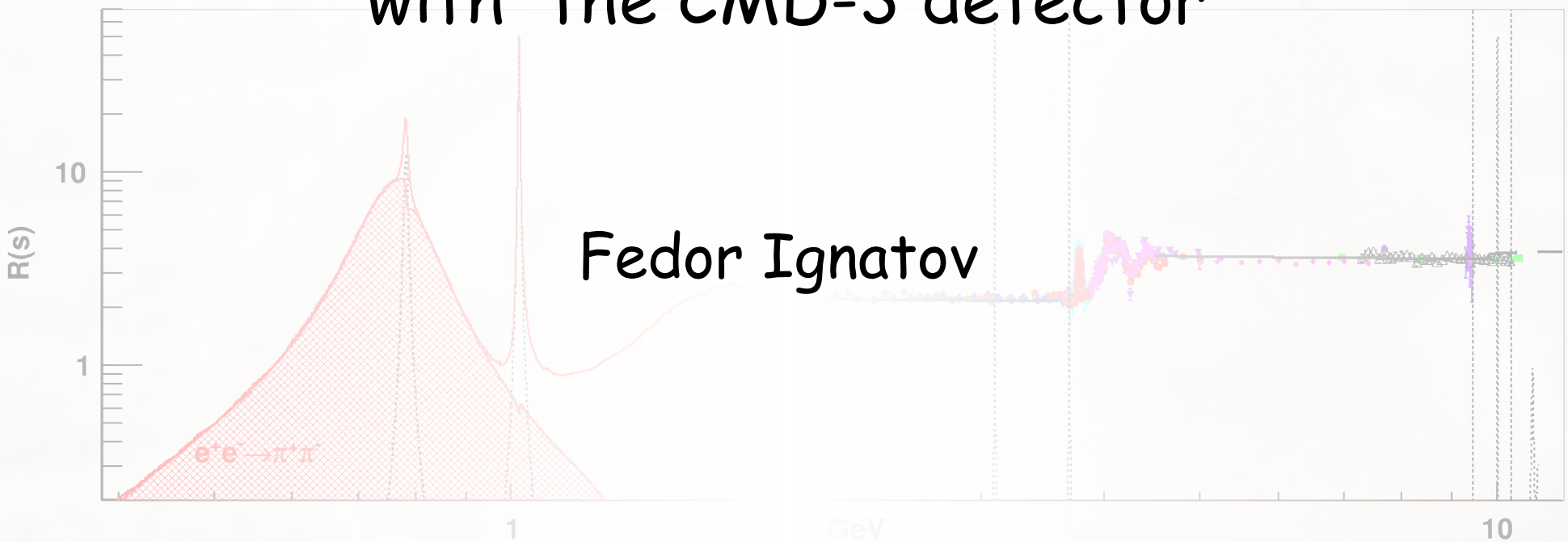
arXiv:2302.08834

Recent e+e- → π+π- measurement with the CMD-3 detector



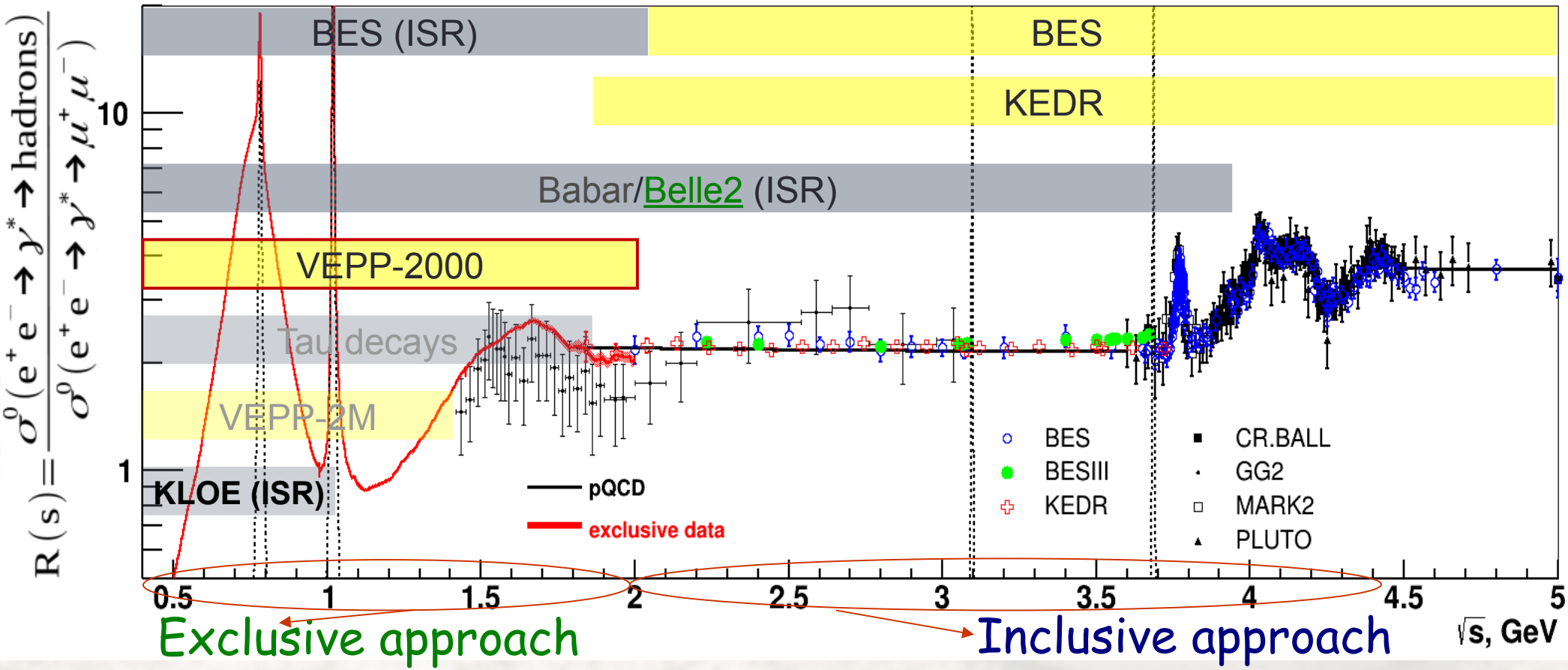
Fedor Ignatov

Radiative corrections and Monte Carlo tools for low energy e^+e^- collisions

Zurich, 7 June 2023

R(s) measurement

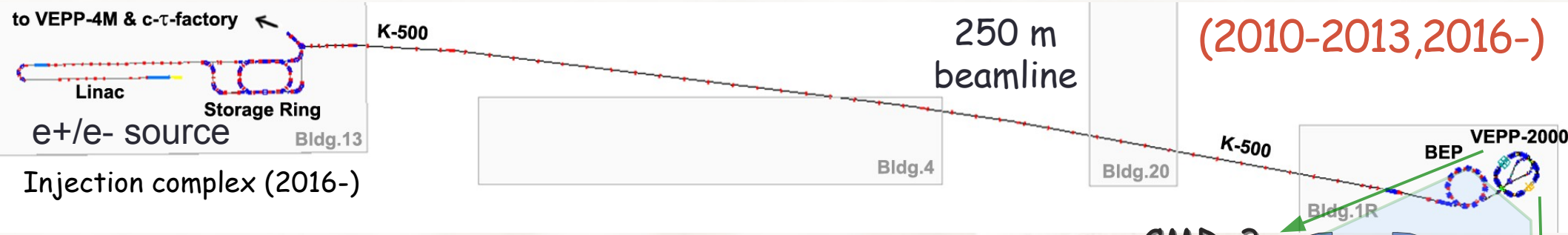
Two techniques: ISR vs Energy scan

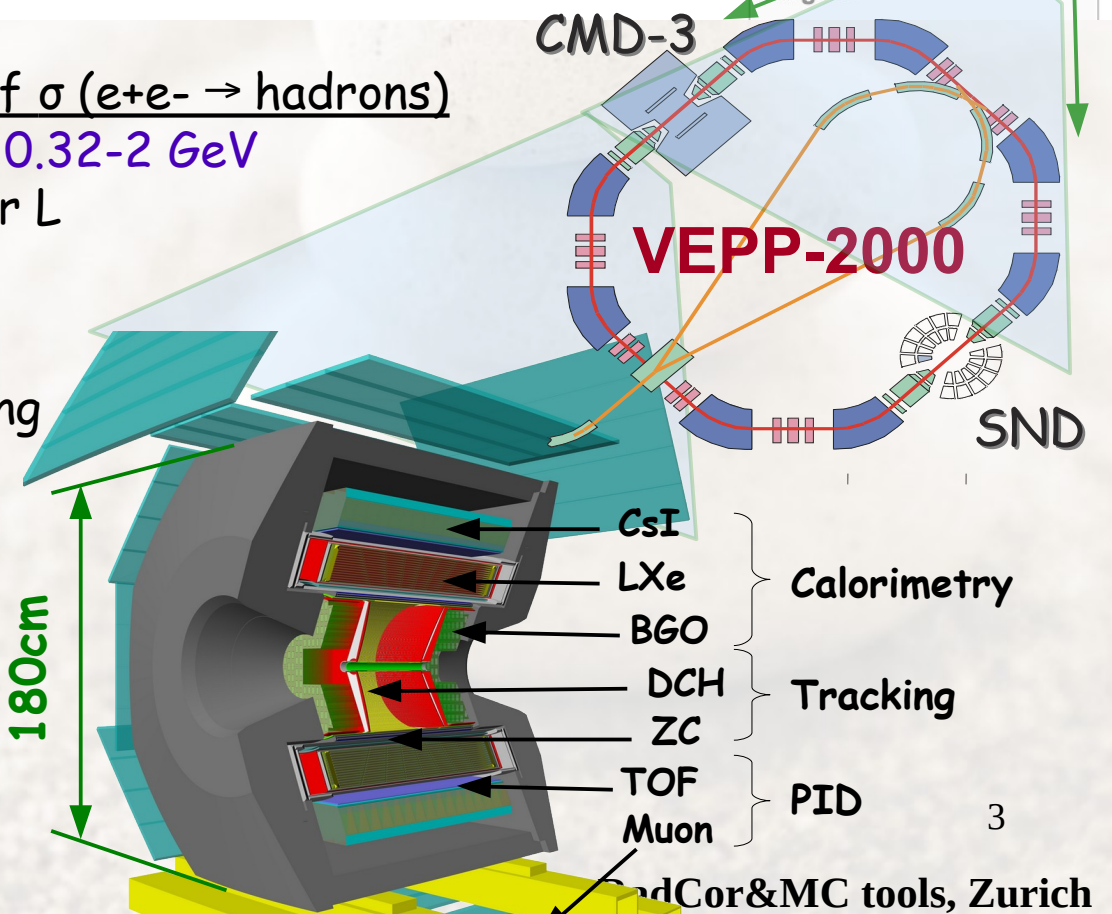


- ✗ Two techniques : Energy scan vs Initial State Radiation (ISR)
- ✗ Two approaches : Exclusive (each channel measured separately) vs Inclusive (total hadronic cross section)

<u>VEPP-2000:</u> Only one working these days on scanning below <2 GeV with world-best luminosity per single bunch at this energies

VEPP-2000 e+e- collider





VEPP-2000: direct exclusive measurement of σ (e+e- → hadrons)

Only one working this days on scanning 2E = 0.32-2 GeV

Unique optics, "round beams" to reach higher L

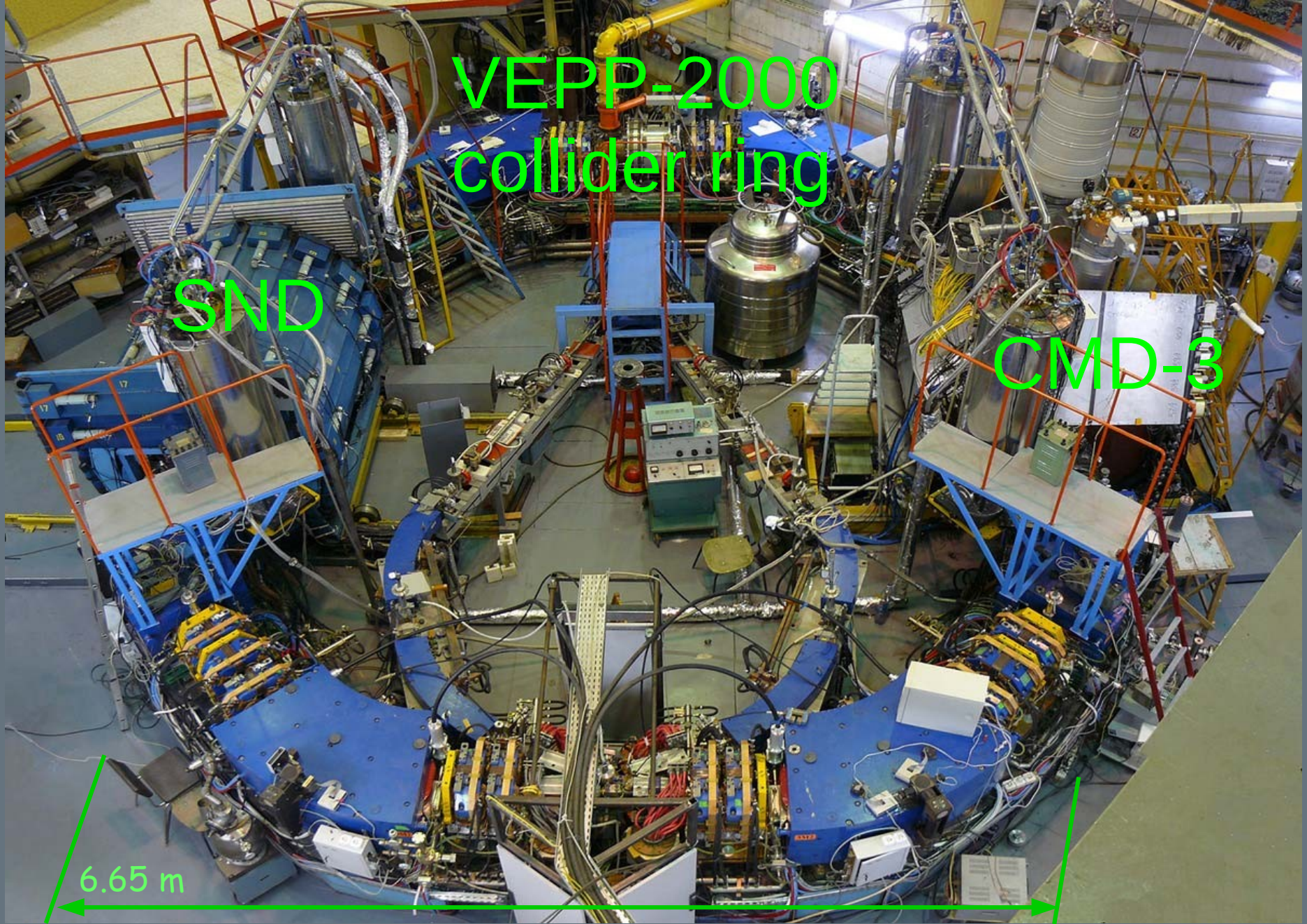
$L = 0.9\times10^{32}\ cm^{-2}s^{-1}$ at 2E= 2 GeV

Energy monitoring by Compton backscattering

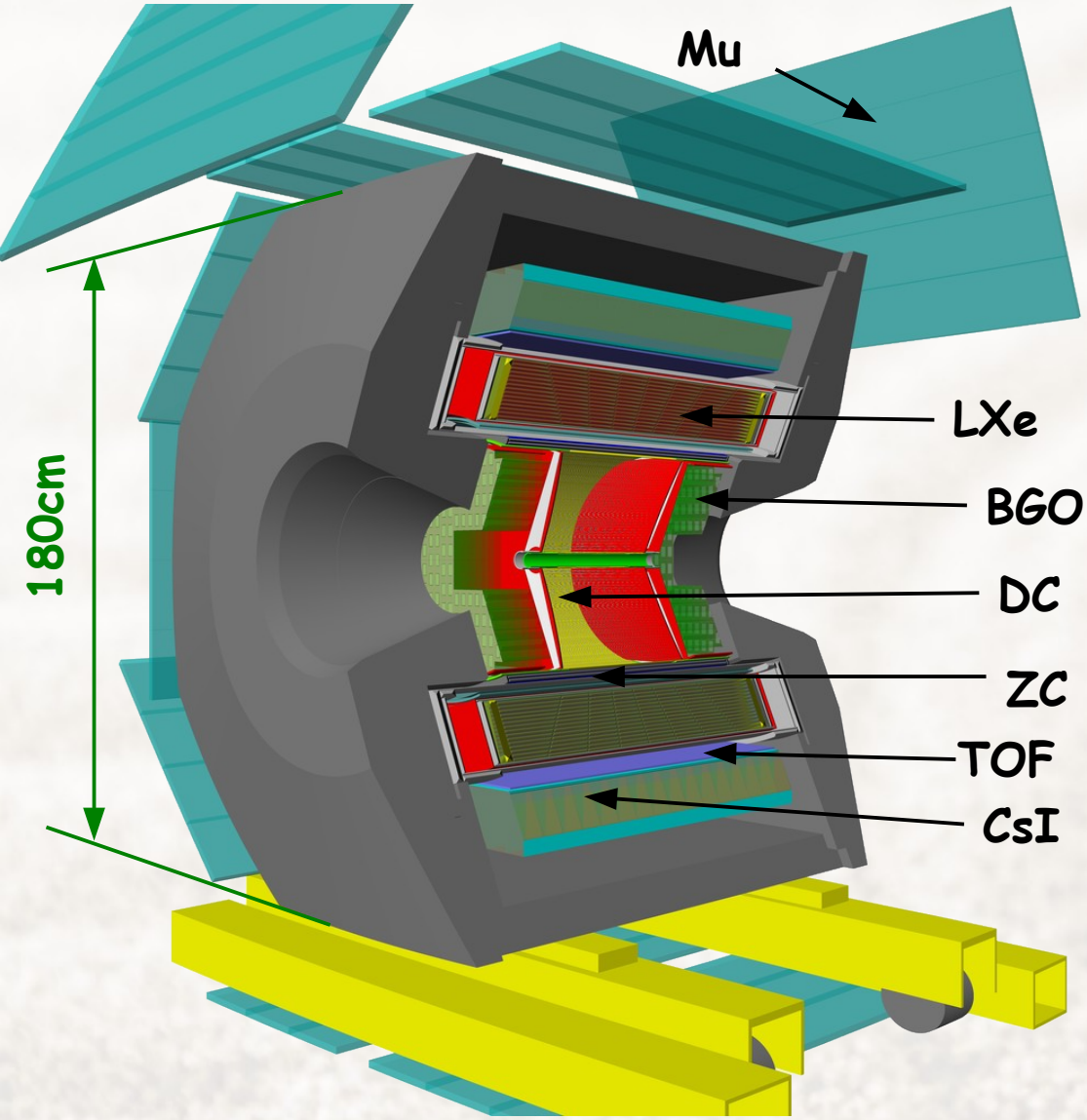
$\sigma_{\sqrt{s}} \approx 0.1$ MeV

Two detectors: CMD-3 and SND

started by the end of 2010


VEPP-2000
collider ring
SND
CMD-3
6.65 m

CMD-3 detector



Tracking:

- Drift Chamber in 1.3 T magnetic field
  $\sigma_{R\varphi}$ ~ 100 μm, σ_Z ~ 2.5mm
  $\sigma_p/P \sim \sqrt{0.6^2+(4.4*p[GeV])^2}$,%
- **ZC-chamber worked until summer 2017**
  σ_Z ~ 0.7mm by strip readout

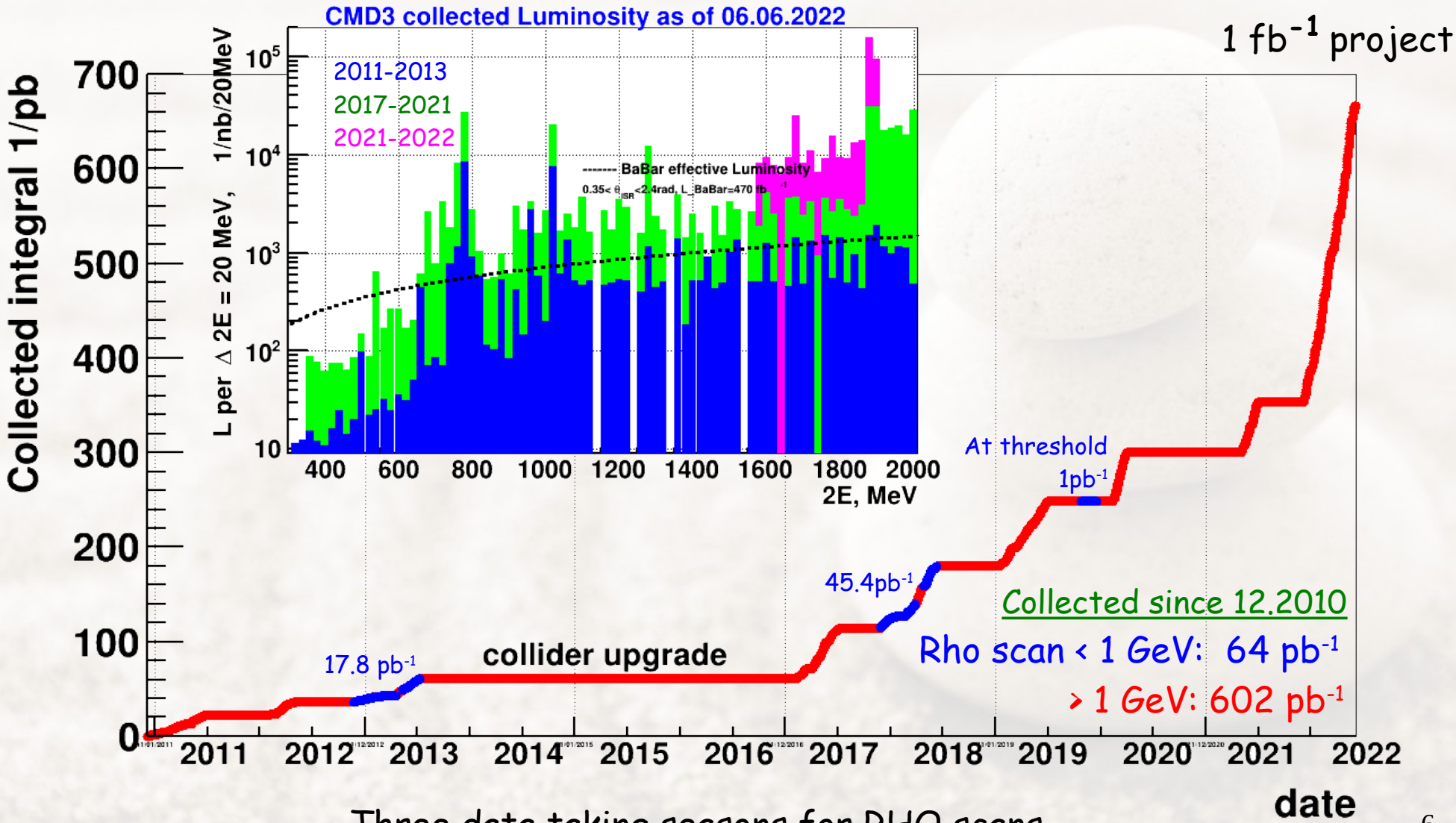
Calorimetry:

- Combined EM calorimeter (LXe,CsI, BGO)
  13.5 X_0 in barrel part
  $\sigma_E/E \sim 0.034/\sqrt{E}$ [GeV] $\oplus$ 0.020 - barrel
  $\sigma_E/E \sim 0.024/\sqrt{E}$ [GeV] $\oplus$ 0.023 - endcap
- LXe calorimeter with 7 ionization layers with strip readout
  ~2mm measurement of conversion point,
  tracking capability,
  shower profile (from 7 layers + CsI)

PID:

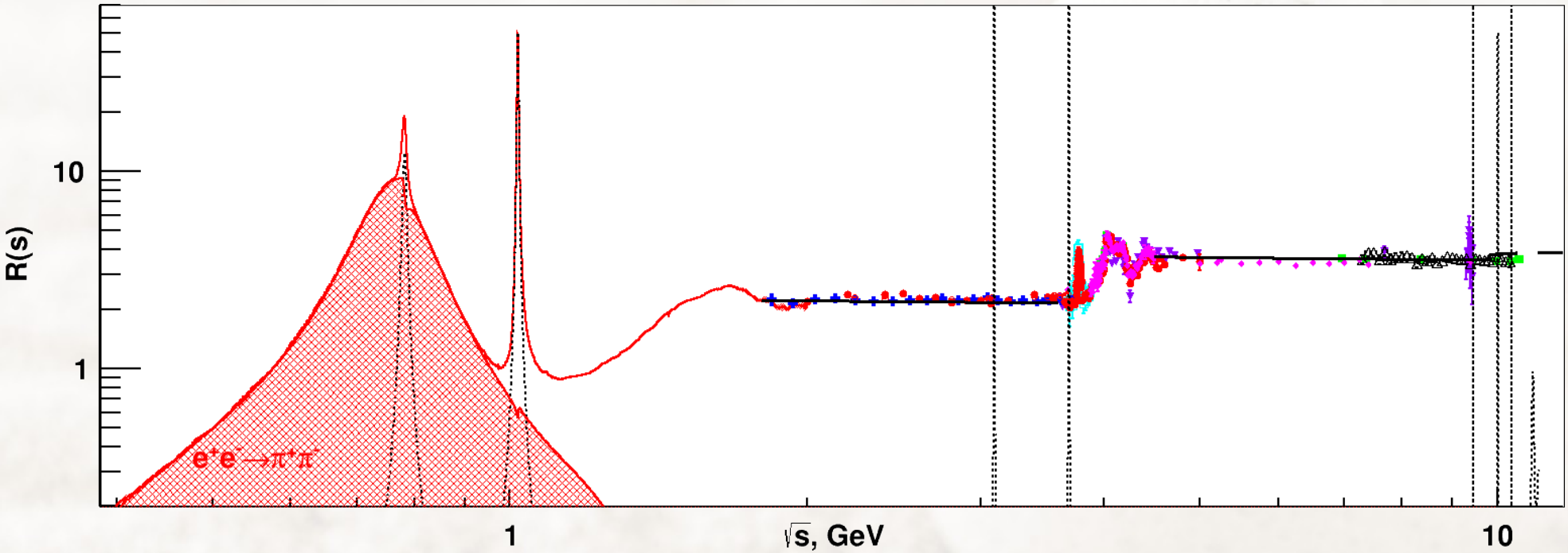
- TOF system (σ_T ~ 0.4 nsec)
  particle id mainly for p, n
- Muon system

Overview of CMD-3 data taking runs



Three data taking seasons for RHO scans

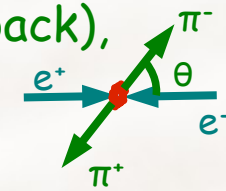
$$R(s)=\frac{\sigma^0(e^+e^-\to\gamma^*\to hadrons)}{\sigma^0(e^+e^-\to\gamma^*\to\mu^+\mu^-)}$$



$e^+e^- \to \pi^+\pi^-$ gives main contribution to R(s) at $\sqrt{s} < 1$ GeV
and this channel is most important for muon (g-2)/2

e+e- → π+π- by CMD3

Very simple topology (just 2 tracks back to back),
but the most challenging channel
due to high precision requirement.



Analysis was performed trying to reach systematic ~0.35-0.5%

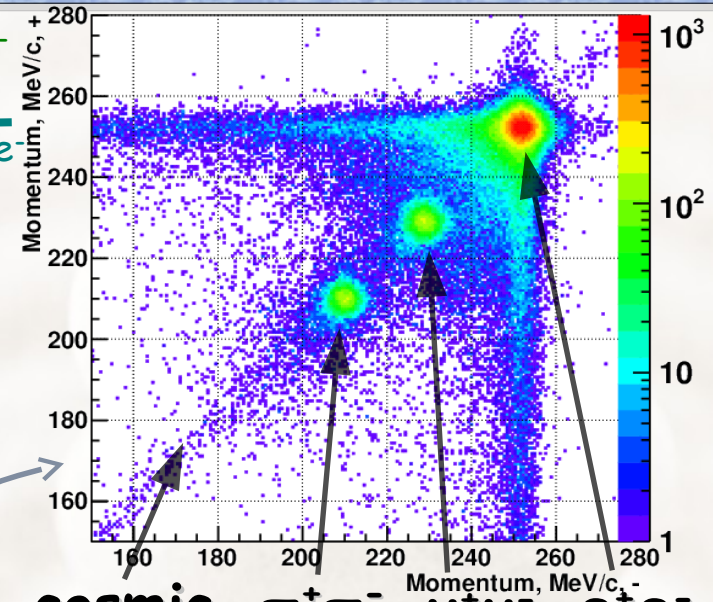
<u>Crucial pieces of analysis:</u>

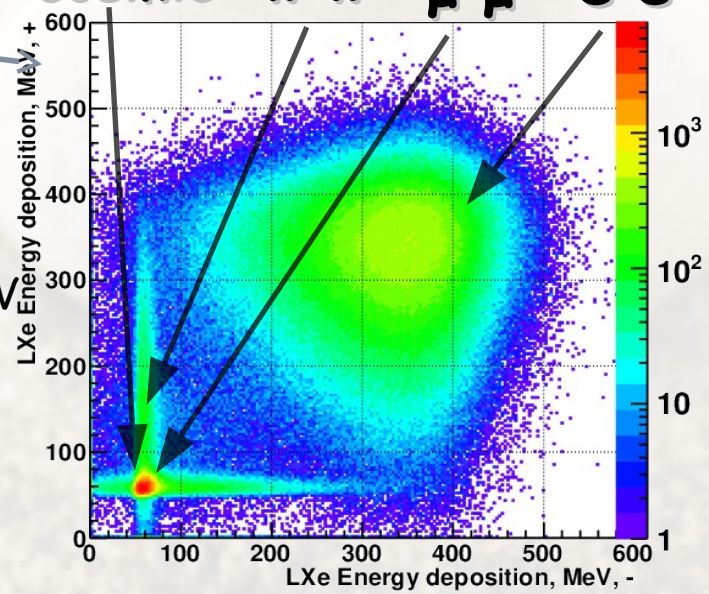
- ✗ e/μ/π separation
- ✗ radiative corrections
- ✗ precise fiducial volume
- ✗ ...

events separation either
1) by momentum
2) or by energy deposition

3) additional cross-check by angle distribution

4) using shower profile at >1GeV





Event separation

events separation is done either

1) by momentum

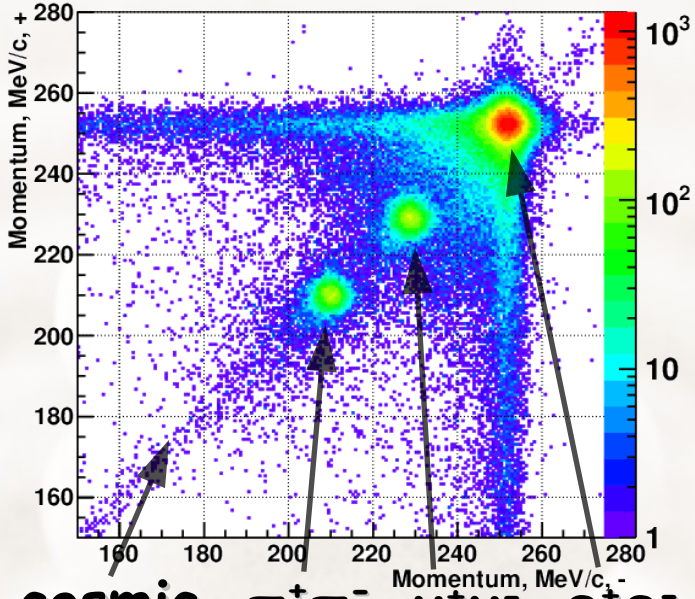
2) or by energy deposition

Separation of $\pi^+\pi^-$, $\mu^+\mu^-$, e^+e^-, …. final states is based on likelihood minimization:

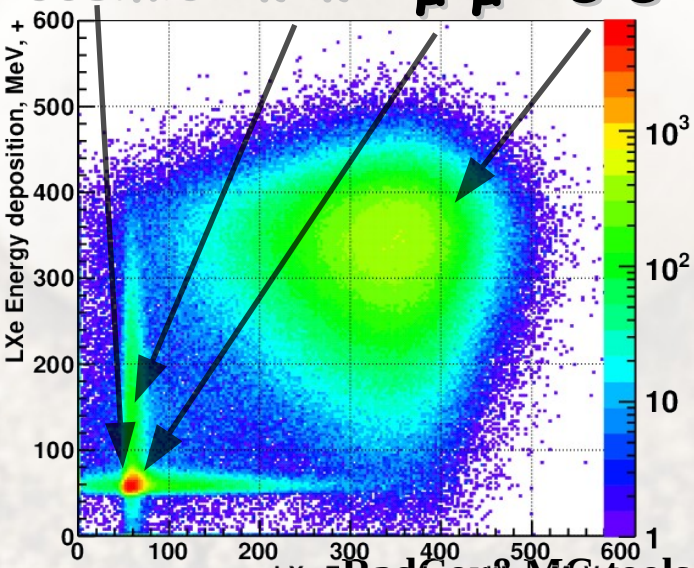
$$-\ln L = -\sum_{events} \ln\left[\sum_i N_i f_i(X^+, X^-)\right] + \sum_i N_i$$

In case of momentum-based separation:
the predicted Momentum spectra from the generators are used as input for PDF construction (+detector effects)

Not the case for energy-deposition based separation – doesn't require knowledge from generators



$P^+ \times P^-$ E_{beam}=250 MeV



$E^+_{LXe} \times E^-_{LXe}$ E_{beam}=480 MeV

Angle distribution fit

dσ/dθ spectra from MC Generators

+ all efficiencies/smearing effects extracted from data and full simulation (cosmic is taken from data itself)

$N_{\mu\mu}$ /N_{ee} - fixed from QED (+efficiencies)

N cosmic, 3π - from momentum based separation

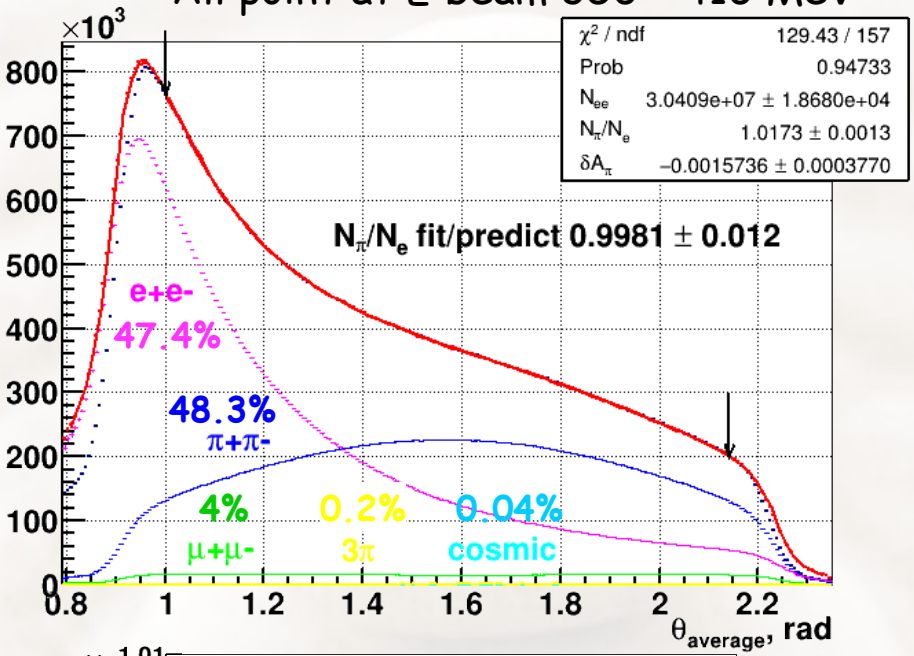
$N_{\pi\pi}$/N_{ee} , δA - free parameters

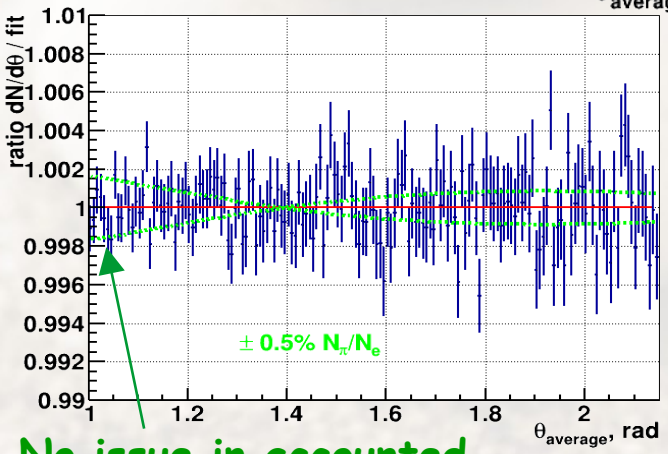
Combined fit on all points around ρ-peak

$\sqrt{s}$ = 0.7 – 0.82 GeV

$N_{\pi\pi}$ /N_{ee} = 1.0173 +- 0.0013

All point at E beam 350 – 410 MeV





No issue in accounted efficiency at θ = 1 rad

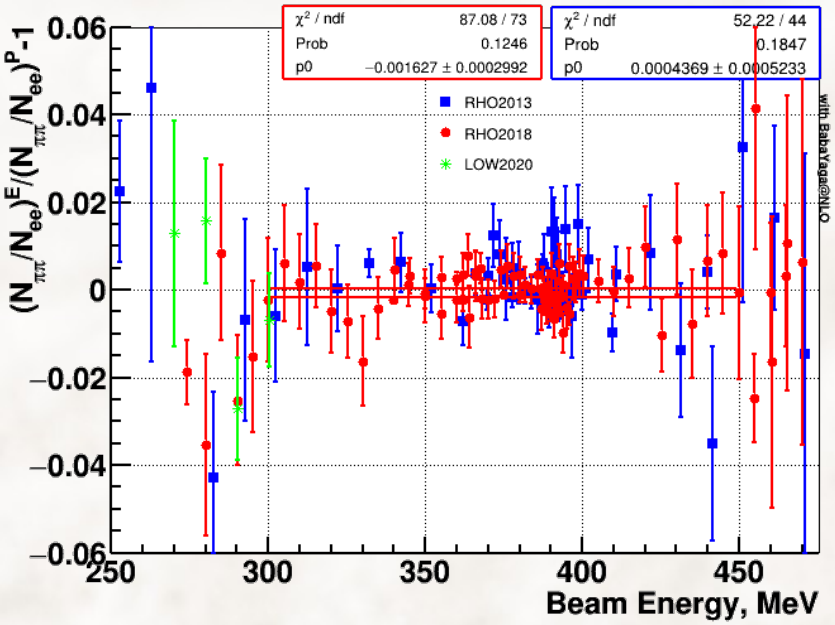
Fit by θ distribution

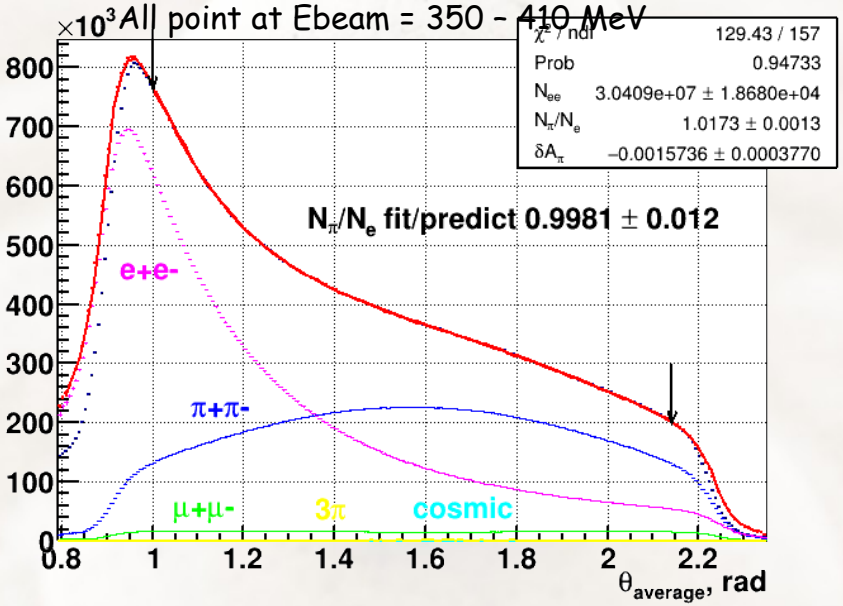
e/μ/π separation

3 methods for $N_{\pi\pi}/N_{ee}$ determination based on independent informations:

1) Momentum from DCH **2)** Energy deposition in LXe **3)** angles in DCH

E vs P separations





Fit by θ distribution

For sum of $\sqrt{s}$ = 0.7 – 0.82 GeV points

<u>by momenta</u> in DCH: $N_{\pi\pi}/N_{ee}$ = 1.0193 +- 0.00030

<u>by energies</u> in LXe $\Delta N_{\pi\pi}/N_{ee}$ = -0.09 +- 0.024%

<u>from theta</u> with free δA: = -0.20 +- 0.12%

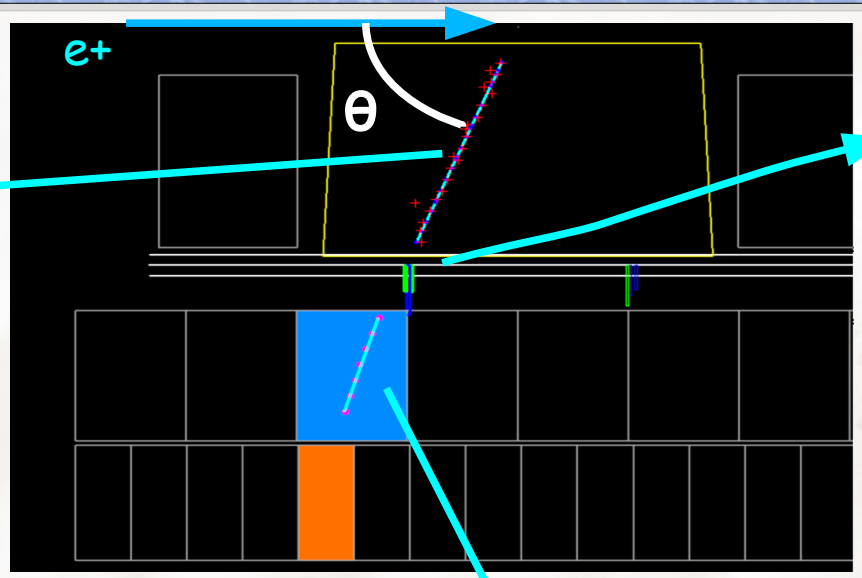
with fixed δA=0: = +0.21 +- 0.07%

Common stat from $\sqrt{N}$: 0.026%

consistency at ~ 0.2%

Precision of fiducial volume

Polar angle measured by
DCH chamber
with help of charge division method
(Z resolution ~ 2mm),
Unstable, depends on calibration and thermal stability of electronic
Calibration done relative to LXe (ZC)



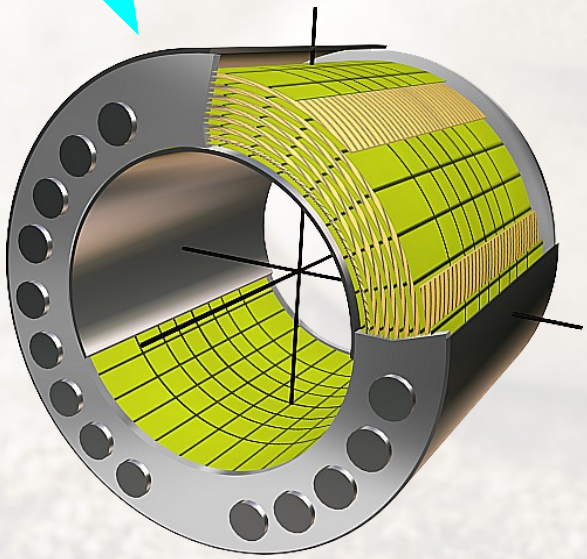
ZC chamber
(was in operation until mid 2017)
multiwire chamber
with 2 layers and with strip readout along Z coordinate

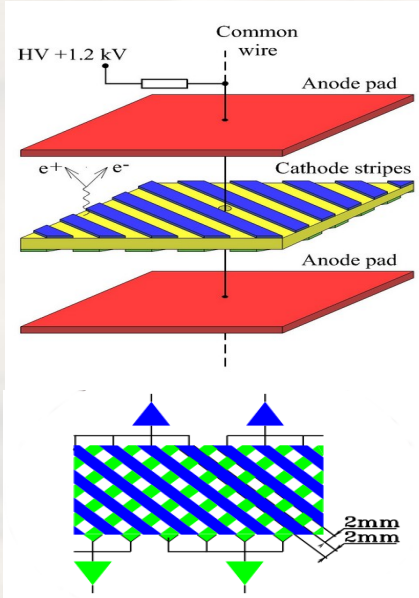
strip size: 6mm
Z coordinate resolution ~ 0.7 mm
(for θ_{track} ~ 1 rad)

LXe calorimeter
ionization collected in 7 layers with cathode strip readout,

combined strip size: 10-15 mm
Coordinate resolution ~ 2mm

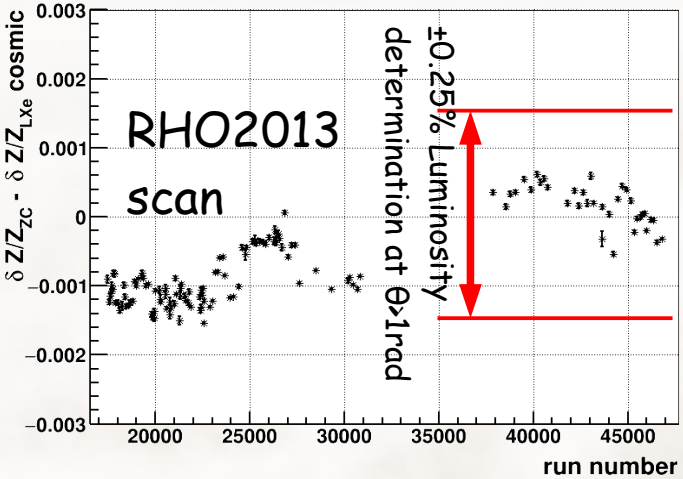
strip precision, coordinate biases ~ 100 μm
should give ~0.1% in Luminosity determination
Can be spoiled by noise environment





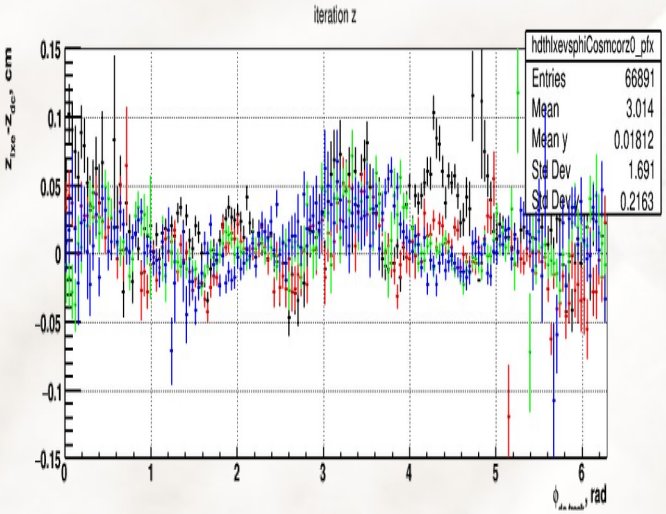
Precision of fiducial volume

Monitoring of z-measurement between ZC vs LXe



Variation because of
DCh instability, different B field,
ZC, LXe noise level

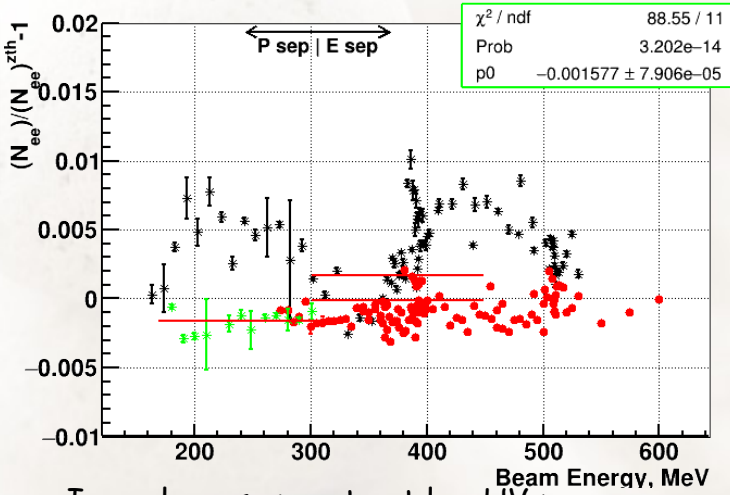
DC tracks vs LXe points



δz ~ 0.5 mm instability over regions at R=40 cm
(by φ, track direction, etc)

N.B. in average <δz> should be better

Inner DC radius effect:
θ – angle with Z vertex constrained vs unconstrained case for 2 tracks



Inner layers operate at low HV→
Low resolution, higher systematics
<u>During RHO2013:</u> 4 middle layers in DCH were switched off
→ higher weights of inner layers

N.B. θ – angle is defined with vertex constrain
→ inner radius biases should be suppressed

Systematic uncertainty to $|F_\pi|^2$

ZC/LXe comparison — LXe/ DC comparison — Inner DC radius effect:

0.25% ⊕ 0.3% ⊕ 0.7%(RHO2013)/0.3%(RHO2018)

= 0.8% (RHO2013) / 0.5%(RHO2018)

Radiative corrections

Measurement of $e^+e^- \to \pi^+\pi^-$ requires high precision calculation of radiative corrections.

Two high precision MC generators were used
MCGPJ(0.2%, e+e-,μ+μ-,π+π-) vs BabaYaga@NLO (0.1%, e+e-,μ+μ-)
They include exact NLO + Higher Order terms in some approximation.

e+e- → e+e-(γ) : great consistency <0.1% in the total cross section
e+e- → μ+μ-(γ) : Mass term in FSR is missed in most of generators
(effect 0.4% at √s=0.32 GeV)
e+e- → π+π-(γ) : only MCGPJ available with 0.2% precision
(for energy scan experiments)

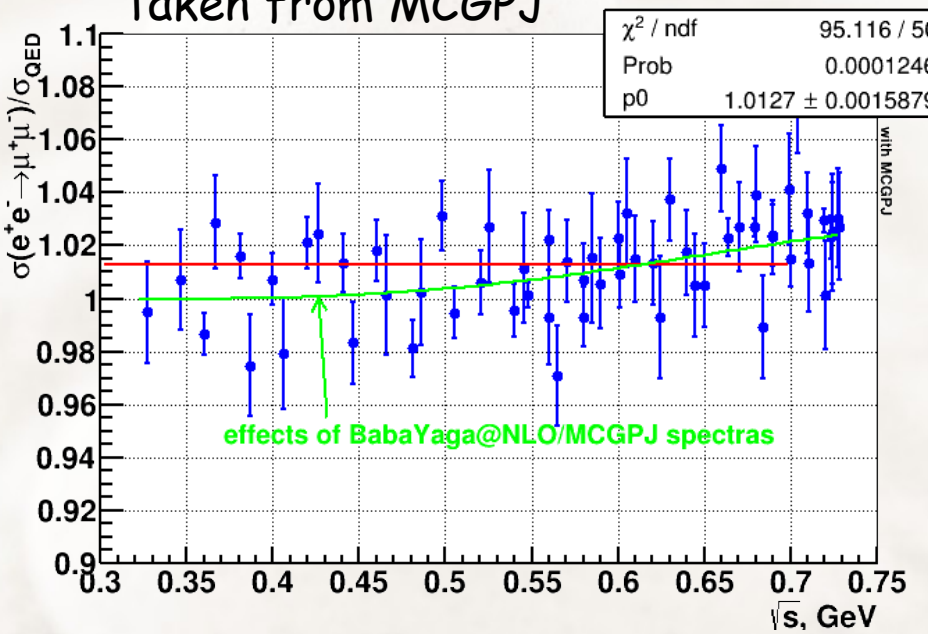
Achieved precision in current analysis is also sensitive
for precision of differential cross sections predictions
e/π separation by momentum requires $d\sigma/dP^+dP^-$ spectra as initial input
Θ-angle (asymmetry) study requires $d\sigma/d\theta$ spectra

Radiative corrections

BaBaYaga@NLO shows better agreement with the data:

1) Momentum spectras better describe data:
gives consistent results in $N_{\mu\mu}$/QED
(effect on $|F_\pi|^2$ ~0.2% at $\sqrt{s}$=0.78 GeV, and rising to 1.5% at 0.9 GeV when using momentum-based separation)

effect on $N_{\mu\mu}$/QED
when input $d\sigma/dP^+dP^-$ spectra
taken from MCGPJ



2) Experimental asymmetry in e+e-

Data vs BabaYaga@NLO:

δA = -0.060 ± 0.026 %

Data vs MCGPJ

δA = -0.140 ± 0.026 %

BabaYaga@NLO consistent with NNLO MCMule

δA = +0.006 ± 0.003 % at $\sqrt{s}$=0.76 GeV

We adopted generators usage in this way:

e+e- : BabaYaga@NLO

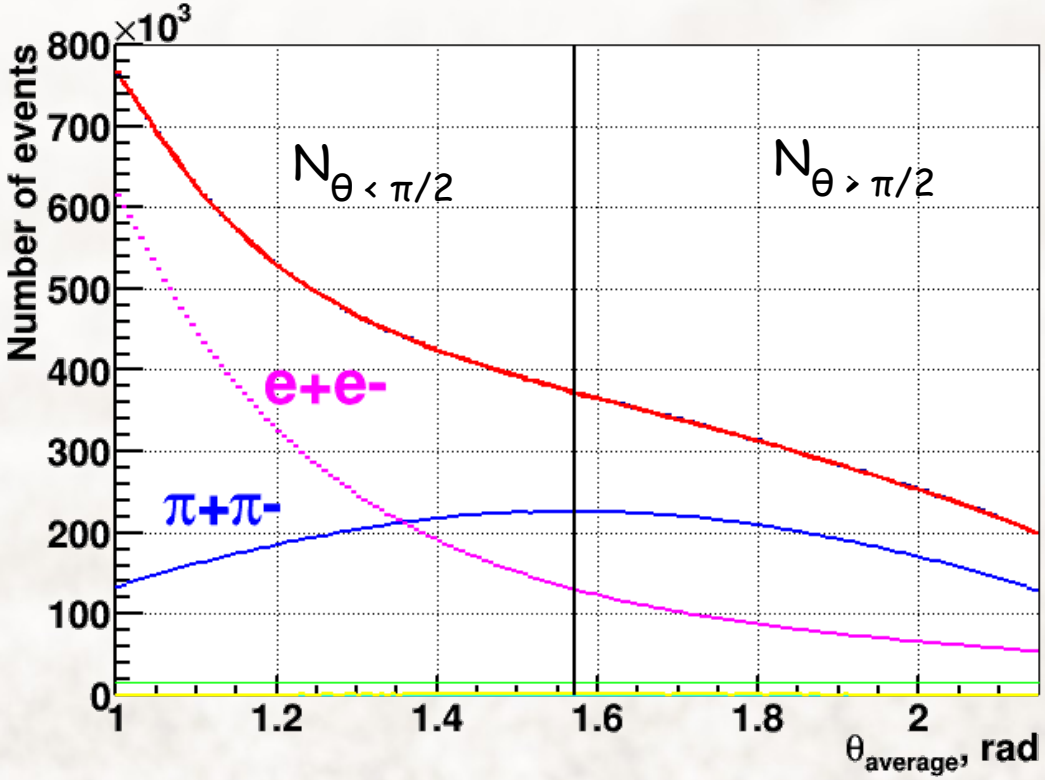
μ+μ- : BabaYaga@NLO (differential cross section)
MCGPJ (integral)

π+π- : MCGPJ

Better NNLO (+VP + next log terms) generators are quite desirable for higher precision

Forward backward charge asymmetry

dσ/dθ spectra



Asymmetry definition:

$A = (N_{\theta < \pi/2} - N_{\theta > \pi/2})/N$

Sensitive to:

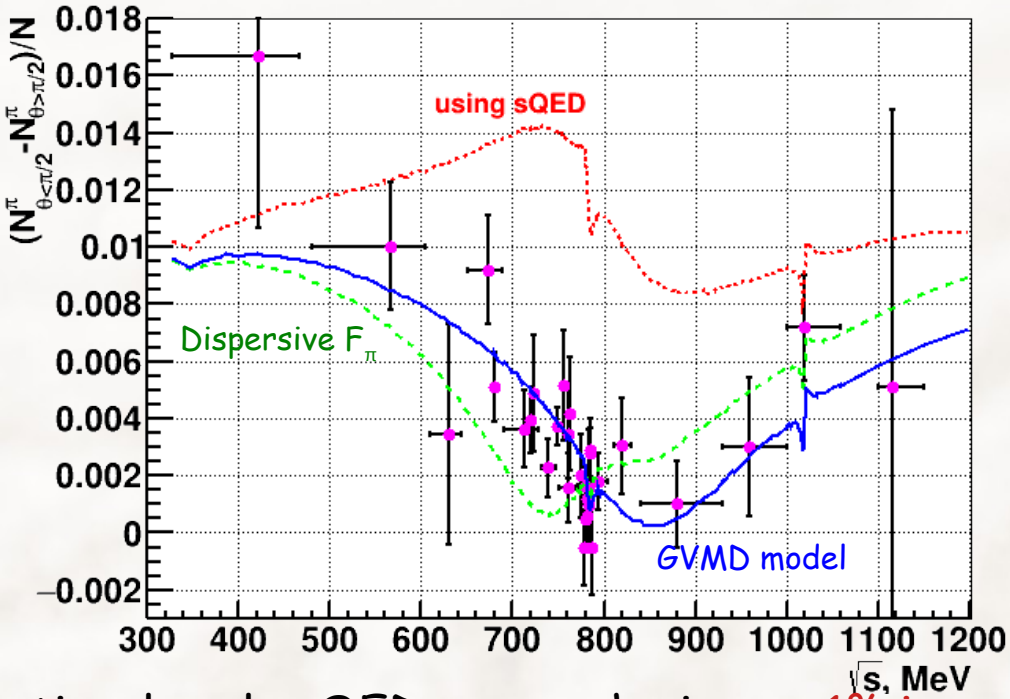
- ✗ angle-related systematics
- ✗ used model of γ-π interaction

At first try:

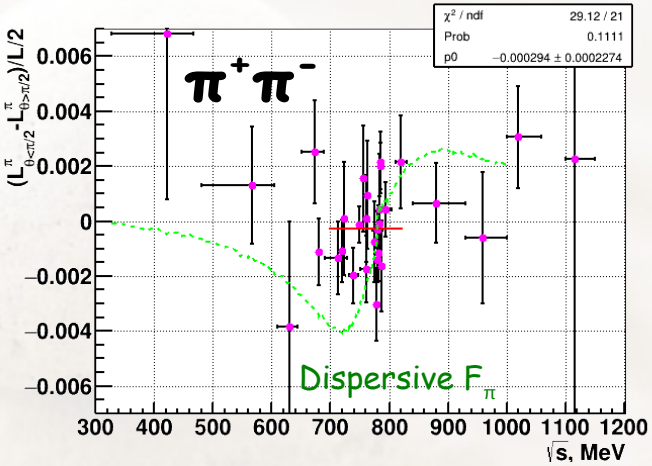
1% inconsistency for π+π- was observed between data and MC prediction

Charge asymmetry in e+e- -> π+π-

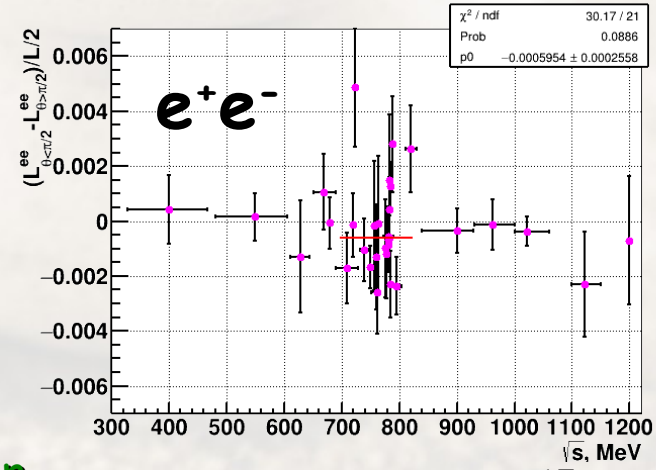
$A = (N_{\theta < \pi/2} - N_{\theta > \pi/2})/N$



Relative to GVMD prediction



to BaBaYaga@NLO



Conventional scalar QED approach gives ~ 1% inconsistency

The theoretical model within GVMD was introduced, describes well the CMD-3 data R.Lee et al., Phys.Lett.B 833 (2022) 137283

was confirmed by calculation in dispersive formalism

M.Hoferichter et al., JHEP 08 (2022) 295

Average at $\sqrt{s}$ = 0.7-0.82 GeV:

$\pi^+\pi^-$: $\langle\delta A\rangle$ = -0.029 ± 0.023 %

e^+e^-: $\langle\delta A\rangle$ = -0.060 ± 0.026 %

Ensure our θ angle systematics estimation for $|F_\pi|^2$

Fπ within different θ selection

Dependence on theta cut $\theta_{cut} < \theta^{event} < \pi - \theta_{cut}$

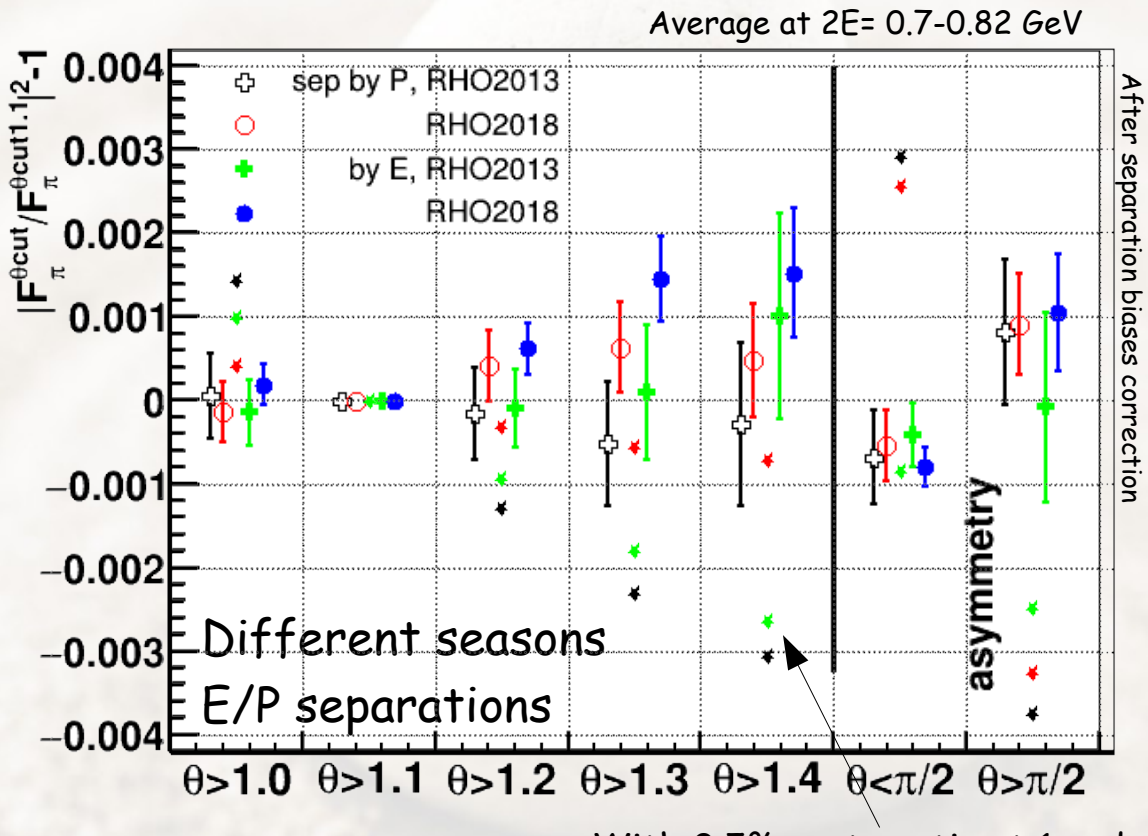
or asymmetrical selection $1 < \theta^{event} < \pi/2$ (or $\pi/2 < \theta^{event} < \pi - 1$)

$|F_\pi|^2$ stable at <0.05-0.1% level within different angle selections

Angle related systematic uncertainty estimation is quite conservative:
0.5% (RHO2018) / 0.8%(RHO2013)

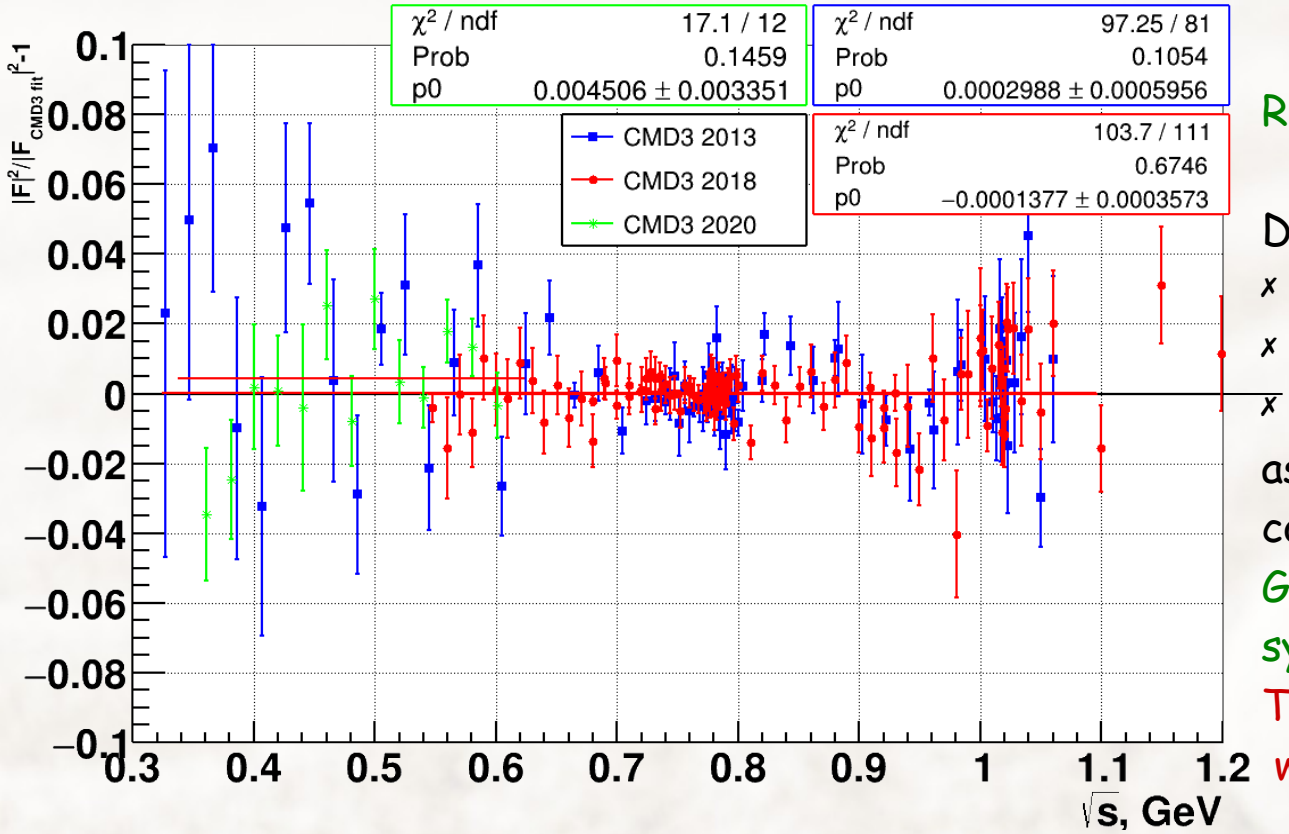
Simplest possible systematics in θ angle:
- Z – length mis-calibration
- Θ^{event} common bias

if gives 0.5% total in $|F_\pi|^2$ at Θ=1 rad should be seen with ~0.3-0.4% on this plot



With 0.5% systematic at 1 rad
★Z-length mis-calibration
★θ bias
★θ bias opposite

Consistency checks



Result consistent between seasons within < 0.1%

DCH was in very different conditions:

- correlated noise
- 4 middle layers off (HV-related) in 2013
- etc….

as result it gives ~x2 difference in some corrections

Good check of angle/tracking related systematics

Total θ-related systematic uncertainty was estimated 0.5%(RHO2018) 0.8%(RHO2013)

$|F_\pi|^2$ RHO2018/RHO2013 Δ = -0.04 ± 0.07 %

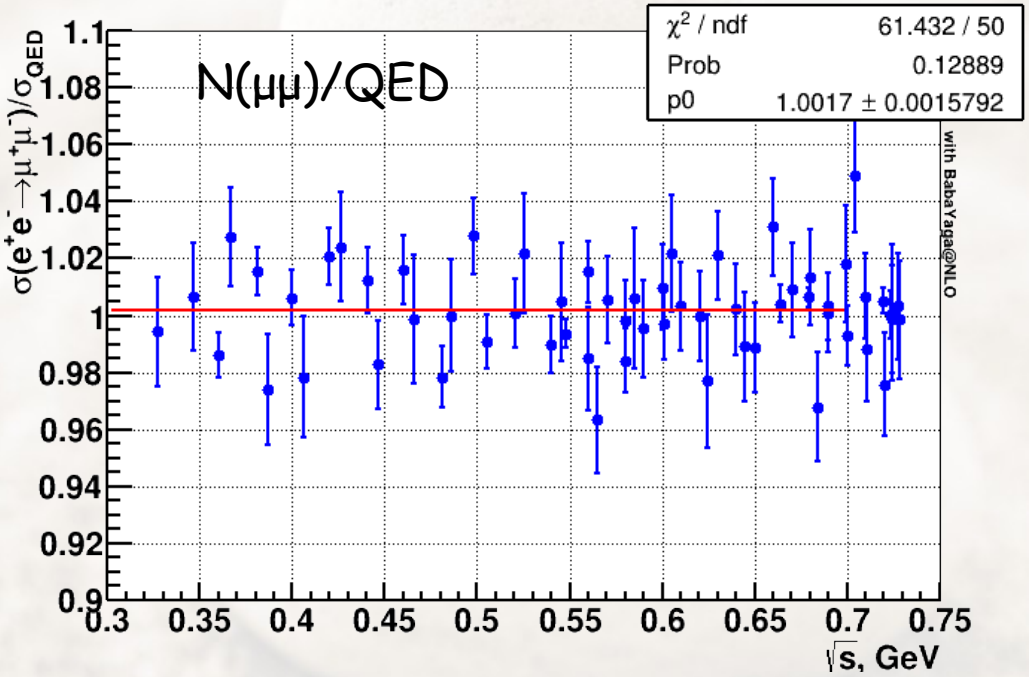
LOW2020/RHO2013 Δ = -0.5 ± 0.6 %

Consistency between seasons can hint that RHO2013 systematic uncertainty should be as good as for RHO2018

e+e- → μ+μ- cross section

One of consistency checks for e+e- → π+π- is provided by comparison of measured e+e- → μ+μ- cross section vs QED prediction

$N_{\mu\mu}$/QED : $\Delta = +0.17 \pm 0.16$ %



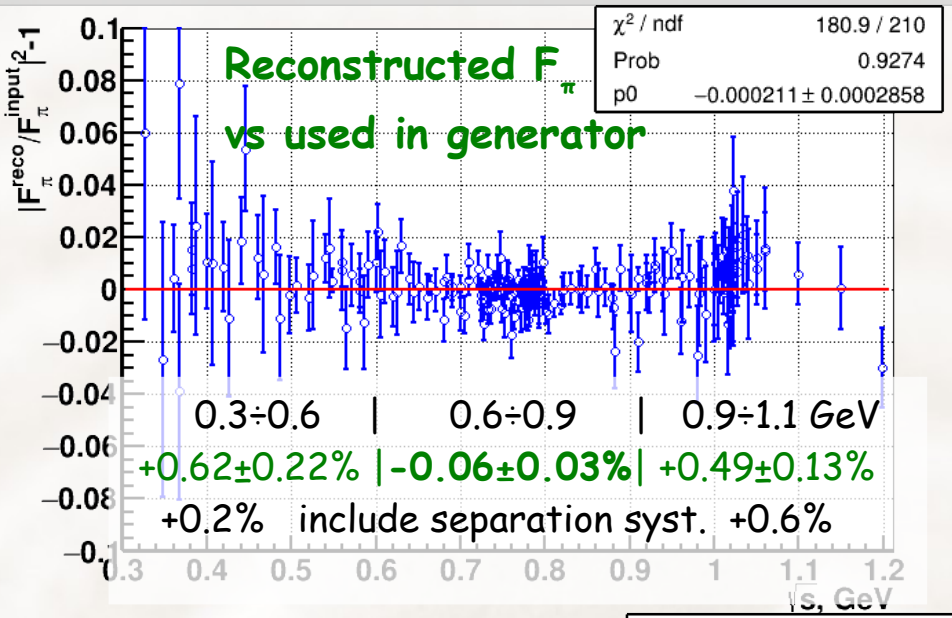
Many others self consistency checks were performed

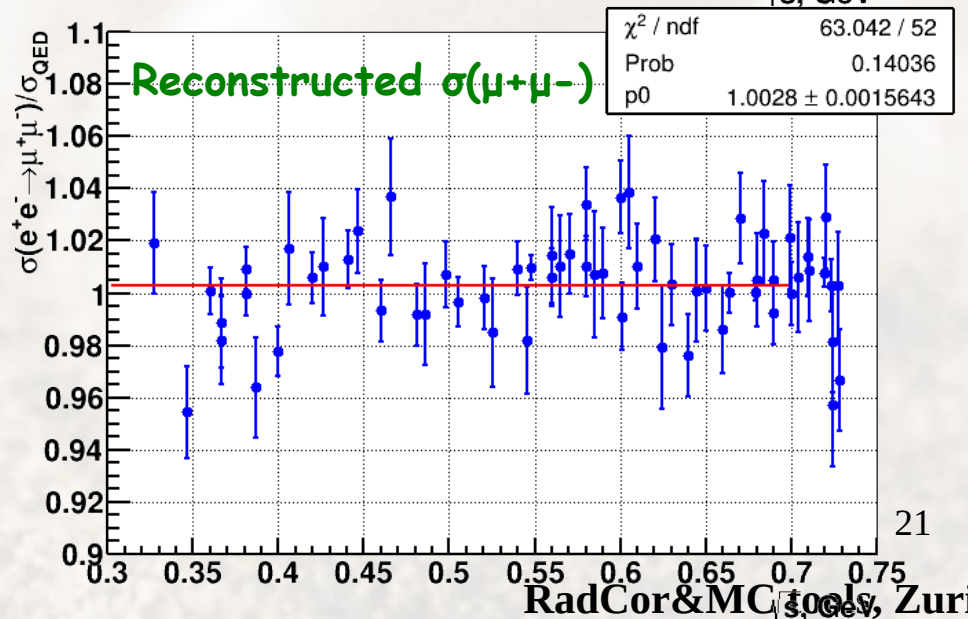
Analysis workflow cross check on MC

Full analysis workflow was checked on mixed full MC data samples
(MC with detector conditioned over time)

Same full analysis as for the data:
- efficiencies reconstructions,
- particle separation, etc
- same scripts,
- same intermediate files, etc

All underneath components (separation, efficiency reconstruction, etc) were also checked with better precision





$|F_\pi|^2$ systematic uncertainty

	At $\sqrt{s}$ near ρ peak (except ω peak)
ˣ Radiative corrections	0.2% (2π) ⊕ 0.2% (Fπ) ⊕ 0.1% (e+e-) = **0.3%**
ˣ e/μ/π separation	**0.2%**
ˣ Fiducial volume	**0.5% / 0.8%** (RHO2013)
ˣ Correlated inefficiency	**0.1%**
ˣ Trigger	**0.05%**
ˣ Beam Energy (by Compton σ_E< 50 keV)	**0.1%**
ˣ Bremsstrahlung loss	**0.05%**
ˣ Pion specific loss	**0.2%** nuclear interaction
	0.1% pion decay
	0.7% / 0.9% (RHO2013)

After quite conservative θ-angle related contribution, the radiative correction is the next biggest part to the systematic table

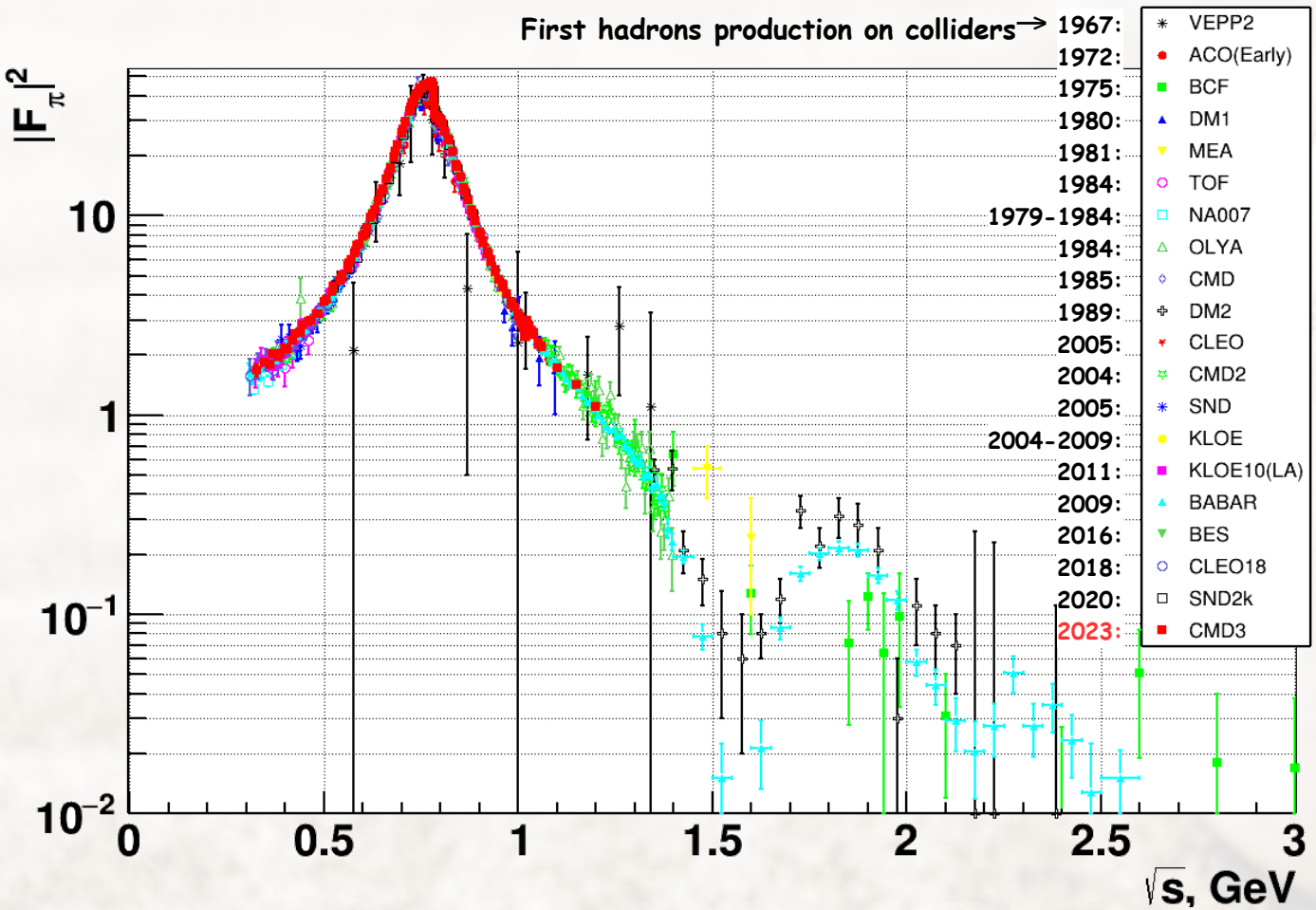
Indirectly theoretical knowledge present in the particle separation and fiducial volume determination as the consistency check

Possible concerns in the analysis related to MC tools:

- ✗ Radiative corrections for the $\pi^+\pi^-$ total cross section
  - ✗ MCGPJ were used by several previous experiments, the cross-check with a new generator will be very valuable
- ✗ Differential cross section over momentum for the particle separation
  - ✔ E/P separations, $\sigma(e^+e^- \to \mu^+\mu^-)$/QED are consistent
- ✗ Differential cross section over polar angle for controlling of systematic uncertainty of the fiducial volume determination
  - ✔ quite remarkable consistency of data (asymmetry, θ – angle distribution, $|F_\pi|^2$ in different cuts) vs prediction

Progress in MC tools can help to give more confidence, or can help to highlight some detector related effects in the obtained CMD-3 result

e+ e− → π+ π− today



Before 1985

Low statistical precision

Systematics >10%

NA7 A few points with >1-5%

1985 - VEPP-2M

with more detailed scan

OLYA systematics 4%

CMD 2%

2004 with CMD2 at VEPP-2M

was boost to systematics: 0.6%

(near same total statistic)

The uncertainty in a_μ(had) was improved by factor 3 as the result of VEPP-2M measurements

New ISR method

e+e- → γ + hadrons (limited only by systematics):

KLOE: 0.8%

BaBar: 0.5%

BES: 0.9%

CLEO: 1.5%

New direct data:

SND2k : 0.8% (with 1./10 of avail Data)

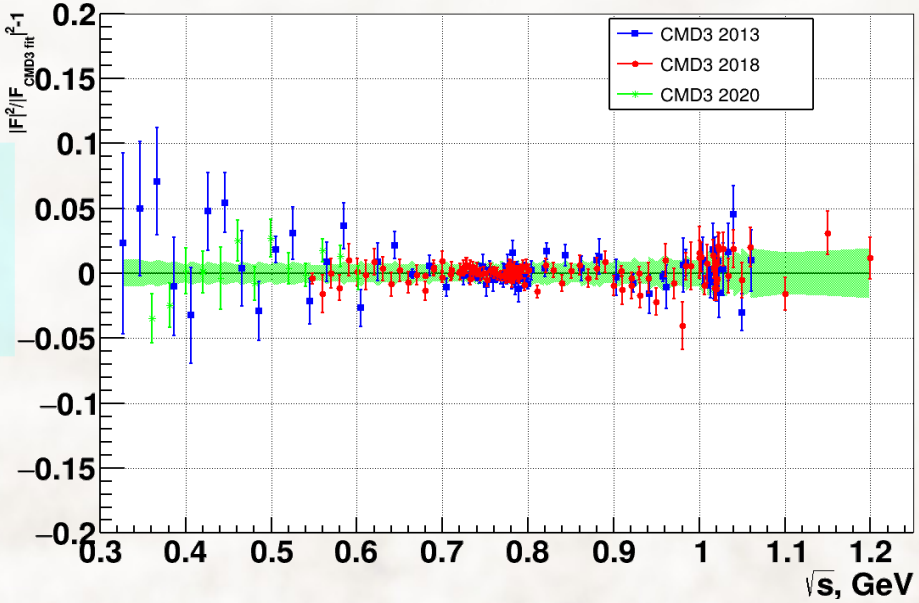
CMD-3: 0.7%

New g-2 experiments and future e+e- as ILC, FCC-ee require average precision ~0.2%

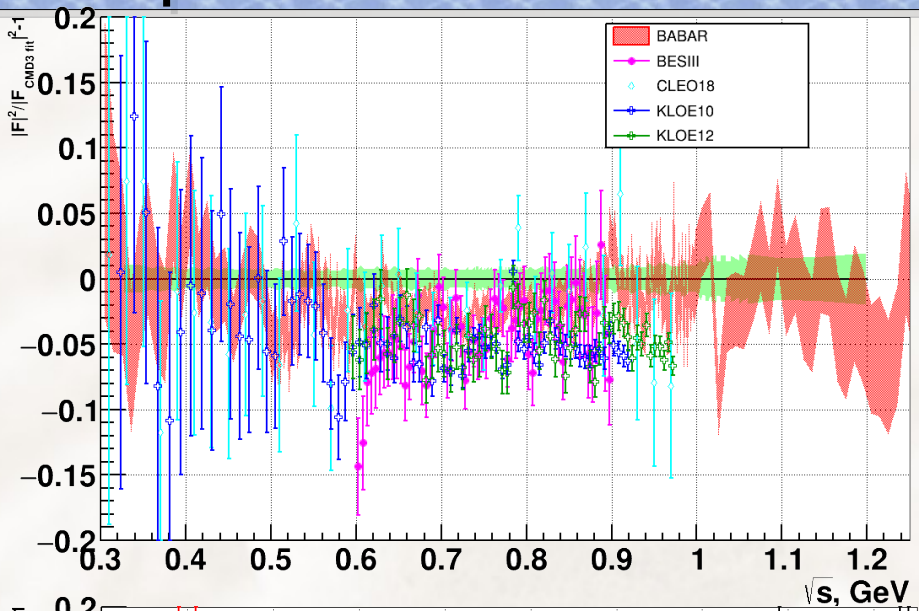
CMD-3 vs other experiments

Relative to CMD-3 fit,
green band – systematic value

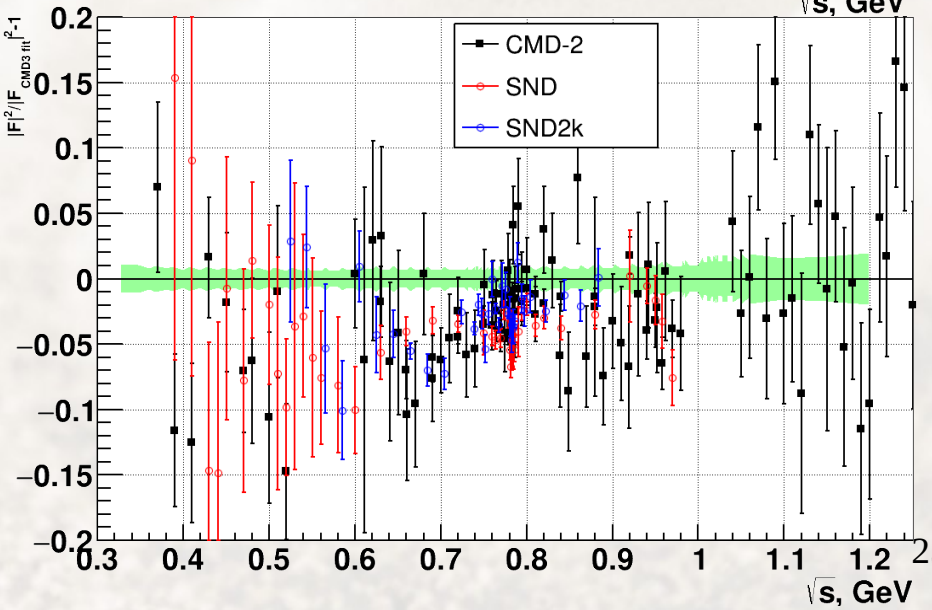
CMD-3



vs ISR



vs direct scan

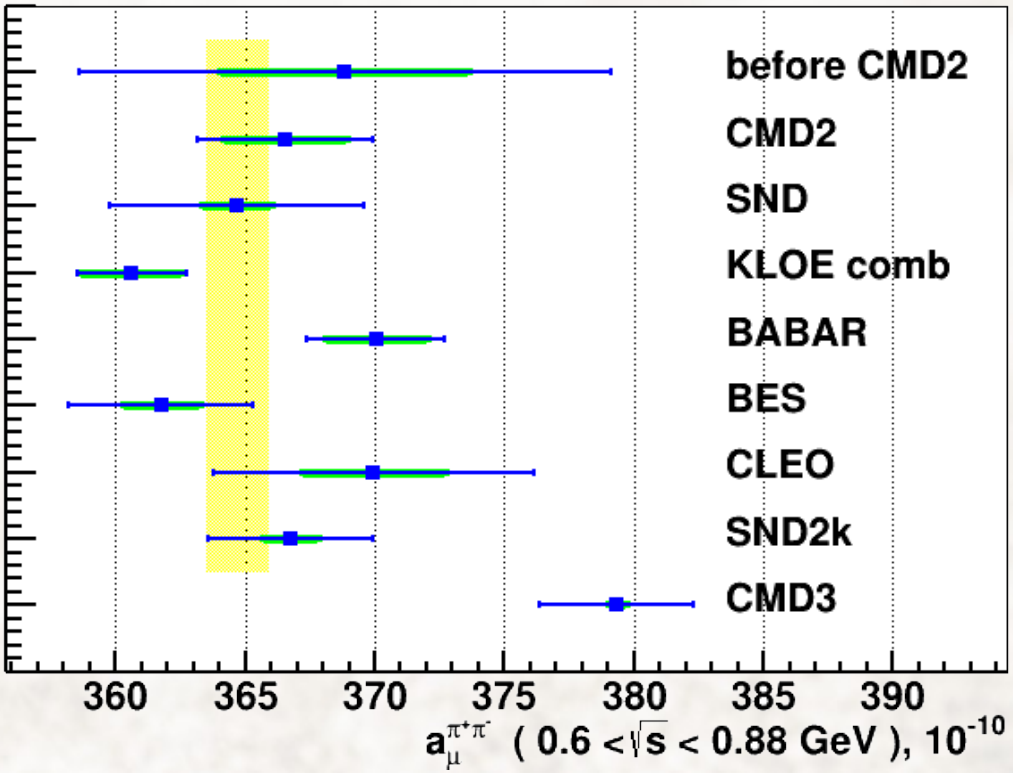


CMD-3

- ✗ Statistical precision is a few times better than any other experiments
- ✗ Cross section is higher by ~ 2-5%

The π+ π− contribution to a_μ^{had}

$$a_\mu^{had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty}\frac{\sigma_{e^+e^-\to\gamma^*\to hadrons}(s)K(s)}{s}ds$$

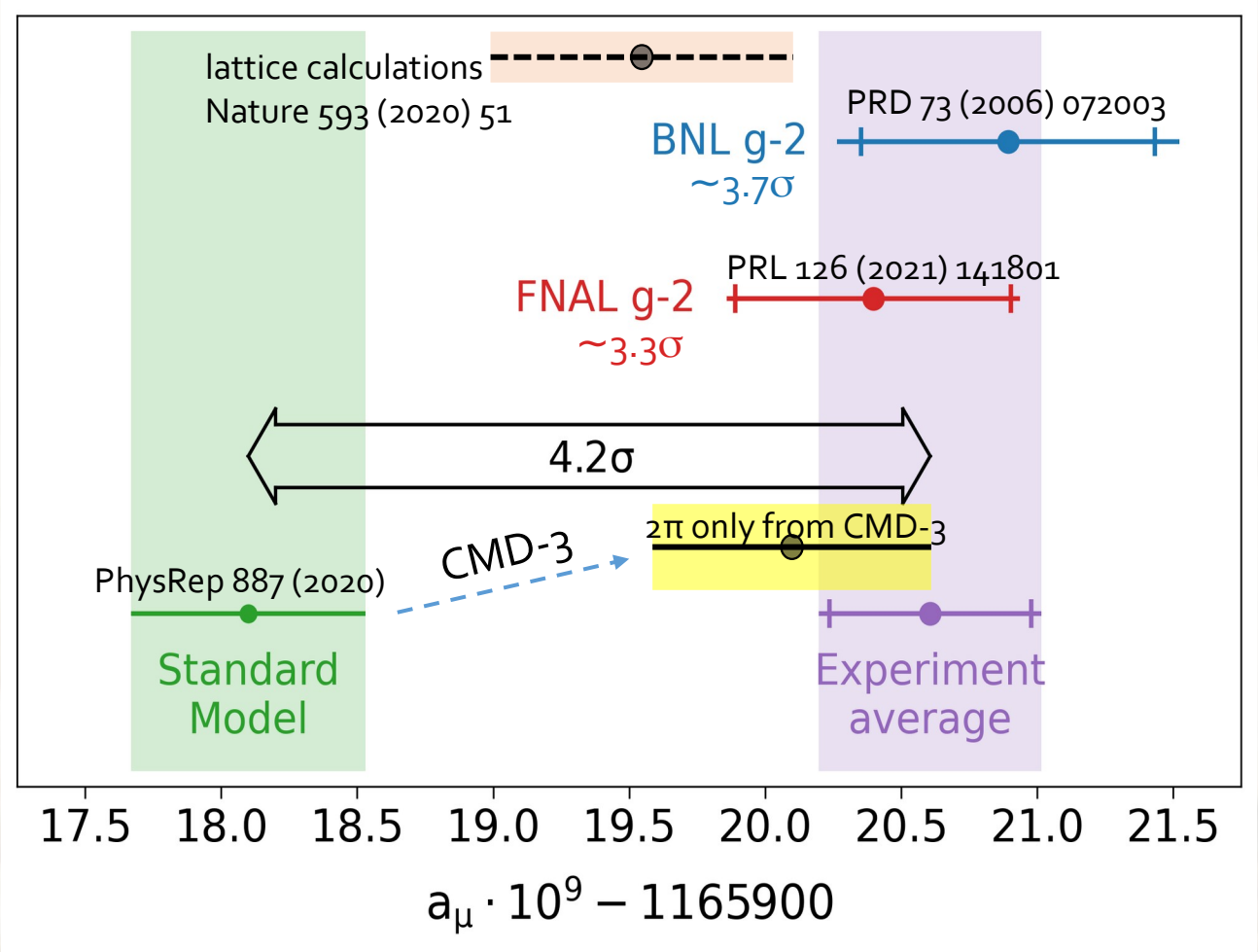


0.6 < √s < 0.88 GeV

	$a_\mu^{\pi\pi,LO}$, 10^{-10}
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

RHO2013	380.06 ± 0.61 ± 3.64
RHO2018	379.30 ± 0.33 ± 2.62 $\times 10^{-10}$
Sum	379.35 ± 0.30 ± 2.95

The impact of CMD-3 on SM prediction of a_μ^{had}



If it will be only CMD-3 than SM will be solved.
But CMD-3 is only one now over many other experiments (BaBar, KLOE, BES, CMD-2, SND, ...)

Unfortunately at the moment, we don't know the reasons of the disagreement between different experiments.

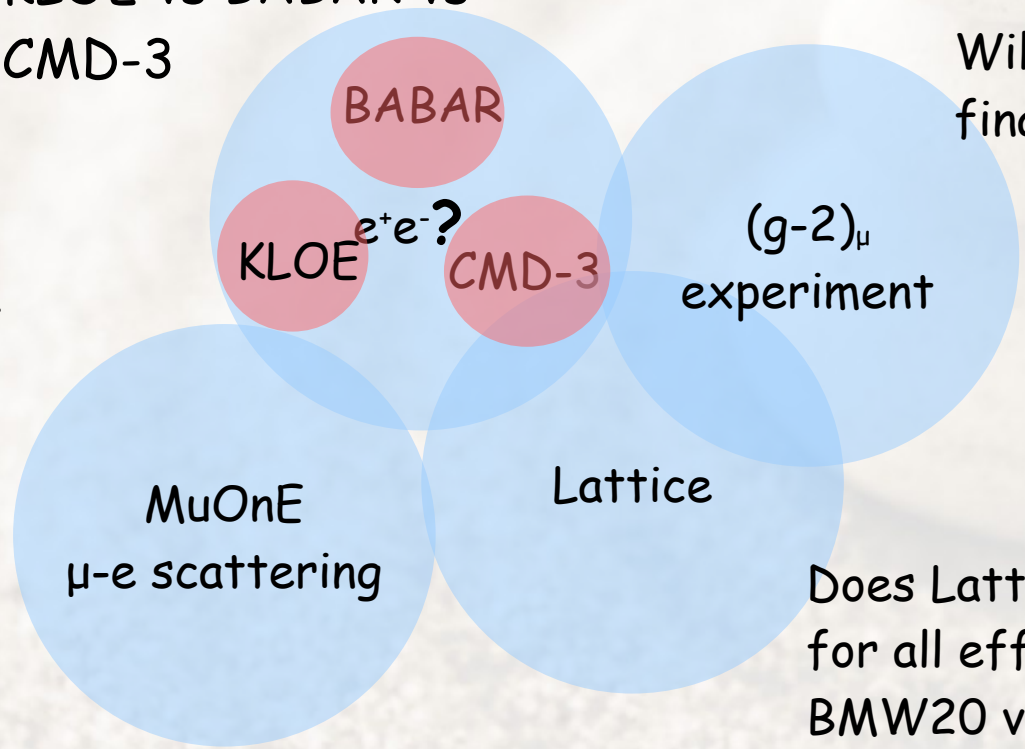
Puzzles in puzzle

Question of comparison:

e+e- vs $(g-2)_\mu$ vs lattice

Where difference comes from:
KLOE vs BABAR vs CMD-3

Will it be confirmed?
final FNAL vs J-PARC



Hard effort against systematics

Does Lattice account for all effects?
BMW20 vs others

backups

More details:

Presentation at the TI seminar, 27 March 2023:

https://indico.fnal.gov/event/59052/

E-Print: 2302.08834 [hep-ex]

55 years of hadron production at colliders

Volume 25B, number 6 PHYSICS LETTERS 2 October 1967

INVESTIGATION OF THE ρ-MESON RESONANCE WITH ELECTRON-POSITRON COLLIDING BEAMS

V. L. AUSLANDER, G. I. BUDKER, Ju. N. PESTOV, V. A. SIDOROV, A. N. SKRINSKY and A. G. KHABAKHPASHEV
Institute of Nuclear Physics, Siberian Branch of the USSR Academy of Sciences, Novosibirsk, USSR

Received 1 September 1967

Preliminary results on the determination of the position and shape of the ρ-meson resonance with electron-positron colliding beams are presented.

When experiments with electron-positron colliding beams were planned [1, 2] investigation of the process

$$e^- + e^+ \to \pi^- + \pi^+$$
$$e^- + e^+ \to K^- + K^+$$

1 September 1967

Start of e+e- → hadrons measurements

Phys.Lett. 25B (1967) no.6, 433-435

VEPP-2, Novosibirsk

Detector was made from different layers of Spark chambers,
readouts by photo camera

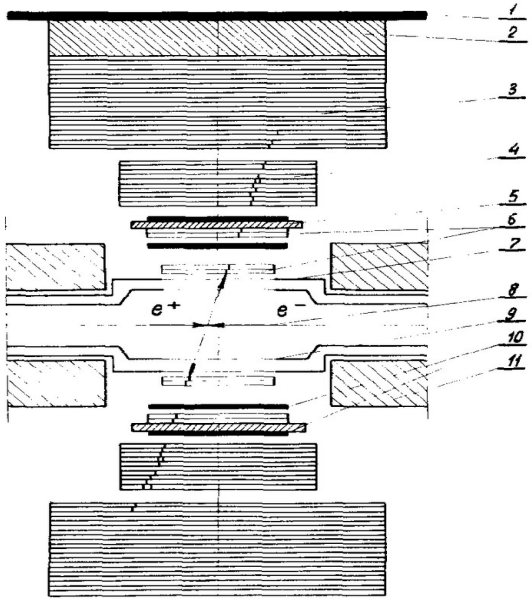


Fig. 1. Spark chambers system:
1) Anticoincidence scintillation counter
2) Lead absorber 20 cm thick
3) "Range" spark chamber
4) "Shower" spark chamber
5) Duraluminium absorber 2 cm thick
6) Thin-plate spark chambers

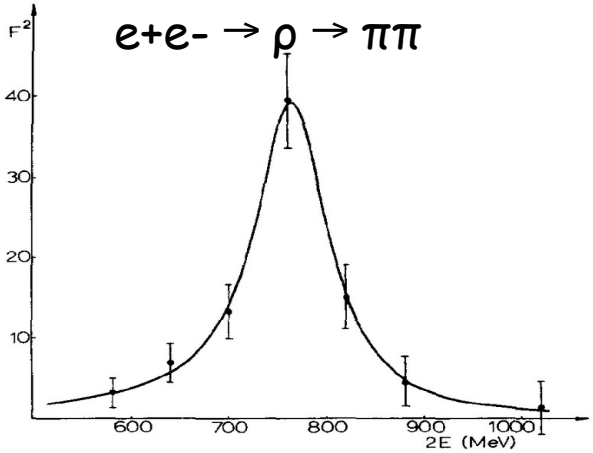
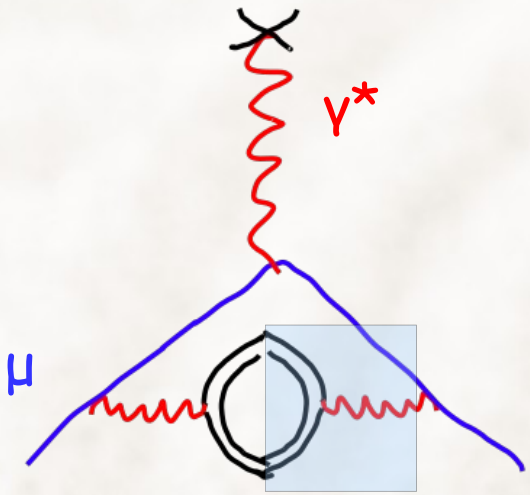


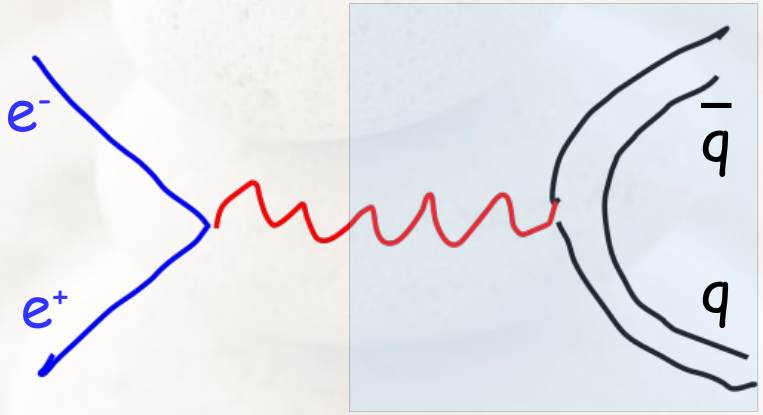
Fig. 2. Experimental values of F^2 (E) approximated by the Breit-Wigner formula.

ment geometry and F- modulus of the form factor for pion pair production [1]. In the case of QED with no other forces F=1. If the particles are produced at the angle 90° with respect to the beam axis then a=18. Integration over the solid angle gives a=20.4.

g-2 and e+e- → hadrons

Muon precession anomaly (g-2)/2 via vacuum polarization is related to e+e- to hadrons production





$$a_\mu^{had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty}\frac{\sigma_{e^+e^-\to\gamma^*\to hadrons}(s)K(s)}{s}ds$$

Dispersion relation is based on analyticity and the optical theorem

SM prediction for muon g-2

White Paper 2020 (e-Print: 2006.04822)

e-Print: 2203.15810

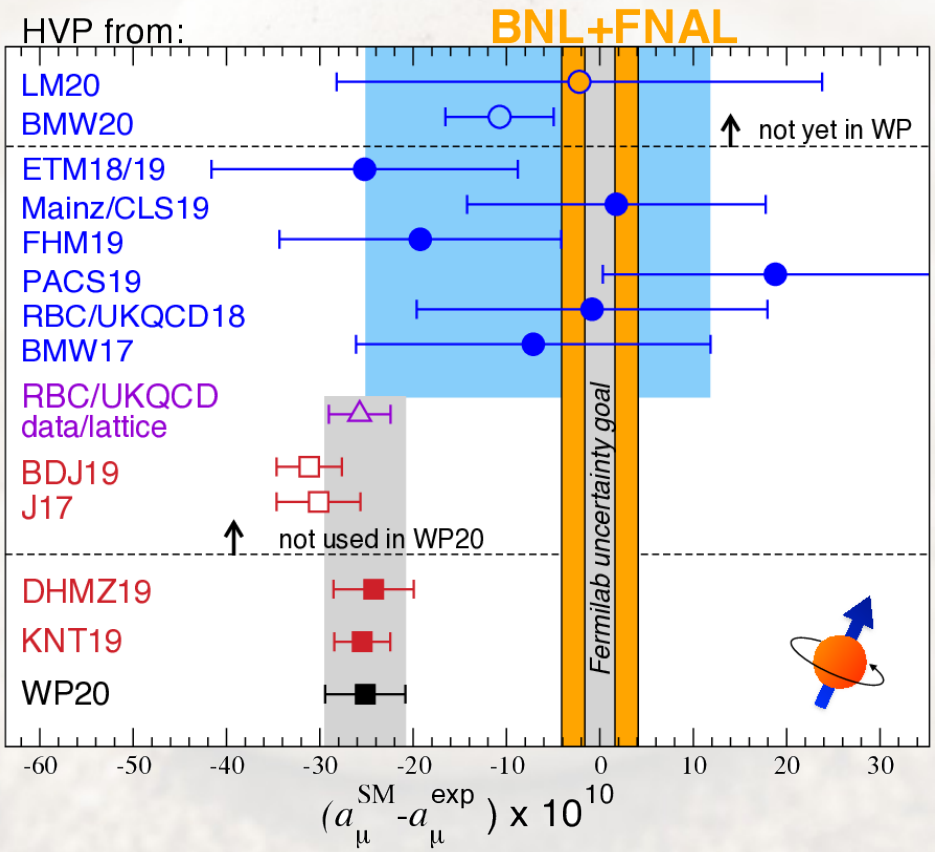
Experimental world average (E821+E989)

a_μ = 11 659 206.1± **4.1** x 10^{-10}

Theoretical prediction data driven

a_μ = 11 659 181.0± **4.3** x 10^{-10} **(WP20)**

Δa_μ = 25.1± **5.9** x 10^{-10}



Hadronic part from e+e- → hadrons:

a_μ(had) = 693.1 ± **4.0** x 10^{-10}

$\pi^+\pi^-$ 506.0 ± 3.4

……

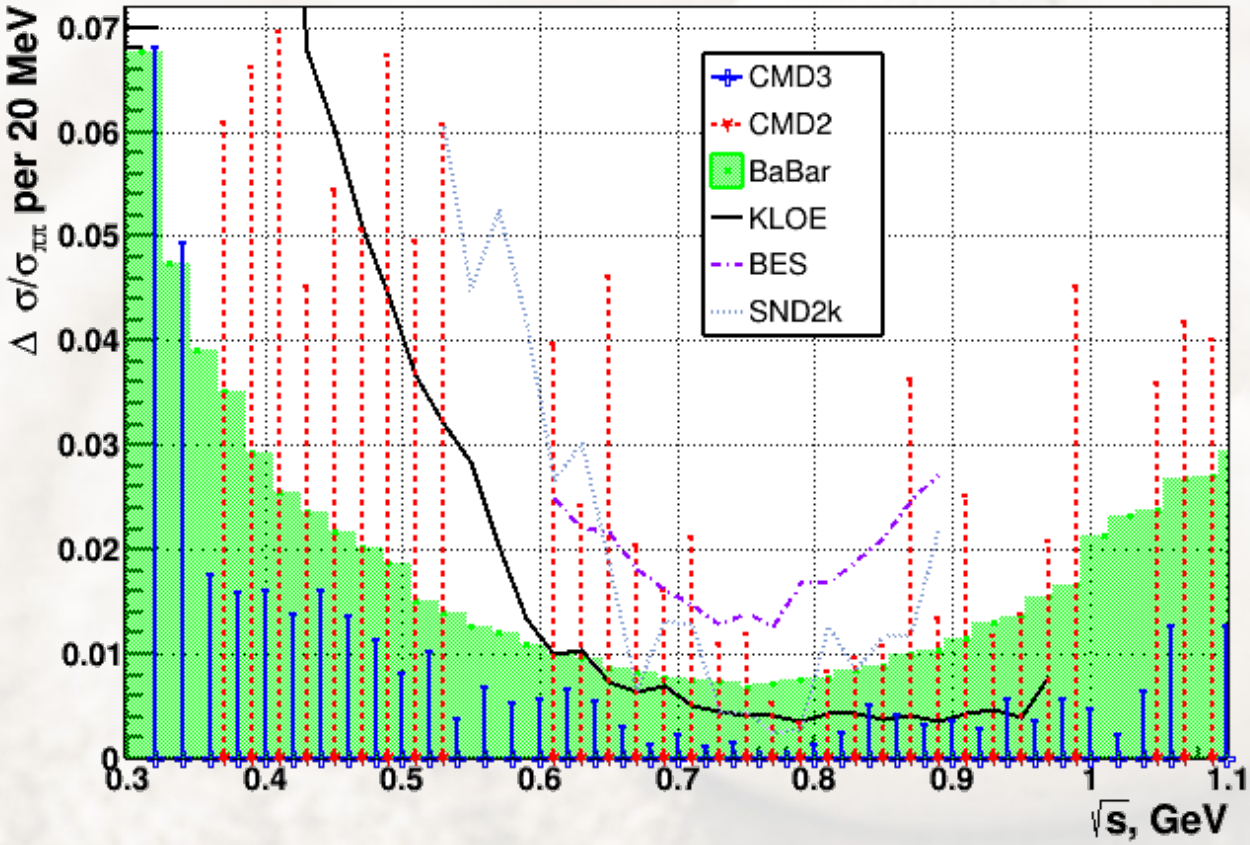
$\pi^+\pi^-$ gives the main contribution (73%) to a_μ^{HAD}

e+e- → π+π- by CMD-3

Statistical precision of CMD-3 cross section measurement
is a few times better than any other experiments

Full statistic is used
collected during ρ scans

3 seasons of data taking:
RHO2013
RHO2018
LOW2020



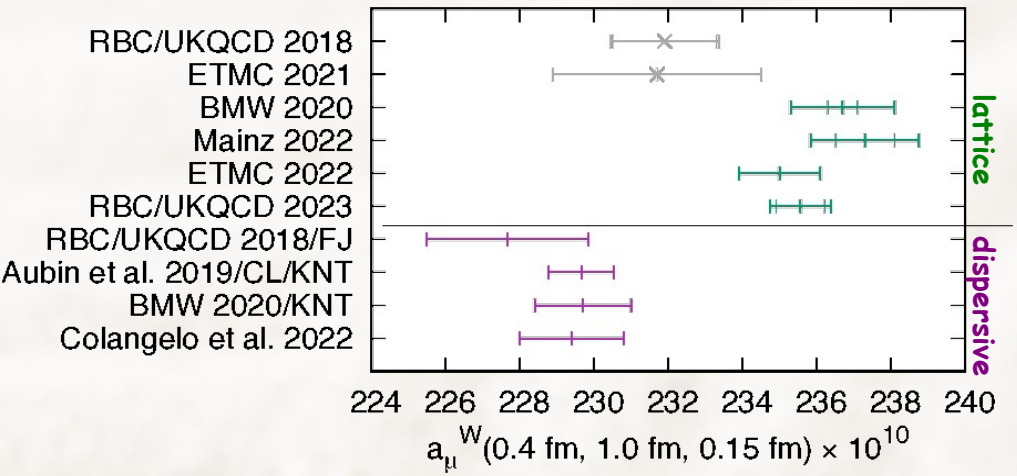
Analysis based on L = 61.9 pb^{-1} at $\sqrt{s}$ < 1 GeV (+25.7 pb^{-1}, 1.0-1.2 GeV)

34×10^6 $\pi^+\pi^-$, 3.7×10^6 $\mu^+\mu^-$, 44×10^6 e^+e^-
events selected at $\sqrt{s}$ < 1 GeV

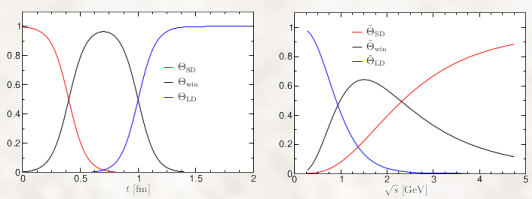
Dispersive vs Lattice

T.Blum et al, e-Print: 2301.08696 [hep-lat]

a^{HVP}_{μ} contribution from intermediate window in Euclidean time



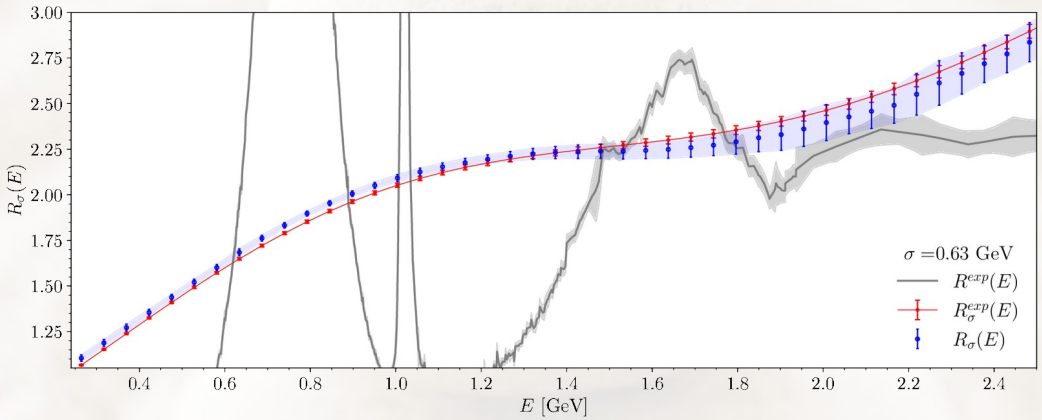
Windows definition

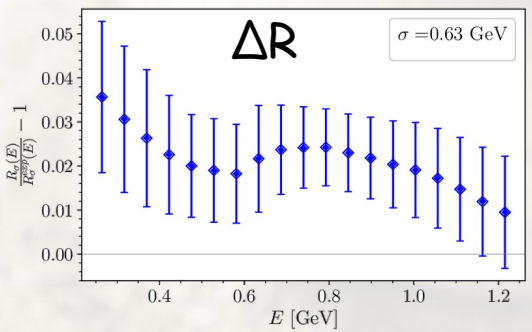


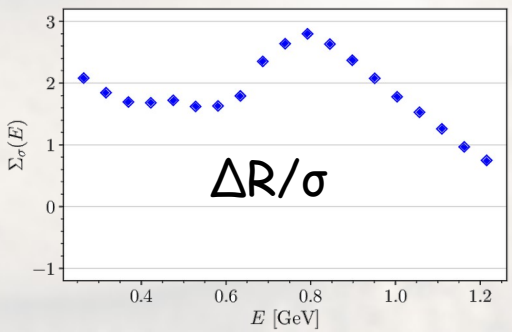
~4σ tension between Lattice/Dispersive

C. Alexandrou et al, e-Print: 2212.08467 [hep-lat]

R(s) is convolved with Gaussian kernel



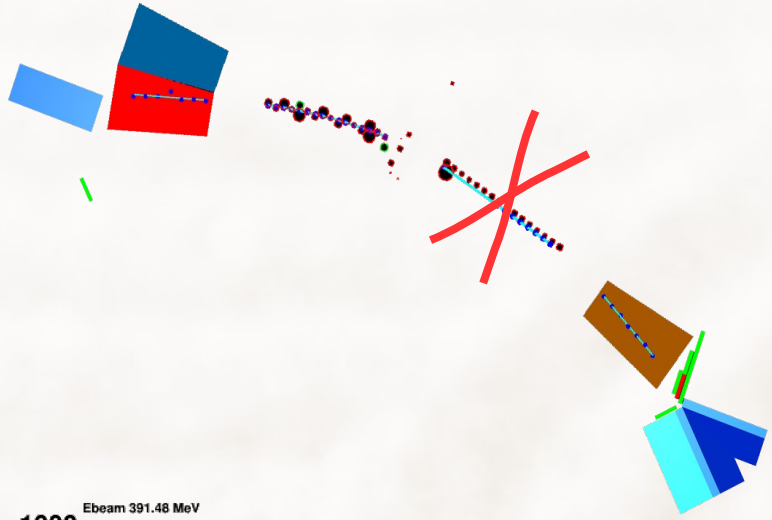


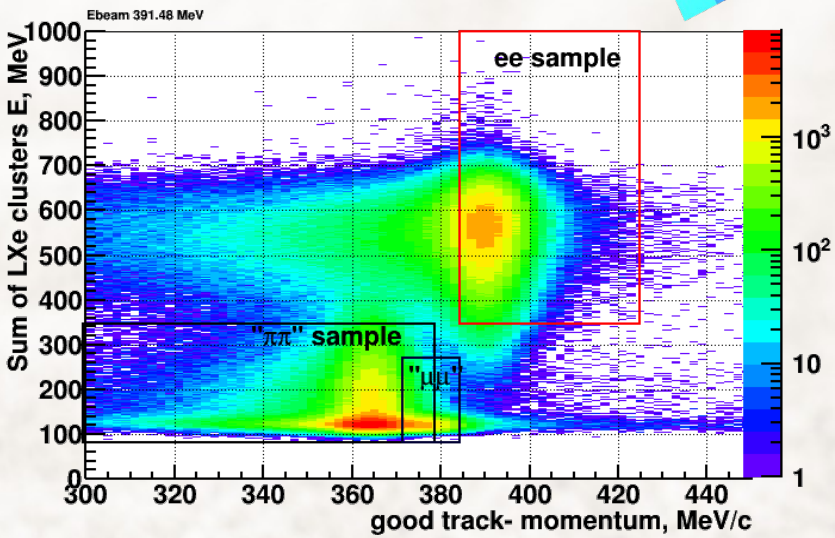


~3σ tension at rho energies

Question of comparison: e+e- vs $(g-2)_\mu$ vs lattice

Efficiency





Assuming independence of Calorimeter & Tracker,
Using the "test" sample based on LXe information:

two collinear clusters are detected + one good track

gives possibility to study track reconstruction inefficiency

Event type is tagged by
energy deposition and momentum of good track

The "test" sample includes only partially some specific losses (when second compatible cluster is not produced):
pion decay, nuclear interaction, .. (~30% ineff. accounted)
electron bremsstrahlung (~5% accounted)

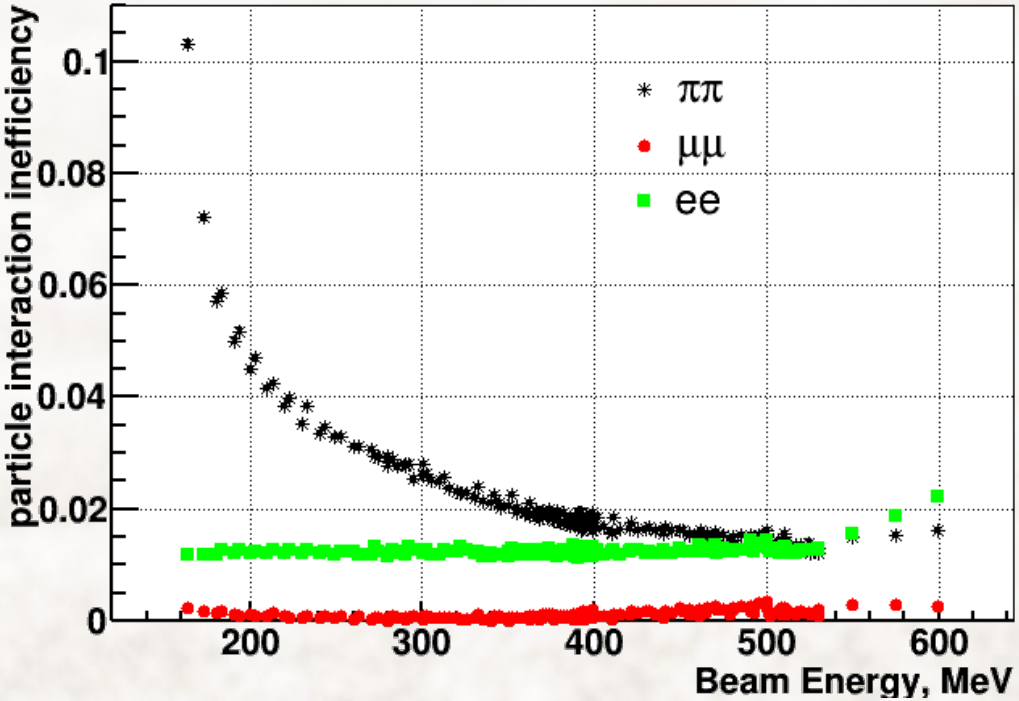
N.B. Correlated inefficiency study was also performed without requirement on detection of one good track

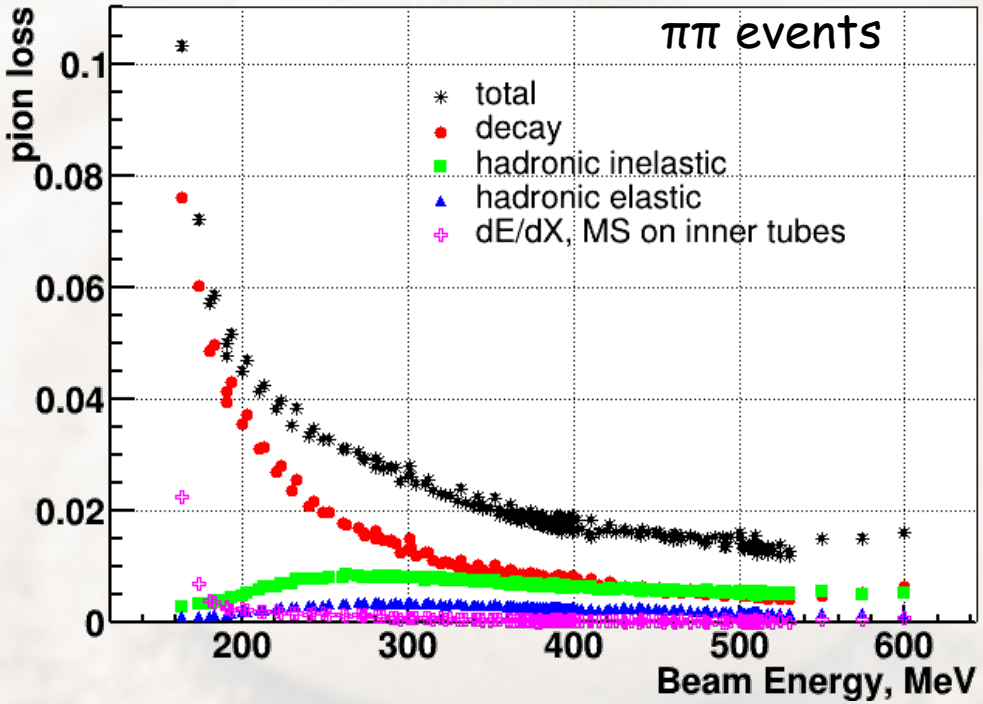
Particle specific losses

bremsstrahlung energy loss, decay in flight, nuclear interaction with materials, MS on the inner vacuum tube, ….

Taken from detailed full MC (includes detector conditions with time)

but it is also controlled by the data



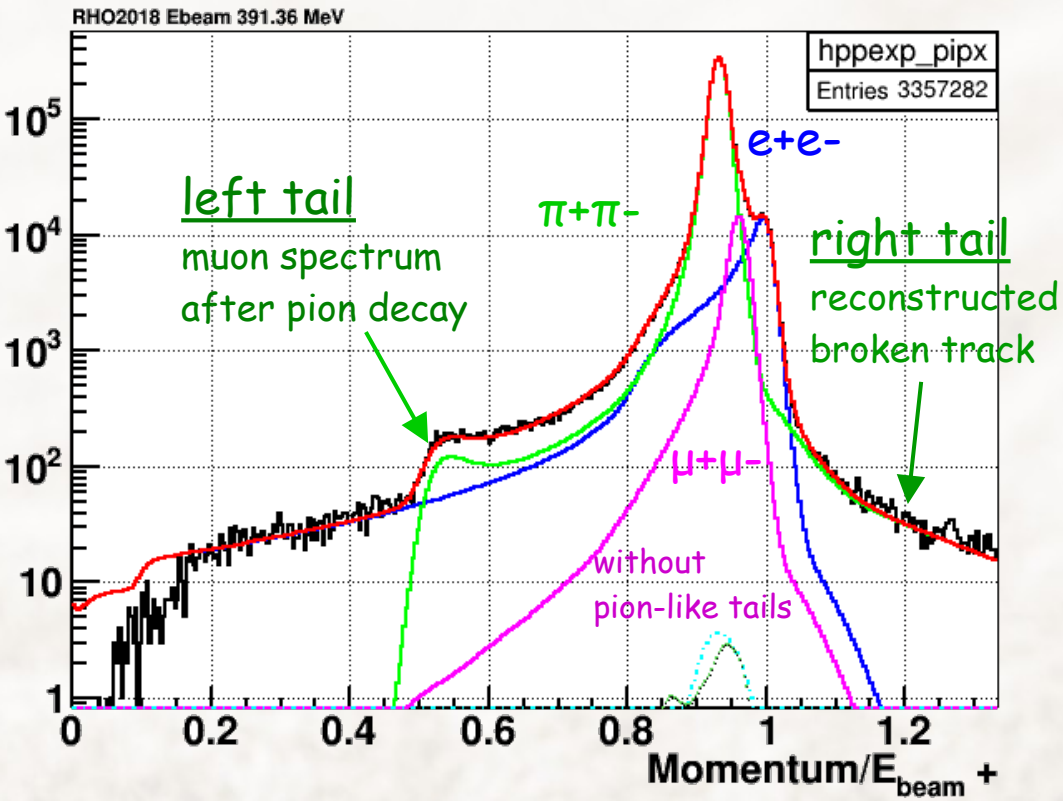


nucler interactions mostly on inner tube (systematics 0.2%)

most dangerous is decay in flight as it depends on detector conditions (syst. 0.2-0.1%)

Pion decay inefficiency

Experimental P+ spectrum
with $|P^- - P_\pi| < 10$ MeV



Decay in flight - depends on DCH efficiency

controlled by number of events in tails
in the data vs simulation

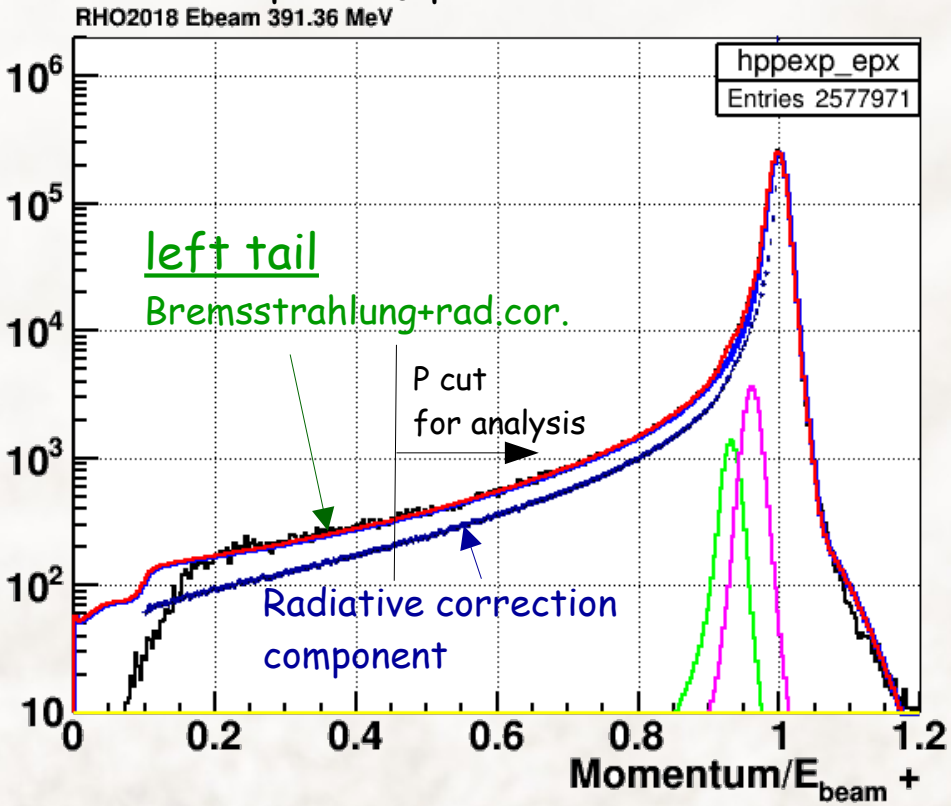
Tails function taken from full MC
(include DCH inefficiencies, resolutions,
amplitudes, correlated noises per layers, etc..)
Number of events in tails are free parameters
in momentum-based separation

$N^{event}_{in\ tails}$ consistent with sim at ~ 3%
→ systematic uncertainty of Nππ
0.2-0.1% (from low to ρ)
(N.B. simplified DCH descriptions gives 15% discrepancies on tails)

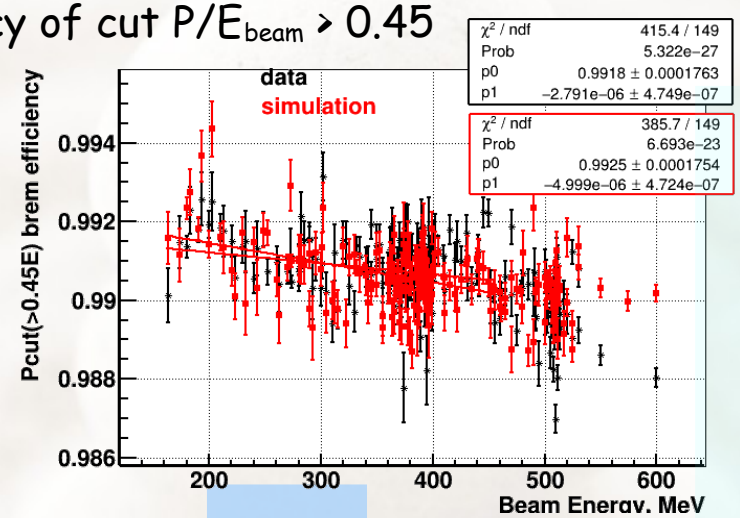
Additional crosscheck with «weak» cuts:
Nhits >= 10 → 8, χ2 < 10 → 20, |Δρ| < 0.3 → 0.6 cm
pion decay inefficiency changes by x1./(2.-2.5)
→ $\Delta|F|^2/|F|^2 < 0.05\%$

Bremsshtrahlung loss on vacuum tube

Experimental P+ spectrum with $|P^- - P_e| < 10$ MeV

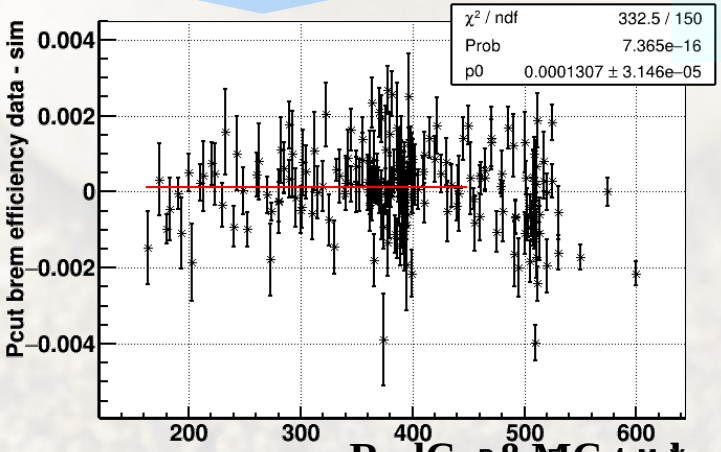


Part of brems. correction (0.9% from 1.2%) can be extracted from fitted spectra: inefficiency of cut $P/E_{beam} > 0.45$



MCGPJ vs BabaYaga spectra gives difference <0.015%

The data vs sim agree ~ 0.02%



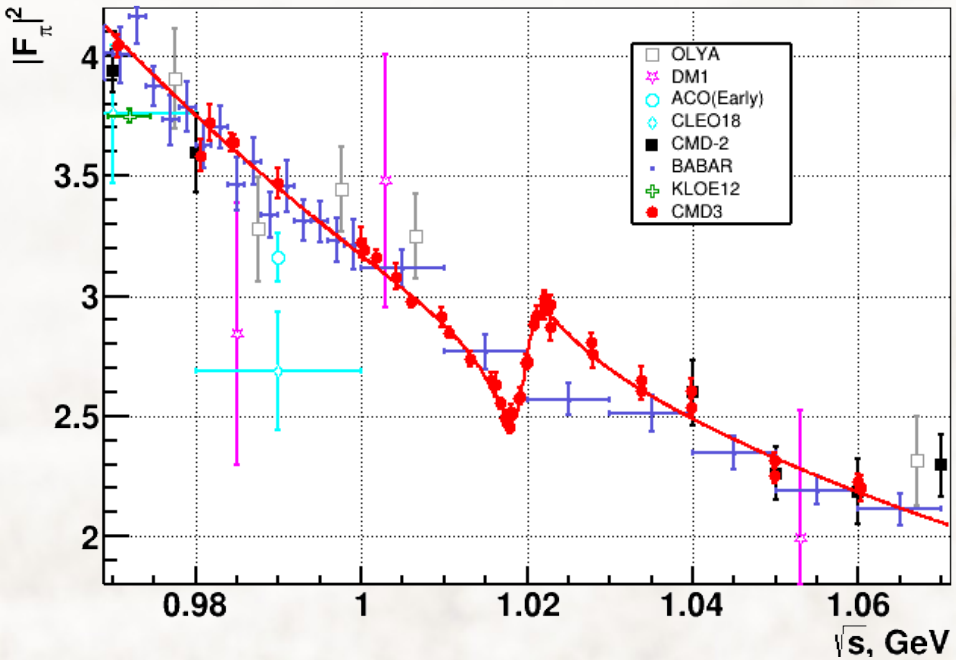
Brems. description is part of detector response function in momentum-based separation (with X/X0 as free param.)

X/X0 of inner wall consistent with sim. within <5%

→ Systematics on $|F_\pi|^2 \sim 0.05\%$

$\varphi \to \pi^+\pi^-$

First direct $|F_\pi|^2$ measurement around φ resonance



CMD-3

$\psi_\pi = (-21.3 \pm 2.0 \pm 10.0)^\circ$

$B(\varphi \to e^+e^-)B(\varphi \to \pi^+\pi^-) = (3.51 \pm 0.33 \pm 0.24)\times 10^{-8}$

Previous measurement using detected $N_{\pi+\pi-}$ or visible cross-section by OLYA, ND, SND (Sergey Burdin et al,Phys.Lett.B474:188-193,2000)

SND

$\psi_\pi = (-34 \pm 5)^\circ$

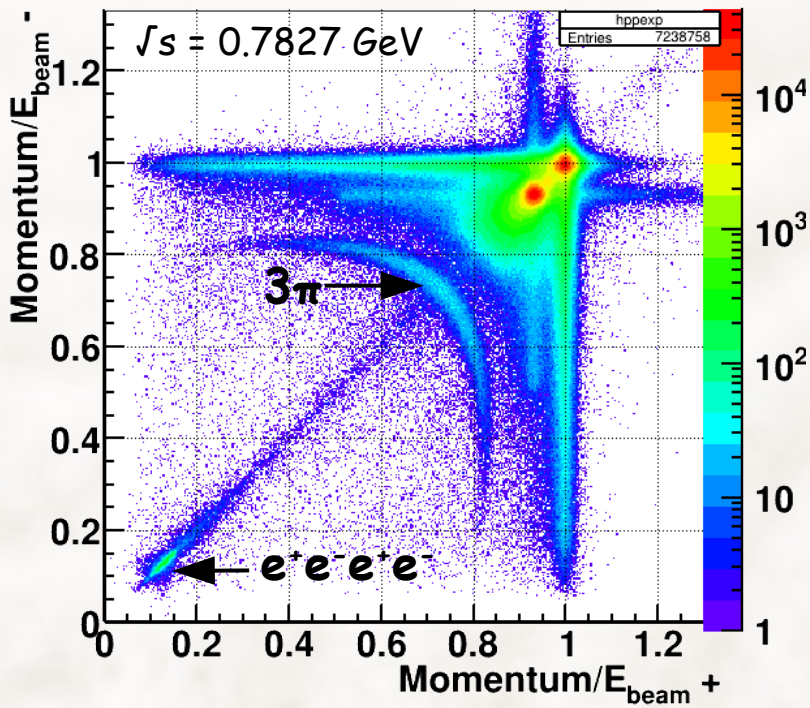
$B(\varphi \to e^+e^-)B(\varphi \to \pi^+\pi^-) = (2.1 \pm 0.4)\times 10^{-8}$

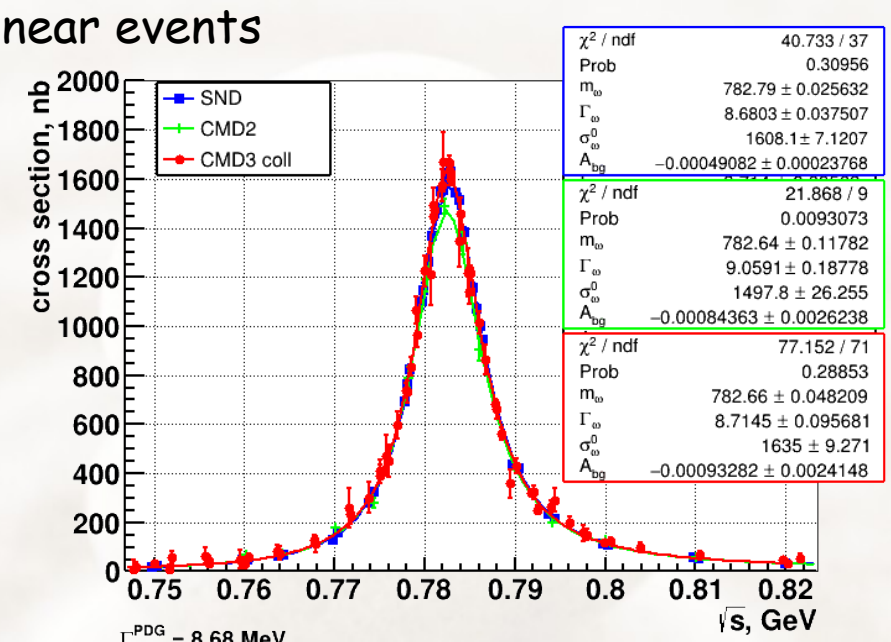
N.B. radiative correction uncertainty (from F_π parametrisation) gives ~1.5 scale factor of total statistical and systematic errors (both for Br and ψ_π)

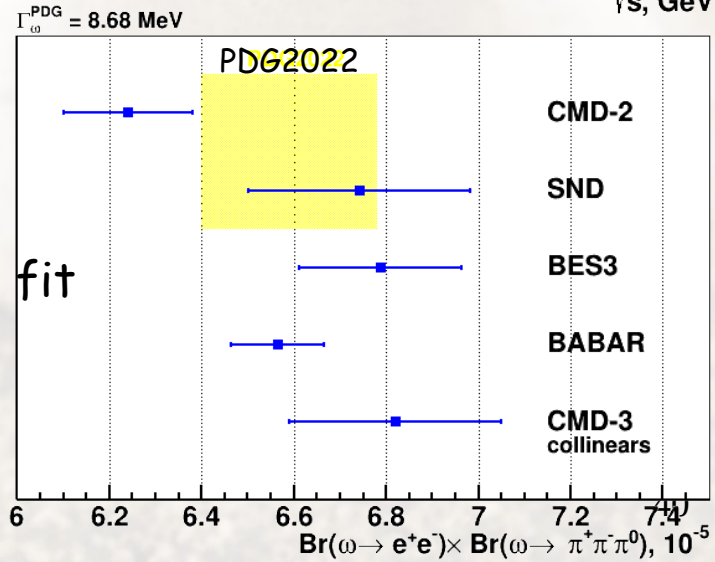
$e^+e^- \to \pi^+\pi^-\pi^0$

$\sigma(e^+e^- \to \pi^+\pi^-\pi^0)$ within collinear events

Collinear events are selected for 2π analysis







$e^+e^- \to \pi^+\pi^-\pi^0$ is background for $\pi^+\pi^-$ analysis (0.8% at ω)

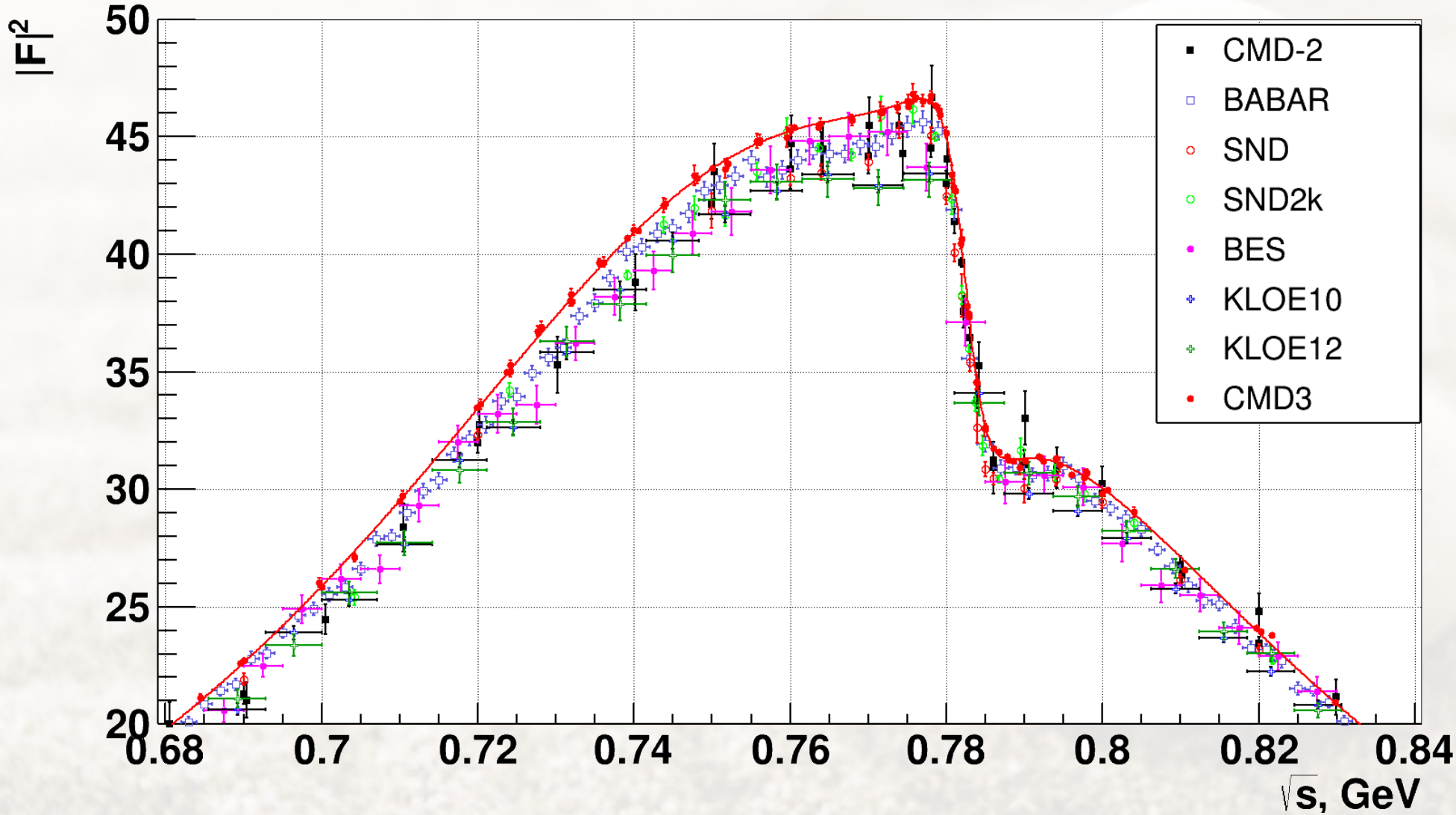
Number of 3π events is additional parameter in likelihood fit

Main systematics (2.4%) inaccuracy of $\rho\pi$ – model for efficiency determination, **total 3.3%**

$B(\omega\to e^+e^-)B(\omega\to\pi^+\pi^-\pi^0) = (6.82 \pm 0.04 \pm 0.23)\times10^{-5}$

confirm SND@VEPP-2M result

Other experiments



$|F_\pi|^2$ systematic uncertainty

× Radiative corrections	**0.2%** (2π) ⊕ **0.2%** (Fπ) ⊕ **0.1%** (e+e-)
× e/μ/π separation	0.5 (low) – **0.2% (ρ)** – 0.6 (φ) %
× Fiducial volume	**0.5% / 0.8%** (RHO2013)
× Correlated inefficiency	**0.1% (ρ)** – 0.15%(>1 ГэВ)
× Trigger	**0.05% (ρ)** – 0.3% (>1 ГэВ)
× Beam Energy (by Compton σ_E< 50 keV)	**0.1%** (out of resonances), 0.5% (at ω, φ -peaks)
× Bremsstrahlung loss	**0.05%**
× Pion specific loss	**0.2%** nuclear interaction
	0.2%(low) - **0.1%** (ρ) pion decay
	0.8% (low) - **0.7%** (ρ) - 1.6% (φ)
	1.1% (low) - **0.9%** (ρ) - 2.0% (φ) (RHO2013)

Fixing of $N_{\mu\mu}$ adds scaling of correspondent sources with ~ (1+ a $N_{\mu\mu}/N_{\pi\pi}$)

at φ with $N_{\mu\mu}/N_{\pi\pi}$ ~ 1 : 1.05% / 1.2%(RHO2013) → 1.6% / 2.0% (RHO2013)

at 1.2 GeV with $N_{\mu\mu}/N_{\pi\pi}$ ~ 2.4 : 1.05% → 1.95% (RHO2018)

Form Factor evaluation

$$\sigma_{e^+e^-\to\gamma\to\pi^+\pi^-} = \frac{\pi\alpha^2}{3s}\beta_\pi^3 |F_\pi|^2$$

$$|F_\pi|^2 = \left(\frac{N_{\pi^+\pi^-}}{N_{e^+e^-}} - \Delta^{bg}\right)\frac{\sigma^0_{e^+e^-}\cdot\left(1+\delta^{rad}_{e^+e^-}\right)}{\sigma^0_{\pi^+\pi^-}\cdot\left(1+\delta^{rad}_{\pi^+\pi^-}\right)}\frac{\epsilon_{e^+e^-}}{\epsilon_{\pi^+\pi^-}}$$

Ratio $N_{\pi\pi}/N_{ee}$ is measured directly -> detector inefficiencies are partially cancelled out

Mostly no background, Applied if not accounted in particle separation

$$\Delta^{BG} = \left(N_{bg}/N_{ee}\right)^{simul}$$

Evaluated as ratio to e+e- by simulation. Both BG and e+e- are taken from sim, inefficiencies cancelled out in same way

Radiative corrections defined in used acceptance, account for ISR and FSR effects, VP included in F_π definition.

Efficiency analysis rely mostly on the data. Important only difference between π+π- / e+e- (common cancelled out)

Form factor