# A frequency-domain finite element model for simulating HTS

## **J-A** and **T-A** formulations

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# Context

Need for a quick method to compute parameters in steady-state for AC regime

- AC losses
- Electrical parameters

What for?

- A quick estimate for pre-design (cooling system for instance)
- A first step before initiating a transient analysis (for electrical machines for instance)

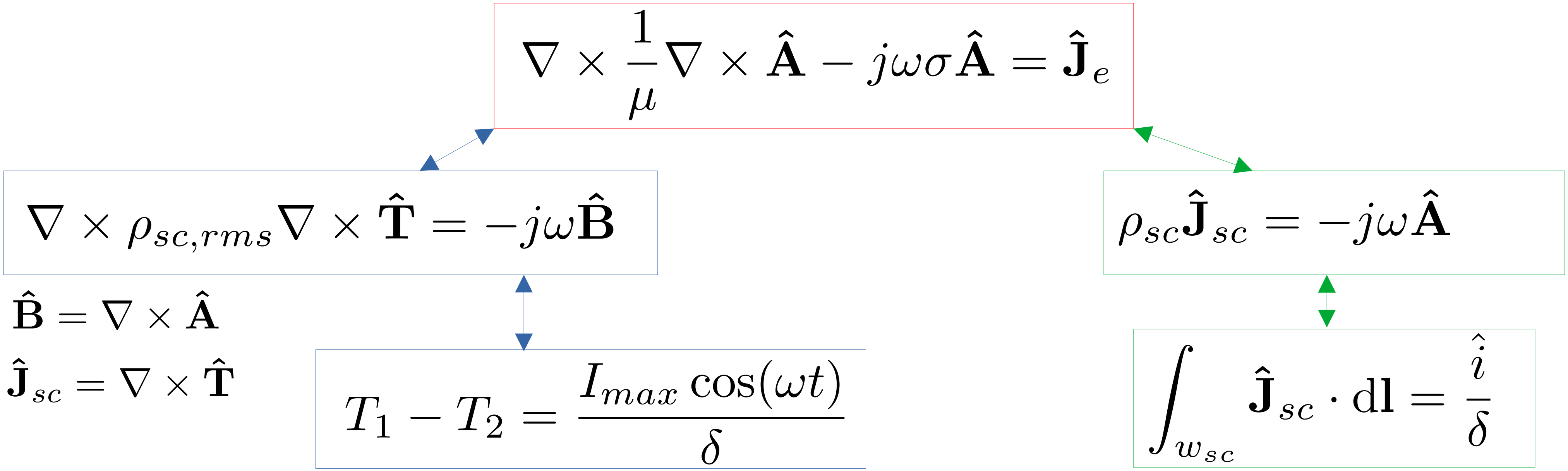
# Maxwell equations in frequency domain

Two newest formulations of the Maxwell equation:

- **T**-**A** formulation (time) → $\hat{\mathbf{T}} - \hat{\mathbf{A}}$ phasor formulation
- **J**-**A** formulation (time) → $\hat{\mathbf{J}} - \hat{\mathbf{A}}$ phasor formulation

$$\hat{\mathbf{X}} = \mathbf{X}_{max} \exp{(j\omega t)}$$
$$\omega = 2\pi f$$
$$j^2 = -1$$

Frequency-domain equations for tape-based model (thin strip):



$$\nabla \times \frac{1}{\mu} \nabla \times \hat{\mathbf{A}} - j\omega\sigma\hat{\mathbf{A}} = \hat{\mathbf{J}}_e$$

$$\nabla \times \rho_{sc,rms} \nabla \times \hat{\mathbf{T}} = -j\omega\hat{\mathbf{B}}$$

$$\rho_{sc}\hat{\mathbf{J}}_{sc} = -j\omega\hat{\mathbf{A}}$$

$$\hat{\mathbf{B}} = \nabla \times \hat{\mathbf{A}}$$
$$\hat{\mathbf{J}}_{sc} = \nabla \times \hat{\mathbf{T}}$$

$$T_1 - T_2 = \frac{I_{max} \cos(\omega t)}{\delta}$$

$$\int_{w_{sc}} \hat{\mathbf{J}}_{sc} \cdot \mathrm{d}\mathbf{l} = \frac{\hat{i}}{\delta}$$

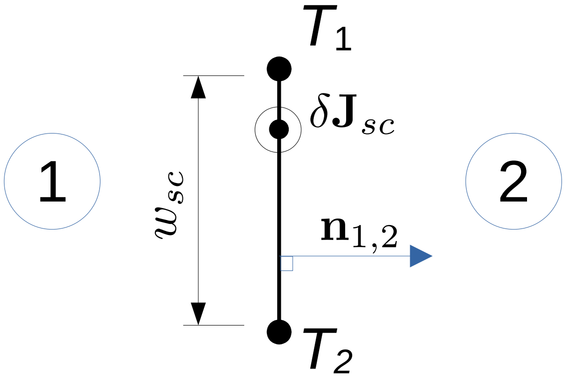
# Tape-based or full model

Each individual tape is simulated given the proper current is impressed as

- A Dirichlet boundary condition for the **T** formulation → $T_1 - T_2 = \frac{I}{\delta}\cos(\omega t)$
- A point-wise constraint in the **J** formulation → $\int_{w_{sc}} \hat{\mathbf{J}}_{sc} \cdot \mathrm{dl} = \frac{\hat{\mathbf{i}}}{\delta}$

In both formulation, the magnetic vector potential is solved assuming a discontinuity of the magnetic field across the 1D line representing the tape:

$$\mathbf{n}_{1,2} \times \left( \frac{1}{\mu} \nabla \times \hat{\mathbf{A}}_2 - \frac{1}{\mu} \nabla \times \hat{\mathbf{A}}_1 \right) = \delta \hat{\mathbf{J}}_{sc}$$



# RMS electrical resistivity

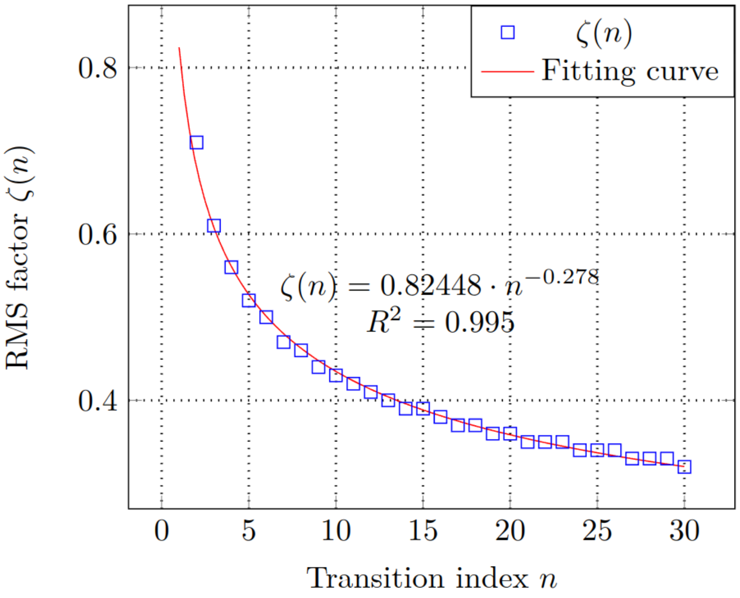
Assuming a cosinusoidal current, the superconductor resistivity is,

$$\rho_{sc} = \frac{E_c}{J_c^n} \left| J_{max} \cos(\omega t) \right|^{n-1}$$

Development in Fourier series and applying the Parseval relation, one can demonstrate that the RMS resistivity can be written as,

$$\rho_{sc,rms} = \frac{E_c}{J_c} \left( \frac{J_{max}}{J_c} \right)^{n-1} \zeta(n)$$

Where the RMS factor $\xi$ has been fitted to be easily implemented in COMSOL



$$J_{c0} = \frac{I_{c0}}{A_{sc}}$$

$$J_c(\mathbf{B}) = \frac{J_{c0}}{\left(1 + \frac{\sqrt{k^2 B_\parallel^2 + B_\perp^2}}{B_0}\right)^{\alpha}}$$

$k$ = 0.0605 , $\alpha$ = 0.7580,
$B_0$ = 0.103 T

# Two case studies

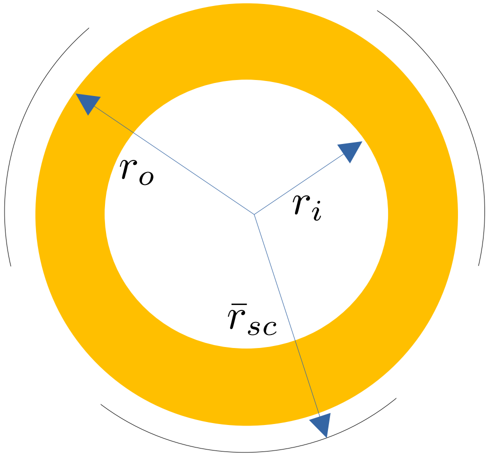
|  | Data | $I_c$ [A] | $n$ |
|---|---|---|---|
| BSCCO | ACT-AC Sumitomo | 68 | 11 |
| REBCO | Superpower | 300 | 30 |
| REBCO (CORC®) | - | 235 | 33 |

Single tape:

- BSCCO
- REBCO (1 µm thick)

CORC® cable:

- REBCO (1 µm thick)



Operating temperature: 77 K

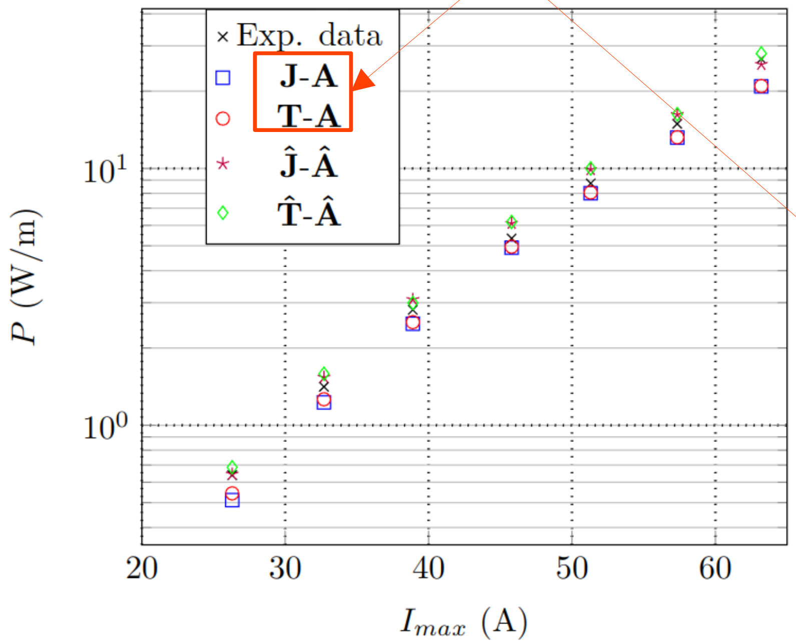
Frequency range: 40 – 1,000 Hz

| $r_i$ (mm) | $r_o$ (mm) | $r_{sc}$ (mm) |
|---|---|---|
| 2.65 | 3.15 | 3.25 |

# Single tape: BSCCO

<u>Data sources</u>: S. Fawaz. Etudes numériques et expérimentales des bobines supraconductrices HTC pour des applications en énergie électrique. PHD thesis, IAEM: Informatique - Automatique - Electronique - Electrotechnique - Mathématiques, Université de Lorraine, France, 2023.

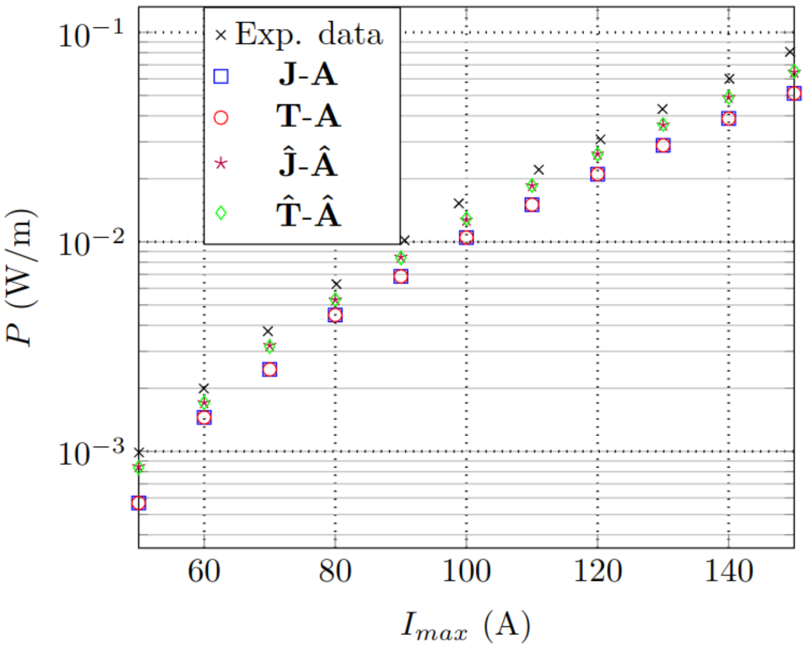
Time-domain model



| Form. | Freq. (Hz) | $R^2$ | $er_{max}/er_{min}$ (%) | Comp. time (s) |
|---|---|---|---|---|
| $\hat{\mathbf{J}}-\hat{\mathbf{A}}$ | 40 | 0.976193 | 17.1 / 4.5 | 20 |
| | 60 | 0.991507 | 14.5 / 0.9 | 26 |
| | 80 | 0.963282 | 18.3 / 1.4 | 13 |
| $\hat{\mathbf{T}}-\hat{\mathbf{A}}$ | 40 | 0.976271 | 19.4 / 0.5 | 20 |
| | 60 | 0.941172 | 21.6 / 7.1 | 26 |
| | 80 | 0.952471 | 22.9 / 2.5 | 22 |
| $\mathbf{J}-\mathbf{A}$ | 40 | 0.889592 | 32 / 11.4 | 65 |
| | 60 | 0.939473 | 25 / 7.9 | 60 |
| | 80 | 0.966140 | 19.7 / 2.3 | 62 |
| $\mathbf{T}-\mathbf{A}$ | 40 | 0.890569 | 28.6 / 11.1 | 66 |
| | 60 | 0.990484 | 16 / 5 | 65 |
| | 80 | 0.967893 | 17 / 1.4 | 63 |

# Single tape: REBCO

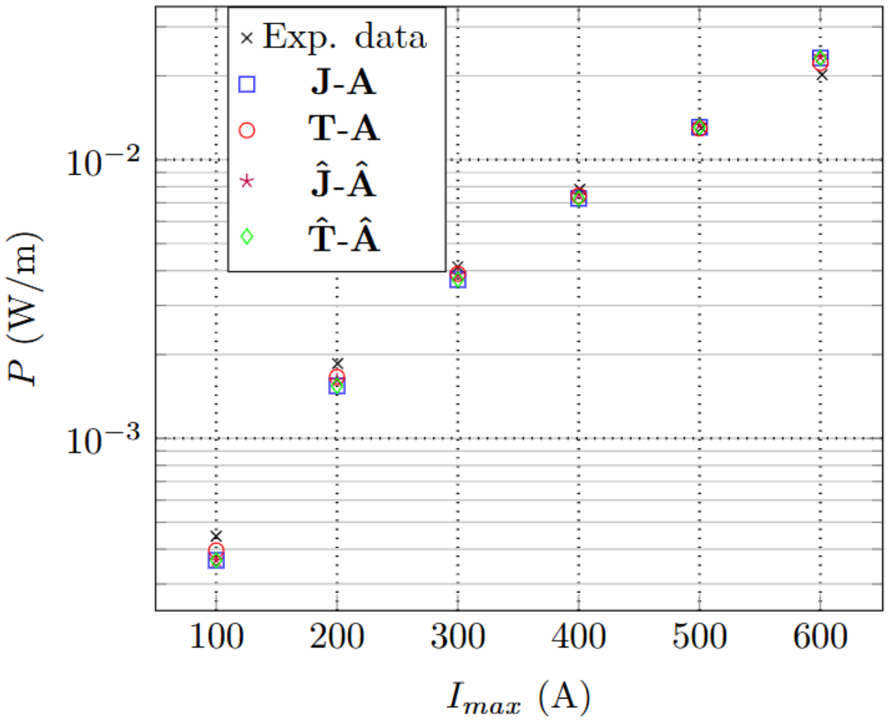
Data sources: B. Shen ert al. Investigation and comparison of AC losses on stabilizer-free and copper stabilizer HTS tapes. Physica C: Superconductivity and its Applications, 541:40–44, 2017.



| Form. | Freq. (Hz) | $R^2$ | $er_{max}/er_{min}$ (%) | Comp. time (s) |
|---|---|---|---|---|
| $\hat{\mathbf{J}}$-$\hat{\mathbf{A}}$ | 10 | 0.961789 | 18.9 /7.3 | 134 |
| | 100 | 0.947192 | 21.7 / 6.5 | 135 |
| | 1000 | 0.819709 | 33.9/21.4 | 137 |
| $\hat{\mathbf{T}}$-$\hat{\mathbf{A}}$ | 10 | 0.960085 | 18.6 / 6.7 | 343 |
| | 100 | 0.949447 | 21.7 / 15.8 | 344 |
| | 1000 | 0.827167 | 33.3/22.6 | 345 |
| **J-A** | 10 | 0.976600 | 15.3 /2.7 | 500 |
| | 100 | 0.782412 | 42.5 / 23.5 | 501 |
| | 1000 | 0.622033 | 71.5 / 35.6 | 502 |
| **T-A** | 10 | 0.985943 | 34.1 / 1.7 | 605 |
| | 100 | 0.762807 | 39.3 / 22.7 | 607 |
| | 1000 | 0.584765 | 48.5 / 44.5 | 609 |

# CORC®-type cable

Data sources: J. Yang, et al. Numerical Study on AC Loss Characteristics of Conductor on Round Core Cables Under Transport Current and Magnetic Field. IEEE Transactions on Applied Superconductivity, 31(8):1–4, 2021.



| Form. | Freq. (Hz) | $R^2$ | $er_{max}/er_{min}$ (%) | Comp. time (s) |
|---|---|---|---|---|
| **Ĵ-Â** | 36 | 0.982973 | 22.7 / 6.6 | 120 |
| | 72 | 0.990254 | 17.5 / 5.6 | 123 |
| | 144 | 0.992968 | 19.3 / 1.6 | 121 |
| **T̂-Â** | 36 | 0.978617 | 22.7 / 8.2 | 125 |
| | 72 | 0.987299 | 17.7 / 7 | 127 |
| | 144 | 0.993072 | 19.5 / 1.5 | 123 |
| **J-A** | 36 | 0.978617 | 22.7 / 8.2 | 1544 |
| | 72 | 0.987299 | 17.4/17.5 | 1542 |
| | 144 | 0.993072 | 19.3/1.5 | 1544 |
| **T-A** | 36 | 0.965897 | 16.2 / 7.9 | 2874 |
| | 72 | 0.992168 | 10.7/3.9 | 2876 |
| | 144 | 0.987674 | 12.9/4.2 | 2871 |

# Discussion

## Single tape:

Very good agreement for BSCCO and CORC®

- Discrepancy measurements and numerical results for on single REBCO tape. The max. error is still reasonable for most of the runs <50%

No clear explanation at the moment on this discrepancy: non-magnetic material, and negligible impact of metallic layers

- Non-magnetic material → guaranteed by manufacturer
- Metallic layers → additional model including the metallic layers not presented here

## Cable:

- Losses dominated by the Cu former
- Same issue as single REBCO tape

# Conclusion

Proposal for a frequency model to get steady-state electromagnetic parameters

Validation via the AC losses against experimental data

For REBCO, the results are fair but systematically underestimate the losses on a single tape → is the substrate not slightly magnetic after all?

The model can be used to predict steady-state conditions or to provide initialization values for a later transient analysis → very useful for superconducting components of the power grid, electrical machines, cables and fault-current limiters amongst other devices.