Introduction of the magnetic scalar potential in the T-A and J-A formulations for efficient electromagnetic simulations of High-Temperature Superconductors

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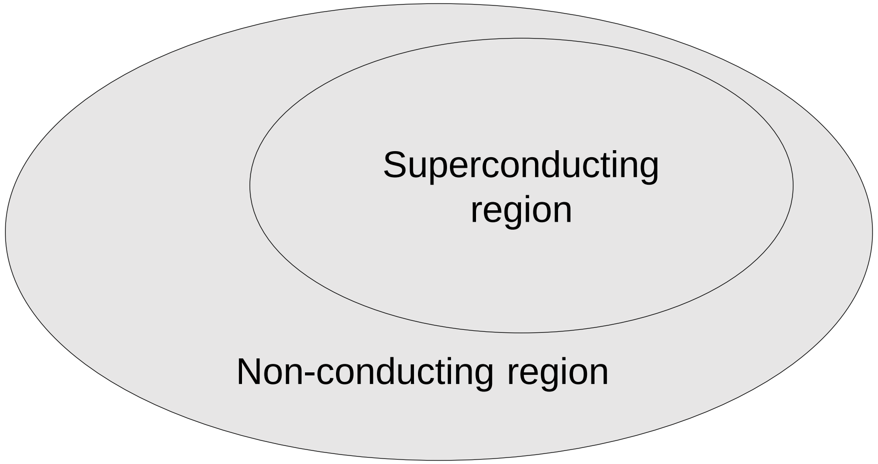
Content

Introduction

When superconducting devices are modeled, it is so common we have:



As presented in $\mathbf{H} - \phi$ formualtion:
For Non-conducting region:
$\nabla \times \mathbf{H} = 0 \Rightarrow -\nabla\phi = \mathbf{H}$, reducing the DoFs and saving the computation time in the problem.

Motivation:
Are there other possibilities to reduce still more the DoFs?

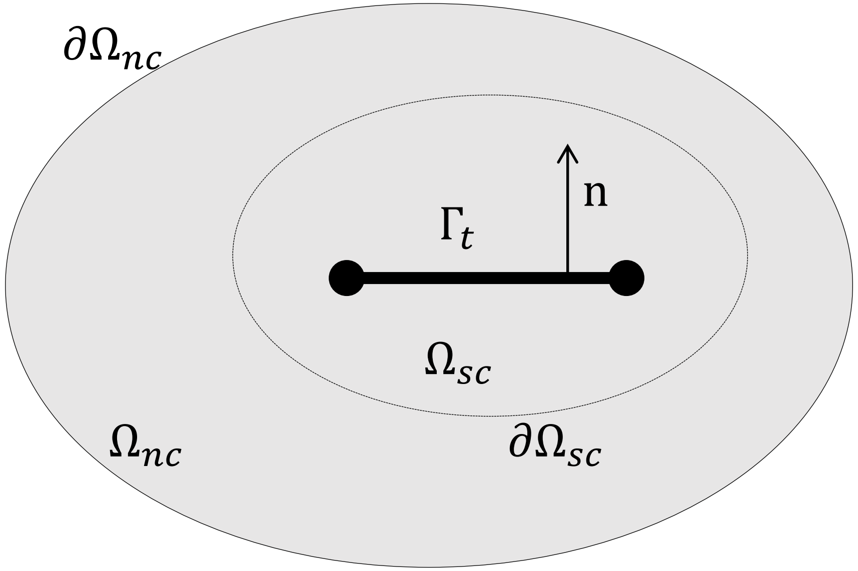
Literature shows that the $\mathbf{T} - \mathbf{A}$ formulation presents faster computation than $\mathbf{H}$ formulation and has accurate results.

More recently, the $\mathbf{J} - \mathbf{A}$ has presented computation time faster than $\mathbf{T} - \mathbf{A}$ with a high accuracy.

We propose introducing the ϕ in the $\mathbf{T} - \mathbf{A}$ and in $\mathbf{J} - \mathbf{A}$ formulation.

Methodology

$\mathbf{T}-\mathbf{A}-\phi$ formulation (thin-shell approximation)



In Ω_{sc}: $\nabla \times (\nu\, \nabla \times \mathbf{A}) = \mathbf{J}$

In Γ_t: $\nabla \times (\rho\, \nabla \times (T\mathbf{n})) = -\partial_t(\mathbf{B}.\mathbf{n})$

In Ω_{nc}: $\nabla(-\nabla\phi) = 0$

Couplings between the formulations:

$\mathbf{T}-\mathbf{A}$ coupling:

$\mathbf{T}$ computes the $\mathbf{J}$ in Γ_t and sends (as laminar current) to $\mathbf{A}$.

$$\mathbf{n} \times (\mathbf{H}^{+} - \mathbf{H}^{-}) = -\big(\nabla \times (T\mathbf{n})\big).\, t_{sc}$$

$\mathbf{A}$ computes the $\mathbf{B}$ in Ω_{sc} and sends (as source) to $\mathbf{T}$.

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$\mathbf{A}-\phi$ coupling:

$\mathbf{A}$ computes the $\mathbf{B}$ on $\partial\Omega_{sc}$ and sends (as BC) to ϕ.

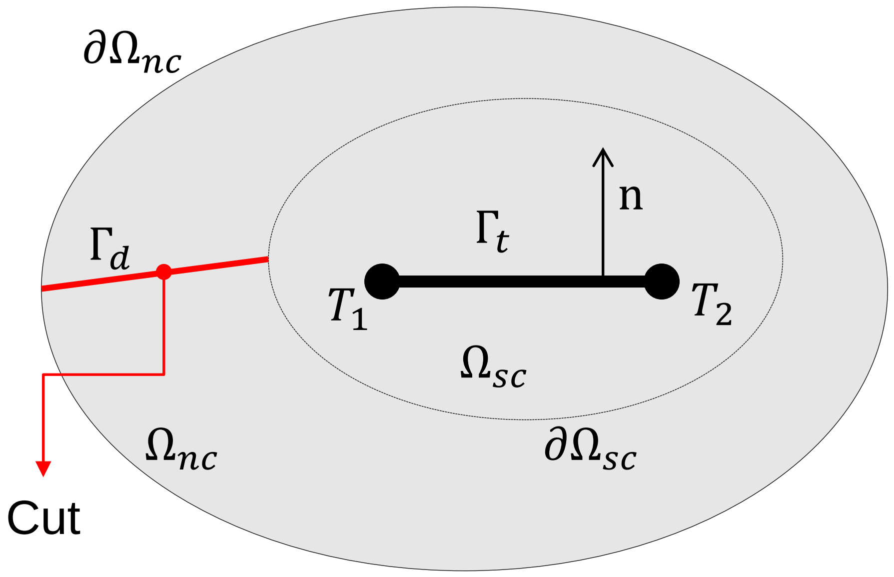
$$\mathbf{n}\,.\,\mathbf{B} = \mathbf{n}\,.\,(\boldsymbol{\nabla} \times \mathbf{A})$$

ϕ computes the $\mathbf{B}$ on $\partial\Omega_{sc}$ and sends (as BC) to $\mathbf{A}$.

$$\mathbf{n} \times \mathbf{H} = \mathbf{n} \times (-\boldsymbol{\nabla}\phi)$$

Methodology

$\mathbf{T}-\mathbf{A}-\phi$ formulation (thin-shell approximation – transport current problem)



In Ω_{sc}: $\nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{J}$

In Γ_t: $\nabla \times (\rho \nabla \times (T\mathbf{n})) = -\partial_t(\mathbf{B}.\mathbf{n})$

In Ω_{nc}: $\nabla(-\nabla\phi) = 0$

Imposition of the current:

As presented in $\mathbf{H}-\phi$, if Ω_{sc} **has non-zero net current, a discontinuity (cut) is needed** to impose the current inside the superconducting domain.

Using the magnetic scalar potential discontinuity,

$$\phi^+ - \phi^- = \phi^\pm = NI(t)$$

Magnetic scalar potential on $\partial\Omega_{nc}$

$$\phi = 0$$

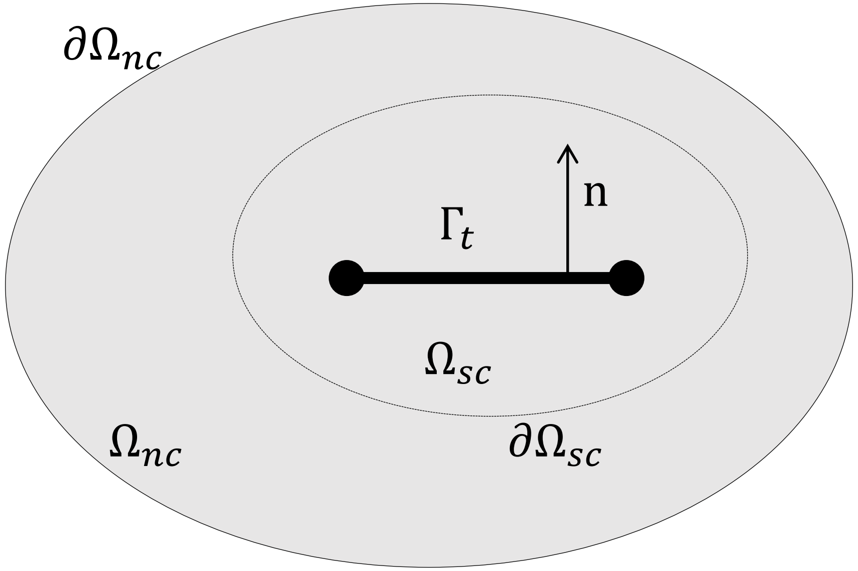
The average of the ϕ on Γ_d **must be** imposed in the $\mathbf{T}$ based on the BDC.

$$T_1 = 0$$

$$T_2 = \frac{\overline{\phi^\pm}}{t_{sc}}$$

Methodology

$\mathbf{J}-\mathbf{A}-\phi$ formulation (thin-shell approximation)



In Ω_{sc}: $\nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{J}$

In Γ_t: $\rho \mathbf{J} = -\partial_t \mathbf{A}$

In Ω_{nc}: $\nabla(-\nabla\phi) = 0$

Couplings between the formulations:

$\mathbf{J}-\mathbf{A}$ coupling:
$\mathbf{J}$ computes the current in Γ_t and sends (as laminar current) to $\mathbf{A}$.

$$\mathbf{n} \times (\mathbf{H}^+ - \mathbf{H}^-) = -\mathbf{J}.t_{sc}$$

$\mathbf{A}$ computes magnetic vector potential in Ω_{sc} and sends (as source) to $\mathbf{J}$.

$\mathbf{A}-\phi$ coupling:
$\mathbf{A}$ computes the $\mathbf{B}$ on $\partial\Omega_{sc}$ and sends (as BC) to ϕ.

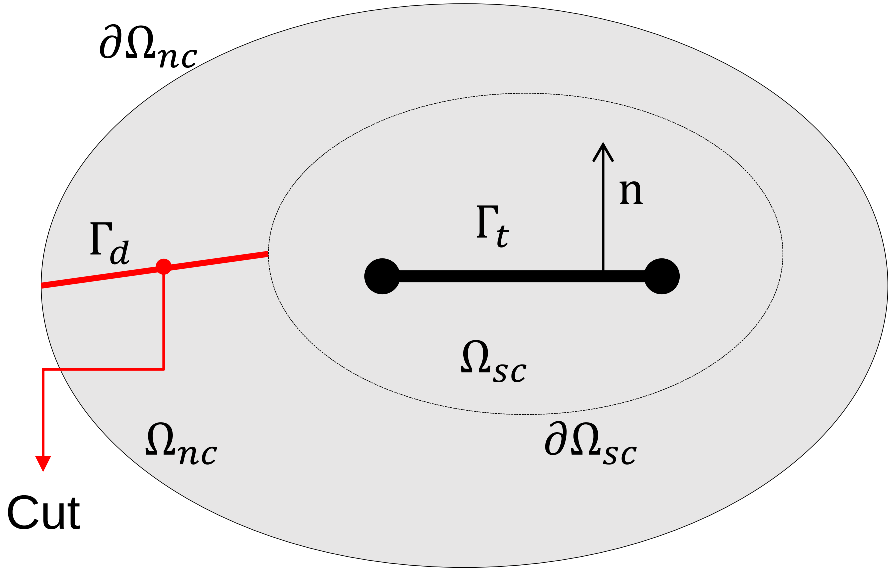
$$\mathbf{n}\,.\,\mathbf{B} = \mathbf{n}\,.\,(\nabla \times \mathbf{A})$$

ϕ computes the $\mathbf{B}$ on $\partial\Omega_{sc}$ and sends (as BC) to $\mathbf{A}$.

$$\mathbf{n} \times \mathbf{H} = \mathbf{n} \times (-\nabla\phi)$$

Methodology

$\mathbf{J} - \mathbf{A} - \phi$ formulation (thin-shell approximation – transport current problem)



In Ω_{sc}: $\nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{J}$

In Γ_t: $\rho \mathbf{J} = -\partial_t \mathbf{A}$

In Ω_{nc}: $\nabla(-\nabla\phi) = 0$

Imposition of the current:

$$\phi^+ - \phi^- = \phi^\pm = NI(t)$$

Magnetic scalar potential on $\partial\Omega_{nc}$

$$\phi = 0$$

The average of the ϕ on Γ_d **can be** imposed in the $\mathbf{J}$ based on a constrain.

$$\int_{\Gamma_t} \mathbf{J} \,.\, t_{sc} \, d\Gamma \pm \overline{\phi^\pm} = 0$$

Case studies

Two models were used to test the formulations:

- REBCO tape carrying current
- CORC cable with 3 tapes carrying current

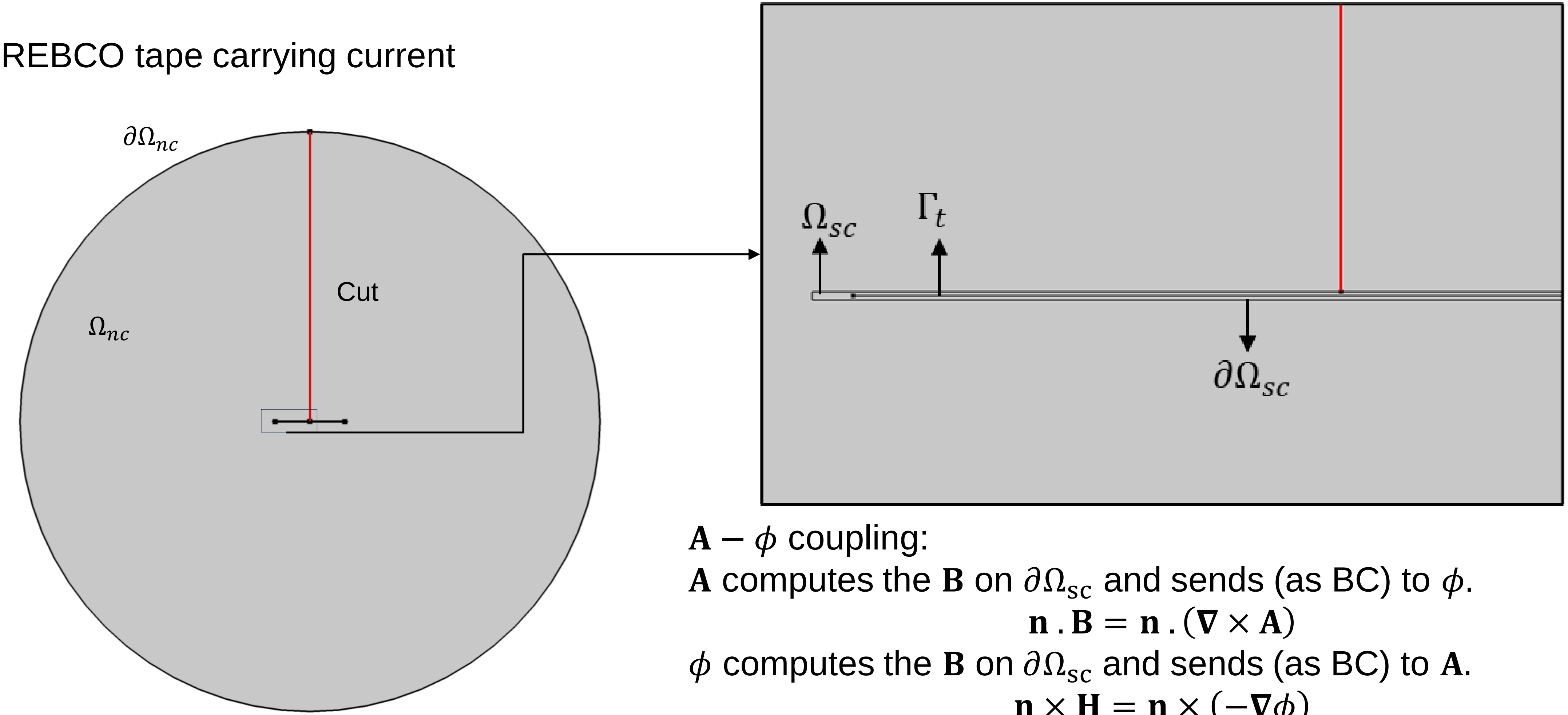
Implementation was done in COMSOL using three modules:
Magnetic Field (mf) → $\mathbf{A}$ formulation
Coefficient Form Boundary PDE (cb) → $\mathbf{T}$ formulation (only in $\mathbf{T}-\mathbf{A}-\phi$)
Boundary ODE & DAE (bode) → $\mathbf{J}$ formulation (only in $\mathbf{J}-\mathbf{A}-\phi$)
Magnetic Field no current (mfnc) → ϕ formulation

Variables analyzed:

- Magnetic Field
- Current density distribution
- Losses

Case studies

REBCO tape carrying current



$$\int_{\Gamma_t} \mathbf{J}\,.\, t_{sc}\, d\Gamma \pm \overline{\phi^{\pm}} = 0 \; (\mathbf{J} - \mathbf{A} - \phi)$$

$$T_1 = 0 \text{ and } T_2 = \frac{\overline{\phi^{\pm}}}{t_{sc}} \; (\mathbf{T} - \mathbf{A} - \phi)$$

$\mathbf{A} - \phi$ coupling:

$\mathbf{A}$ computes the $\mathbf{B}$ on $\partial\Omega_{sc}$ and sends (as BC) to ϕ.

$$\mathbf{n}\,.\,\mathbf{B} = \mathbf{n}\,.\,(\nabla \times \mathbf{A})$$

ϕ computes the $\mathbf{B}$ on $\partial\Omega_{sc}$ and sends (as BC) to $\mathbf{A}$.

$$\mathbf{n} \times \mathbf{H} = \mathbf{n} \times (-\nabla\phi)$$

* The shape function order in ϕ can be linear → save computation time in 2D.

Case studies

REBCO tape carrying current

Material characterization:

- Ω_{ns}:

$$\mu = \mu_0 \text{ [H/m]}$$
$$\sigma = 0 \text{ [S/m]}$$

- Ω_{sc}:

$$\mu = \mu_0 \text{ [H/m]}$$
$$\sigma = 0 \text{ [S/m]}$$

- Γ_t:

$$\mu = \mu_0 \text{ [H/m]}$$
$$\rho = \frac{Ec}{Jc}\left(\frac{\mathrm{J}}{J_c}\right)^{\mathrm{n}} [\Omega.\mathrm{m}]$$

Superconductor parameters:

$E_c = 1$ [μV/cm]
$I_c = 300$ [A]
w_{tp} = 12 [mm]
t_{sc} = 1[μm]
$n = 30$

Time solver configuration:
$f = 10$ Hz
Simulation with 1 cycle with a 200 time steps.

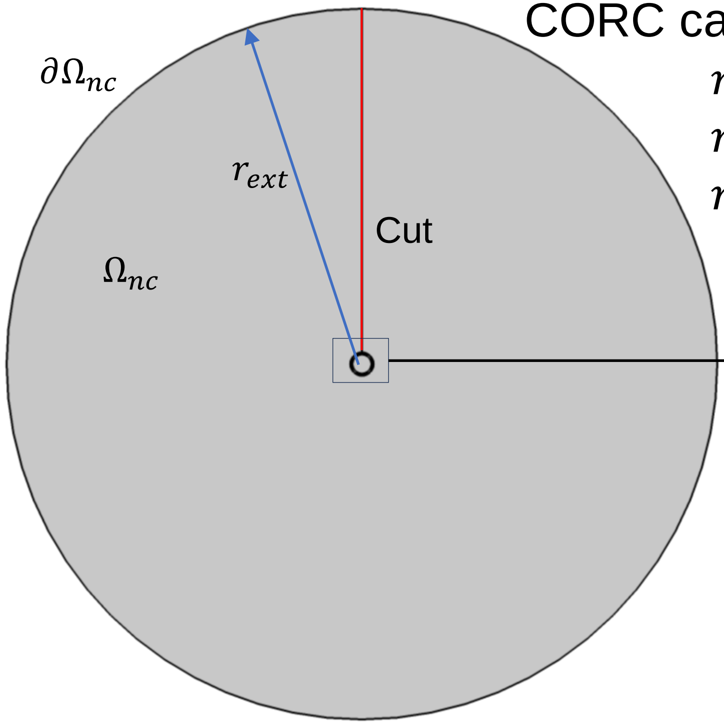
Applied Current:

$$f(t) = \text{Iap}.\sin(\omega t)$$
$$\text{Iap} = \{50,60,70,80,90,100,110,120,130,140,150\}$$

Mesh:
Γ_t: 100 elements
$\partial\Omega_{sc}$: 202 elements
Other domains: Free triangular mesh with maximum element size 6.7 mm and minimum element size 0.03 mm

Case studies

CORC cable carrying current

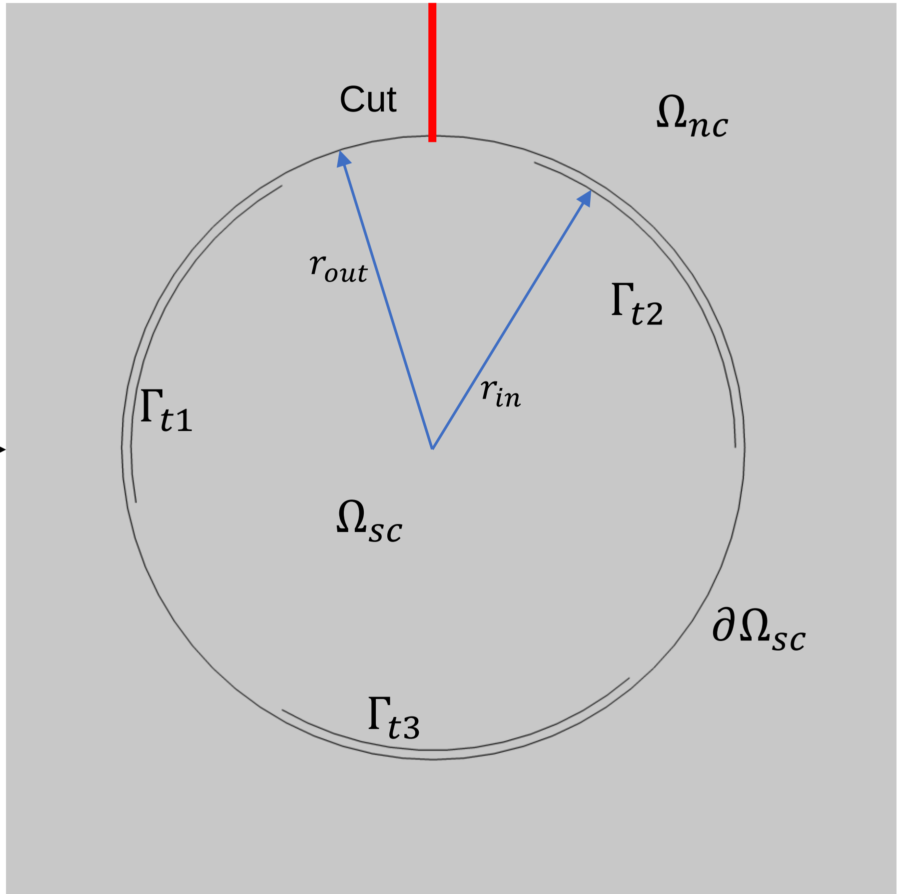


CORC cable geometry:

$r_{out} = 3.35\ [mm];$

$r_{int} = 3.25\ [mm];$

$r_{ext} = 100\ [mm]$



$$\int_{\Gamma_t} \mathbf{J} \, . \, t_{sc} \, d\Gamma \pm \overline{\phi^{\pm}} = 0 \ (\mathbf{J} - \mathbf{A} - \phi)$$

$$T_1 = 0 \text{ and } T_2 = \frac{\overline{\phi^{\pm}}}{t_{sc}} \ (\mathbf{T} - \mathbf{A} - \phi)$$

$\mathbf{A} - \phi$ coupling:

$\mathbf{A}$ computes the $\mathbf{B}$ on $\partial\Omega_{sc}$ and sends (as BC) to ϕ.

$$\mathbf{n} \, . \, \mathbf{B} = \mathbf{n} \, . \, (\nabla \times \mathbf{A})$$

ϕ computes the $\mathbf{B}$ on $\partial\Omega_{sc}$ and sends (as BC) to $\mathbf{A}$.

$$\mathbf{n} \times \mathbf{H} = \mathbf{n} \times (-\nabla\phi)$$

* The shape function order in ϕ can be linear → save computation time in 2D.

Case studies

CORC cable carrying current

Material characterization:

- Ω_{ns}:

$$\mu = \mu_0 \text{ [H/m]}$$
$$\sigma = 0 \text{ [S/m]}$$

- Ω_{sc}:

$$\mu = \mu_0 \text{ [H/m]}$$
$$\sigma = 0 \text{ [S/m]}$$

- Γ_{t1} and Γ_{t2} and Γ_{t3} :

$$\mu = \mu_0 \text{ [H/m]}$$
$$\rho = \frac{Ec}{Jc}\left(\frac{\mathbf{J}}{J_c(\mathbf{B})}\right)^{\mathrm{n}} [\Omega.\text{m}]$$

Superconductor parameters:

$E_c = 1$ [μV/cm]

$I_c = 235$ [A]

$w_{tp} = 4$ [mm]

$t_{sc} = 1$[μm]

$n = 33$

Time solver configuration:

$f = 36$ Hz

Simulation with 1 cycle with a 300 time steps.

Applied Current:

$$f(t) = \mathrm{N_{tapes}Iap.}\sin(\omega t)$$
$$\mathrm{N_{tapes}Iap} = \{100,110,120,130,140,150\}$$

Mesh:

Γ_t: 200 elements

Other domains: Free triangular mesh with maximum element size 13.4 mm and minimum element size 1.3 mm

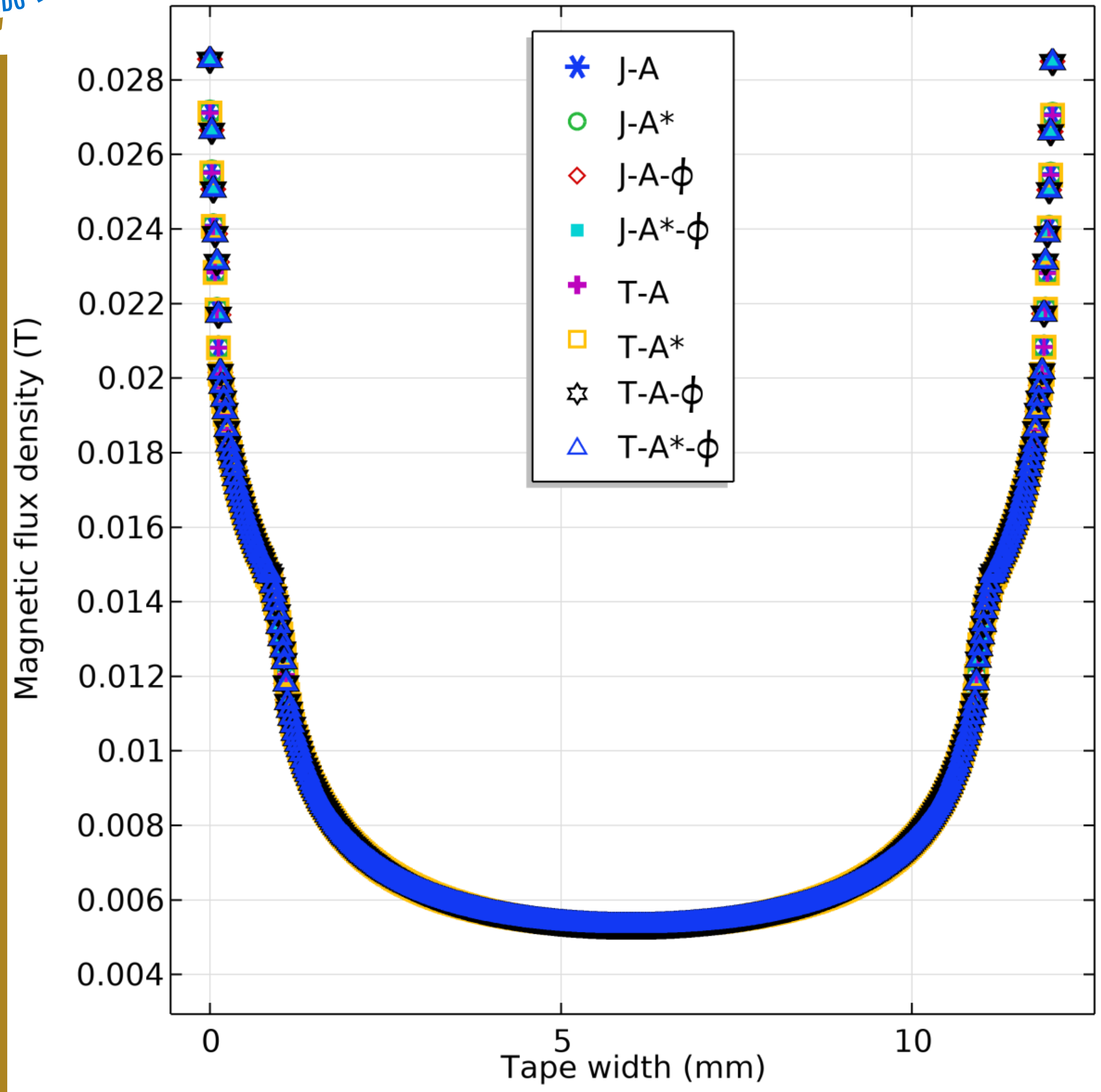
Anderson-Kim model

$$J_c(\mathbf{B}) = \frac{J_{c0}}{\left(1 + \sqrt{\frac{\kappa^2 B_\parallel^2 + B_\perp^2}{B_0^2}}\right)^b}$$

Results

REBCO tape carrying current

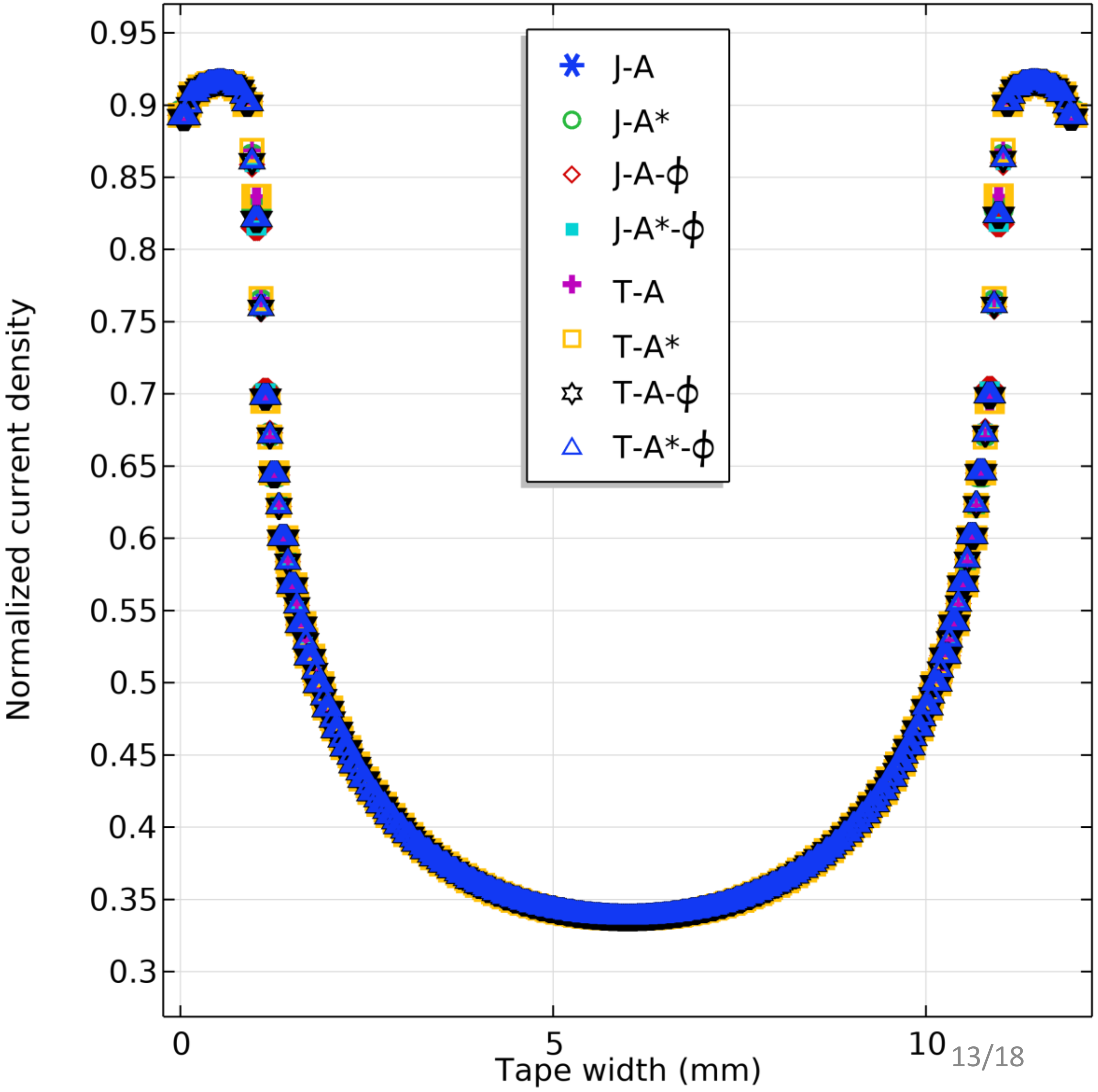
$t = 25\ ms$



$\mathbf{A}$ – only $\mathbf{Az}$ is computed

$\mathbf{A}^*$ – $\mathbf{Ax}$, $\mathbf{Ay}$ and $\mathbf{Az}$ are computed

$t = 25\ ms$



Results

REBCO tape carrying current

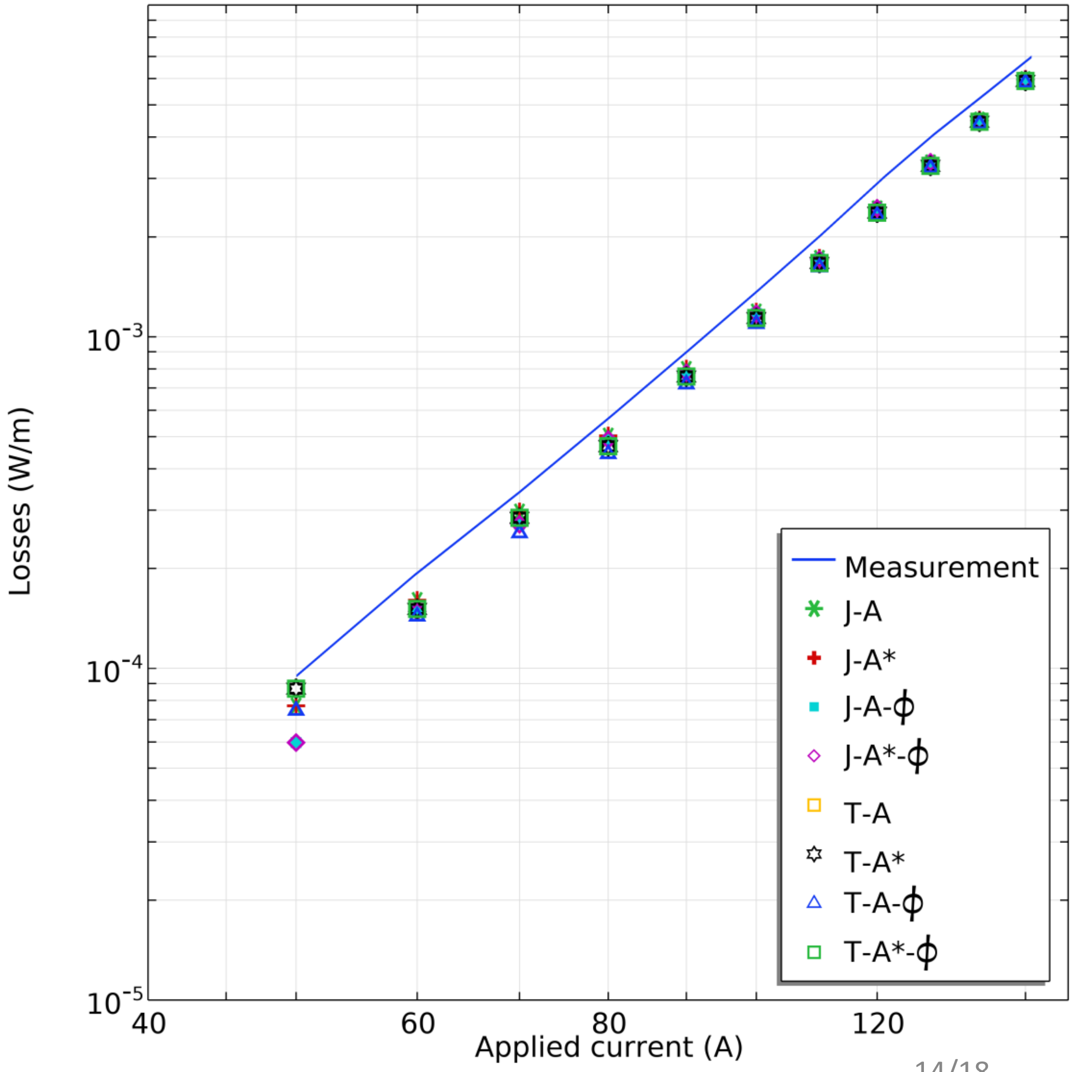
Measurements: B. Shen ert al. Investigation and comparison of AC losses on stabilizer-free and copper stabilizer HTS tapes. Physica C: Superconductivity and its Applications, 541:40–44, 2017.

Formulation	R^2 Losses (%)	DoFs	Computation times (s)
$\mathbf{J}-\mathbf{A}$	95.781	7969	80
$\mathbf{J}-\mathbf{A}^*$	95.786	27567	218
$\mathbf{J}-\mathbf{A}-\phi$	95.581	3125	40
$\mathbf{J}-\mathbf{A}^*-\phi$	95.579	5347	41
$\mathbf{T}-\mathbf{A}$	94.642	7970	125
$\mathbf{T}-\mathbf{A}^*$	94.642	27568	368
$\mathbf{T}-\mathbf{A}-\phi$	93.648	3126	44
$\mathbf{T}-\mathbf{A}^*-\phi$	93.723	5348	52

Considering a 3-component case (3D applications) $\mathbf{J}-\mathbf{A}^*-\phi$ is faster than $\mathbf{J}-\mathbf{A}^*$ 81.2%.

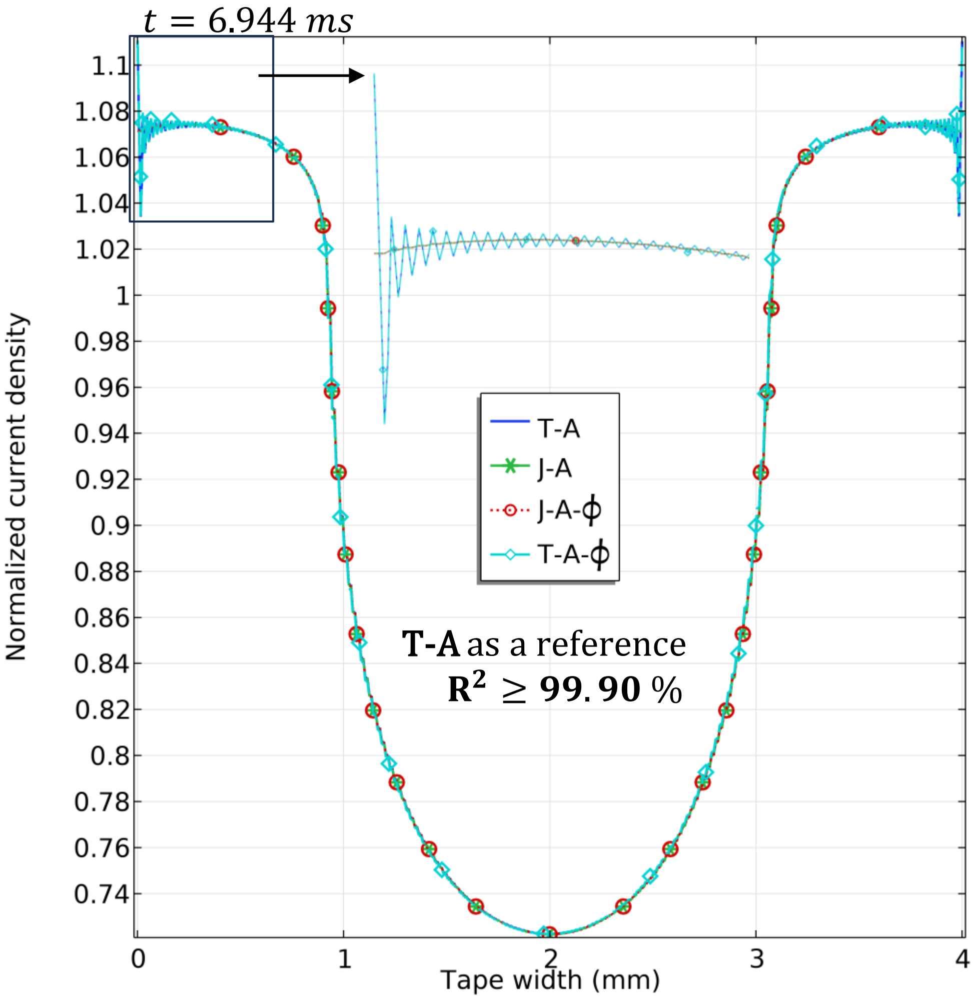
Considering a 3-component case (3D applications) $\mathbf{T}-\mathbf{A}^*-\phi$ is faster than $\mathbf{T}-\mathbf{A}^*$ 85.06%

$\mathbf{A}$ – only $\mathbf{Az}$ is computed
$\mathbf{A}^*$ – $\mathbf{Ax}$, $\mathbf{Ay}$ and $\mathbf{Az}$ are computed

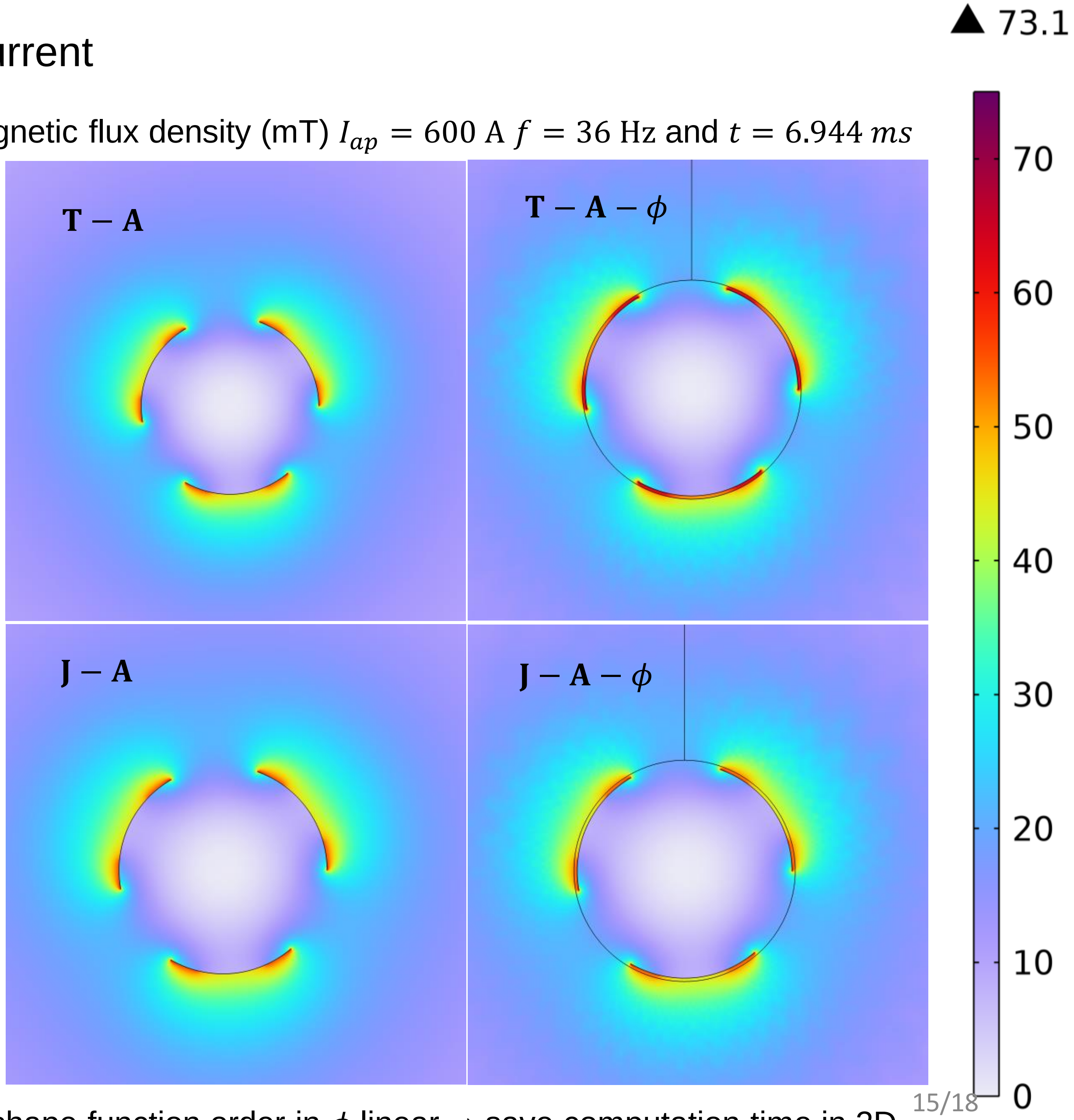


Results

CORC cable carrying current



Magnetic flux density (mT) $I_{ap} = 600$ A $f = 36$ Hz and $t = 6.944\ ms$



* The shape function order in ϕ linear → save computation time in 2D.

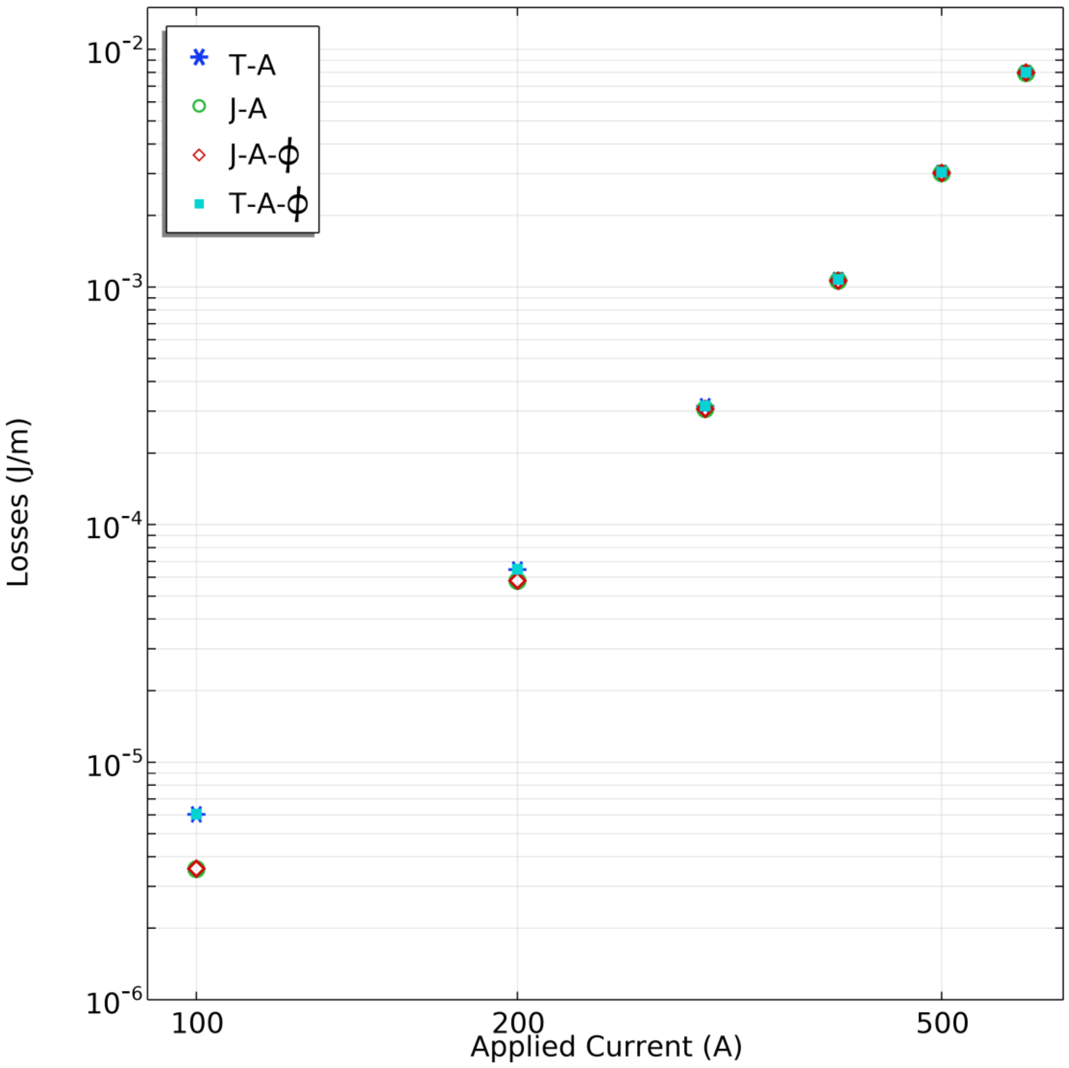
Results

CORC cable carrying current

Formulation	R^2 Losses (%)	DoFs	Computation times (s)
$\mathbf{J-A}$	99.999	43333	494
$\mathbf{J-A}-\phi$	99.998	29219	224
$\mathbf{T-A}$	Reference	43336	889
$\mathbf{T-A}-\phi$	99.995	29326	716

$\mathbf{J-A}-\phi$ is faster than $\mathbf{J-A}$ 54.65%.

$\mathbf{T-A}-\phi$ is faster than $\mathbf{T-A}$ 19.46%



Conclusion

1. The introduction of the magnetic scalar potential in the T-A and J-A formulation was presented.
2. For REBCO:
   A good agreement in the losses computing was obtained for all formulations (All results with $R^2 \geq 93\%$).

   An excellent agreement between the magnetic flux density and current density ($R^2 \geq 95\%$)

   Considering the three components of the magnetic vector potential the $\mathbf{J}-\mathbf{A}-\phi$ formulation is 81% faster than the $\mathbf{J}-\mathbf{A}$, on the same way the $\mathbf{T}-\mathbf{A}-\phi$ is faster 85% than the $\mathbf{T}-\mathbf{A}$. Comparing the $\mathbf{J}-\mathbf{A}-\phi$ and the $\mathbf{T}-\mathbf{A}-\phi$, the $\mathbf{J}-\mathbf{A}-\phi$ is faster 21%.
3. For CORC cable
   Using the T-A formulation as a reference, an agreement higher 99% was obtained for all compared variables.
   $\mathbf{J}-\mathbf{A}-\phi$ is faster than $\mathbf{J}-\mathbf{A}$ 54.65%.
   $\mathbf{T}-\mathbf{A}-\phi$ is faster than $\mathbf{T}-\mathbf{A}$ 19.46%.

Conclusion

Thank you so much!

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