ	Syst. errors (70)	$\Delta^{\mu} a_{\mu} abs [4]$	Δa_{μ} ratio	
	Background Filter (FILFO)	negligible	negligible	
Ι	Background subtraction	0.3	0.6	
Т	Trackmass	0.2	0.2	_
	Particle ID	negligible	negligible	
Ι	Tracking	0.3	0.1	
Ι	Trigger	0.1	0.1	
	Unfolding	negligible	negligible	
Т	Acceptance $(\theta_{\pi\pi})$	0.2	negligible	
	Acceptance (θ_{π})	negligible	negligible	
	Software Trigger $(L3)$	0.1	0.1	possible
	Luminosity	$0.3(0.1_{th}\oplus 0.3_{exp})$	-	ISR-FSR
	\sqrt{s} dep. of H	0.2	-	factorization for
	Total exp systematics	0.6	0.7	
Π	Vacuum Polarization	0.1	-	
	FSR treatment	0.3	0.2	
	Rad. function H	0.5	-	
	Total theory systematics	0.6	0.2	
	Total systematic error	0.9	0.7	

KLOE 2π activities

- New effort to analyse the full statistics KLOE 2π data (integrated $L \gtrsim 2$ fb⁻¹)
- New **blind analysis**, unbiased from previous results of KLOE & other experiments
- Significant involvement from theoretical groups
 => improvement of MC(s) to describe ISR and FSR events (PHOKHARA, ...)
- Goal: sub-percent accuracy: improvement of a factor of ~2 on the total uncertainty => $\Delta a_{\mu}^{HLO} \leq 0.4\%$
- Challenges and opportunities to get a clearer understanding of the puzzles
- The Liverpool + externals team:
 - Leverhulme International Professorship: G. Venanzoni
 F. Ignatov, P. Beltrame, E. Zaid; A. Kumari, N. Vestergaard, C. Devanne
 - > Theory efforts: T. Teubner; W. Torres Bobadilla, J. Paltrinieri; T. Dave, P. Petit Rosas

+ contributors from the wider Liverpool Theoretical Physics group

External collaborators: A. Kupsc, S. Müller, L. Punzi, O. Shekhovstova, A. Keshavarzi, W. Wislicki, A. Lusiani, J. Wiechnik

Outlook / Conclusions

- The still **unresolved muon g-2 discrepancy** has triggered a lot of experimental & theory activities, including experiments, the Muon g-2 Theory Initiative & **lattice**
- Much progress has been made for HLbL (disp. & lattice), previously the bottleneck
- For HVP dispersive, the TI published a conservative consensus (WP20)
 - -- no significant changes since WP20 yet, but
 - > the resolution of the puzzles in the crucial 2π channel requires further new data
 - -- expected/puzzling new σ_{had} data for 2π and other channels from

BaBar, CMD-3, SND, BES III, Belle II, and KLOE (Liverpool analysis has started)

- > if precise data agree, the $a_{\mu}^{HVP, LO}$ (dispersive) puzzle will go away and the error down
- -- but further theory input (NNLO⁺ rad. corrs. & MCs) will be crucial
- > may solve the puzzle w. lattice HVP predictions. Longer term, 3rd way: MUonE
- There is a lot to do in the field of RCs and MCs beyond/before the HL LHC ...

Extras

Why HVP: g-2 exp vs theory - sensitivity chart



Plot from Fred Jegerlehner

 $a_{\mu} = a_{\mu}^{\text{CLD}} + a_{\mu}^{\text{HCAR}} + a_{$

Muon g-2 Theory Initiative est. 2017

- "... map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental result."
- Organised 8 int. workshops in 2017-2022, 15tr plenary workshop 5-9.9.2022 @ Higgscentre in Edinburgh
- Next workshop 4-8.9.2023 in Bern
- White Paper posted 10 June 2020 (132 authors, from 82 institutions, in 21 countries)

``The anomalous magnetic moment of the muon in the Standard Model'' [T. Aoyama et al., arXiv:2006.04822, *Phys. Rept.* 887 (2020) 1-166 985 cites to date]

Group photo from the Seattle workshop in September 2019



Channel	Energy range [GeV]	$a_{\mu}^{\mathrm{had,LOVP}} imes 10^{10}$	$\Delta lpha_{ m had}^{(5)}(M_Z^2) imes 10^4$	New data
	Chiral perturbation th	eory (ChPT) threshold conti	ributions	
$\pi^0\gamma$	$m_{\pi} \leq \sqrt{s} \leq 0.600$	0.12 ± 0.01	0.00 ± 0.00	
$\pi^+\pi^-$	$2m_{\pi} \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	
$\pi^+\pi^-\pi^0$	$3m_{\pi} \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	
ηγ	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	
	Data based c	channels ($\sqrt{s} \le 1.937$ GeV)		
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+ 2\pi^- \pi^0)_{non}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{norm}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	
K^+K^-	$0.988 < \sqrt{s} < 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K^0 K^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
ΚΚπ	$1260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
ny	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$n\pi^+\pi^-$	$1.091 < \sqrt{s} < 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(n\pi^+\pi^-\pi^0)$	$1.333 < \sqrt{s} < 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$n^2\pi^+ 2\pi^-$	$1338 \le \sqrt{s} \le 1937$	0.08 ± 0.01	0.03 ± 0.00	
n(i)	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70.71]
$\omega(\rightarrow \pi^0 \gamma) \pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$n\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	
$\phi \rightarrow \text{unaccounted}$	$0.988 < \sqrt{s} < 1.029$	0.04 ± 0.04	0.01 ± 0.01	
$n\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$n(\rightarrow \text{npp})K\bar{K}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$n\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
nn	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.00 0.03 ± 0.01	0.01 ± 0.00 0.01 ± 0.00	[77]
	Estimated con	tributions ($\sqrt{s} < 1.937$ GeV)	
$(\pi^+\pi^-3\pi^0)$	$1.013 < \sqrt{s} < 1.937$	0.50 ± 0.04	0.16 ± 0.01	
$(\pi^+\pi^-4\pi^0)$	$1313 < \sqrt{s} < 1037$	0.21 ± 0.21	0.08 ± 0.08	
$(\kappa \kappa \eta \eta)_{no\eta}$	$1.515 \le \sqrt{3} \le 1.957$	0.03 ± 0.021	0.02 ± 0.01	
$\omega(\rightarrow \text{nnn})2\pi$	$1.309 \le \sqrt{s} \le 1.937$ $1.285 \le \sqrt{s} \le 1.037$	0.05 ± 0.02 0.10 + 0.02	0.02 ± 0.01	
$\omega(\rightarrow npp)2\pi$	$1.203 \le \sqrt{s} \le 1.937$	0.10 ± 0.02 0.17 ± 0.03	0.05 ± 0.01	•••
$\omega(\rightarrow npp)S\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.00 ± 0.01	•••
$w \rightarrow npp/KK$	$1.309 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	•••
$\eta \pi^+ \pi^- 2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.05 ± 0.02	•••
	Other contri	butions ($\sqrt{s} > 1.937$ GeV)		556 69 693
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ		6.26 ± 0.19	7.07 ± 0.22	
ψ $\gamma(1 - 4 - 6)$		1.58 ± 0.04	2.51 ± 0.06	
1(15 - 45)		0.09 ± 0.00	1.00 ± 0.02	•••
pqcD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	
Total	$m_{\pi} \leq \sqrt{s} \leq \infty$	693.26 ± 2.46	276.11 ± 1.11	

Table from KNT18, PRD 97(2018)114025

Update: KNT19 LO+NLO HVP for

a_{e,μ,τ} & hyperfine splitting of muonium PRD101(2020)014029

Breakdown of HVP contributions in ~35 hadronic channels

From 2-11 GeV, use of inclusive data, pQCD only beyond 11 GeV

a_µ (SM): White Paper Mtps://adi.yrg/18.1916/j.physrep.2020.07.006

White Paper [T. Aoyama et al, arXiv:2006.04822], 132 authors, 82 institutions, 21 countries

Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	-98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [<mark>8</mark>]
HVP LO (lattice, udsc)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, <i>uds</i>)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^- , LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_{\mu} := a_{\mu}^{\exp} - a_{\mu}^{SM}$	279(76)	

w.r.t. BNL only

Rad Corrs: HVP for running $\alpha(q^2)$. Accuracy

• Typical accuracy $\delta\left(\Delta \alpha_{\rm had}^{(5)}(s)\right)$

Error of VP in the timelike regime at low and higher energies (HLMNT compilation):



 \rightarrow Below one per-mille (and typically $\sim 5 \cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.

HVP: $\pi^+\pi^-$ channel. Error inflation in KNT

• Inflation of error with local χ^2_{min} accounts for tensions, leading to a ~14% error inflation, with overlay of 2π cross section fit (blue markers) and global χ^2_{min} (dash-dotted line)



HVP: $\pi^+\pi^-$ channel

- Combination of same three KLOE data sets by DHMZ (left) and KNT (right), leading to
- different results, depending on use of long-range correlations through systematic errors;
 - -- DHMZ: restricted to error estimate, but not used to determine combination mean values
 - -- KNT: full use of correlated errors in fit, allowing change of mean values within errors



a^{HVP}: Hadronic tau decay data

- Historically, hadronic tau decay data, e.g. $\tau^- \to \pi^0 \pi^- \nu_{\tau}$, were used to improve precision of e⁺e⁻ based evaluations
- However, with the increased precision of the e⁺e⁻ data there is now limited merit in this (there are some conflicting evaluations, DHMZ have dropped it)
- The required iso-spin breaking corrections re-introduce a model-dependence and connected systematic uncertainty (there is, e.g., no $\rho-\omega$ mixing in τ decays)
- Quote from the WP, where this approach is discussed in detail:

"Concluding this part, it appears that, at the required precision to match the e^+e^- data, the present understanding of the IB corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. It remains a possibility, however, that the alternate lattice approach, discussed in Sec. 3.4.2, may provide a solution to this problem."

New contribution to the discussion by Masjuan, Miranda, Roig: arXiv:2305.20005
 τ data-driven evaluation of Euclidean windows for the hadronic vacuum polarization'