Parton Showers (WP4)
a.k.a. QED resummation in Monte Carlos

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Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions
STRONG2020

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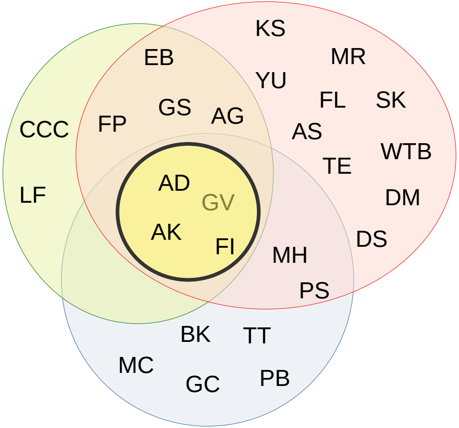
Outline

✓ Our **green** session was organized with an introductory talk, followed by a brain-storming discussion.
No other talks were scheduled



⇝ Overview of QED resummation approaches: QED Parton Shower & Yennie-Frautschi-Suura

⇝ Inclusion of exact matrix elements

⇝ Comparisons

⇝ Outcome of WP4 discussion & plans for the future

QED resummation

- Instead of focussing on calculating order-by-order exact corrections in α for a given process, **QED Parton Shower** and **Yennie-Frautschi-Suura resummation** take a different point of view as starting point:

  they aim at calculating **approximate** and **"universal"** corrections **up to all orders**, by including (the important, leading) contributions arising from soft and/or collinear regions

- They rely on the general properties of **factorization of soft/collinear divergencies** (enhancements) in QED, which lead to **exponentiation**

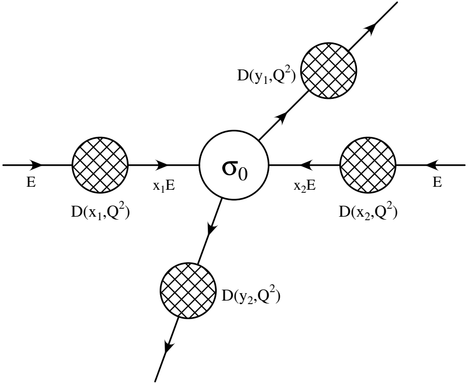
- Dealing with "universal" properties of QED, the underlying process to be dressed with photons can be generic

$\rightsquigarrow$ Sometimes, in some phase-space regions, for some observables, for certain experimental cuts, you better have an approximate resummed result than a fixed-order one

$$\alpha < \alpha^2 L^2 \quad \text{somewhere, with } L = \log \frac{s}{m^2}$$

PS: QED collinear Structure Functions

- PS algorithms rely on QCD-inspired Structure Function approach to radiative corrections (it's still called ***Parton*** Shower although here it describes multiple photon emissions. . .)



- If we are interested only in photon radiation, $D(x, Q^2)$ are the **Leading-Log non-singlet QED SF**

QED DGLAP equation

$D(x,Q^2)$ is the solution of the QED DGLAP equation

$$Q^2 \frac{\partial}{\partial Q^2} D(x,Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} P_+(y) D(\frac{x}{y},Q^2)$$

$$P_+(x) = \frac{1+x^2}{1-x} - \delta(1-x) \int_0^1 \frac{1+t^2}{1-t} dt$$

which can be solved analytically (but some “exclusive” information is lost because integrated out) or by a **Monte Carlo iterative solution** (the **Parton Shower**, which is “exclusive”)

$$\begin{aligned} D(x,Q^2) &= \Pi(Q^2,m^2)\delta(1-x) \\ &+ \int_{m^2}^{Q^2} \Pi(Q^2,s') \frac{ds'}{s'} \Pi(s',m^2) \frac{\alpha}{2\pi} \int_0^{x_+} dy P(y) \delta(x-y) \\ &+ \int_{m^2}^{Q^2} \Pi(Q^2,s') \frac{ds'}{s'} \int_{m^2}^{s'} \Pi(s',s'') \frac{ds''}{s''} \Pi(s'',m^2) \times \\ &\left(\frac{\alpha}{2\pi}\right)^2 \int_0^{x_+} dx_1 \int_0^{x_+} dx_2 P(x_1) P(x_2) \delta(x - x_1 x_2) + \cdots \end{aligned}$$

PS: pros and cons

$$\Pi(Q^2, m^2)_\epsilon = e^{-\frac{\alpha}{2\pi}\log\frac{Q^2}{m^2}\int_0^{1-\epsilon} dx P(x)} = e^{-\frac{\alpha}{2\pi}\log\frac{Q^2}{m^2} I_+}$$

is the Sudakov Form Factor, which exponentiates approximate virtual and soft emission up to all orders

✓ Advantages:

- ⇝ the number of emitted photons is not limited (shower)
- ⇝ at each branching, kinematical variables are generated and photons' momenta can be reconstructed
  $\rightarrow$ **fully exclusive event generation**
- ⇝ it can be truncated at $\mathcal{O}(\alpha^n)$ and consistently compared to fixed-order N^{n}LO calculations.

✗ Disadvantages:

- ⇝ initial-final state radiation interference effects are not naturally included, but they can be recovered by choosing an appropriate photons' angular distribution (eikonal, YFS-inspired)

Carloni Calame, PLB 520 (2001) 16

$$I(k) = \sum_{i,j} \eta_i \eta_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} E_\gamma^2$$

- ⇝ at its LL level, it misses already corrections at $\mathcal{O}(\alpha)$: a matching to NLO is needed

- Firstly, the corrected LL cross section can be cast in the form

$$d\sigma_{LL}^{\infty} = \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \, |\mathcal{M}_{n,LL}|^2 \, d\Phi_n,$$

$$|\mathcal{M}_{1,LL}|^2 = \frac{\alpha}{2\pi} \frac{1+z^2}{1-z} I(k) \, |\mathcal{M}_0|^2 \, \frac{8\pi^2}{E^2 z(1-z)}$$

⇝ The multi-differential phase-space is kept **exact** (differently to what is usually done in QCD showers)

⇝ Any approximation is shifted on matrix elements

⇝ A mapping of momenta is needed: this is a delicate and ambiguous job.
You hope ambiguities are effects beyond your working accuracy...

PS: matching (Pavia solution)

- A LL PS-corrected differential cross section can be expanded at $\mathcal{O}(\alpha)$

$$d\sigma^{\alpha}_{LL} = \left[1 - \frac{\alpha}{2\pi} I_+ \log\frac{Q^2}{m^2}\right] |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1$$
$$\equiv [1 + C_{\alpha,LL}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1,$$

while the NLO cross section can be always cast as

$$d\sigma^{\alpha} = [1 + C_{\alpha}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1.$$

By defining the factors

$$F_{SV} = 1 + (C_{\alpha} - C_{\alpha,LL}), \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$$

the NLO cross section can be re-written (up to terms of $\mathcal{O}(\alpha^2)$) as

$$d\sigma^{\alpha} = F_{SV} (1 + C_{\alpha,LL}) |\mathcal{M}_0|^2 d\Phi_0 + F_H |\mathcal{M}_{1,LL}|^2 d\Phi_1$$

which brings to the master formula. . .

PS: matching (Pavia solution)

$$d\sigma^{\infty}_{matched} = F_{SV}\, \Pi(Q^2,\varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=0}^{n} F_{H,i} \right) |\mathcal{M}_{n,LL}|^2 \, d\Phi_n,$$

- ⇝ it's based on LO and NLO building blocks
- ⇝ F_{SV} and F_H are collinear and infrared safe, no double counting of LL terms
- ⇝ the cross-section is still fully differential
- ⇝ its $\mathcal{O}(\alpha)$ expansion coincides with NLO
- ⇝ resummation of LL higher-orders, beyond NLO, is preserved
- ⇝ it can be expanded at $\mathcal{O}(\alpha^2)$ and compared to exact NNLO corrections

✓ Successfully applied to match QED NLO to PS in **BabaYaga@NLO**, EWK NLO to PS in **Horace** (neutral and charged Drell-Yan) and **Hto4l** ($H \to 4\ell$)

✗ generalization to NNLO?

Yennie-Frautschi-Suura soft exponentiation

✓ It all started in this beautiful work, full of insights and clever tricks

D. R. Yennie, S. C. Frautschi and H. Suura
“The infrared divergence phenomena and high-energy processes”, Ann. Phys. 13, 379 (1961)

⇝ Many Monte Carlos for LEP (and LHC) developed by S. Jadach and colleagues on this framework (**`Koral[W/Z]`**, **`BH[LUMI/WIDE]`**, **`YFS[WW3/ZZ]`**, **`WINAC`**, **`KKMC`**)

⇝ Nowadays YFS is the basis for QED radiation resummation in **`Sherpa`**.
Applied also to (future) e^+e^- machines

Krauss, Price, Schönherr, SciPost Phys. 13, 026 (2022)

YFS: master formulae

- As usual, the full perturbative series for the emission of an arbitrary number of photons in a given LO process can be written as

$$\mathrm{d}\sigma = \sum_{n_\gamma=0}^{\infty} \frac{1}{n_\gamma!} \mathrm{d}\Phi_Q \left[\prod_{i=1}^{n_\gamma} \mathrm{d}\Phi_i^\gamma\right] (2\pi)^4\, \delta^4\left(\sum_{i=1}^{2} p_i - \sum_{j=3}^{N+2} q_j - \sum_{k=1}^{n_\gamma} k_k\right) \left|\sum_{\bar{n}_\gamma=0}^{\infty} \mathcal{M}_{n_\gamma}^{\bar{n}_\gamma+\frac{1}{2}n_\gamma}\right|^2 ,$$

- After factorizing out all soft virtual and soft real corrections, you end up with something like

$$\mathrm{d}\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} \mathrm{d}\Phi_Q \left[\prod_{i=1}^{n_\gamma} \mathrm{d}\Phi_i^\gamma\, \tilde{S}\,(k_i)\, \Theta(k_i,\Omega)\right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{S}\,(k_j)} + \sum_{\substack{j,k=1\\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j,k_k)}{\tilde{S}\,(k_j)\tilde{S}\,(k_k)} + \cdots\right)$$

where

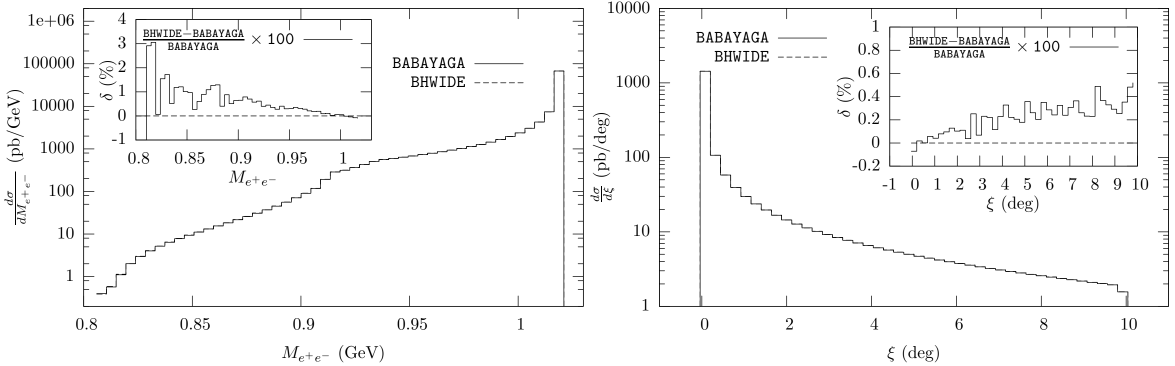
- $e^{Y(\Omega)}$ resums all soft virtual and soft real emissions
- $\tilde{S}(k_i)$ are eikonal factors
- $\tilde{\beta}_n$ are IR-subtracted matrix elements remnants (with n photons)

YFS properties

- $\rightsquigarrow$ it relies on factorization of soft virtual and real photon emissions
- $\rightsquigarrow$ fully exclusive event generation
- $\rightsquigarrow$ inclusion of exact higher-order matrix elements more “natural” than in PS
  $\rightarrow \tilde{\beta}_1 \neq 0$ matches to NLO, $\tilde{\beta}_2 \neq 0$ matches to NNLO, . . . (I think)
- $\rightsquigarrow$ two flavours:
  - **EEX**
    exclusive exponentiation: based on YFS original paper, works at $|\mathcal{M}|^2$ level
  - **CEEX**
    coherent exclusive exponentiation: works at $\mathcal{M}$ level. Only in **KKMC**, drastically more difficult to implement
- $\rightsquigarrow$ a mapping of momenta still necessary

NLO matched PS vs NLO YFS

- distributions: **BabaYaga@NLO** vs. **BHWIDE** (Bhabha $e^+e^- \to e^+e^-(+n\gamma)$, at KLOE)

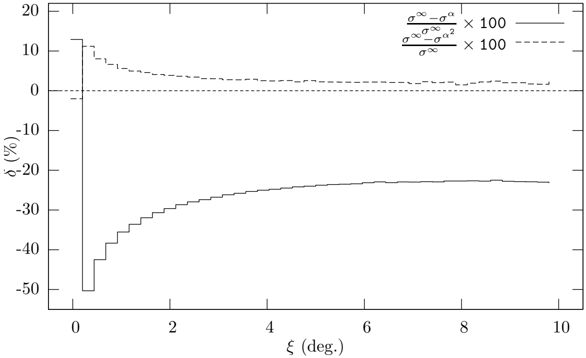


ξ = acollinearity

from Balossini et al., NPB 758 (2006) 227

Resummation beyond α^2

- With a complete NNLO generator at hand, can LL resummation beyond α^2 be neglected (again Bhabha at KLOE)?



$\xi =$ acollinearity

- Resummation beyond α^2 still important (at least for some distributions)!

Workstop/Thinkstart outcome for WP4

Phokhara

$\pi^+\pi^-\gamma, \mu^+\mu^-\gamma$ [NLO]

BabaYaga@NLO

$e^+e^-, \mu^+\mu^-, \gamma\gamma$ [NLO+PS]

MCGPJ

$\pi^+\pi^-, e^+e^-, \mu^+\mu^-$ [NLO+SF]

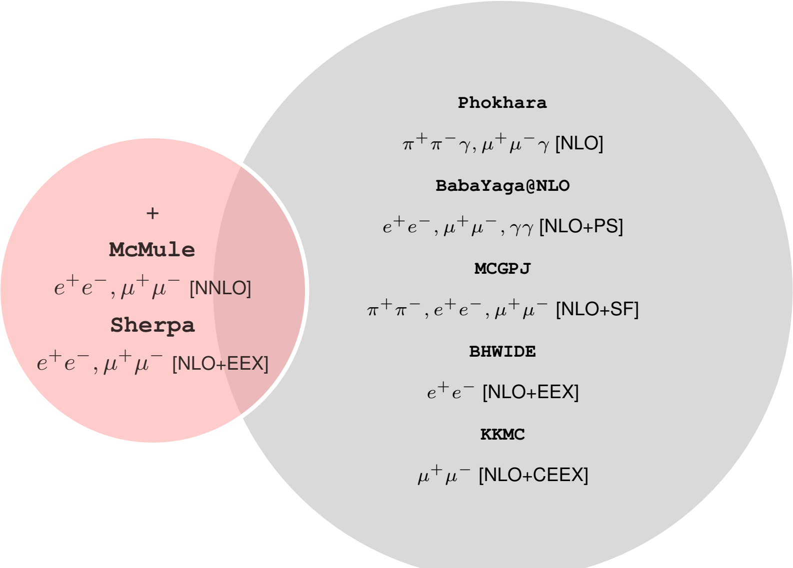
BHWIDE

e^+e^- [NLO+EEX]

KKMC

$\mu^+\mu^-$ [NLO+CEEX]

Workstop/Thinkstart outcome for WP4



Workstop/Thinkstart outcome for WP4

+

McMule

$e^+e^-, \mu^+\mu^-$ [NNLO]

Sherpa

$e^+e^-, \mu^+\mu^-$ [NLO+EEX]

Phokhara

$\pi^+\pi^-\gamma, \mu^+\mu^-\gamma$ [NLO]

BabaYaga@NLO

$e^+e^-, \mu^+\mu^-, \gamma\gamma$ [NLO+PS]

MCGPJ

$\pi^+\pi^-, e^+e^-, \mu^+\mu^-$ [NLO+SF]

BHWIDE

e^+e^- [NLO+EEX]

KKMC

$\mu^+\mu^-$ [NLO+CEEX]

+

McMule

$\gamma\gamma$ [NNLO]

$\pi^+\pi^-\gamma, \mu^+\mu^-\gamma$ [ISR NNLO]

Sherpa

$\pi^+\pi^-$ [NLO+EEX]

BabaYaga@NLO

$\mu^+\mu^-\gamma$

$\pi^+\pi^-, \pi^+\pi^-\gamma$ [NLO+PS]

Workstop/Thinkstart outcome for WP4

- We focussed on leptonic and $\pi^+\pi^-[\gamma]$ final states.
  We do not see any show-stopper to implement QED resummation on π's, at least in $F_\pi \times$sQED (after due tests and cross-checks)

- New actors on the low-energy MC scene

  `McMule` and **`Sherpa`**. **`BabaYaga`** commits to play with pions

✓ Considered processes will be simulated by at least 2 generators
  ⇝ Technical accuracy will be much more under control

✓ Considered processes available at different accuracies (NLO+PS vs NNLO) and/or in different resummation schemes (PS vs YFS)
  ⇝ Estimate of theoretical uncertainties much more robust