Radiative corrections and MC tools for low-energy hadronic cross sections in e^+e^- collisions

Theoretical MUonE

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Based on 2212.06481 [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Ronca, Rocco, Signer, Torres Bobadilla, Ulrich, Zoller]

- higher-order predictions and comparison with precision experiments
- focus on low-energy QED scattering processes
- theoretical background for lepton experiments (Mu3e, MUSE, MUonE...)
- all this in

McMule

Monte Carlo for MUons and other LEptons
`https://mule-tools.gitlab.io/`

- fully-differential Monte Carlo integrator, not an event generator (yet)

problems & solutions

- MUonE's signal is $\sim 10^{-3}$
  $\rightarrow$ at least 10 ppm precision
- require excellent control on background
- read: higher-order QED corrections to $e - \mu$ scattering
- probably something more: resummation, PS...

[mules by A. Signer]

- steal QCD@LHC techniques
  - dim.reg.
  - MIs, automation, EFT
  - subtraction method
  - (future) match FO w/ PS
- realise massive fermions yield simpler IR structure but harder loop amplitudes and log-enhanc.
- let the mule trot [McMule 20, 22]

a few names

- *photonic* and *fermionic* corrections
- photonic are split into three parts at NNLO:

$$\mathrm{d}\sigma^{(2)} \quad = \quad \int \mathrm{d}\Phi_n \, \mathcal{M}_n^{(2)} + \int \mathrm{d}\Phi_{n+1} \, \mathcal{M}_{n+1}^{(1)} + \int \mathrm{d}\Phi_{n+2} \, \mathcal{M}_{n+2}^{(0)}$$

- for each part identify gauge-invariant subsets based on lepton charges (q for electron, Q for muon)

  - ◇ $q^6\, Q^2$ :: *electronic*

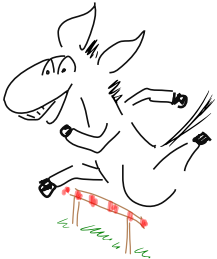
  - ◇ $\{\, q^5\, Q^3,\ q^4\, Q^4,\ q^3\, Q^5 \,\}$ :: *mixed*

  - ◇ $q^2\, Q^6$ :: *muonic*

2-loop (mixed) diagrams

full muone 2-loop amplitude with $M \neq 0,\ m = 0 \rightarrow$ [Bonciani et al. 21]

full muone 2-loop amplitude with $M \neq 0,\ m \neq 0 \rightarrow$ [??]



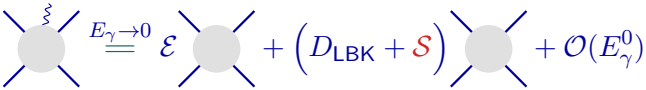
$\rightarrow$ exploit scale hierarchy $m^2 \ll M^2, Q^2$

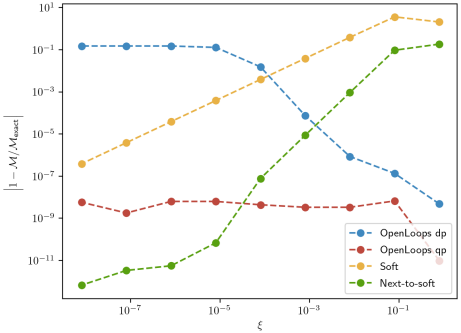
- massification: $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$

  [Penin 06, Becher, Melnikov 07; Engel, Gnendiger, Signer, Ulrich 18]

OpenLoops [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19]

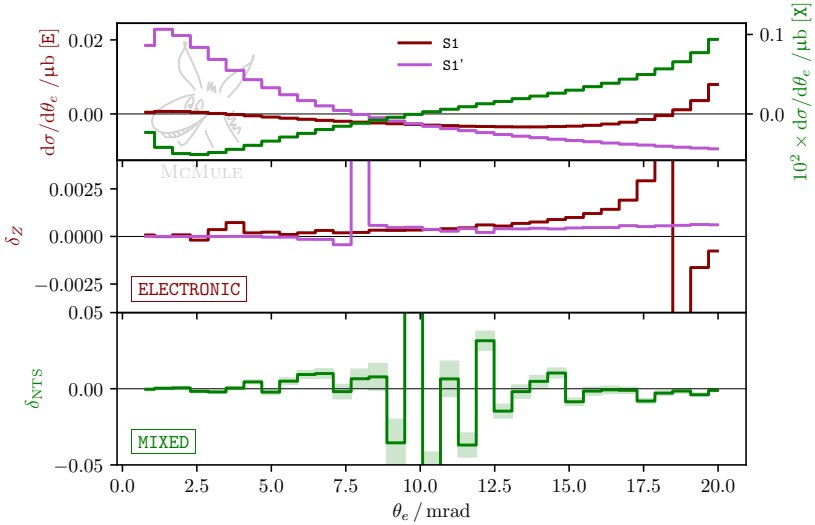
LBK theorem [LBK 58-61, Engel, Signer, Ulrich 21, Engel 23]





◇ introduce NTS stabilisation [McMule 21, 22]

validation: massification and NTS stabilisation



finally, results

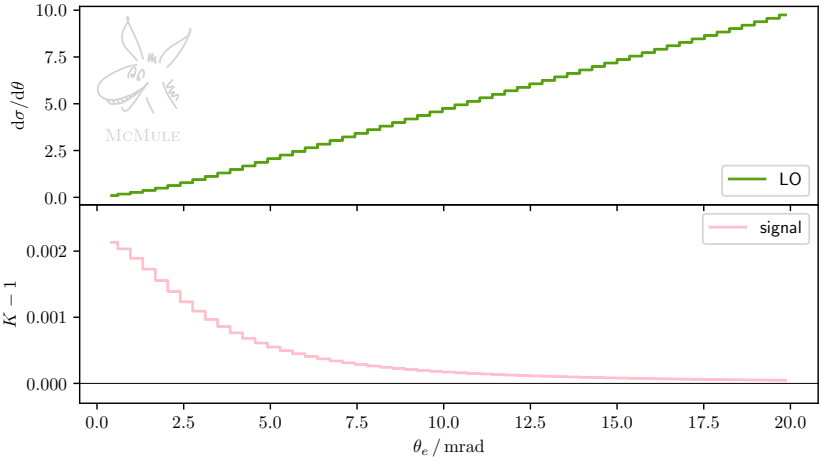
$$\mu\, e \to \mu\, e \quad @\ \mathrm{NNLO}$$

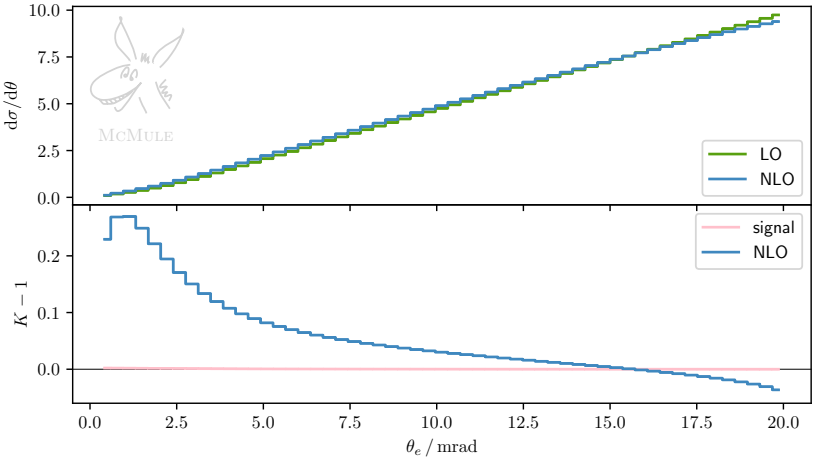
- kinematical setup mimics MUonE:

$$E_{\mu,i} = 160\,\mathrm{GeV} \qquad E_{e,f} > 1\,\mathrm{GeV} \qquad \theta_{\mu,f} > 0.3\,\mathrm{mrad}$$

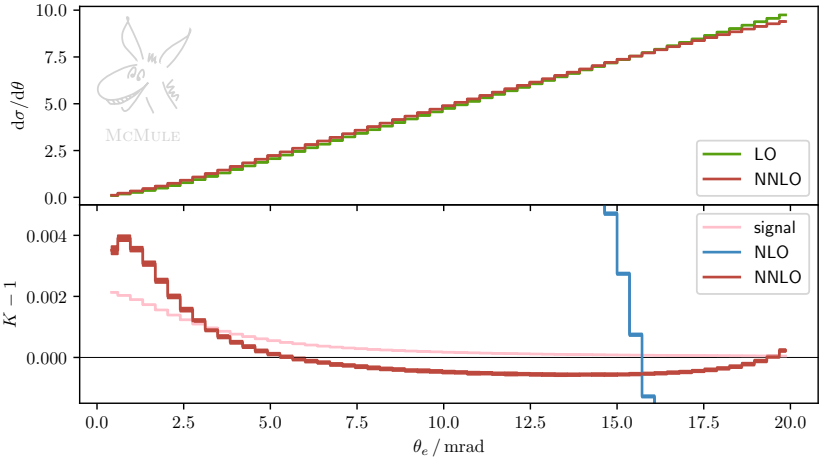
- results for different kinematical scenarios and any IR safe observable
- no mass is neglected

S1 θ_e spectrum



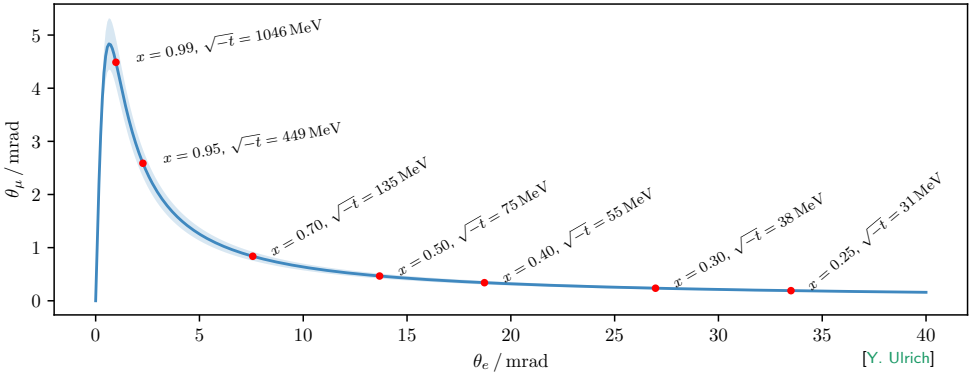

10.0
7.5
5.0
2.5
0.0
$d\sigma/d\theta$
McMule
LO
NLO
0.2
0.1
0.0
$K-1$
signal
NLO
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad

S1 θ_e spectrum



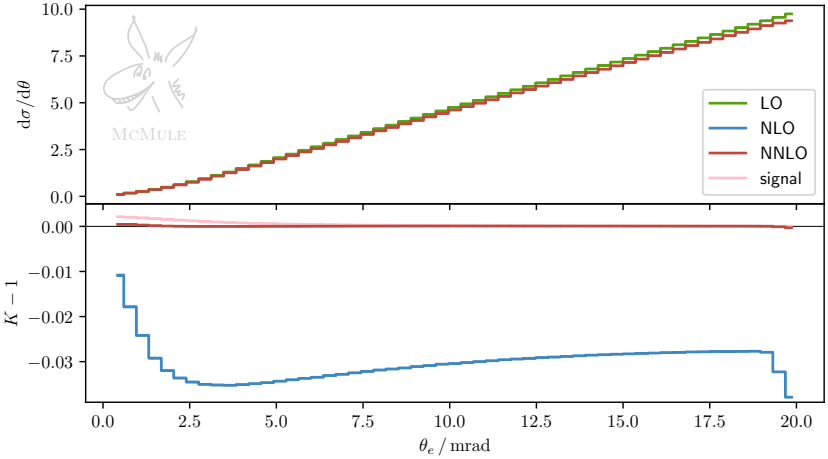
how to handle hard radiation?

- elasticity veto! $\rightarrow 0.9 < \frac{\theta_{\mu,f}}{\theta^{\mathrm{el}}_{\mu,f}} < 1.1$ (S2)
- ...

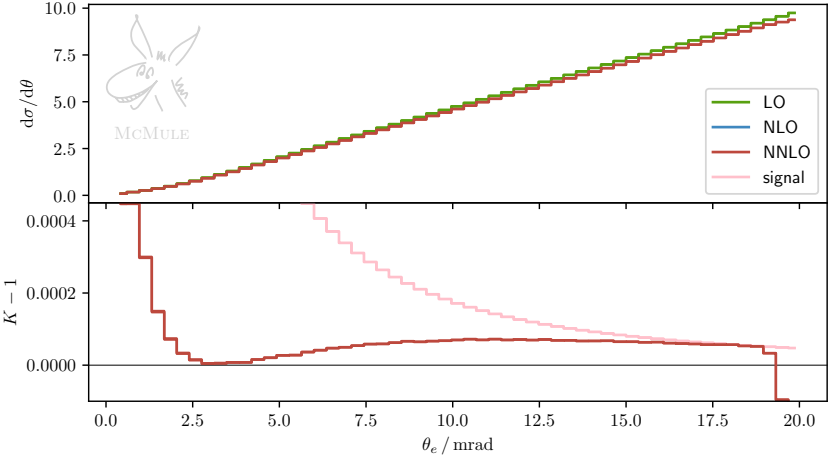


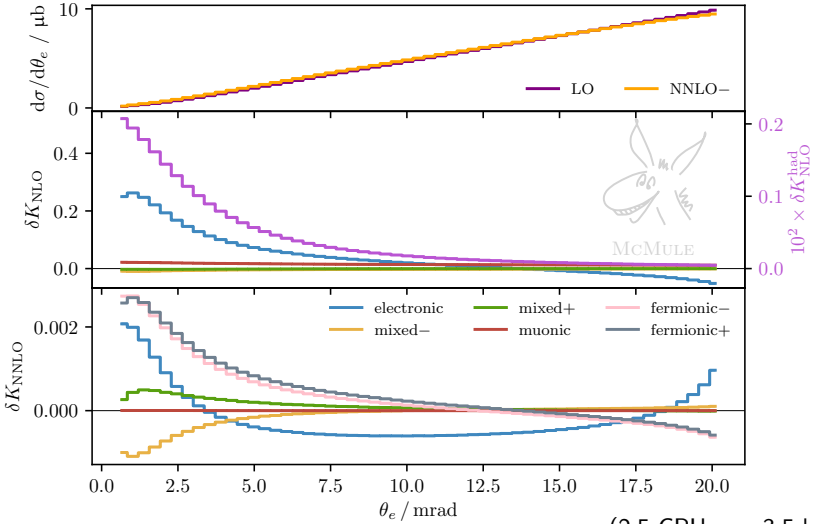
[Y. Ulrich]

S2 θ_e spectrum



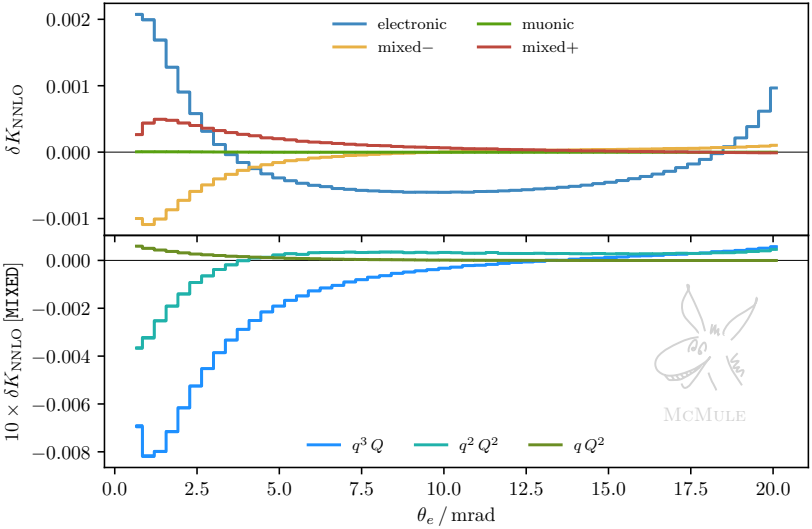
S2 θ_e spectrum





(2.5 CPU yr – 3.5 kgCO2e)

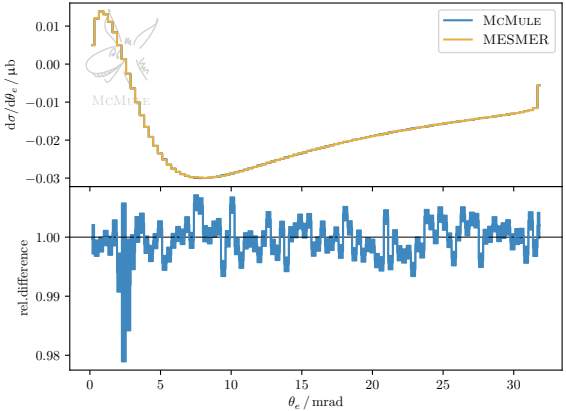
zoom on trotting mule



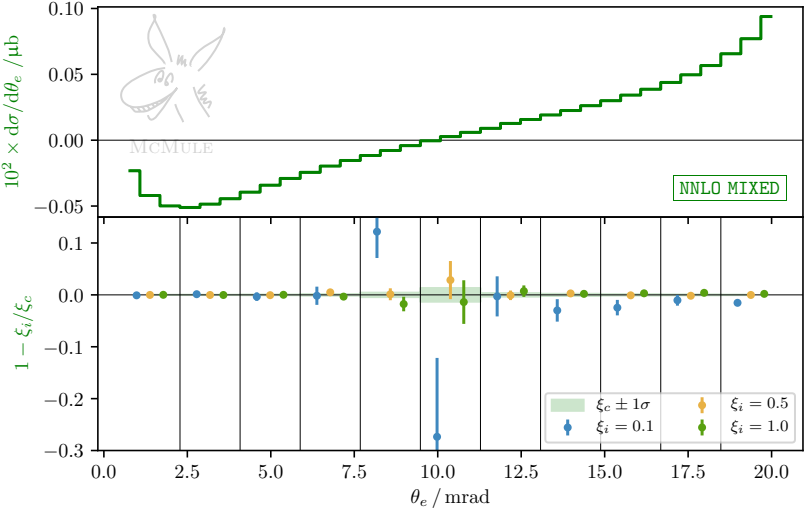
- first NNLO with different external masses [2212.06481]
- precision now $\mathcal{O}(10^{-3/-4})$, would like (need) to reach more
- stay tuned for mule updates after the workstop!

(FKS$^\ell$ + DIMREG) vs (slicing + m_γ)

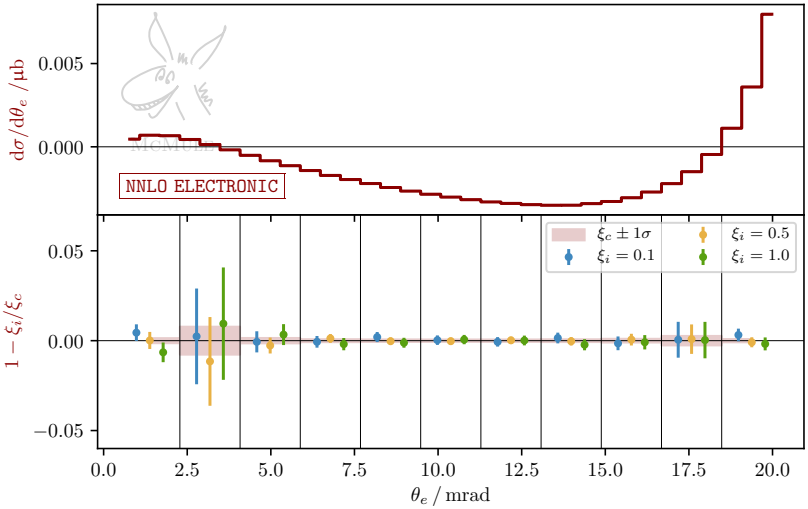
$e\,\mu \to e\,\mu\,\gamma$ @ NLO with $\xi_c = \omega_s = 10^{-\{6,5,4\}}$ (Mesmer as in [Carloni et al. 20])



validation: ξ_c (in)dependence for mixed corrections

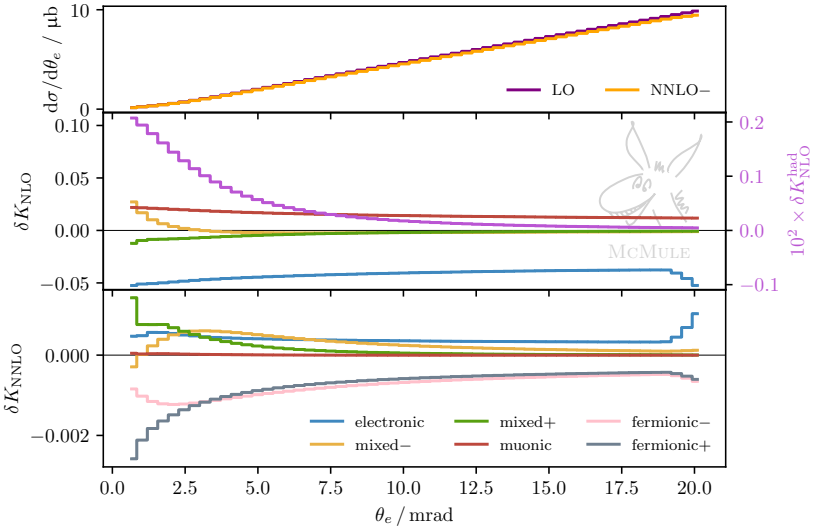


validation: ξ_c (in)dependence for electronic corrections



total cross sections

	$\sigma/\mu\text{b}$		$\delta K^{(i)}/\%$	
	S1	S2	S1	S2
σ_0	106.44356	106.44356		
$\sigma_1 \begin{cases} - \\ + \end{cases}$	106.99038(3) 107.41847(3)	102.86304(3) 103.18338(3)	0.51372(3) 0.91589(3)	−3.36377(3) −3.06283(3)
$\sigma_e^{(2)}$	0.00090	0.06595	0.00084	0.06411
$\sigma_{e\mu}^{(2)} \begin{cases} - \\ + \end{cases}$	0.00097(1) 0.00328(1)	0.01926 −0.01768	0.00091(1) 0.00305(1)	0.01872 −0.01713
$\sigma_\mu^{(2)}$	−0.00005	0.00002	−0.00005	0.00002
$\sigma_{\text{lep}}^{(2)} \begin{cases} - \\ + \end{cases}$	−0.01195 −0.00424	−0.06568 −0.05959	−0.01117 −0.00395	−0.06385 −0.05775
$\sigma_{\text{had}}^{(2)} \begin{cases} - \\ + \end{cases}$	−0.00045 −0.00004	−0.00104 −0.00068	−0.00042 −0.00004	−0.00101 −0.00066
$\sigma_2 \begin{cases} - \\ + \end{cases}$	106.97977(3) 107.41832(3)	102.88154(3) 103.19386(3)	−0.00992(4) −0.00013(4)	0.01799(4) −0.01016(4)


$d\sigma/d\theta_e$ / μb
10
0
LO
NNLO−
$\delta K_{\rm NLO}$
0.10
0.05
0.00
−0.05
$10^2 \times \delta K^{\rm bad}_{\rm NLO}$
0.2
0.1
0.0
−0.1
McMule
$\delta K_{\rm NNLO}$
0.000
−0.002
electronic
mixed−
mixed+
muonic
fermionic−
fermionic+
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad

let the mule trot [2212.06481]: S1 — θ_μ