



Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

June 7th - 9th, 2023

The Hadron Vacuum Polarization (HVP) from lattice QCD

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June 8, 2023

Outline

1 Motivations

2 How to define the HVP on the lattice

- Lattice QCD
- Time momentum representation
- Euclidean window

3 Common challenges

- Statistical Noise
- Setting the scale
- Finite volume effects
- Isospin breaking corrections
- Continuum extrapolation

4 State of the art

- BMW 2020
- Results on the window observable

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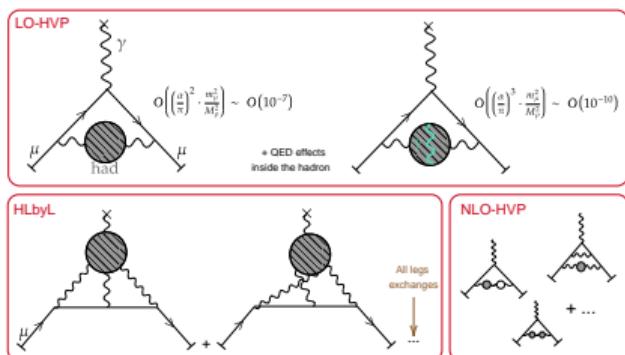
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Reference SM value of $a_\mu = (g - 2)_\mu / 2$

$$a_\mu^{\text{SM}} = \underbrace{a_\mu^{\text{QED}}}_{\mathcal{O}(10^{-3})} + \underbrace{a_\mu^{\text{had}}}_{\mathcal{O}(10^{-7})} + \underbrace{a_\mu^{\text{weak}}}_{\mathcal{O}(10^{-9})}$$

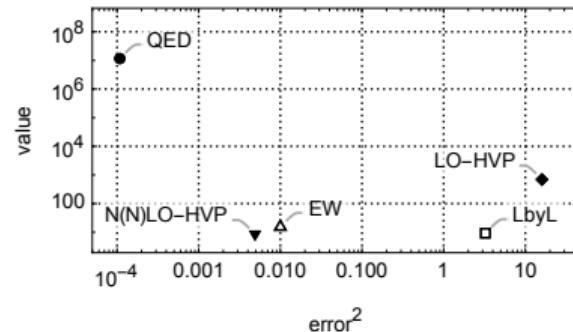
$$a_\mu^{\text{had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HLbyL}} + a_\mu^{\text{NLO-HVP}} + \dots$$



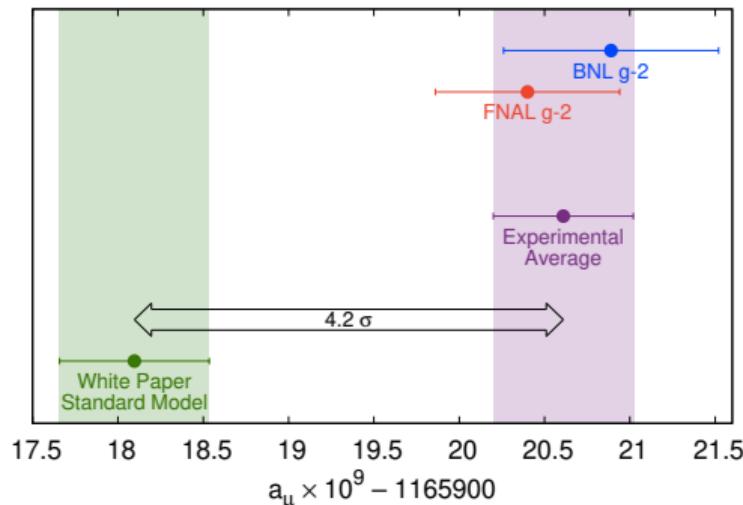
- Non-perturbative QCD ($q^2 = 0$ and $m_\mu \ll 1 \text{ GeV}$)
 - phenomenological approach
 - lattice QCD
- Consensus (pheno+lat) for hadronic LbyL

SM contrib.	$a_\mu^{\text{contrib.}} \times 10^{10}$	
HVP-LO ($e^+ e^-$)	693.1	± 4.0
HVP-NLO ($e^+ e^-$)	-9.83	± 0.07
HVP-NNLO ($e^+ e^-$)	1.24	± 0.01
HLbL-LO (pheno)	9.2	± 1.9
HLbL (lattice usd)	7.8	± 3.4
HLbL (pheno+lattice)	9.0	± 1.7
HLbL-NLO (pheno)	0.2	± 0.1
QED (5 loops)	11 658 471.8931	± 0.0104
EW (2 loops)	15.36	± 0.10
HVP ($e^+ e^-$, LO + N(N)LO)	684.5	± 4.0
HLbL (pheno + lattice + NLO)	9.2	± 1.8
SM Total	11 659 181.0	± 4.3

SM results as stated in the White Paper (Aoyama et al. 2020)

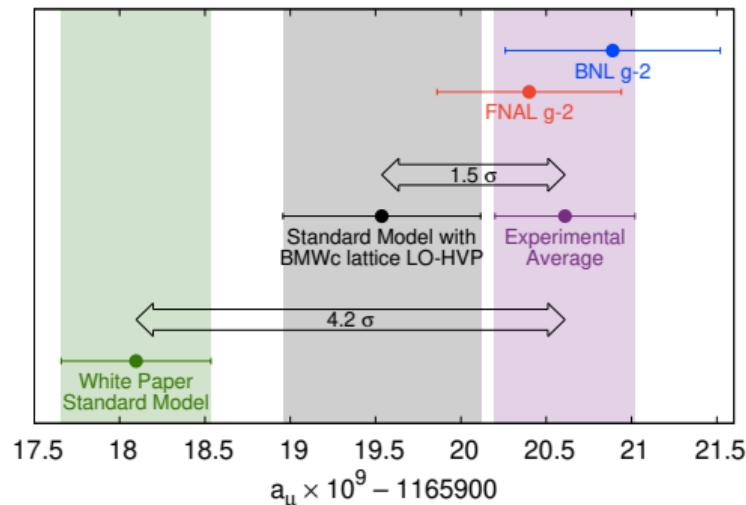


Reference SM result vs experiment



- Reference SM computation and experimental average (Abi et al. 2021) have \sim errors but 4.2σ disagreement:
$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 0.00116592061(41) - 0.00116591810(43) = 0.00000000251(59)$$
- Important to check the most uncertain contribution (HVP) with a fully independent method → lattice QCD.
- First lattice result of a_μ below 1% precision (BMWc'20) between WP and measurement.

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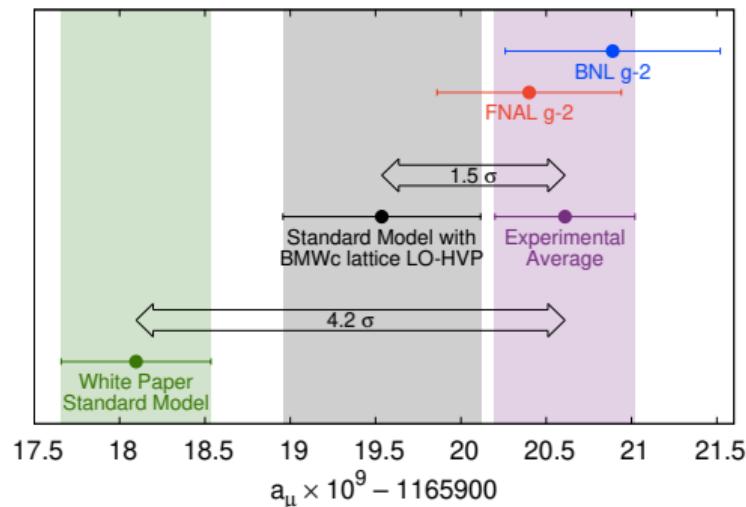


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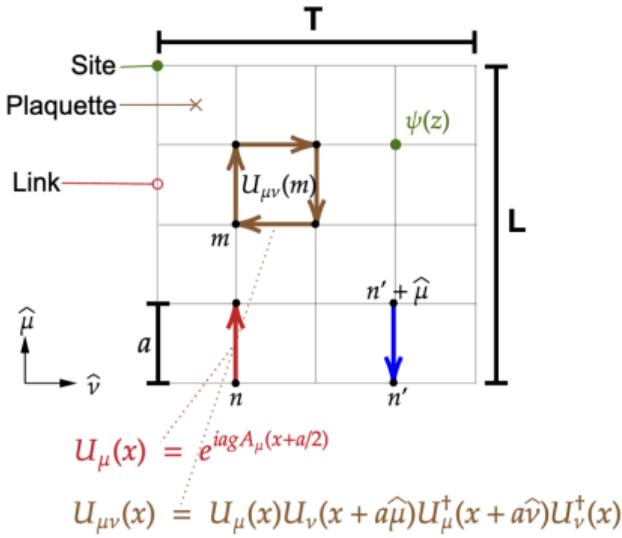
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Lattice-QCD's key points

- Lattice = **euclidean tool** $\Rightarrow t \rightarrow -ix_4$
 - Spacetime is a **discretized** on a lattice with
 - size $L^3 \times T$
 - spacing a
- \Rightarrow no **UV** divergences.



- ① Define discretized fields and actions s.t. $\mathcal{S}_{\text{lat}} \xrightarrow{a \rightarrow 0} \mathcal{S}_E$, e.g.

$$S_W = -\frac{\beta}{3} \sum_{x \in \Lambda_E} \sum_{\mu < \nu} \text{Re} \operatorname{Tr}\{\mathbb{1} - U_{\mu\nu}(x)\} \xrightarrow{a \rightarrow 0} \frac{1}{4} \int d^4x (F_{\mu\nu}^a)^2$$

iff $\beta = 6/g^2$.

- ② At finite a, L, T , define obs. of interest $\hat{O}|_{a;T,L}$ with same quantum numbers as in continuum.
- ③ Expectation value $\langle \hat{O} \rangle$ is well-defined:

$$\begin{aligned} \langle \hat{O} \rangle &= \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-S_G} \int \bar{\psi} D[M] \psi \hat{O}[U, \psi, \bar{\psi}] \\ &= \int \mathcal{D}U e^{-S_G} \det(D[M]) \hat{O}[U] \end{aligned}$$

- ④ $\mathcal{D}U e^{-S_G} \det(D(M)) > 0$ and dofs being finite, $\langle \hat{O} \rangle$ can be computed numerically using stochastic methods.
- ⑤ LQCD \rightarrow QCD when $m_q \rightarrow m_q^{\text{phys}}$, $a \rightarrow 0$, and $L \rightarrow \infty$.

Lattice-QCD's many recipes for...

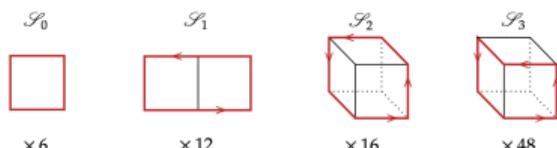
- **Set of physical observable** used to fix lattice parameters m_u, m_d, m_s, m_c, g, e
- **Improved actions**

Cutoff effects depend on definition lattice action \hat{S} and fields $\hat{\phi}$. One can define *improved actions* and fields

$$S_{\text{imp}} = S + \int d^4x \sum_{j=1}^r c_j^{(S)} O_j^{(S)}(x)$$

and tune coefficients such that $\mathcal{O}(a^r)$ discretization effects in a given observable disappear.

E. g. **Luscher-Weisz action** = improved Wilson action to remove $\mathcal{O}(a^2)$ effects.



- **Fermions**

Naive discretization of Dirac action leads to propagator with 16 poles!

$$\tilde{S}_F(p) = \frac{m\mathbb{1} - ia^{-1} \sum_\mu \gamma_\mu \sin(p_\mu a)}{m^2 + a^{-2} \sum_\mu \sin^2(p_\mu a)}$$

Some solutions:

- Staggered fermions (reduced number of doublers)
- Wilson fermions (modify chiral symmetry)
- Domain wall fermion (4 space dimensions)
- Overlap fermions (similar to Wilson)
- Twisted-mass fermions (explicitly violate chiral symmetry)

each with \neq discretization effects, computational costs, systematics, ...

- **Boundary conditions** (periodic, C^* , ...)
- **Smearing gauge fields** (4Stout, Hex, ...)
- **Adding QED** (QED_L , QED_m , QED_{C^*} , ...)
- ...

All prescriptions must lead to the same physical results

Lattice definition of a_ℓ^{LO-HVP}

- ① From (Blum 2003)

$$a_\mu = \alpha^2 \int_0^\infty \frac{dQ^2}{m_\mu^2} w\left(\frac{Q^2}{m_\mu^2}\right) \underbrace{\Pi(Q^2) - \Pi(0)}_{\hat{n}(Q^2)} \quad w(r) \equiv \frac{[r + 2 - \sqrt{r(r+4)}]^2}{\sqrt{r(r+4)}}$$

- ② In Euclidean space the polarization tensor is a real function of $Q^2 \geq 0$:

$$\Pi_{\mu\nu}(Q) = \frac{1}{e^2} \int d^4x e^{iQ \cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \underbrace{(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2) \Pi(Q^2)}_{O(4) \text{ inv. and current conservation}}$$

where $\langle \dots \rangle$ means **1pl**, QCD+QED expectation value.

- ③ Electromagnetic current: $J_\mu/e \equiv \sum_f Q_f \bar{q}_f(x) \gamma_\mu q(x)$. Consider $f = u, d, s, c$.

- $a_\mu^{\text{charm}} \sim 1\%$ of total $a_\mu \rightarrow$ must be included
- a_μ^{bottom} computed in (Colquhoun et al. 2015)
- a_μ^{top} negligible.

- ④ Chose $\vec{Q} = 0$ and average over space positions, $\Pi(Q^2) = -\frac{1}{3Q^2} \sum_{i=1}^3 \Pi_{ii}(Q)$

- ⑤ In a finite volume $V = L^4$: $\Pi_{\mu\nu}(0) \neq 0 \rightarrow$

$$\Pi_L(Q^2) \equiv -\frac{1}{3Q^2} \sum_{i=1}^3 [\Pi_{L,ii}(Q) - \Pi_{L,ii}(0)]$$

Lattice definition of a_ℓ^{LO-HVP} : time-momentum representation

- ⑥ Define current-current time correlator ($\ell = u, d$ and L denotes finite volume)

$$C_L(t) \equiv \frac{a^3}{3} \sum_{i=1}^3 \sum_{\vec{x}} \langle J_i(x) J_i(0) \rangle \equiv C_L^\ell(t) + C_L^s(t) + C_L^c(t) + C_L^{\text{disc}}(t)$$

- ⑦ Perform a Fourier transformation and (Bernecker and Meyer 2011) obtain

$$\Pi_L(Q^2) = \int_0^\infty dt \left(\frac{e^{iQ_0 t} - 1}{Q^2} \right) C_L(t) \Rightarrow \hat{\Pi}_L(Q^2) = \int_0^\infty dt \left[\frac{t^2}{2} - \frac{2}{Q^2} \sin^2 \left(\frac{Qt}{2} \right) \right] C_L(t).$$

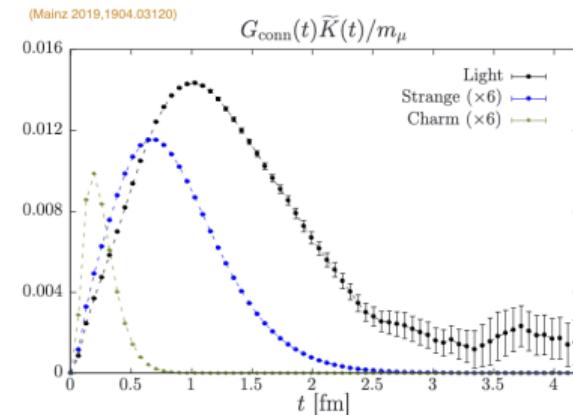
- ⑧ Separate $a_\mu = \underbrace{a_\mu(Q^2 \leq Q_{\max}^2)}_{\text{lattice}} + \underbrace{a_\mu(Q^2 > Q_{\max}^2)}_{\text{pQCD}} + \underbrace{\gamma_\mu(Q_{\max}^2) \hat{\Pi}(Q_{\max}^2)}_{\text{c.t.}}$

- ⑨ For a given component $f = u, d, s, c, \text{disc}$

$$a_\mu^f(Q^2 \leq Q_{\max}^2) = \lim_{\substack{a \rightarrow 0 \\ L, T \rightarrow \infty}} \alpha^2 a \sum_t^{T/2} K(t, Q_{\max}^2) C_{TL}^f(t)$$

$$\text{with } K(t, Q_{\max}^2) = \int_0^{Q_{\max}^2} \frac{dQ^2}{m_\mu^2} w\left(\frac{Q^2}{m_\mu^2}\right) \left[\frac{t^2}{2} - \frac{2}{Q^2} \sin^2\left(\frac{Qt}{2}\right) \right].$$

- ⑩ Sum over f and add remaining contributions.



Euclidean window

In (RBC/UKQCD 2018) authors propose to split $a_\mu = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$

$$a_\mu^{\text{SD}} = \alpha^2 \int_0^\infty dt [1 - \Theta(t; t_0, \Delta)] K(t) C(t)$$

$$a_\mu^{\text{W}} = \alpha^2 \int_0^\infty dt W(t; t_0, t_1, \Delta) K(t) C(t)$$

$$a_\mu^{\text{LD}} = \alpha^2 \int_0^\infty dt \Theta(t; t_1, \Delta) K(t) C(t)$$

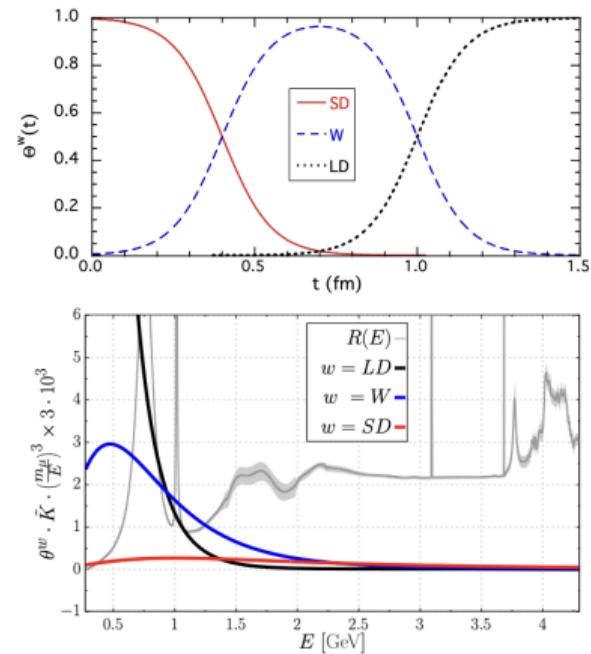
where

$$W(t; t_a, t_b, \Delta) \equiv \Theta(t; t_a, \Delta) - \Theta(t; t_b, \Delta)$$

$$\Theta(t; t_a, \Delta) \equiv \frac{1}{2} + \frac{1}{2} \tanh [(t - t_a)/\Delta]$$

These contributions can be compared to $R(s)$ via

$$C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) e^{-\sqrt{s}t}$$



(arXiv:2212.10490, ETMC)

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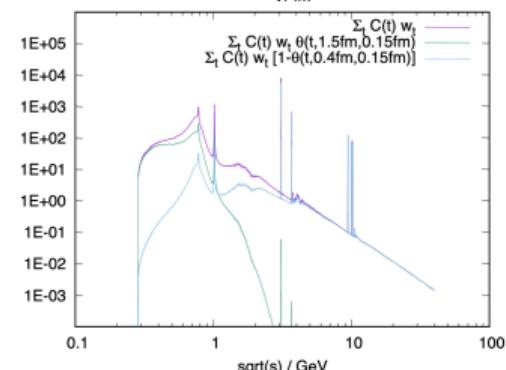
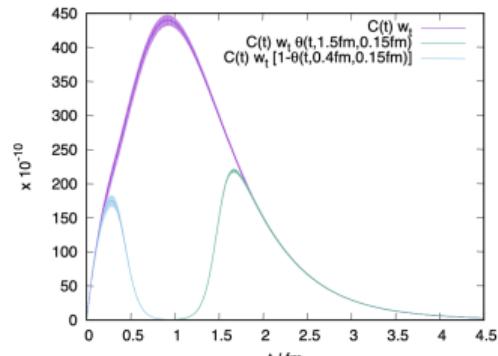
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(RBC/UKQCD 2018)

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Major challenges of a lattice computation of a_μ

The high precision required for a_μ poses some common problems:

- ① **Long distance contributions and noise reduction** – Statistical noise in $C_L^{ud}(t)$ and $C_L^{disc}(t)$ increases exponentially with $t \Rightarrow$ error at large t must be reduced.
- ② **Scale determination** – Relative error in the lattice spacing propagates into \sim twice relative error in a_μ . \Rightarrow Severe requirements for scale setting.
- ③ **Infinite volume effects** – a_μ is very sensitive to the lattice size L : the general rule $M_\pi L > 4$ is not satisfactory.
- ④ **Continuum extrapolations** – Continuum limit very challenging at this level of precision.
- ⑤ **QED and strong isospin breaking** – Unquenched QCD in the isospin limit $m_u = m_d$ is not satisfactory for the desired level of precision \Rightarrow isospin breaking effects must be included up to first order in isospin breaking parameters $\delta m_I \equiv m_d - m_u$ and α .

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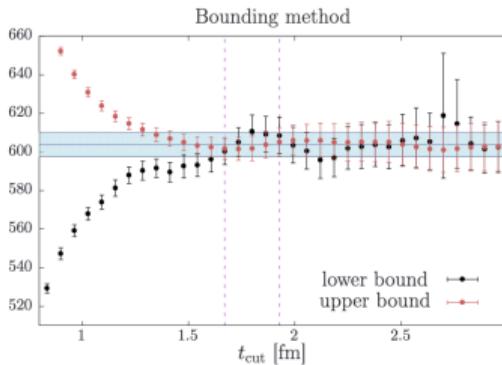
The next slides will discuss *mainly* the solutions adopted in (BMW 2020)

Long distance contributions and noise reduction

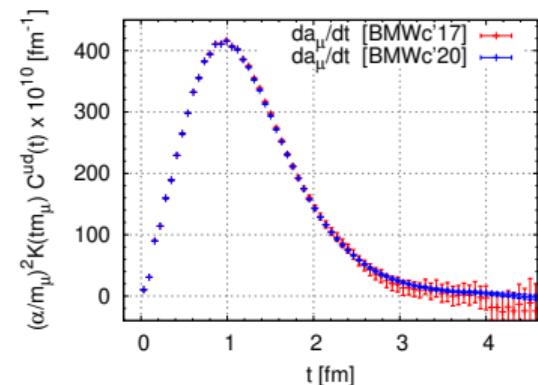
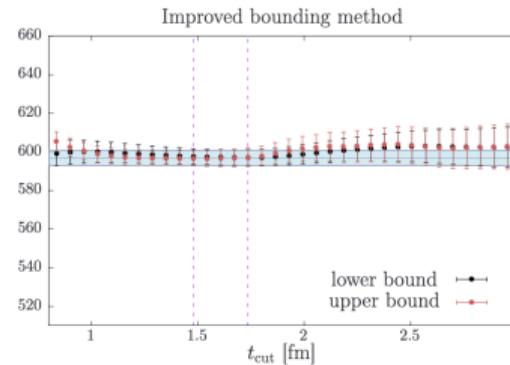
Problem: Exponentially increasing noise-to-signal ratio in $C^{\text{light}}(t)$ and $C^{\text{disc}}(t)$, with $N/S \sim e^{(M_\rho - M_\pi)t}$ (Parisi'84, Lepage'89).

Solutions:

- Low/All Mode Averaging (LMA/AMA) with n large enough to cover essential physics.
- Modeling/bounding the tail:
 - Bounding method: $0 \leq C^{\text{light}}(t) \leq C^{\text{light}}(t_c) e^{-E_{2\pi}(t-t_c)}$ (BMW 2017, RBC/UKQCD 2018, BMW 2020)
Chose t_c such that it minimizes statistical error in the range where upper/lower bounds become consistent.
 - Improved bounding method: $\tilde{C}^{\text{light}}(t) = C^{\text{light}}(t) - \sum_{n=1}^N \frac{|A_n|^2}{2E_n} e^{-E_n t}$, then $0 \leq \tilde{C}^{\text{light}}(t) \leq C^{\text{light}}(t_c) e^{-E_{N+1}(t-t_c)}$ (Mainz 2019)
- Use large statistics



(Mainz 2019)



(BMW 2020)

Tuning of the parameters of QCD

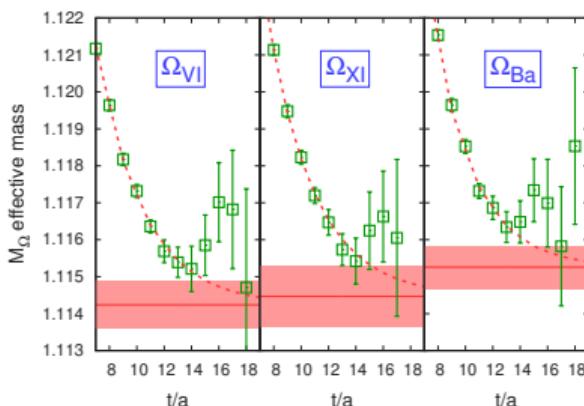
Lattice computation with 2+1+1 flavours, including IB effects $\mathcal{O}(m_d - m_u)$ and $\mathcal{O}(\alpha) \rightarrow$ 6 parameters need to be fixed in order for any prediction to be a physical one:

- quark masses m_u, m_d, m_s, m_c ,
- the overall mass scale (lattice spacing a),
- electric charge e .

The uncertainty on the lattice scale Λ significantly affects the uncertainty of a_μ :

$$\Delta a_\mu = |\Lambda \frac{da_\mu}{d\Lambda}| \cdot \frac{\Delta \Lambda}{\Lambda} \rightarrow \frac{\Delta a_\mu}{a_\mu} \approx 1.8 \frac{\Delta \Lambda}{\Lambda} \quad (\text{Mainz 2017})$$

⇒ to reach 1% precision on a_μ the scale must be determined at few permil uncertainty.



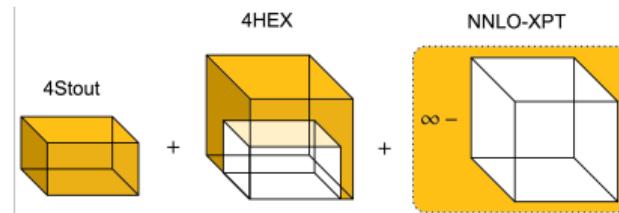
Lattice spacing in (BMW 2020)

- $a = (\hat{M}_{\Omega^-})/M_{\Omega^-}^\phi$, with \hat{M}_{Ω^-} computed in lattice units
- Three different operators for Ω (different taste formulations): masses consistent in the error
- Fit windows chosen by a Kolmogorov-Smirnov test
- Omega masses extracted via *four-state fit* or *GEVP*
- Same methods used to fit the masses of pseudoscalar mesons needed to fix the quark masses: $M_{\pi^\pm}, M_{\pi^0}, M_{K^\pm}, M_{K^0}$

Finite-volume effects

- Finite-volume (FV) effects are dominated by long distance effects from $\pi\pi$ and ρ
- Largest source of uncertainty (1.9%) in (BMW 2017) → dropped to 0.4% in (BMW 2020)
- Even in large lattices, $L \sim 6$ fm, $T \sim 9$ fm, the distortion attributable to FV effects is estimated (using NLO-XPT) to be about 2% (Aubin et al. 2015)
- $[a_\mu(L_{\text{big}}, T_{\text{big}}) - a_\mu(L_{\text{ref}}, T_{\text{ref}})]$ is computed in lattice QCD, using a coarse lattice spacing, to bridge the gap between reference volume and larger volume with $L_{\text{big}} = T_{\text{big}} = 10.572$ fm.
- Phenomenological models (NLO XPT, NNLO XPT, MLLGS, HP, RHO) are verified by comparing to lattice result above, then used to compute $[a_\mu(\infty, \infty) - a_\mu(L_{\text{big}}, T_{\text{big}})]$.

$$\begin{aligned} a_\mu(\infty, \infty) - a_\mu(L_{\text{ref}}, T_{\text{ref}}) &= \\ &+ [a_\mu(L_{\text{big}}, T_{\text{big}}) - a_\mu(L_{\text{ref}}, T_{\text{ref}})]_{\text{4HEX}} \\ &+ [a_\mu(\infty, \infty) - a_\mu(L_{\text{big}}, T_{\text{big}})]_{\text{NNLO-XPT}} \end{aligned}$$



	NLO XPT	NNLO XPT	MLLGS	HP	RHO
$a_\mu(L_{\text{big}}, T_{\text{big}}) - a_\mu(L_{\text{ref}}, T_{\text{ref}})$	11.6	15.7	17.8	—	—
$a_\mu(L_{\text{big}}, \infty) - a_\mu(L_{\text{ref}}, \infty)$	11.2	15.3	17.4	16.3	14.8
$a_\mu(\infty, \infty) - a_\mu(L_{\text{big}}, T_{\text{big}})$	0.3	0.6	—	—	—
$a_\mu(\infty, \infty) - a_\mu(L_{\text{big}}, \infty)$	1.2	1.4	—	1.4	1.4

Inclusion of isospin breaking effects

Computed by expanding $\langle O(e, \delta m) \rangle$ at first order in $\alpha = \frac{e^2}{4\pi}$ and $\delta m = m_d - m_u$ (de Divitiis 2013), with dynamical QED included in the QED_L Hayakawa and Uno 2008 scheme:

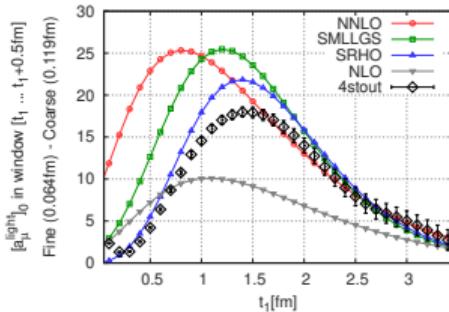
$$\langle O \rangle = \frac{\int [dU][dA] e^{-S[U,A]} \text{det}_{s_0} \left(1 + e_s \frac{\text{det}'_1}{\text{det}_{s_0}} + e_s^2 \frac{\text{det}''_2}{\text{det}_{s_0}} \right) \left(O_0 + \frac{\delta m}{m_l} O'_m + e_v O'_1 + e_v^2 O''_2 \right)}{\int [dU] e^{-S_g[U]} \int [dA] e^{-S_\gamma[A]} \text{det}_{s_0} \left(1 + e_s \frac{\text{det}'_1}{\text{det}_{s_0}} + e_s^2 \frac{\text{det}''_2}{\text{det}_{s_0}} \right)}$$

	Connected	Disconnected
$= Z_0^{-1} \int [dU] e^{-S[U]} \text{det}_{s_0} O_0$	$\langle O_0 \rangle_0$	
$+ \frac{\delta m}{m_l} \langle O'_m \rangle_0$	$\equiv \langle O \rangle'_m$	
$+ e_v^2 \langle O''_{20} \rangle_0$	$\equiv \langle O \rangle''_{20}$	
$+ e_v e_s \left\langle O'_1 \frac{\text{det}'_1}{\text{det}_{s_0}} \right\rangle_0$	$\equiv \langle O \rangle''_{11}$	
$+ e_s^2 \left\langle [O_0 - \langle O_0 \rangle_0] \frac{\text{det}''_2}{\text{det}_{s_0}} \right\rangle_0$	$\equiv \langle O \rangle''_{02}$	

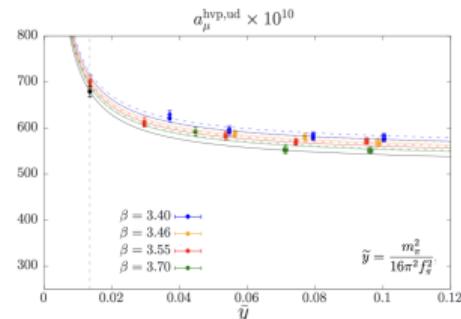
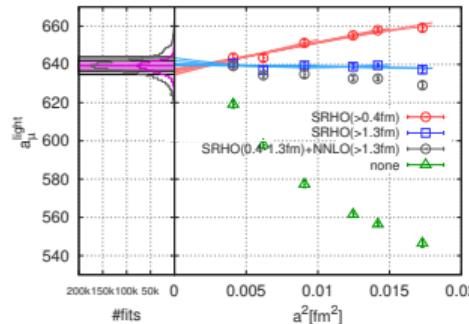
where $\text{det}[U, A; \{m_f\}, \{q_f\}, e] = \prod_f \det M_f [V_U e^{ie q_f A}, m_f]^{1/4}$ (M_f fermionic matrix), $O_0 = O(0, 0)$, $O'_m = m_l \partial_{\delta m} O|_{(0,0)}$, $O'_1 = \partial_{e_v} O|_{(0,0)}$ and $O''_2 = \frac{1}{2} \partial_{e_v}^2 O|_{(0,0)}$.

Taste breaking effects and extrapolation to the continuum

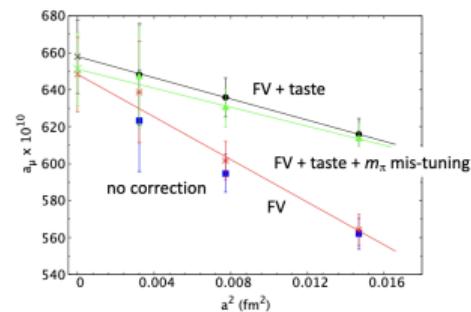
- Most important discretization errors: taste-breaking effects in the pion sector.
- Taste breaking effects as a function of a scale more like a^4 , faster than expected $\alpha_s a^2$ (Lepage 1998).
- Correct effects using staggered versions of the effective theories that work well to describe FV effects. (NNLO-XPT and the MLLGS and RHO models + staggered analogues: NNLO-SXPT, SMLLGS and SRHO).
- Improve the continuum extrapolation without modifying the continuum-limit value (they vanish in the continuum limit).
- Applied to $C^{\text{light}}(t)$ and $C^{\text{disc}}(t)$.



(BMW 2020)



Chiral + continuum limit
(Mainz 2019)



FV effects corrected point by point
(Aubin et al. 2020)

Outline

1 Motivations

2 How to define the HVP on the lattice

- Lattice QCD
- Time momentum representation
- Euclidean window

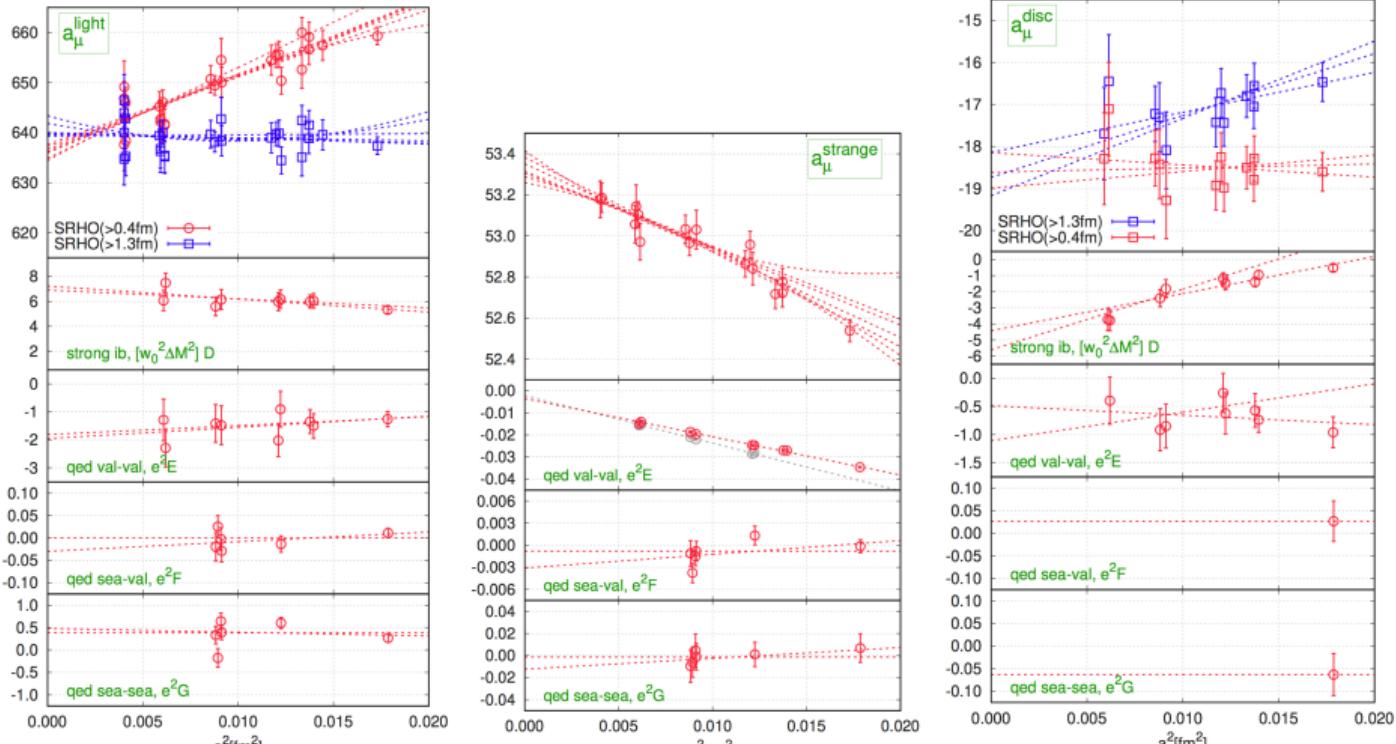
3 Common challenges

- Statistical Noise
- Setting the scale
- Finite volume effects
- Isospin breaking corrections
- Continuum extrapolation

4 State of the art

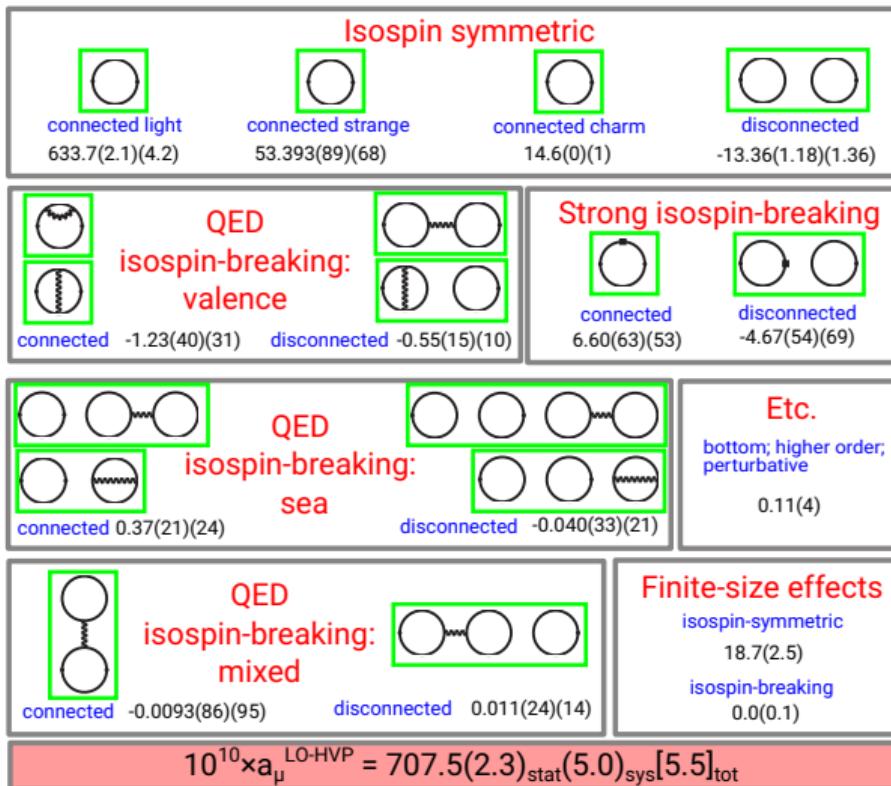
- BMW 2020
- Results on the window observable

Results for a_μ^{light} , a_μ^{strange} and a_μ^{disc}



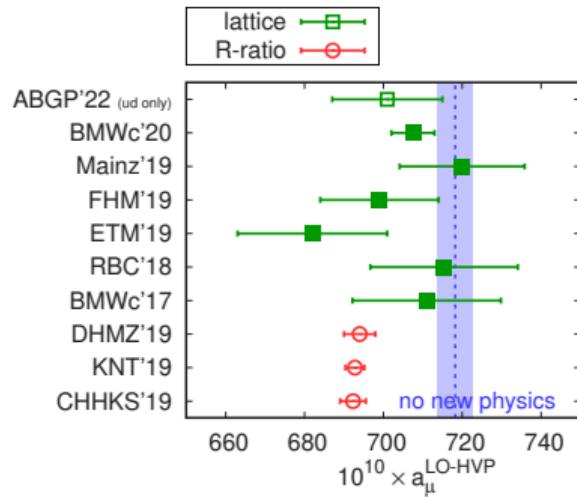
Figures 25, 26 and 27 in (BMW 2020): continuum extrapolation to $a_\mu^{\text{light}}(L_{\text{ref}}, T_{\text{ref}})$, $a_\mu^{\text{strange}}(L_{\text{ref}}, T_{\text{ref}})$ and $a_\mu^{\text{disc}}(L_{\text{ref}}, T_{\text{ref}})$. First window is total result form type-I fit, other windows are IB decompositions from type-II fits.

Results for all the components of a_μ



(BMW 2020)

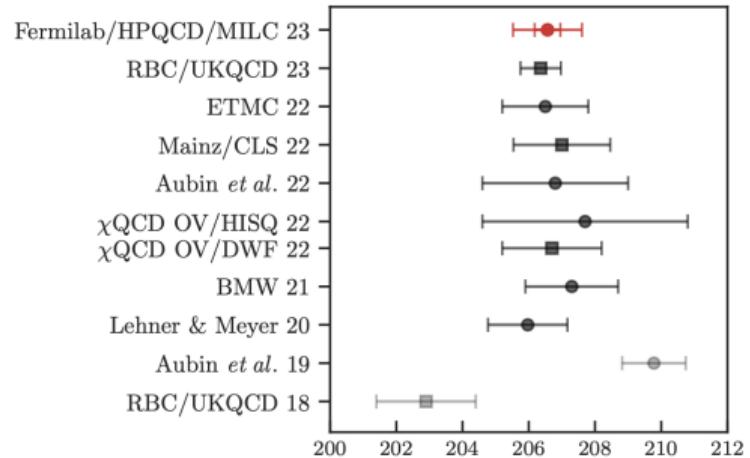
Comparison



- (BMW 2020) Consistent with other lattice results
- 2.1σ larger than R-ratio average value (White Paper 2020)
- Consistent with a_μ measurement at 1.5σ level (possible “no new physics” scenario)

Window results

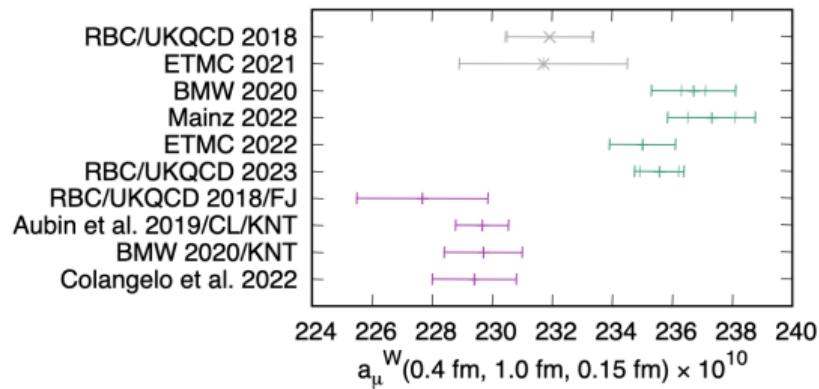
- Less challenging than full $a_\mu^{\text{LO-HVP}}$
 - much better signal/noise → stat. err. ($\leq 0.2\%$)
 - much smaller FV effects
 - much smaller discretization effects (long & short distance) ($\lesssim 2.7\%$ for $a \leq 0.1 \text{ fm}$)
 - (BMW 2020) tot. err. $\sim 0.7\%$ of which 88% comes from $a \rightarrow 0$
- Other lattice QCD collaborations have comparable errors
- $\sim 3.7\sigma$ tension with R-ratio
- (RBC/UKQCD 2023), (Mainz 2022), (ETMC 2022) confirm (BMW 2020) $a_{\mu,\text{win}}^{\text{LO-HVP}}$ and $a_{\mu,\text{ud},\text{win}}^{\text{LO-HVP}}$ using \neq fermion discretizations (DW, Wilson).



Isosymmetric intermediate window $a_\mu^{\text{light},W}(\text{conn})$
(Fermilab/HPQCD/MILC 2023)

Window results

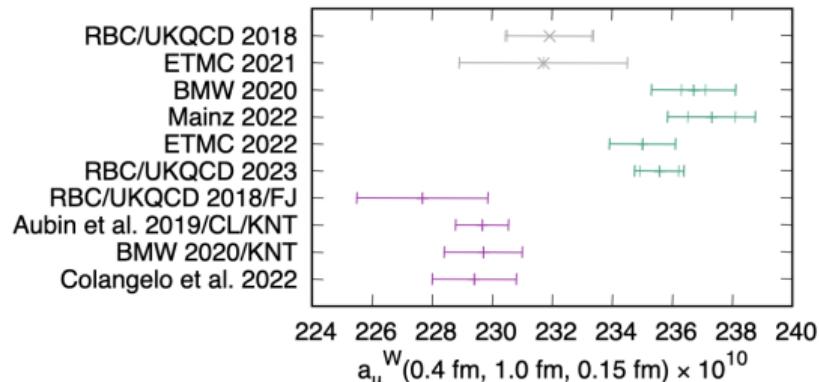
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Total intermediate window $a_\mu^{\text{light},W}(\text{conn})$
(RBC/UKQCD 2023)

Window results

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Total intermediate window $a_\mu^{\text{light,W}}(\text{conn})$
(RBC/UKQCD 2023)

Thank you!

For more detailed comparisons among the various lattice results see [talk](#) by Mattia Bruno at Hadron2023