©AWM

Leptonic processes at NNLO & beyond

William J. Torres Bobadilla

Max-Planck-Institut Für Physik

Radiative corrections and Monte Carlo tools for Strong 2020

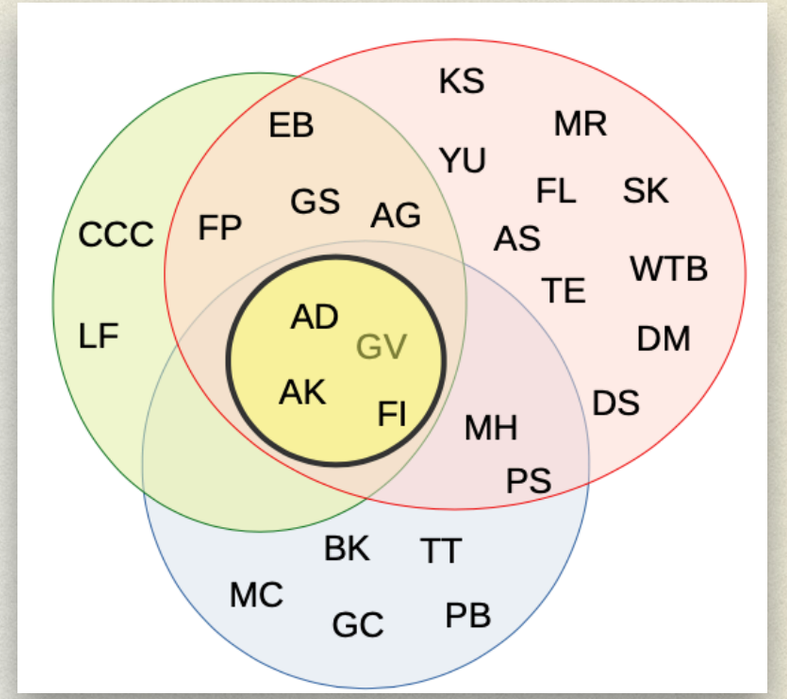
June 7 — 9, 2029

Zurich, Switzerland

Outline

- Summary of WP1
- Perspectives on electron-muon scattering w/ $m_e^2 \neq 0$
- Future directions

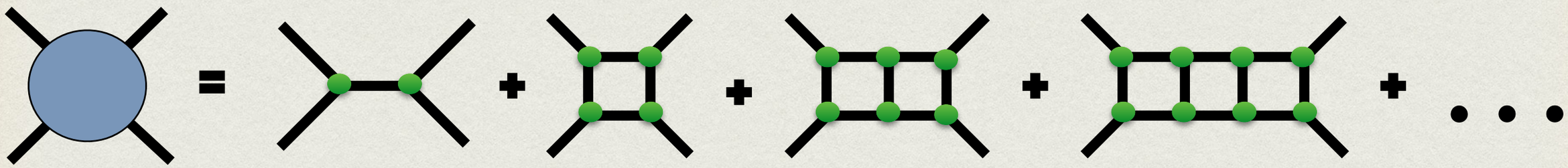
WP1



- From Diagrams to Amplitudes :: WTB
- From Amplitudes to Cross sections :: TE
- From Cross sections to Event :: YU

- Dispersive Approach to Massive two-loop Amplitudes :: AG

Standard approach @multi-loop level



Draw all Feynman diagrams

Generate integrands

Profit of DimReg

Sector Decomposition

Use Integration-By-Parts identities

Tropical geometry

Numerically

LTD approach

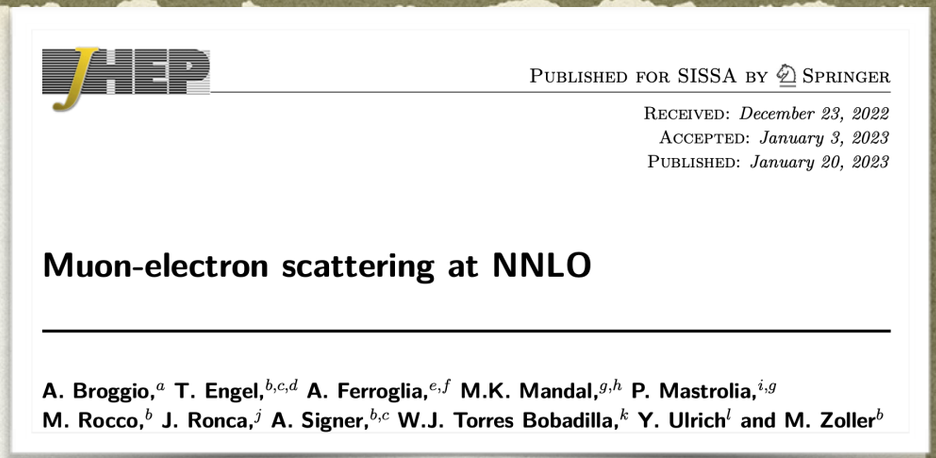
Evaluate integrals

Diff. Eqs.

Analytically

$e\mu$-scattering @ NNLO

$$\mu^+(p_1) + e^-(p_2) \to \mu^+(p_3) + e^-(p_4)$$


JHEP

Published for SISSA by Springer

Received: December 23, 2022
Accepted: January 3, 2023
Published: January 20, 2023

Muon-electron scattering at NNLO

A. Broggio,[a] T. Engel,[b,c,d] A. Ferroglia,[e,f] M.K. Mandal,[g,h] P. Mastrolia,[i,g]
M. Rocco,[b] J. Ronca,[j] A. Signer,[b,c] W.J. Torres Bobadilla,[k] Y. Ulrich[l] and M. Zoller[b]

Anatomy

$$s = (p_1 + p_2)^2 \,, \quad t = (p_2 - p_3)^2 \,,$$
$$u = (p_1 - p_3)^2 \,, \quad s + t + u = 2M^2 \,.$$

- Real-Real contribution
  Tree-level (n+2)-particles

[OpenLoops framework]

- Real-Virtual Contribution
  one-loop (n+1)-particles

- Virtual-Virtual Contribution
  two-loop n-particles

[Mandal, Mastrolia, Ronca, WJT *et al* (2021)]

$$\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

+ Subtractions & MC integrations

[McMule framework]

eμ-scattering @ NNLO

<<Engel

assume $A^{(0),(1)}_{n,n+1,n+2}$ known, $A^{(2)}_n$ known with $m_e = 0$

steps

1. massification
2. HVP corrections
3. IR divergences
4. numerical stability

QED vs. QCD

⊖ $\log m_e$ physical

⊕ only soft div.

⊖ $E_\gamma \ll m_e \ll Q$

① Massification [Mitov, Moch 07; Becher, Melnikov 07; McMule 18]

@2-loop

heavy particle

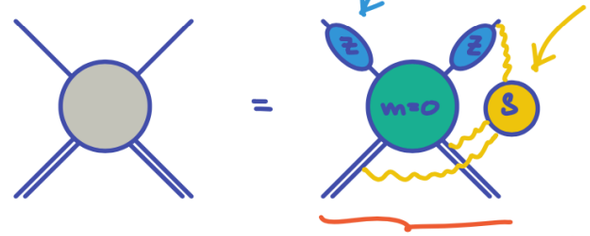
3/7

hierarchy $p_j^2 = m_j^2 \ll Q^2$

process indep. ⇝ extract from simple process with MoR

1 + fermion loops

factorisation



$= \quad + \mathcal{O}(m)$ ⇝ more details in WP2

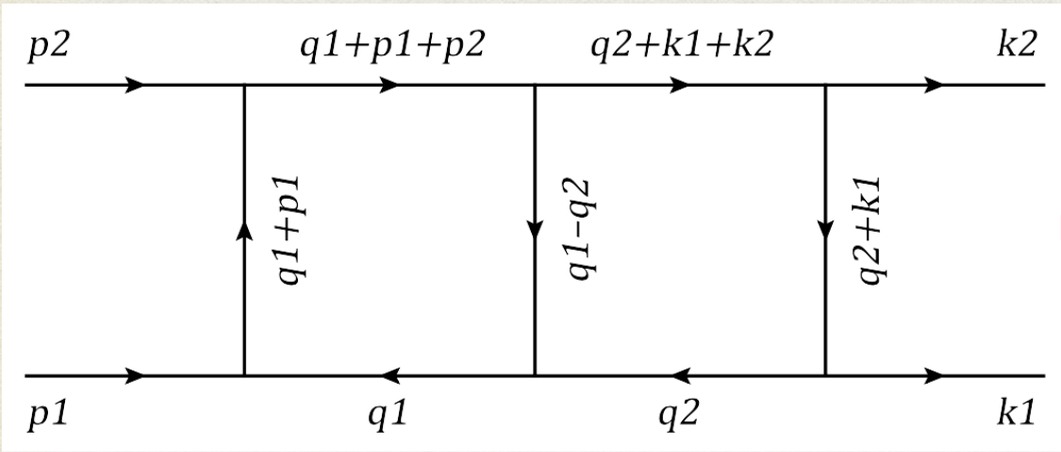
logs and const.

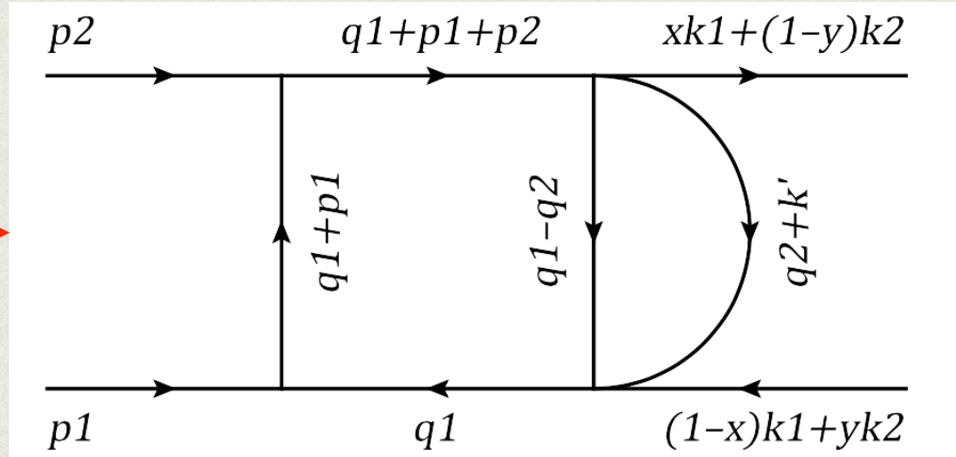
cancels $1/\eta$ in $Z^{(2)}$

eμ-scattering @ NNLO

<<Gurgone

- Dispersive Approach to Massive Two-Loop Amplitudes





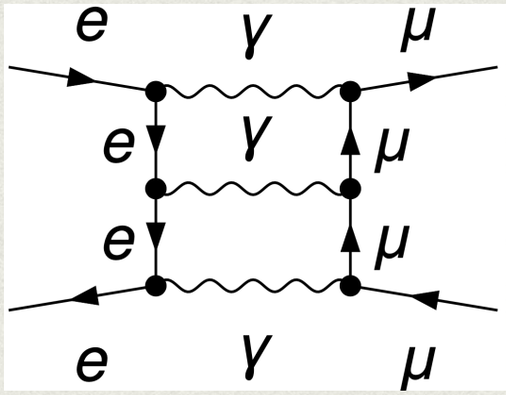
$$= -\int dx\, dy \left\{ \int_{\sigma_0}^{\infty} d\sigma\, \partial^2_{m'}\, \Delta B_0\left(\sigma;\, m'^2, m^2\right) \left[D_0(\sigma) - \frac{\sigma_0}{\sigma} D_0(\sigma_0) \right] \right.$$
$$\left. + \sigma_0\, \partial^2_{m'}\, \Delta B_0\left(0;\, m'^2, m^2\right) D_0(\sigma_0) \right\}$$

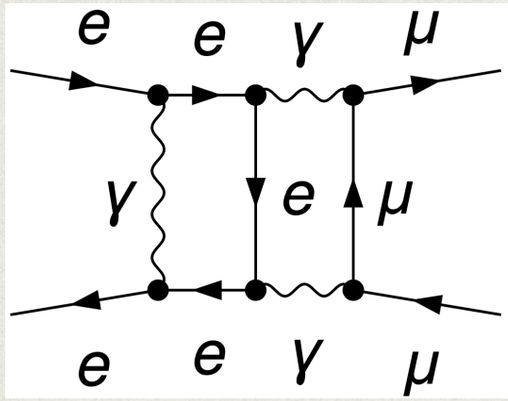
- Use of Feynman parametrisation
- Dispersion relations
- Use 2- & 4-pt one-loop scalar integral

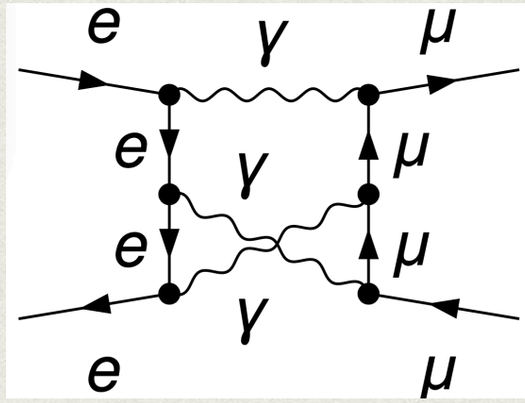
electron-muon scattering w/ $m_e^2 \neq 0$

What is new & problematic?

- Follow diagrammatic approach







[Heller 2021]

- Integration-by-parts identities

`FiniteFlow`

`Kira & FireFly`

☑ All IBP reductions easily handed by reconstruction over finite fields

- Numerical evaluation of loop integrals
  - — Sector Decomposition —> `PySecDec`, `Fiesta`
  - — Auxiliary Mass Flow :: DEQ in $x \sim \imath 0$
  - — Series expansions :: solve DEQ along path —> `DiffExp`, `SeaSyde`
  - — Tropical integration :: `FeynTrop`

eμ-scattering @ NNLO

<<Ulrich

origin of negative weights

$$\sigma_{\mathrm{NLO}} = \int \mathrm{d}\sigma_n^{(0)} + \frac{\alpha}{4\pi}\int \mathrm{d}\sigma_n^{(1)} + \frac{\alpha}{4\pi}\int \mathrm{d}\sigma_{n+1}^{(0)}$$

$$= \int \underbrace{\left(\mathrm{d}\sigma_n^{(0)} + \frac{\alpha}{4\pi}\mathrm{d}\sigma_n^{(1)} + \mathrm{soft}(\omega_c)\right)}_{\text{mostly} > 0} + \frac{\alpha}{4\pi}\int_{\omega>\omega_c} \underbrace{\mathrm{d}\sigma_{n+1}^{(0)}}_{>0}$$

$$= \int \underbrace{\left(\mathrm{d}\sigma_n^{(0)} + \frac{\alpha}{4\pi}\mathrm{d}\sigma_n^{(1)} + \frac{\alpha}{4\pi}\int \mathrm{dCT}\right)}_{\text{mostly} > 0} + \frac{\alpha}{4\pi}\int \underbrace{\left(\mathrm{d}\sigma_{n+1}^{(0)} - \mathrm{dCT}\right)}_{\text{whatever}}$$

slicing: fairly few negative weights **but** numerically construct $\log(\omega_c)$

subtraction: stable integration **but** lots and lots of negative weights

if $r \times N$ events are negative, you need $\propto 1/(1-2r)^2$ events

origin of negative weights

$$\sigma_{\mathrm{NLO}} = \int \mathrm{d}\sigma_n^{(0)} + \frac{\alpha}{4\pi}\int \mathrm{d}\sigma_n^{(1)} + \frac{\alpha}{4\pi}\int \mathrm{d}\sigma_{n+1}^{(0)}$$

$$= \int \underbrace{\left(\mathrm{d}\sigma_n^{(0)} + \frac{\alpha}{4\pi}\mathrm{d}\sigma_n^{(1)} + \mathrm{soft}(\omega_c)\right)}_{\text{mostly} > 0} + \frac{\alpha}{4\pi}\int_{\omega}$$

$$= \int \underbrace{\left(\mathrm{d}\sigma_n^{(0)} + \frac{\alpha}{4\pi}\mathrm{d}\sigma_n^{(1)} + \frac{\alpha}{4\pi}\int \mathrm{dCT}\right)}_{\text{mostly} > 0} + \frac{\alpha}{4\pi}$$

slicing: fairly few negative weights **but** r

subtraction: stable integration **but** lots

if $r \times N$ *events are negative, you need* $\propto 1/(1 -$

cross sections are positive & experiments have finite resolution

algorithm: [Andersen, Maier 2021]

- choose *seed event* with $w < 0$ to define a cell $\mathcal{C}$
- add nearby events to $\mathcal{C}$ until $\sum_{i\in\mathcal{C}} w_i > 0$
- reweight $w_i \to \frac{\sum_{j\in\mathcal{C}} w_j}{\sum_{j\in\mathcal{C}} |w_j|}|w_i| > 0$
- repeat until all $w_i \geq 0$

if largest cell size is bigger than experimental resolution, need more events

needs proper metric in event space. MUonE example: $d(e_1, e_2) = \sqrt{|\theta_1^e - \theta_2^e|^2 + |\theta_1^\mu - \theta_2^\mu|^2}$

Recap

- ☆ Straightforward generation of integrand from Feynman diagrams
  + available tool for numerical evaluations.

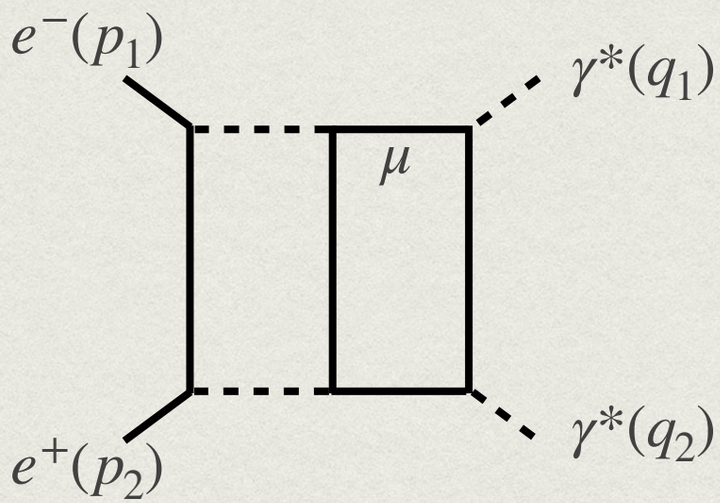
- ☆ Clear path for the transition between massless to massive particles.

- ☆ Strategies to deal with event generators.

Target processes

- e+e- —> y* y*

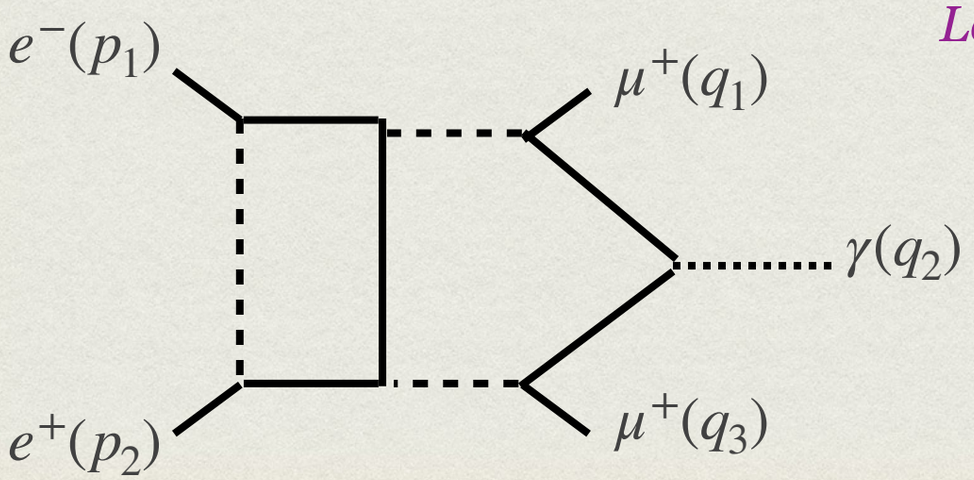
Loop integrals known —> Assemble pieces



- Massless electrons & on-shell photons ($p_1^2 = p_2^2 = q_1^2 = q_2^2 = 0$) and $m_\mu^2 \neq 0$.
- Massless electrons, one on-shell photon & one off-shell photon ($p_1^2 = p_2^2 = q_1^2 = 0\,, q_2^2 \neq 0$) and $m_\mu^2 \neq 0$.
- Massless electrons and two off-shell photons ($p_1^2 = p_2^2 = 0\,, q_1^2 \neq q_2^2, q_1^2, q_2^2 \neq 0$) and $m_\mu^2 = 0$.

- e+e- —> μ+ μ- y

Loop integrals known —> Assemble pieces



- Complete massless 5-point process.

Conclusions

- Cross sections @NNLO from Amplitudes —> Understood!

- The high-energy community has done a lot of work —> Let's use it!

- If you are interested to contribute, you are welcome —> This means work!