$e^+e^- \to \gamma\gamma^*$ at two loops in massless QED

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Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

Loops in QED

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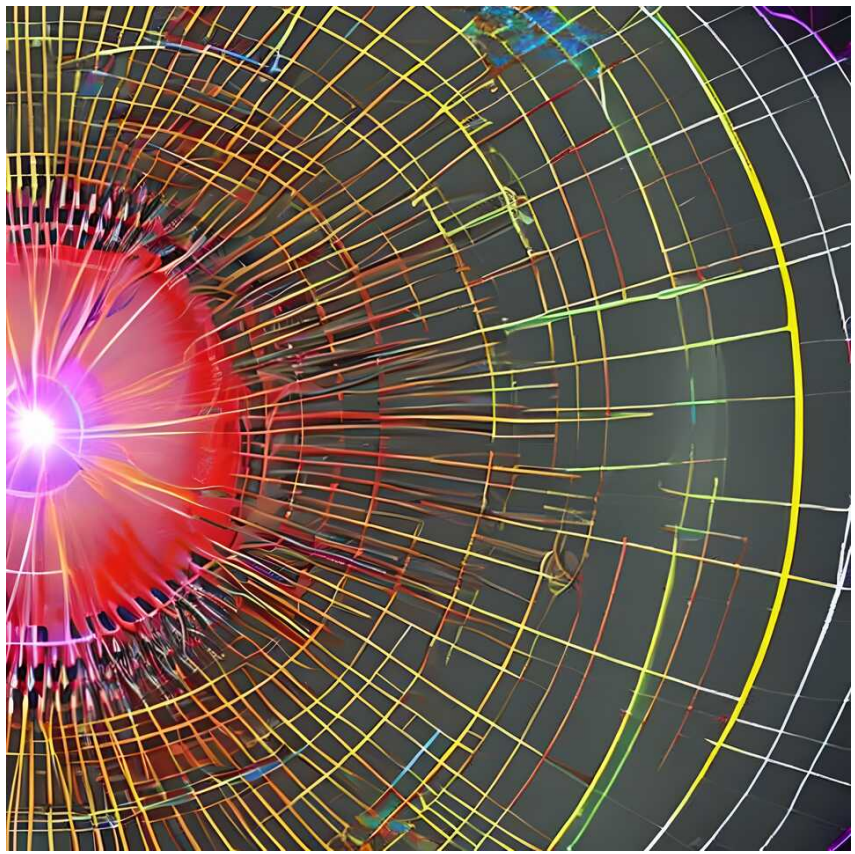
Outline

Introduction:
precision frontier

Amplitudes:
computation and result

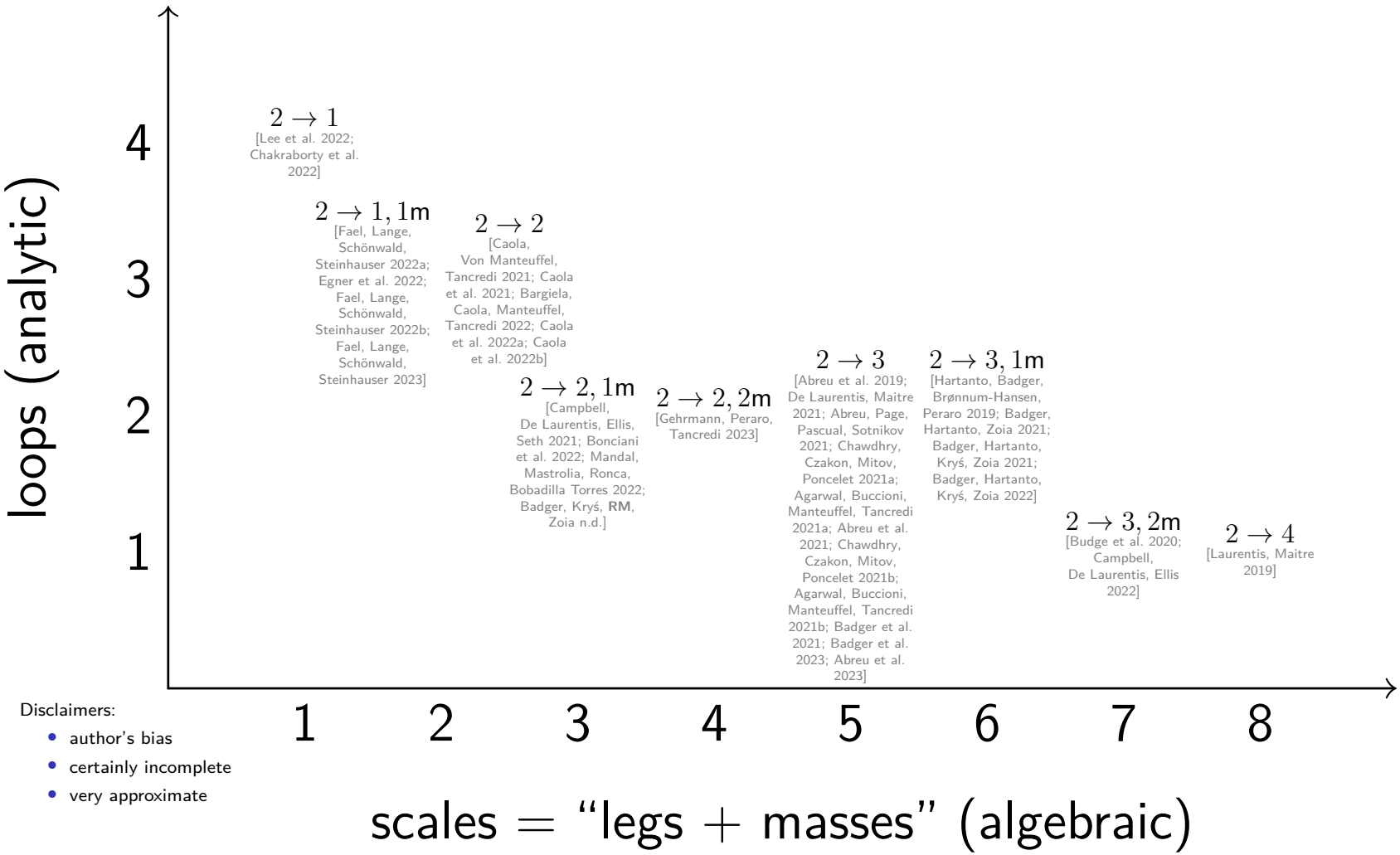
Phenomenology:
a_μ^{HVP}

Conclusion



Q*D amplitudes

Recent analytic results

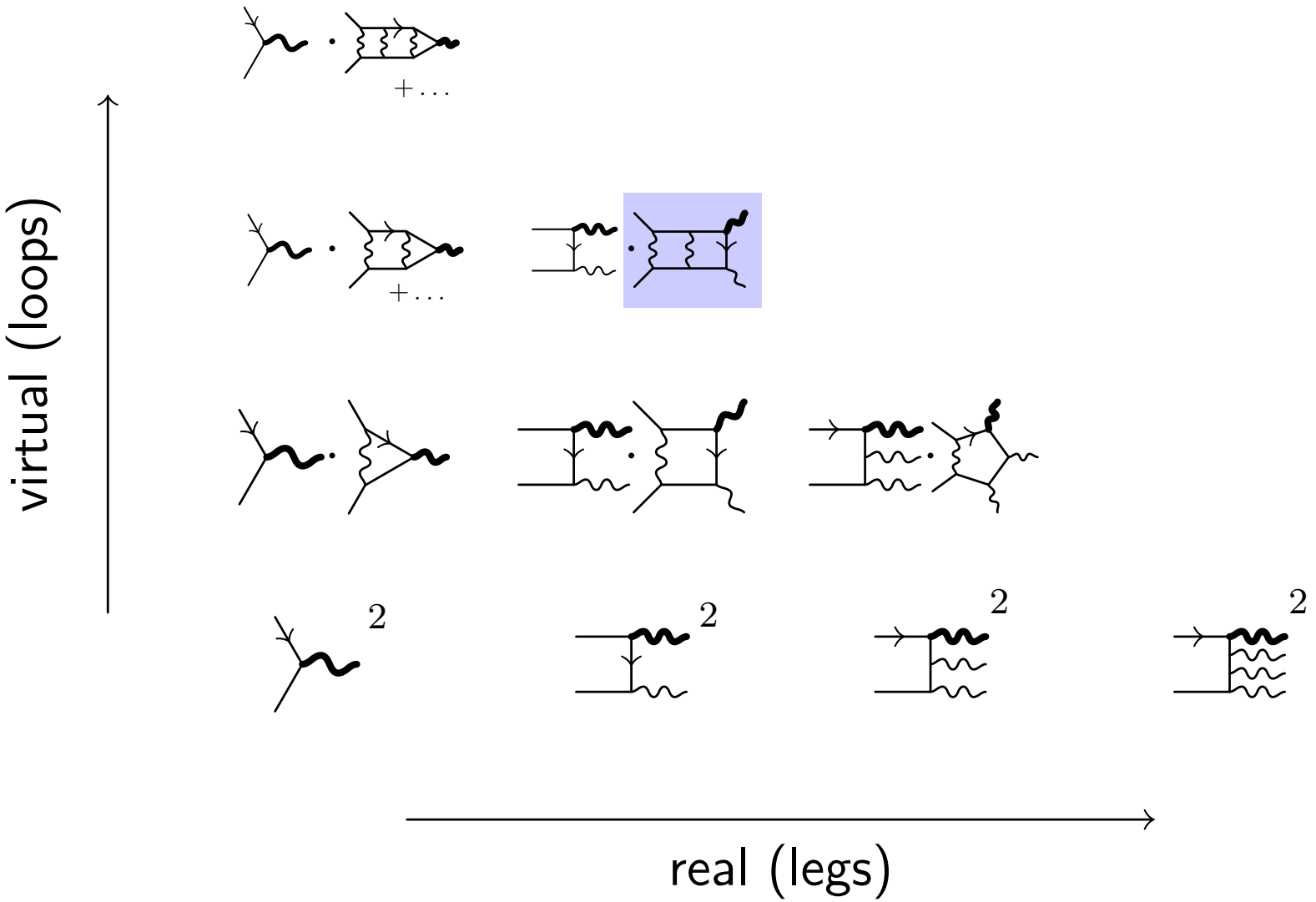


Disclaimers:

- author's bias
- certainly incomplete
- very approximate

Matrix elements

Fixed order



Computation

Summary

$$\mathcal{A}(x) = \sum_i c_i(x) f_i(x)$$

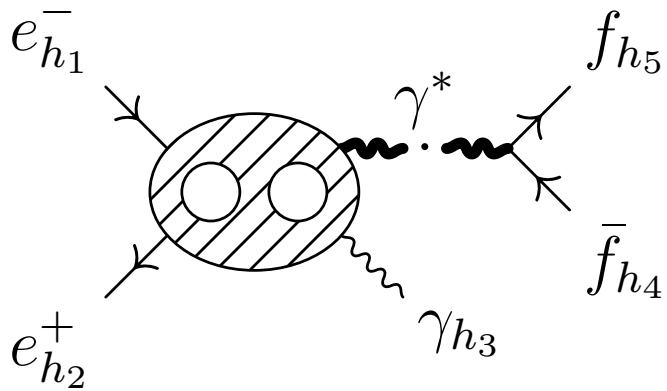
- Rational coefficients
- Reconstruct compact analytic form from numerical evaluations over finite fields

- Feynman integrals
- "Special function" basis of Goncharov polylogarithms (GPLs)

Result

[Badger, Kryś, **RM**, Zoia n.d.]

- Helicity amplitudes
- Current $\mathcal{A}^{\mu}(h_1, h_2, h_3)$



- Running in `McMule` ($e\mu \to e\mu\gamma$) at $\sim$130 ev/s [N^{3}LO kick-off 2022]
- Massification [Penin 2006; Becher, Melnikov 2007; Engel, Gnendiger, Signer, Ulrich 2019]

Measurements

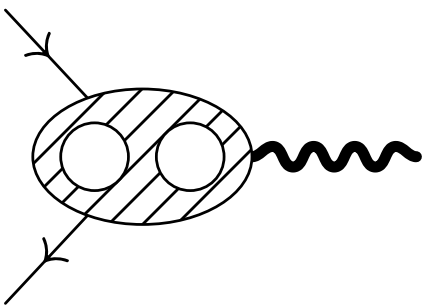
See other talks!

- a_μ
  - most theoretical uncertainty: a_μ^{HVP} [Abbiendi et al. 2022]
  - ee (timelike)
    - dispersive integral method [Aoyama et al. 2020]
    - requires $\sigma(ee \to \text{hadrons}/\mu\mu(+\gamma))$
    - data-driven $\gamma^* \to$ hadrons [PrecisionSM Group 2023]
    - amplitude ingredient $ee \to \gamma^*(+\gamma)$
  - $e\mu$ (spacelike)
    - $e\mu \to e\mu$
    - MUonE [Abbiendi et al. 2017; Balzani, Laporta, Passera 2022]
- a_τ

Corrections

Direct scan

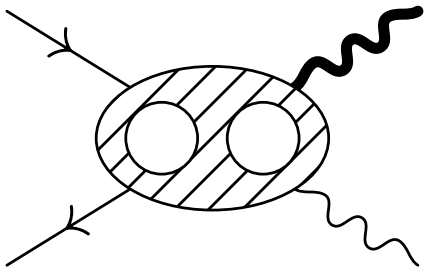
$ee \to \gamma^*$



- Scan over $\sqrt{s}$
- Initial-state corrections
- RVV for N^{3}LO

Radiative return

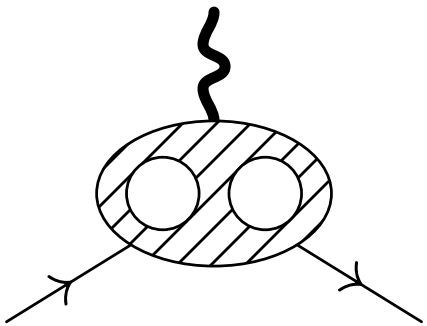
$ee \to \gamma\gamma^*$



- Scan over ISR
- Initial-state corrections
- VV for NNLO

MUonE

$e \to e\gamma^*$



- Electron-line corrections
- RVV for N^{3}LO

Conclusion

- Massless $ee \to \gamma\gamma^*$ two-loop helicity amplitude currents
- Fast and stable evaluation
  - Compact analytic form
  - GPL integral basis
- Access next order
  - RVV for N^{3}LO $ee \to \gamma^*$ and $e \to e\gamma^*$
  - VV for NNLO $ee \to \gamma\gamma^*$

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