A frequency-domain finite element model for simulating HTS

J-A and **T-A** formulations

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Context

Need for a quick method to compute parameters in steady-state for AC regime

- AC losses
- Electrical parameters

What for?

- A quick estimate for pre-design (cooling system for instance)
- A first step before initiating a transient analysis (for electrical machines for instance)

Maxwell equations in frequency domain

Two newest formulations of the Maxwell equation:

T-A formulation (time) →$\hat{\mathbf{T}} - \hat{\mathbf{A}}$ phasor formulation

J-A formulation (time) →$\hat{\mathbf{J}} - \hat{\mathbf{A}}$ phasor formulation

$$\left.\begin{array}{l}\\ \\ \end{array}\right\} \quad \begin{aligned} \hat{\mathbf{X}} &= \mathbf{X}_a \exp(j\omega t) \\ \omega &= 2\pi f \\ j^2 &= -1 \end{aligned}$$

Frequency-domain equations for tape-based model (thin strip):

$$\begin{tikzcd}
 & \boxed{\nabla \times \frac{1}{\mu} \nabla \times \hat{\mathbf{A}} - j\omega\sigma\hat{\mathbf{A}} = \hat{\mathbf{J}}_e} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\
\boxed{\nabla \times \rho_{sc,rms} \nabla \times \hat{\mathbf{T}} = -j\omega\hat{\mathbf{B}}} \arrow[d, leftrightarrow] & & \boxed{\rho_{sc}\hat{\mathbf{J}}_{sc} = -j\omega\hat{\mathbf{A}}} \arrow[d, leftrightarrow] \\
\boxed{T_1 - T_2 = \frac{|\hat{i}|\cos(\omega t))}{\delta}} & & \boxed{\int_{w_{sc}} \hat{\mathbf{J}}_{sc} \cdot \mathrm{d}\mathbf{l} = \frac{\hat{i}}{\delta}}
\end{tikzcd}$$

$$\hat{\mathbf{B}} = \nabla \times \hat{\mathbf{A}}$$

$$\hat{\mathbf{J}}_{sc} = \nabla \times \hat{\mathbf{T}}$$

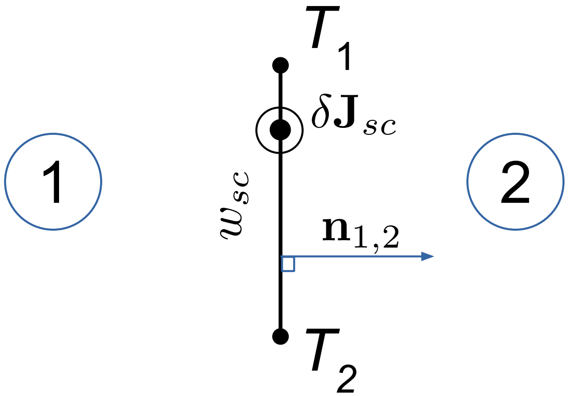
Tape-based or full model

Each individual tape is simulated given the proper current is impressed as

- A Dirichlet boundary condition for the **T** formulation $T_1 - T_2 = \frac{I}{\delta}\cos(\omega t)$
- A point-wise constraint in the **J** formulation $\int_{w_{sc}} \hat{\mathbf{J}}_{sc} \cdot \mathrm{d}\mathbf{l} = \frac{\hat{\mathbf{i}}}{\delta}$

In both formulation, the magnetic vector potential is solved assuming a discontinuity of the magnetic field across the 1D line representing the tape:

$$\mathbf{n}_{1,2} \times \left(\frac{1}{\mu} \nabla \times \hat{\mathbf{A}}_2 - \frac{1}{\mu} \nabla \times \hat{\mathbf{A}}_1 \right) = \delta \hat{\mathbf{J}}_{sc}$$



RMS electrical resistivity

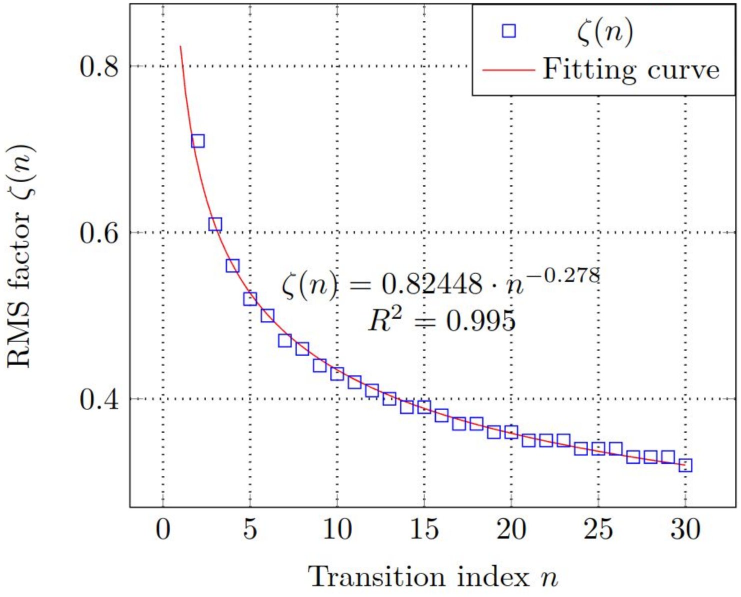
Assuming a cosinusoidal current, the superconductor resistivity is,

$$\rho_{sc} = \frac{E_c}{J_c^n} \left| J_{max} \cos(\omega t) \right|^{n-1}$$

Development in Fourier series and applying the Parseval relation, one can demonstrate that the RMS resistivity can be written as,

$$\rho_{sc,rms} = \frac{E_c}{J_c} \left(\frac{J_{max}}{J_c} \right)^{n-1} \zeta(n)$$

Where the RMS factor ζ has been fitted to be easily implemented in COMSOL



$$J_{c0} = \frac{I_{c0}}{A_{sc}}$$

$$J_c(\mathbf{B}) = \frac{J_{c0}}{\left(1 + \frac{\sqrt{k^2 B_\parallel^2 + B_\perp^2}}{B_0}\right)^{\alpha}}$$

k = 0.0605 , a = 0.7580,
B_0 = 0.103 T

Two case studies

Single tape:

- BSCCO
- REBCO (1 μm thick)

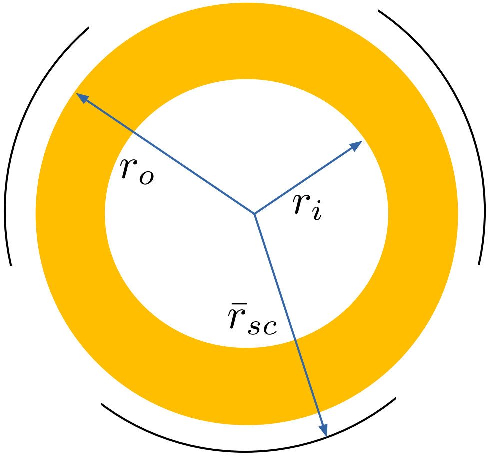
CORC® cable:

- REBCO (1 μm thick)

	Data	I_c [A]	n
BSCCO	ACT-AC Sumitomo	68	11
REBCO	Superpower	300	30
REBCO (CORC®)	-	235	33

Operating temperature: 77 K

Frequency range: 40 – 1000 Hz

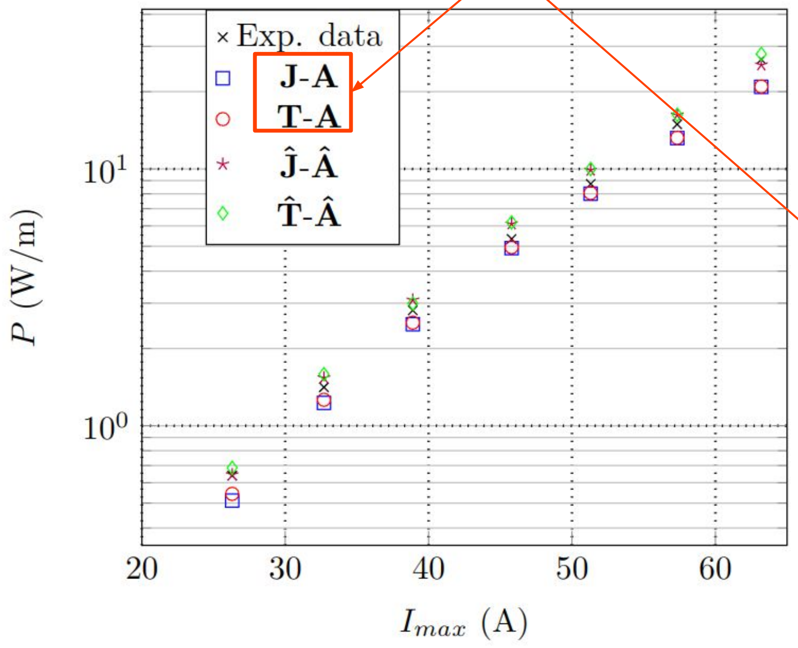


r_i (mm)	r_o (mm)	r_{sc} (mm)
2.65	3.15	3.25

Single tape: BSCCO

<u>Data sources</u>: S. Fawaz. Etudes numériques et expérimentales des bobines supraconductrices HTC pour des applications en énergie électrique. PHD thesis, IAEM: Informatique - Automatique - Electronique - Electrotechnique - Mathématiques, Université de Lorraine, France, 2023.

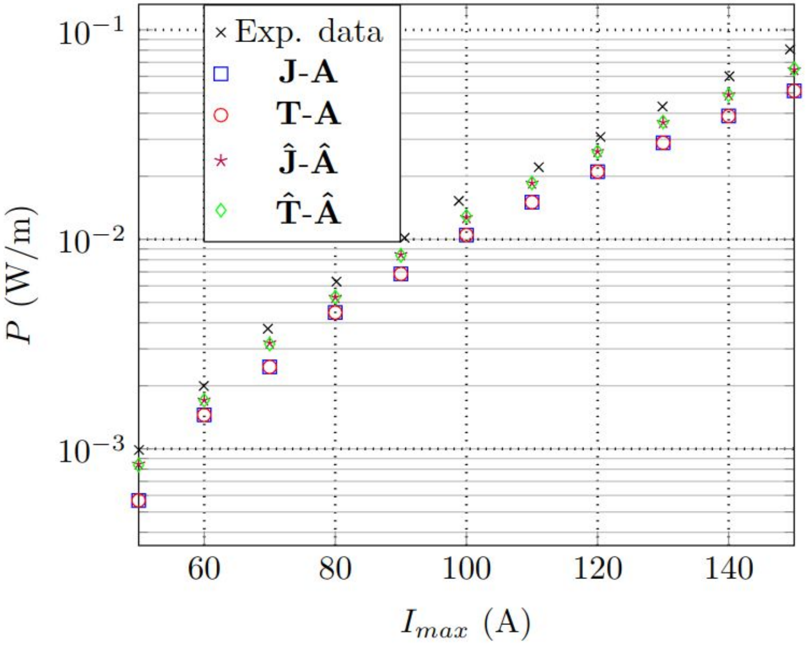
Time-domain model



Form.	Freq. (Hz)	R^2	er_{max}/er_{min} (%)	Comp. time (s)
$\hat{\mathbf{J}}-\hat{\mathbf{A}}$	40	0.976193	17.1 / 4.5	20
	60	0.991507	14.5 / 0.9	26
	80	0.963282	18.3 / 1.4	13
$\hat{\mathbf{T}}-\hat{\mathbf{A}}$	40	0.976271	19.4 / 0.5	20
	60	0.941172	21.6 / 7.1	26
	80	0.952471	22.9 / 2.5	22
$\mathbf{J}-\mathbf{A}$	40	0.889592	32 / 11.4	65
	60	0.939473	25 / 7.9	60
	80	0.966140	19.7 / 2.3	62
$\mathbf{T}-\mathbf{A}$	40	0.890569	28.6 / 11.1	66
	60	0.990484	16 / 5	65
	80	0.967893	17 / 1.4	63

Single tape: REBCO

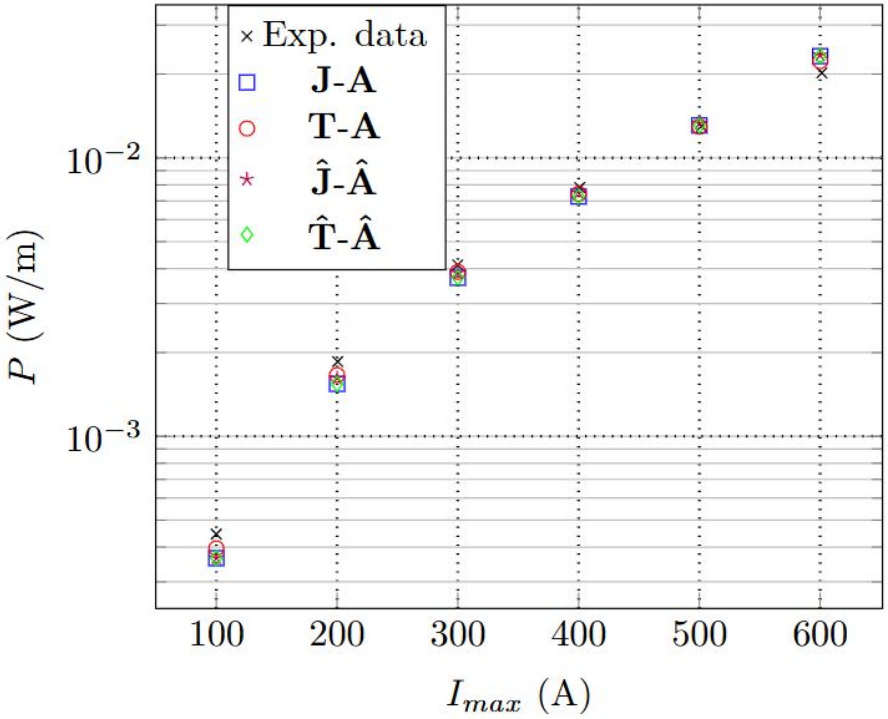
Data sources: B. Shen ert al. Investigation and comparison of AC losses on stabilizer-free and copper stabilizer HTS tapes. Physica C: Superconductivity and its Applications, 541:40–44, 2017.



Form.	Freq. (Hz)	R^2	er_{max}/er_{min} (%)	Comp. time (s)
$\hat{\mathbf{J}}$-$\hat{\mathbf{A}}$	10	0.961789	18.9 /7.3	134
	100	0.947192	21.7 / 6.5	135
	1000	0.819709	33.9/21.4	137
$\hat{\mathbf{T}}$-$\hat{\mathbf{A}}$	10	0.960085	18.6 / 6.7	343
	100	0.949447	21.7 / 15.8	344
	1000	0.827167	33.3/22.6	345
J-A	10	0.976600	15.3 /2.7	500
	100	0.782412	42.5 / 23.5	501
	1000	0.622033	71.5 / 35.6	502
T-A	10	0.985943	34.1 / 1.7	605
	100	0.762807	39.3 / 22.7	607
	1000	0.584765	48.5 / 44.5	609

CORC®-type cable

Data sources: J. Yang, et al. Numerical Study on AC Loss Characteristics of Conductor on Round Core Cables Under Transport Current and Magnetic Field. IEEE Transactions on Applied Superconductivity, 31(8):1–4, 2021.



Form.	Freq. (Hz)	R^2	er_{max}/er_{min} (%)	Comp. time (s)
Ĵ-Â	36	0.982973	22.7 / 6.6	120
	72	0.990254	17.5 / 5.6	123
	144	0.992968	19.3 / 1.6	121
T̂-Â	36	0.978617	22.7 / 8.2	125
	72	0.987299	17.7 / 7	127
	144	0.993072	19.5 / 1.5	123
J-A	36	0.978617	22.7 / 8.2	1544
	72	0.987299	17.4/17.5	1542
	144	0.993072	19.3/1.5	1544
T-A	36	0.965897	16.2 / 7.9	2874
	72	0.992168	10.7/3.9	2876
	144	0.987674	12.9/4.2	2871

Discussion

Single tape:

- Very good agreement for BSCCO
- Discrepancy measurements and numerical results for REBCO (no thorough analysis in the literature however this difference seems to be expected!)

No clear explanation at the moment on this discrepancy: non-magnetic material, and negligible impact of metallic layers

- Non-magnetic material → guaranteed by manufacturer
- Metallic layers → additional model including the metallic layers not presented here

Cable:

- Losses dominated by the Cu former
- Very good agreement for CORC

Conclusion

Proposal for a frequency model to get steady-state electromagnetic parameters

Validation via the AC losses against experimental data

For REBCO, the results are fair but systematically underestimate the losses → is the substrate not slightly magnetic after all?

The model can be used to predict steady-state conditions or to provide initialization values for a later transient analysis → very useful for superconducting components of the power grid, electrical machines, cables and fault-current limiters amongst other devices.

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