HTS MODELLING WORKGROUP

KENER

# Parameter-free reconstruction of HTS critical current magnetic field angular dependency with sparse measurements

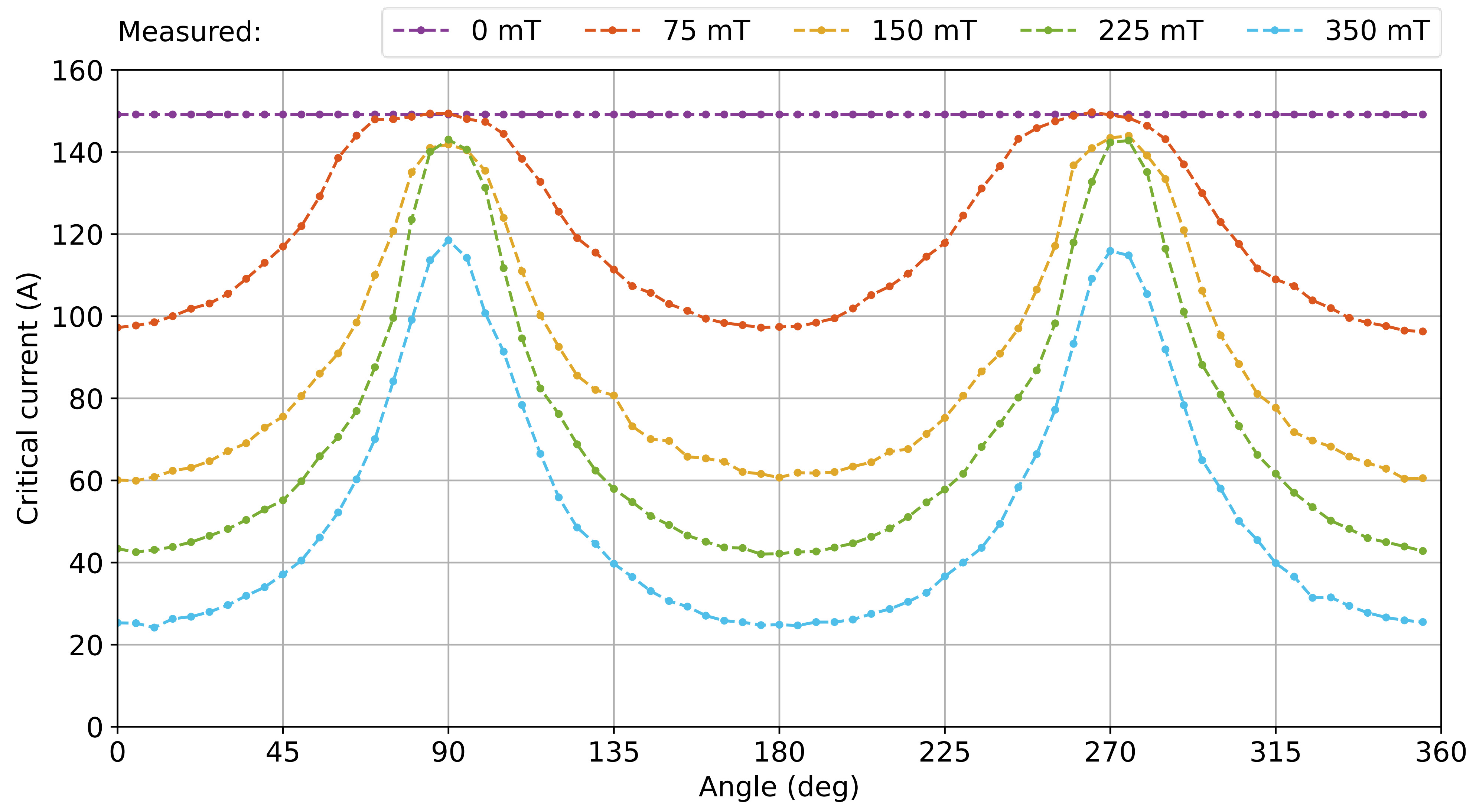
**Paweł Lasek,** Krzysztof Habelok, Kamil Gruszczyk, Mariusz Stępień

Silesian University of Technology

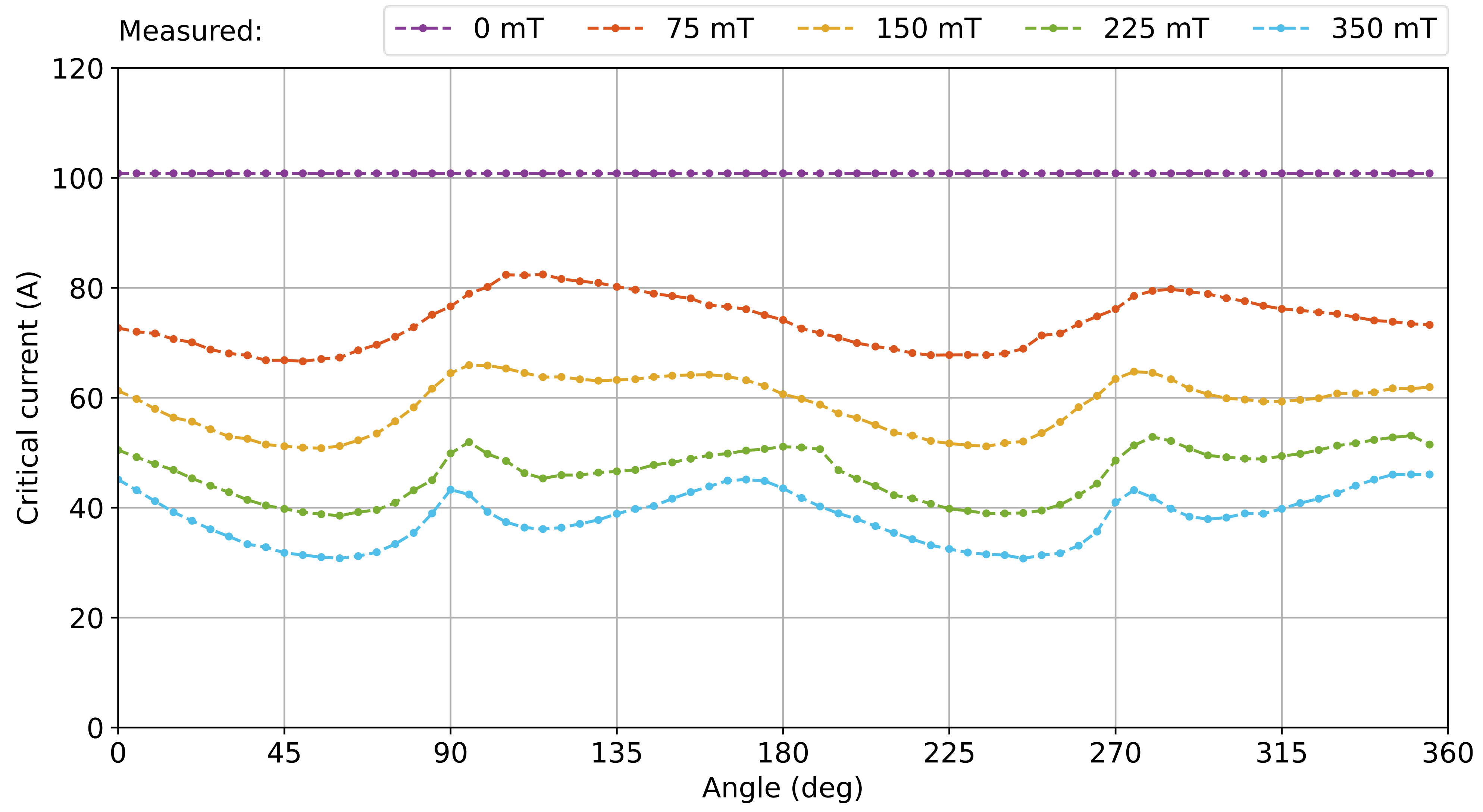
Department of Power Electronics, Electrical Drives and Robotics,
Faculty of Electrical Engineering, Gliwice, Poland kener.elektr.polsl.pl | kener.eu | kener.pl

# Problem

### 1G: AMSC BiSCCO 150A




### 2G: Super Power SCS4050-AP 99A

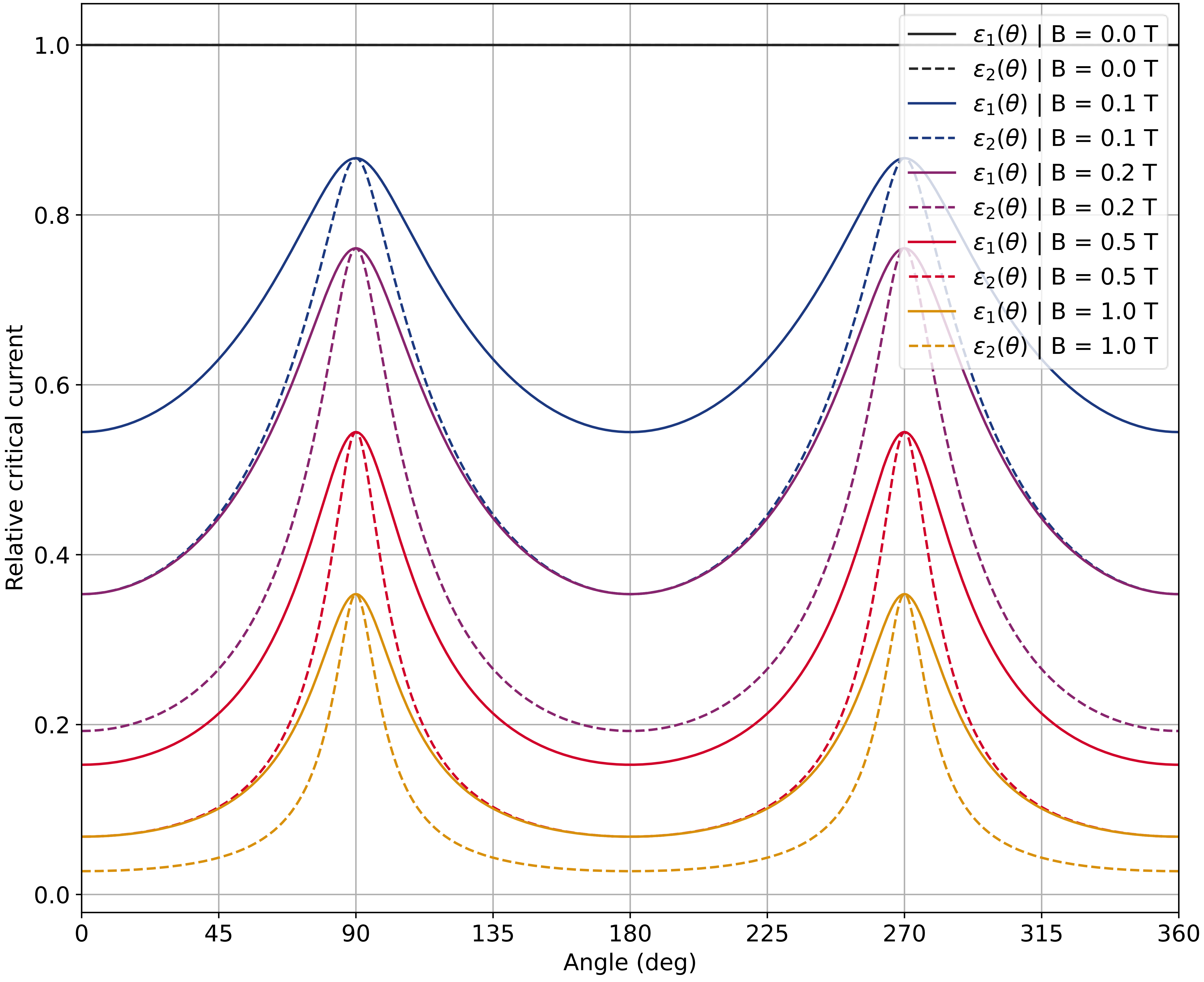



| No. | Model | Equation |
| --- | --- | --- |
| 1 | Kim-like | $\mathrm{I_c}(\theta, \mathbf{B}) = I_{c0}\left(\frac{1}{1+\frac{\lvert\mathbf{B}\rvert}{B_{c0}}\varepsilon(\theta)}\right)^b$ |
| 2 | Magneto-angular anisotropy | $\mathrm{I_c}(\theta, \mathbf{B}) = I_{c0}\left(\frac{1}{1+\left(\frac{\lvert\mathbf{B}\rvert}{B_{c0}}\right)^\alpha \varepsilon(\theta)}\right)^b$ |
| 3 | Percolation | $\mathrm{I_c}(\theta, \mathbf{B}) = I_{c0}\cdot\exp\left(-\left(\frac{\lvert\mathbf{B}\rvert}{B_{c0}}\right)^\alpha \varepsilon^b(\theta)\right)$ |

| $\boldsymbol{\varepsilon(\theta)}$ |
| --- |
| $\varepsilon_1(\theta) = \sqrt{k^2\cos^2\theta + \sin^2\theta}$ |
| $\varepsilon_2(\theta) = \sqrt{k^2\cos^2\theta + l^2\sin^2\theta}$ |

## Problem

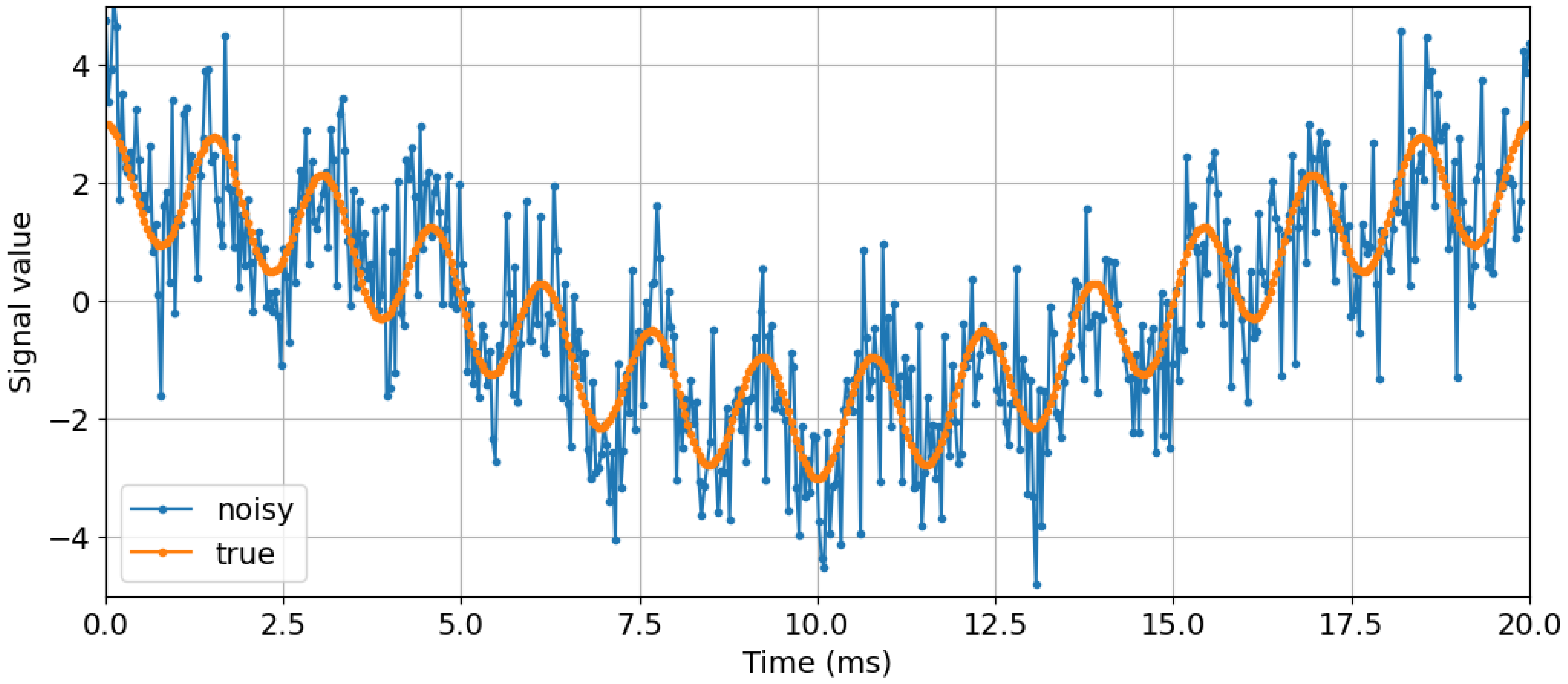
### Kim-like model



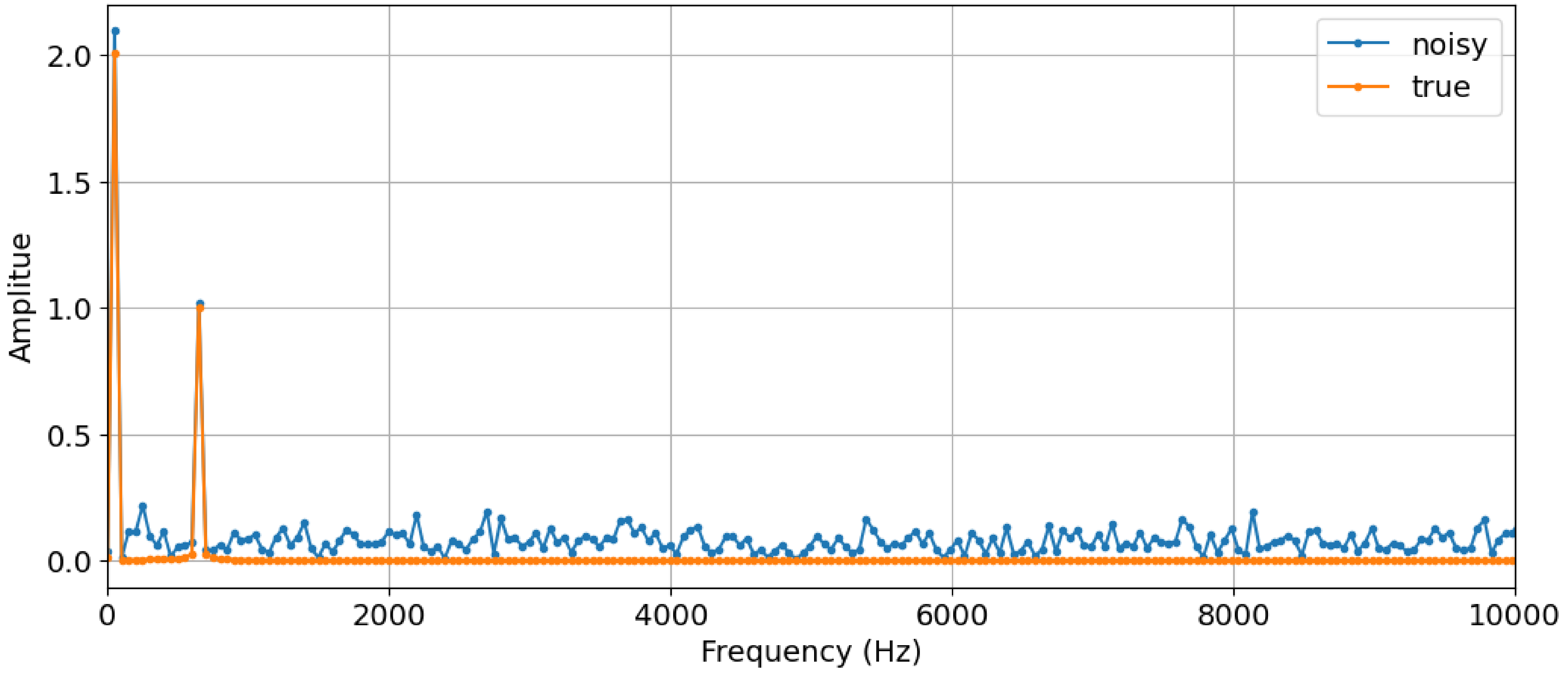

| No. | Model | Equation |
|---|---|---|
| 1 | Kim-like | $\mathrm{I_c}(\theta,\mathbf{B}) = I_{c0}\left(\frac{1}{1+\frac{\lvert\mathbf{B}\rvert}{B_{c0}}\varepsilon(\theta)}\right)^{b}$ |
| 2 | Magneto-angular anisotropy | $\mathrm{I_c}(\theta,\mathbf{B}) = I_{c0}\left(\frac{1}{1+\left(\frac{\lvert\mathbf{B}\rvert}{B_{c0}}\right)^{\alpha}\varepsilon(\theta)}\right)^{b}$ |
| 3 | Percolation | $\mathrm{I_c}(\theta,\mathbf{B}) = I_{c0}\cdot\exp\left(-\left(\frac{\lvert\mathbf{B}\rvert}{B_{c0}}\right)^{\alpha}\varepsilon^{b}(\theta)\right)$ |

| $\boldsymbol{\varepsilon(\theta)}$ |
|---|
| $\varepsilon_1(\theta) = \sqrt{k^2\cos^2\theta+\sin^2\theta}$ |
| $\varepsilon_2(\theta) = \sqrt{k^2\cos^2\theta+l^2\sin^2\theta}$ |

# Oscillatory signal




# Frequency domain




# Signal decomposition

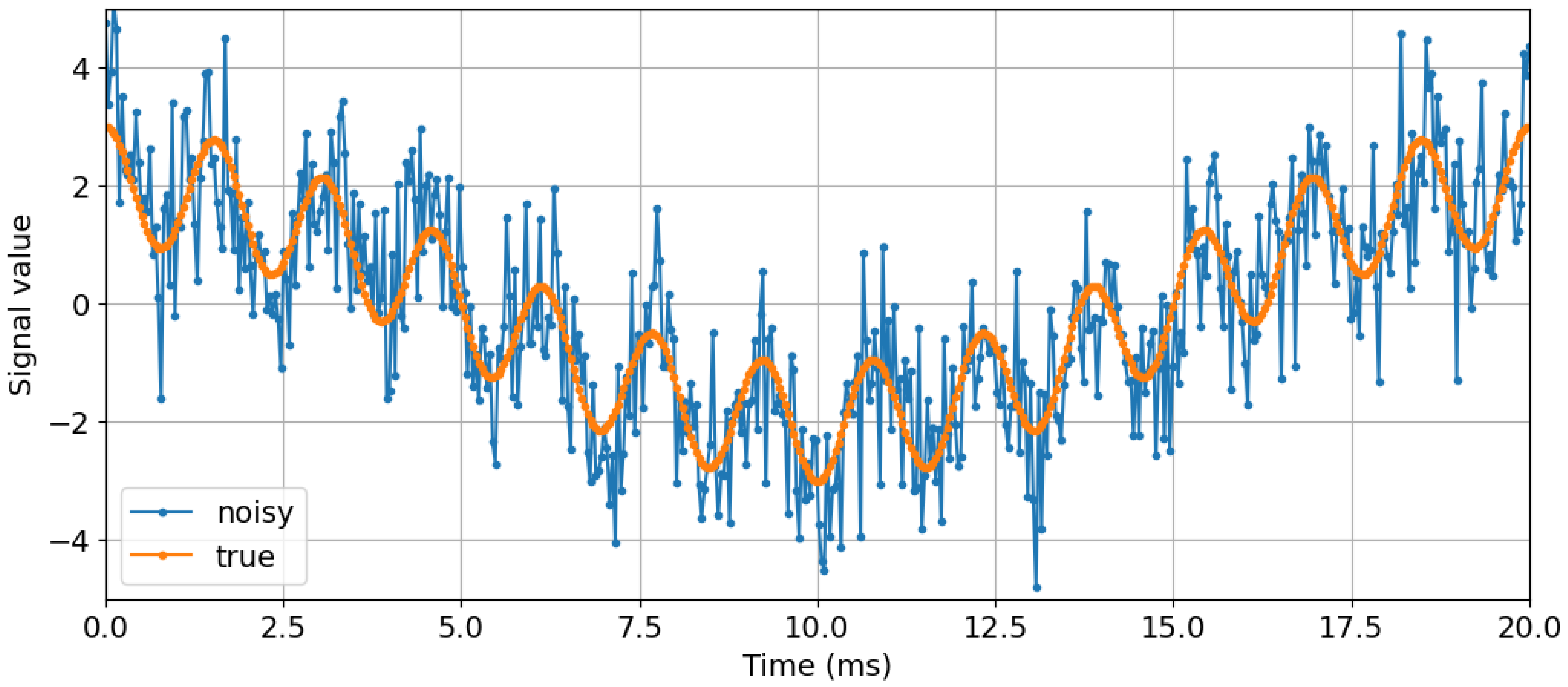
$$y(t) = 2 \cdot \cos(2\pi \cdot 50 \cdot t) + \cos(2\pi \cdot 650 \cdot t)$$

## Fourier Series (FS)

$$y(t) = Y_0 + \sum_{n=1}^{\infty} [A_n \cos(n\omega t) + B_n \sin(n\omega t)]$$

$$y[k] = Y_0 + \sum_{n=1}^{\infty} [A_n \cos(n\omega \cdot \Delta t \cdot k) + B_n \sin(n\omega \cdot \Delta t \cdot k)]$$

## Oscillatory signal




## Signal decomposition

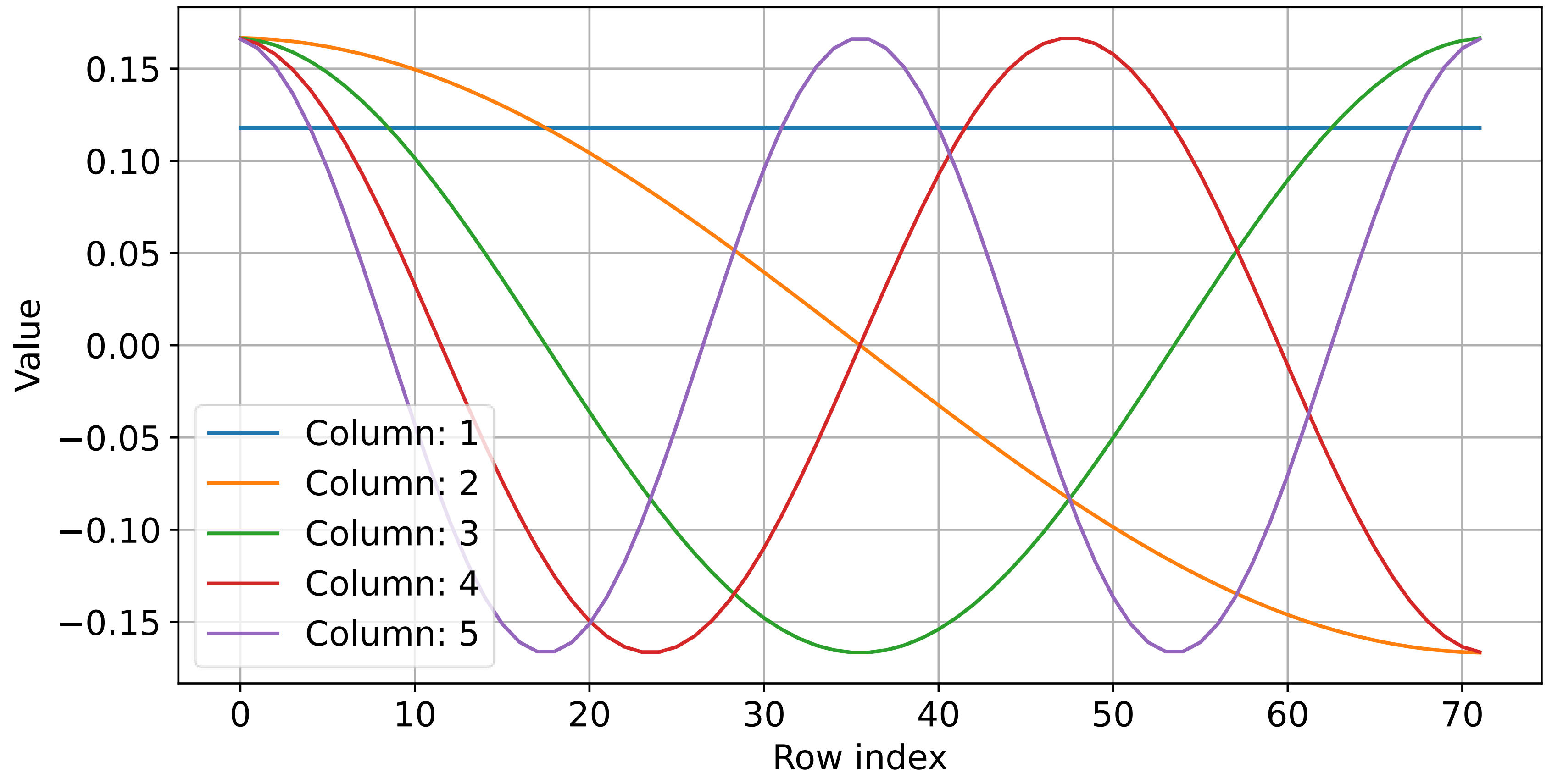
$$y(t) = 2 \cdot \cos(2\pi \cdot 50 \cdot t) + \cos(2\pi \cdot 650 \cdot t)$$

## Fourier Series (FS)

$$y(t) = Y_0 + \sum_{n=1}^{\infty} [A_n \cos(n\omega t) + B_n \sin(n\omega t)]$$

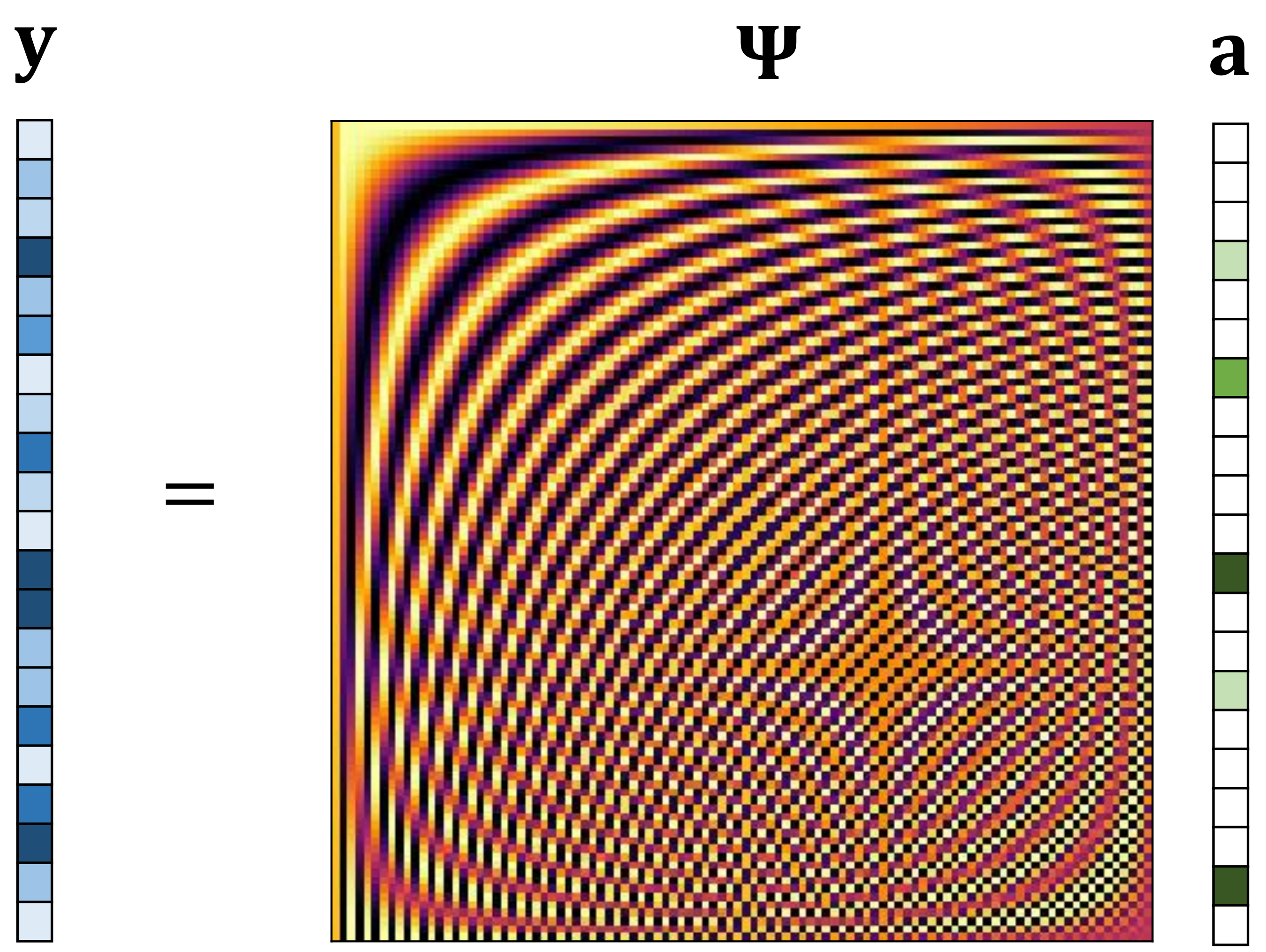
$$y[k] = Y_0 + \sum_{n=1}^{\infty} [A_n \cos(n\omega \cdot \Delta t \cdot k) + B_n \sin(n\omega \cdot \Delta t \cdot k)]$$

## Discrete Cosine Transform (DCT) basis: $\boldsymbol{\Psi}$

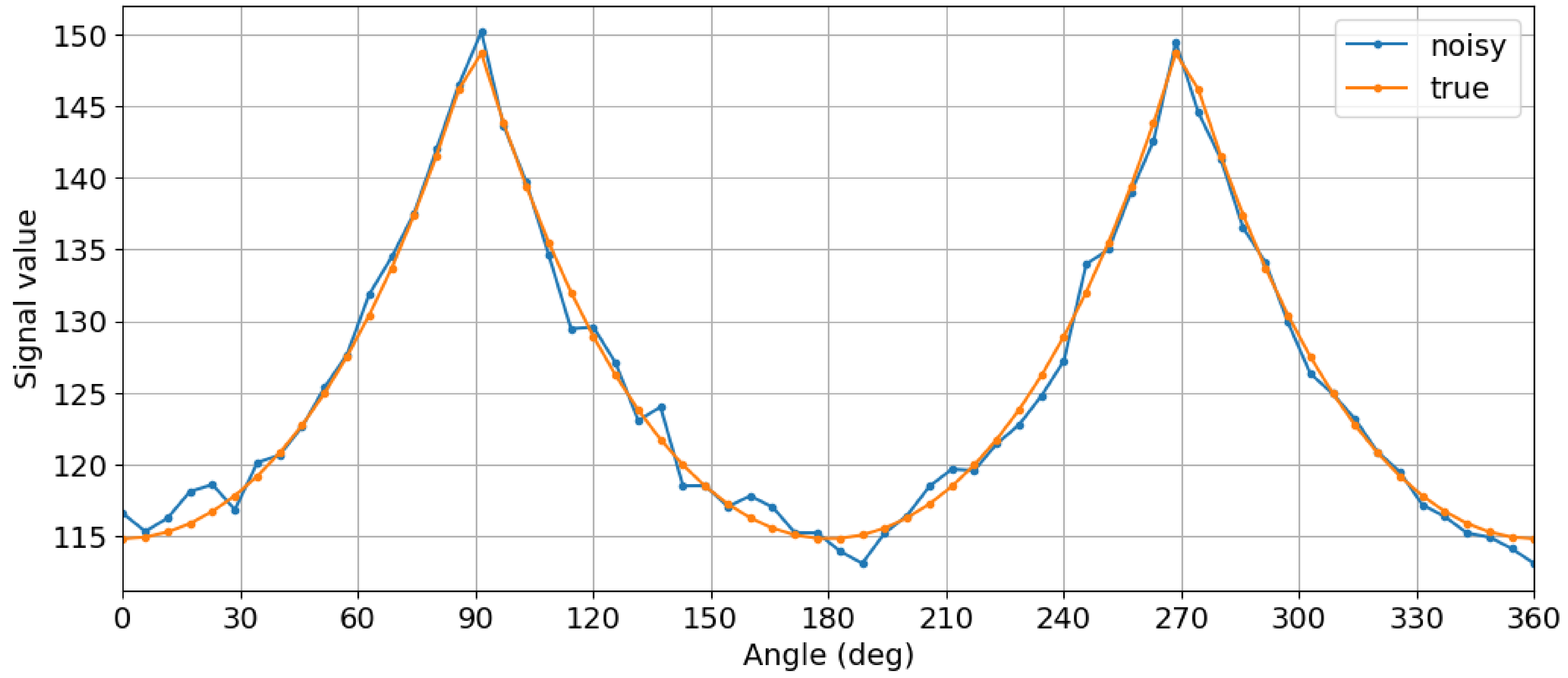



Signal reconstruction:

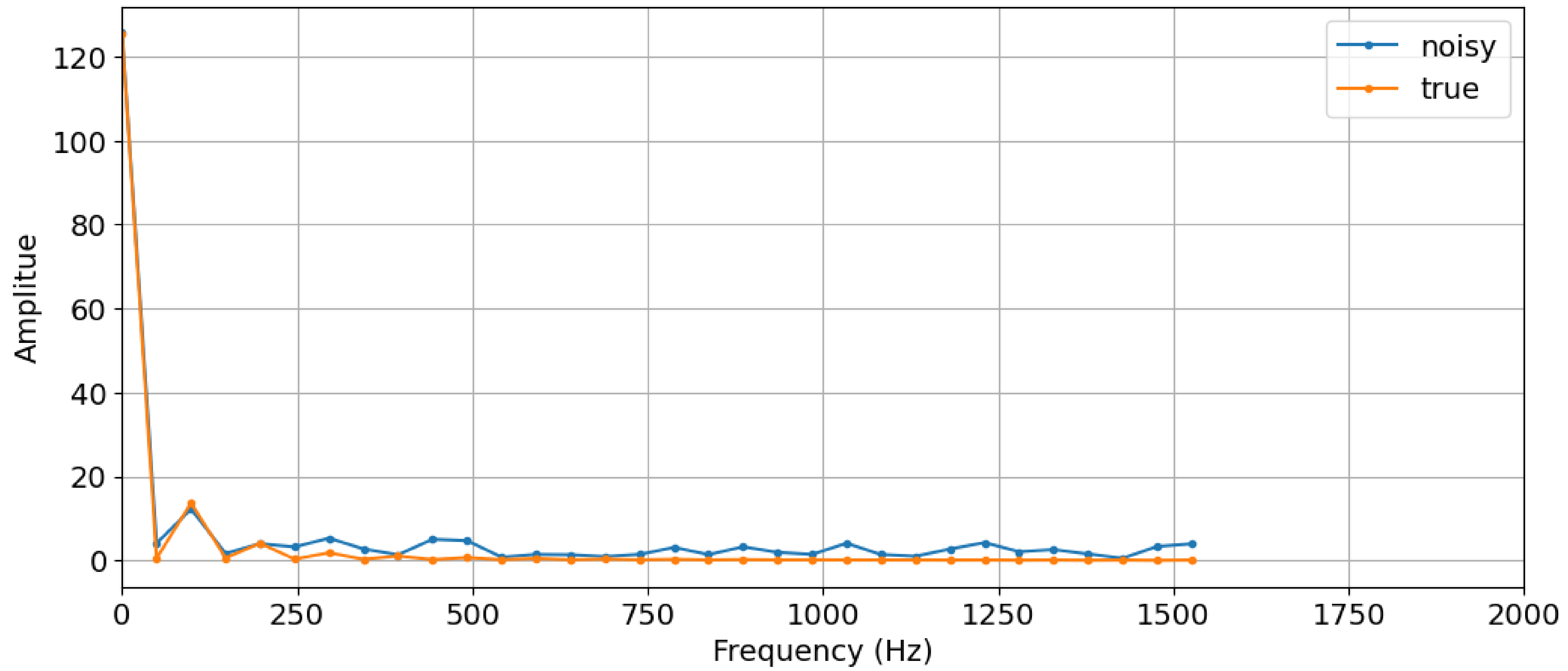
$$\mathbf{y} = \boldsymbol{\Psi}\mathbf{a}$$



## HTS tape measurement    Signal decomposition




## Frequency domain




HTS MODELLING WORKGROUP

KENER

Some observations:

- Difficulty of making fine measurements.
- Nyquist-Shannon sampling theorem limits amount of information, however...
- The amplitudes are not sparse at all!
- What if sines and cosines are not the best functions (basis) for describing the problem?

# Finding optimal basis – snapshot matrix



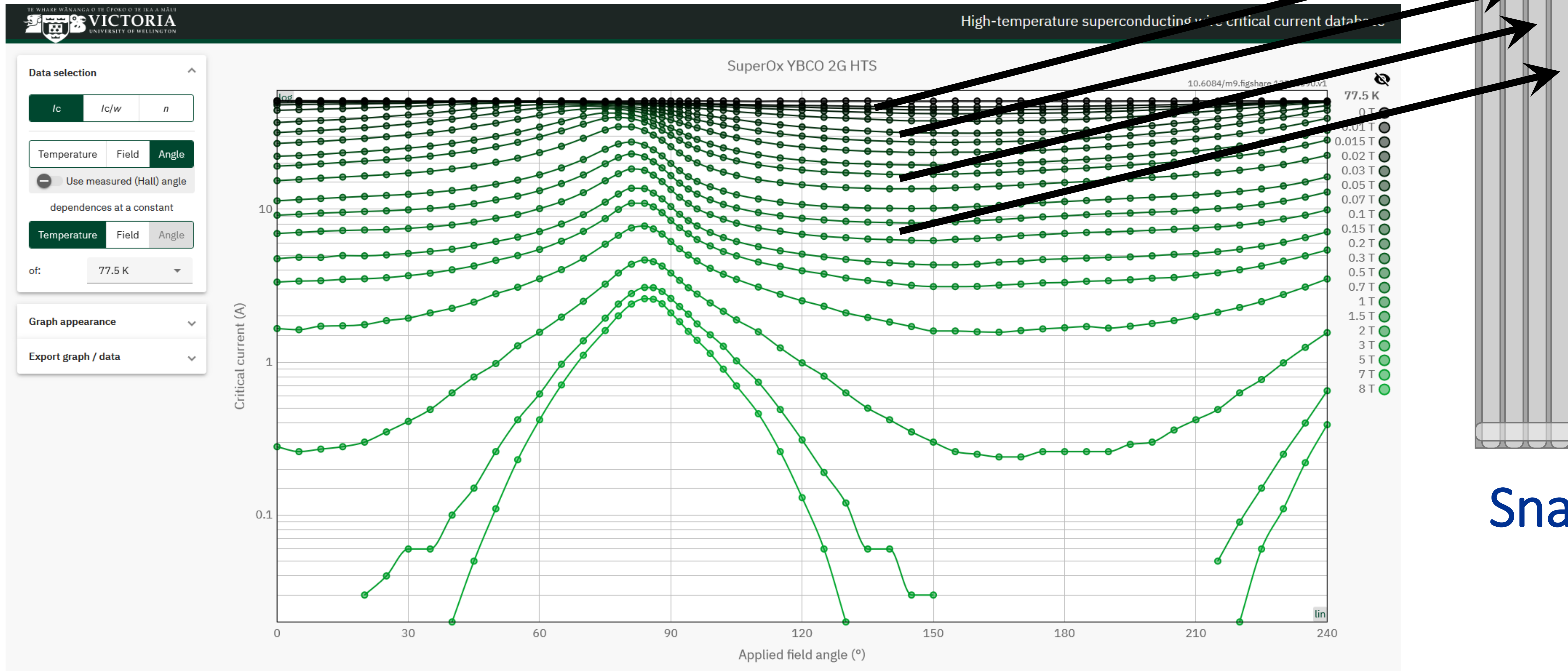
https://htsdb.wimbush.eu/ - **High-temperature superconducting wire critical current database**

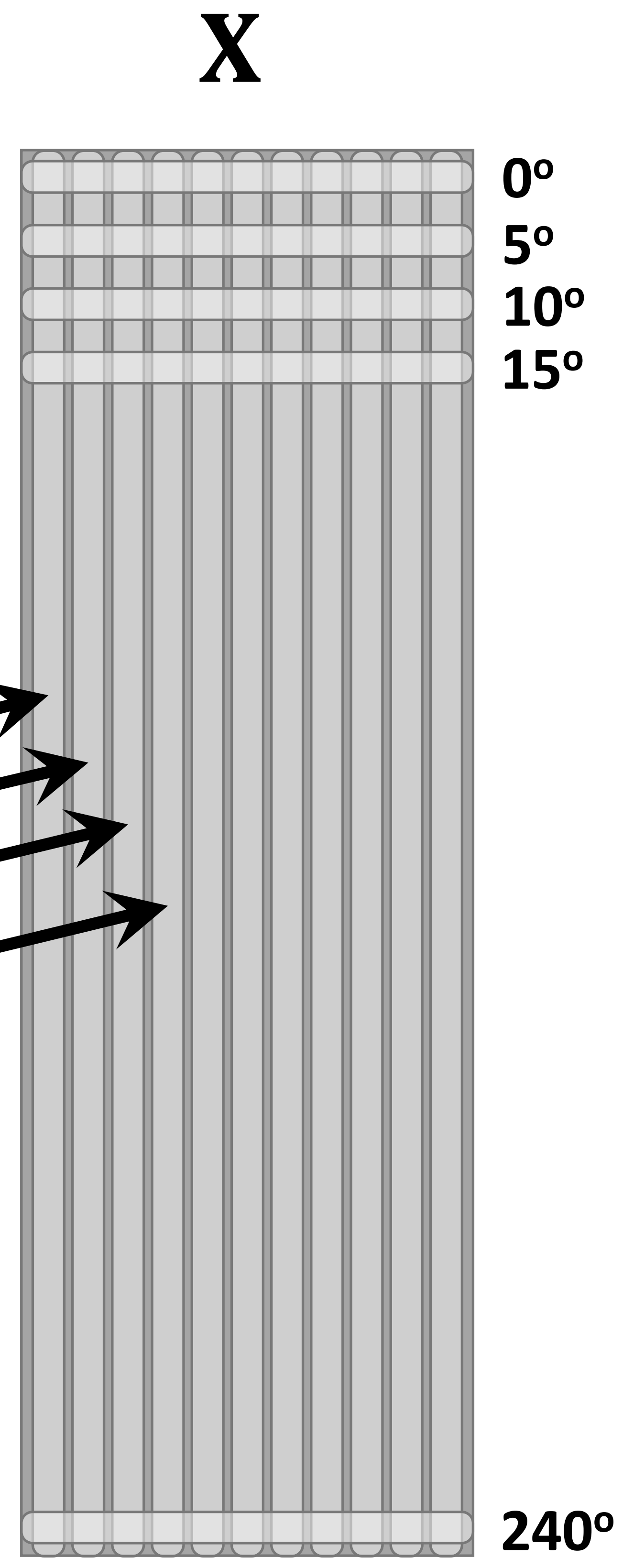
## Data base content:

- 22 HTS tapes – 1G and 2G
- $I_c(\theta; \mathbf{B}; T)$
- $n(\theta; \mathbf{B}; T)$
- Temperature range: 15K – 90K
- Field range: 0T – 8T
- Angle range: 0° – 240° (49 points)
- 5000 valid data sets

## Constitutive law:

$$U = U_c \left( \frac{I}{I_c(\theta; \mathbf{B}; T)} \right)^{n(\theta; \mathbf{B}; T)}$$




Snapshots

# Singular value decomposition (SVD)

$$\mathbf{X} = \mathbf{U} \cdot \mathbf{\Sigma} \cdot \mathbf{V}^{\mathrm{T}}$$

HTS MODELLING WORKGROUP

KENER

Properties:

$$\mathbf{U}^{\mathrm{T}}\mathbf{U} = \mathbf{I}$$

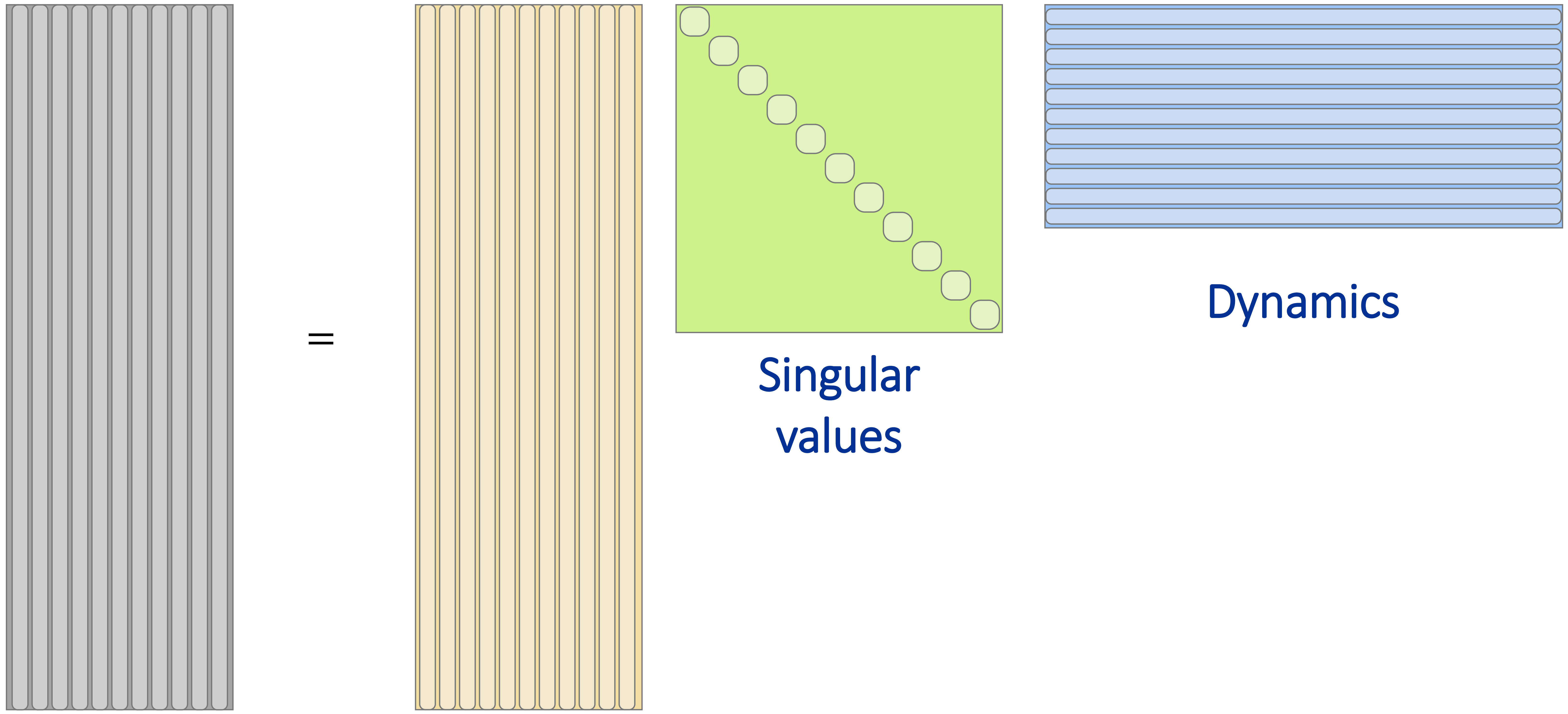
$$\mathbf{U}\mathbf{U}^{\mathrm{T}} = \mathbf{I}$$

$$\mathbf{V}^{\mathrm{T}}\mathbf{V} = \mathbf{I}$$

$$\mathbf{V}\mathbf{V}^{\mathrm{T}} = \mathbf{I}$$

$\mathbf{U}$ and $\mathbf{V}$ are orthonormal!

$\mathbf{X} \in \mathbb{R}^{n \times m}$ = $\mathbf{U} \in \mathbb{R}^{n \times m}$ $\mathbf{\Sigma} \in \mathbb{R}^{n \times n}$ $\mathbf{V}^{\mathrm{T}} \in \mathbb{R}^{m \times n}$



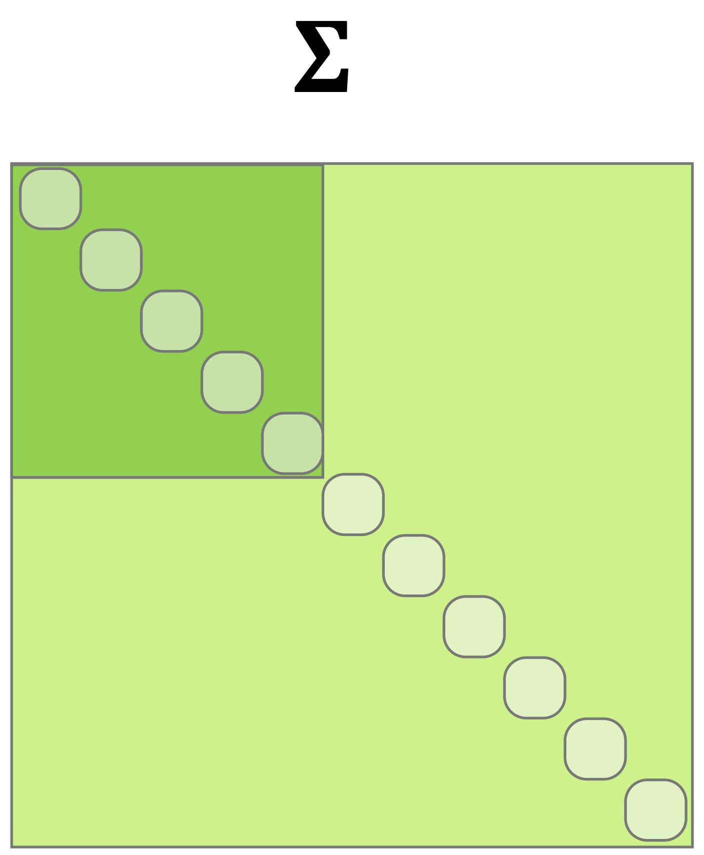
Snapshots

Modes

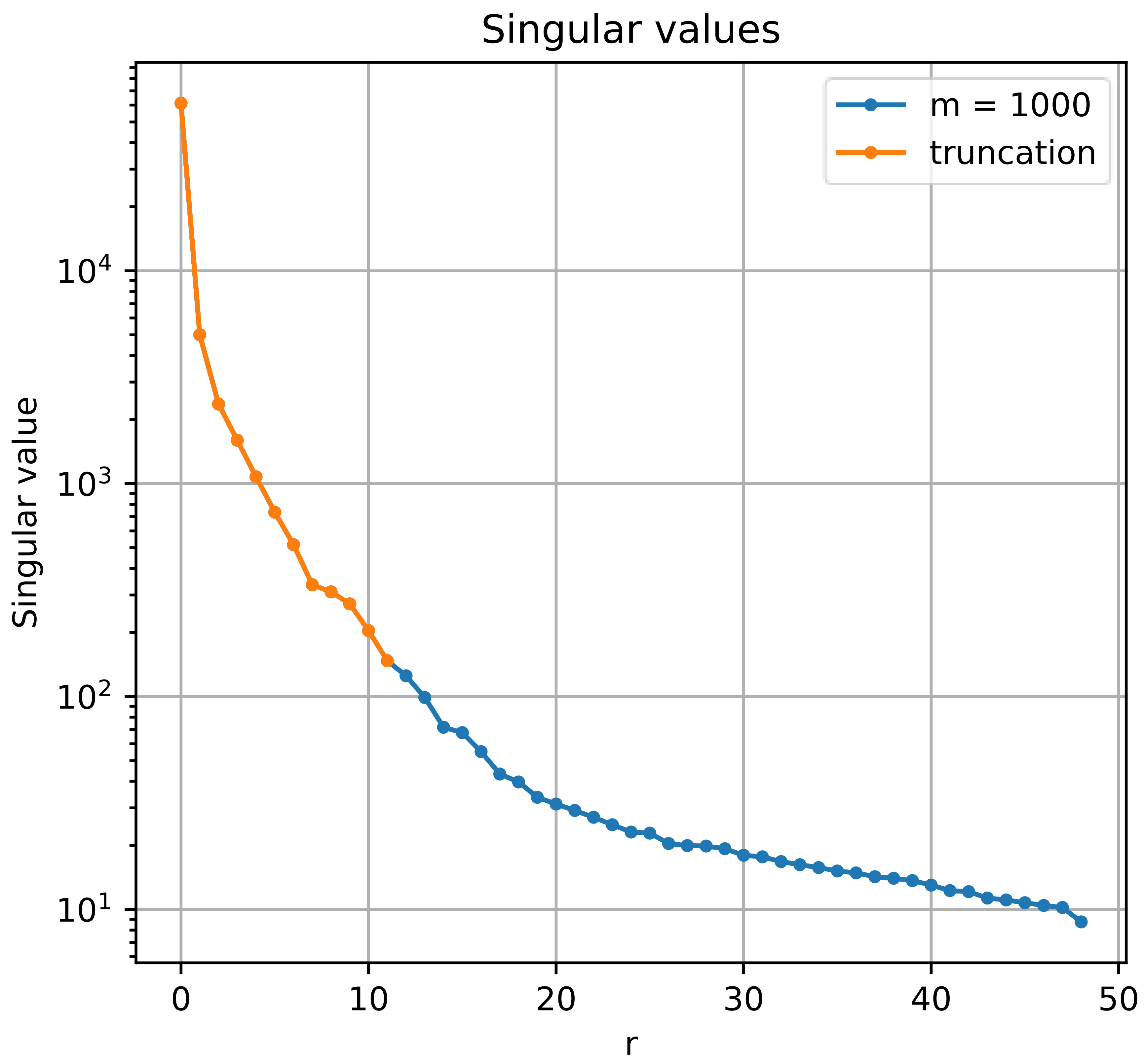
Singular values

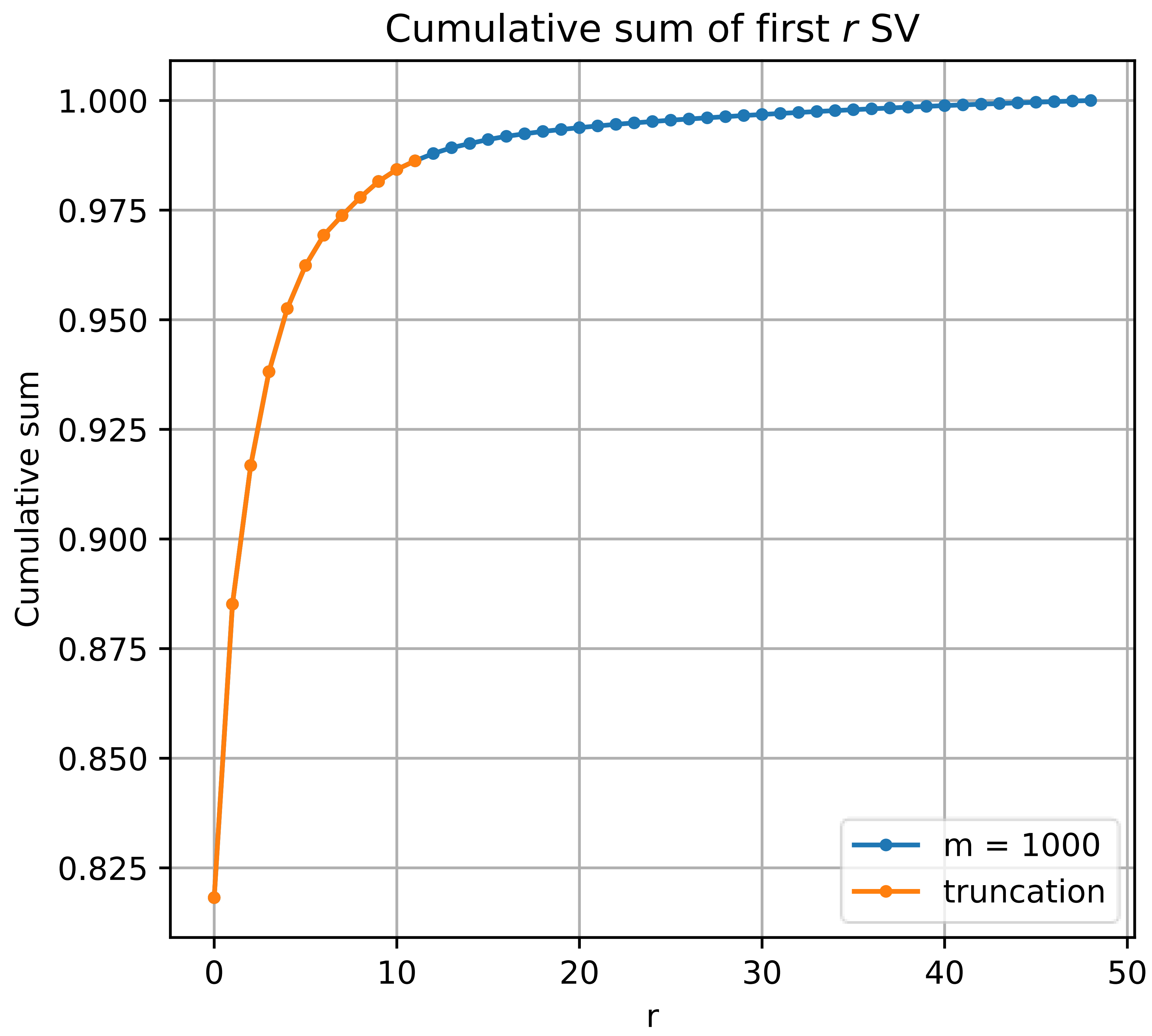
Dynamics

# Singular values truncation




Singular values



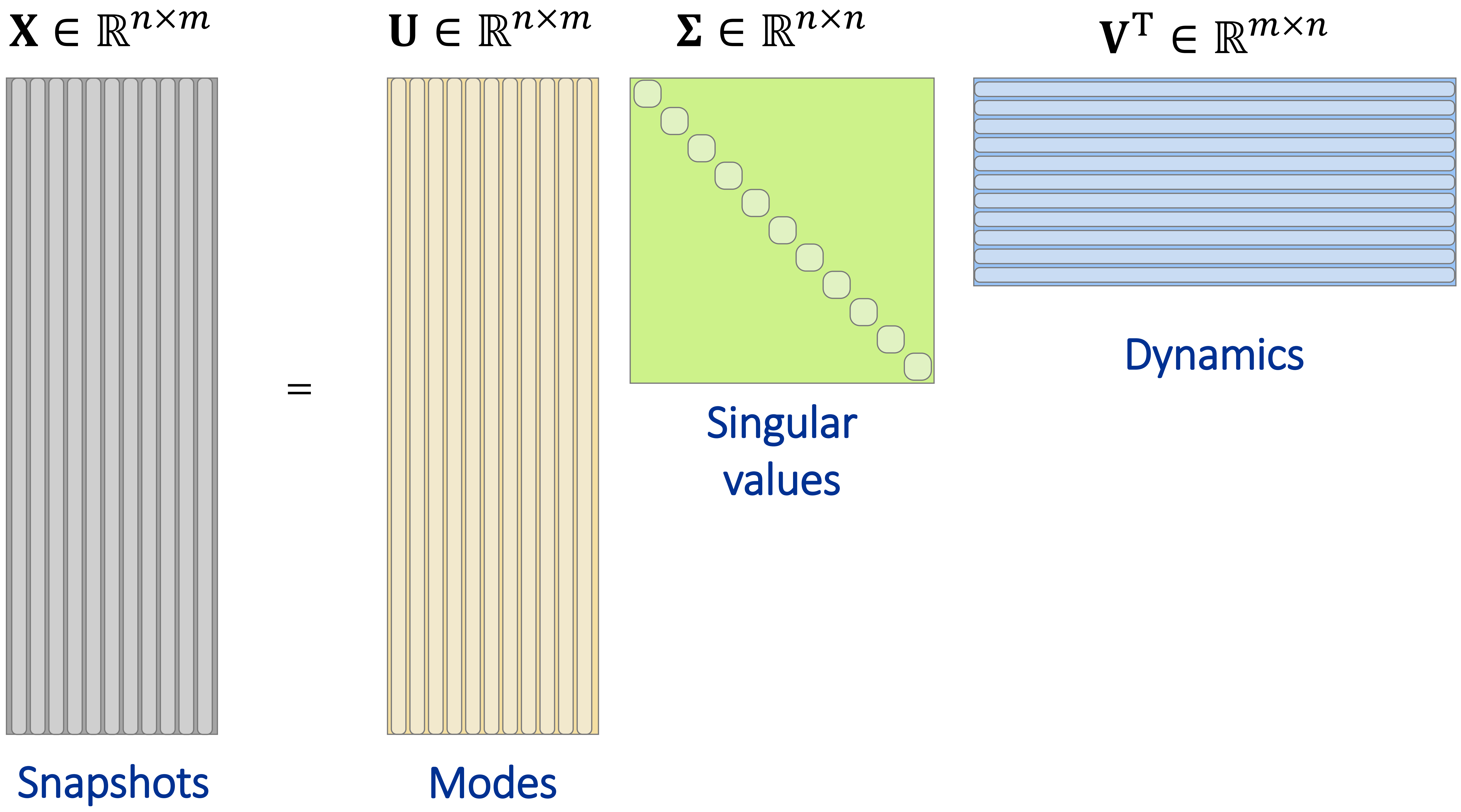




$$S(r) = \frac{\sum_{i=0}^{r-1} \sigma_i}{\sum_{i=0}^{n-1} \sigma_i}$$

**The optimal hard threshold for singular values is 4/√3**, by M. Gavish and D. L. Donoho, IEEE Transactions on Information Theory, 2014

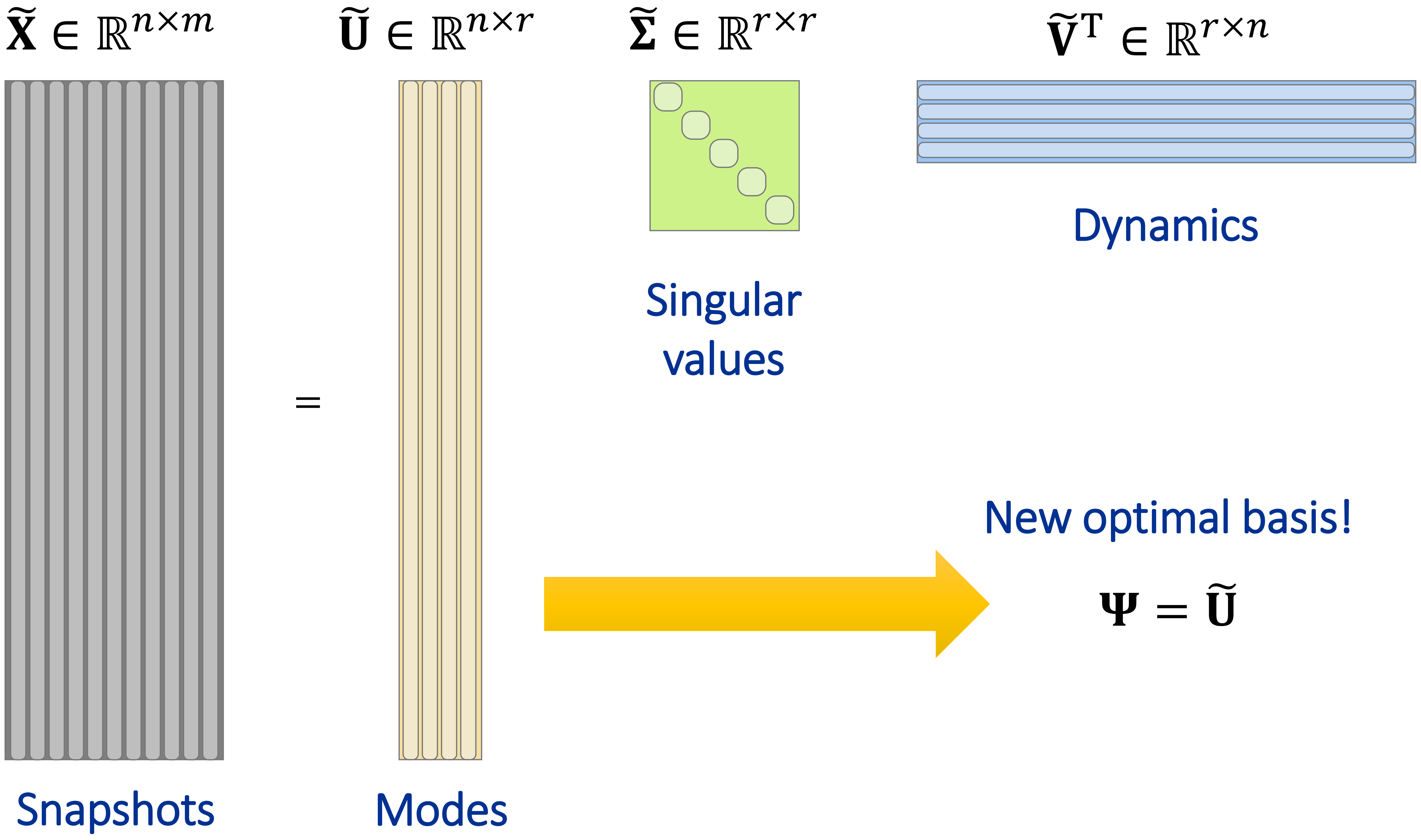
# Singular value decomposition (SVD)

$$\mathbf{X} = \mathbf{U} \cdot \mathbf{\Sigma} \cdot \mathbf{V}^{\mathrm{T}}$$



# Singular value decomposition (SVD)

HTS MODELLING WORKGROUP
KENER




New optimal basis!

$$\mathbf{\Psi} = \tilde{\mathbf{U}}$$

$$\mathbf{X} = \mathbf{U}\cdot\mathbf{\Sigma}\cdot\mathbf{V}^{\mathrm{T}}$$ Full matrix

$$\tilde{\mathbf{X}} = \tilde{\mathbf{U}}\cdot\tilde{\mathbf{\Sigma}}\cdot\tilde{\mathbf{V}}^{\mathrm{T}}$$ Full matrix approximation

$$\mathbf{A} = \tilde{\mathbf{\Sigma}}\cdot\tilde{\mathbf{V}}^{\mathrm{T}}$$

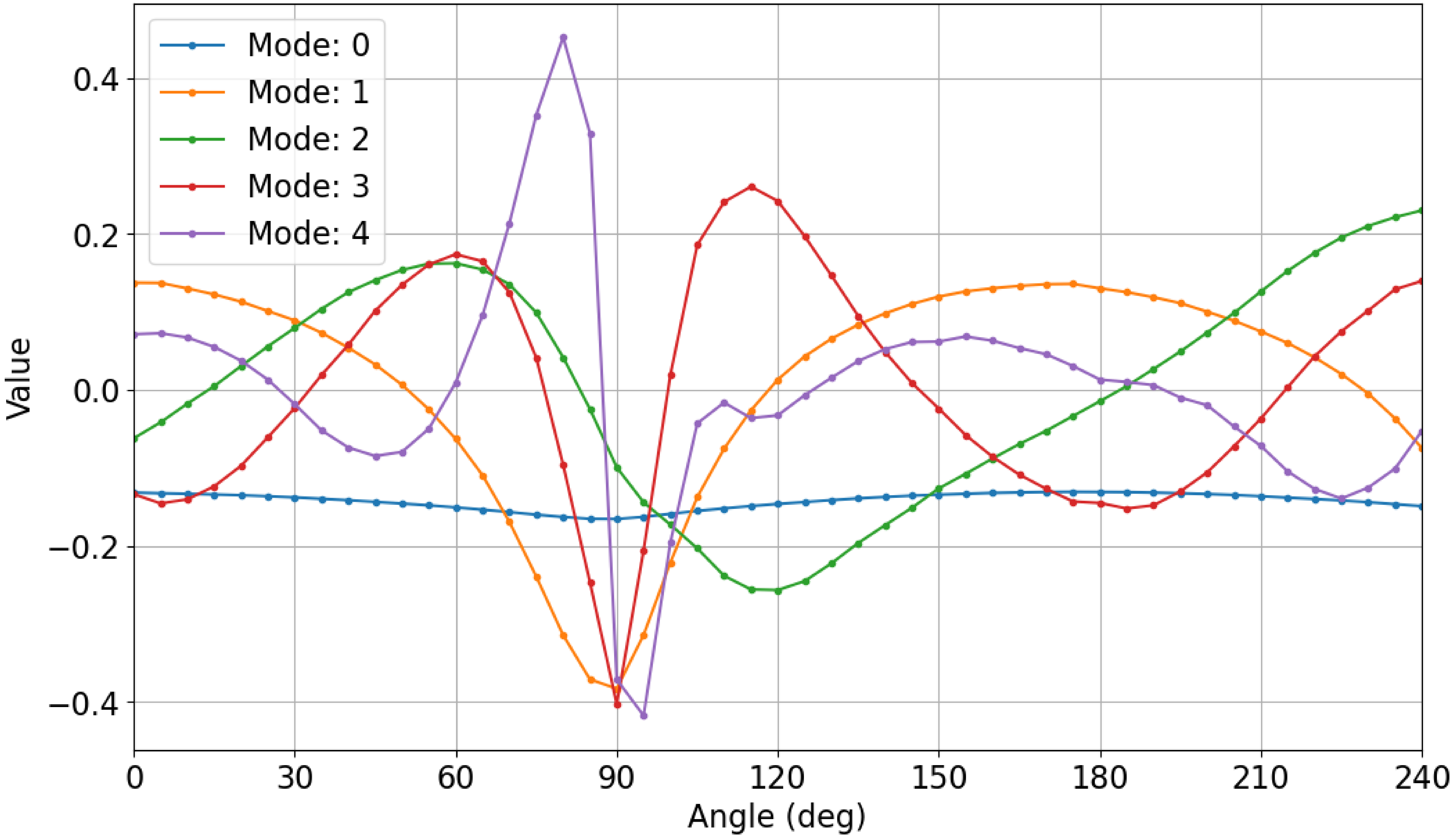
$$\tilde{\mathbf{X}} = \tilde{\mathbf{U}}\cdot\mathbf{A}$$ Approximation by amplitudes

$$\tilde{\mathbf{x}} = \tilde{\mathbf{U}}\cdot\mathbf{a}$$ Single column approximation

We've seen that before…

$$\mathbf{y} = \mathbf{\Psi}\cdot\mathbf{a}$$

# Tailored (optimal) basis: $\Psi$



# Sparse measurements
## optimal measurement points

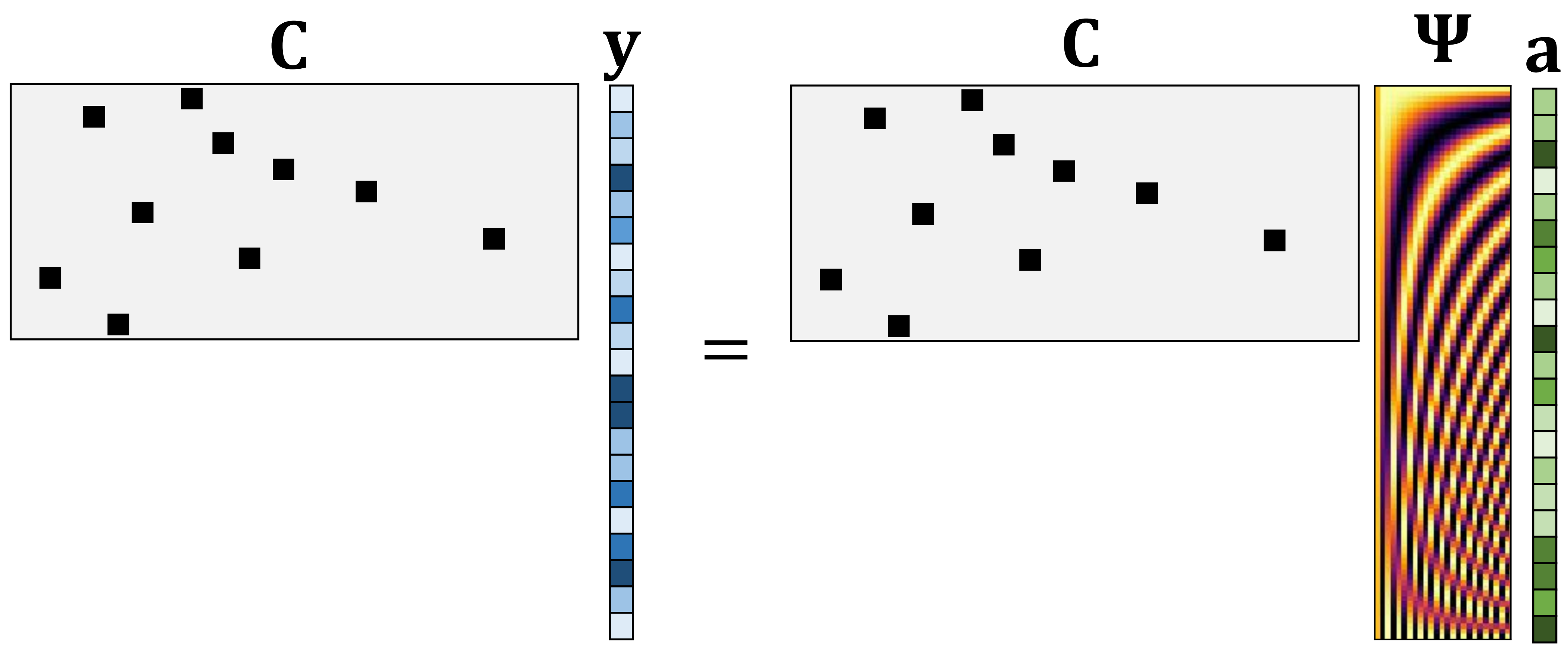
Full measurements $y$: $\mathbf{y} = \mathbf{\Psi} \cdot \mathbf{a}$

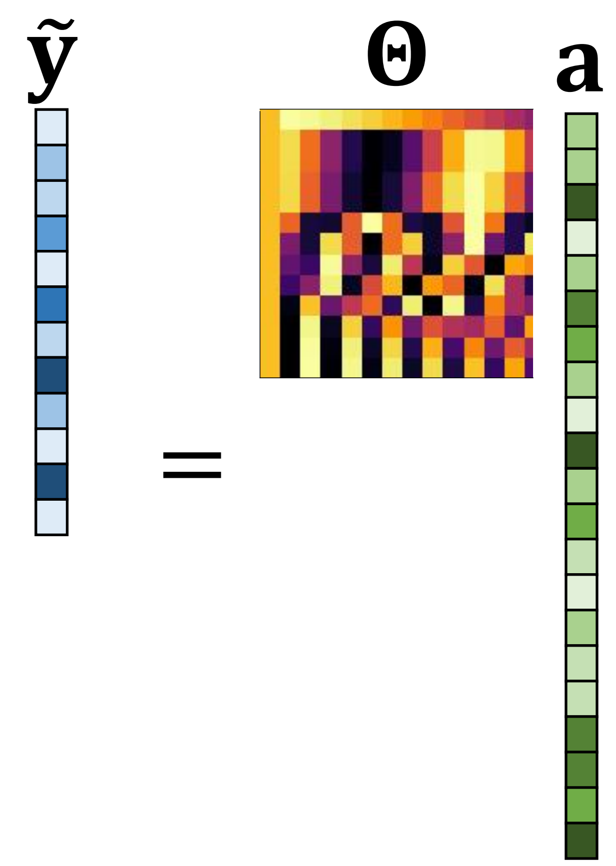
Primary task: Find measurement matrix $\mathbf{C}$ that is sparse and rank $r$.

Matrix $\mathbf{\Theta}$ is a square matrix!

$$\mathbf{C} \cdot \mathbf{y} = \mathbf{C} \cdot \mathbf{\Psi} \cdot \mathbf{a}$$

$$\tilde{\mathbf{y}} = \mathbf{\Theta} \cdot \mathbf{a}$$



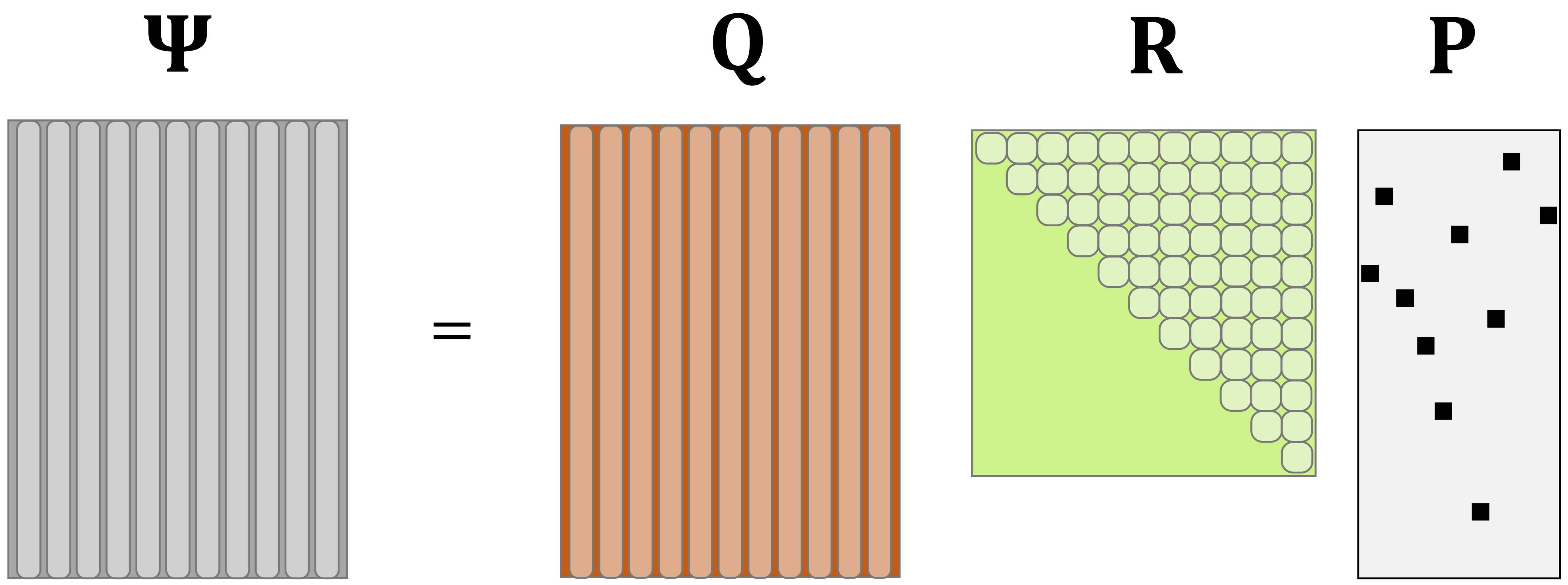




Matrix $\mathbf{\Theta}$ is a square matrix!

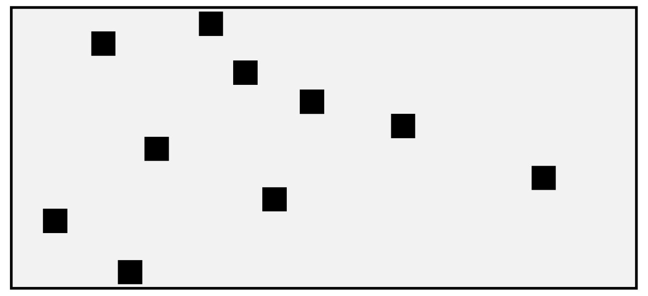
# Sparse measurements
# optimal measurement points

$$\mathbf{\Psi} = \mathbf{Q} \cdot \mathbf{R} \cdot \mathbf{P}$$

QR-decomposition with pivoting
(rank-revealing QR-decomposition)




$$\mathbf{C} = \mathbf{P}^T$$



Number of points equal to $r$

0º, 40º, 75º, 85º, 90º, 95º, 100º, 105º, 115º, 135º, 195º, 240º

Calculate amplitudes of modes

$$\mathbf{a} = \mathbf{\Theta}^{-1} \cdot \tilde{\mathbf{y}}$$

Reconstruct signal

$$\mathbf{y} = \mathbf{\Psi} \cdot \mathbf{a}$$

# Sparse measurements – algorithm

1. Create snapshot martix $\mathbf{X}$
2. Singular Value Decomposition $\mathbf{X} = \mathbf{U} \cdot \boldsymbol{\Sigma} \cdot \mathbf{V}^{\mathrm{T}}$
3. SVD truncation $\tilde{\mathbf{X}} = \tilde{\mathbf{U}} \cdot \tilde{\boldsymbol{\Sigma}} \cdot \tilde{\mathbf{V}}^{\mathrm{T}}$
4. Make new basis $\boldsymbol{\Psi} = \tilde{\mathbf{U}}$
5. Find optimal measurement points $\boldsymbol{\Psi} = \mathbf{Q} \cdot \mathbf{R} \cdot \mathbf{P}$ → $\mathbf{C} = \mathbf{P}^{\mathrm{T}}$
6. Calculate reconstruction matrix $\boldsymbol{\Theta} = \mathbf{C} \cdot \boldsymbol{\Psi}$
7. Perform measurements at optimal angles $\tilde{\mathbf{y}}$
8. Find amplitudes $\mathbf{a} = \boldsymbol{\Theta}^{-1}\tilde{\mathbf{y}}$
9. Reconstruct $\mathbf{y} = \boldsymbol{\Psi}\mathbf{a}$

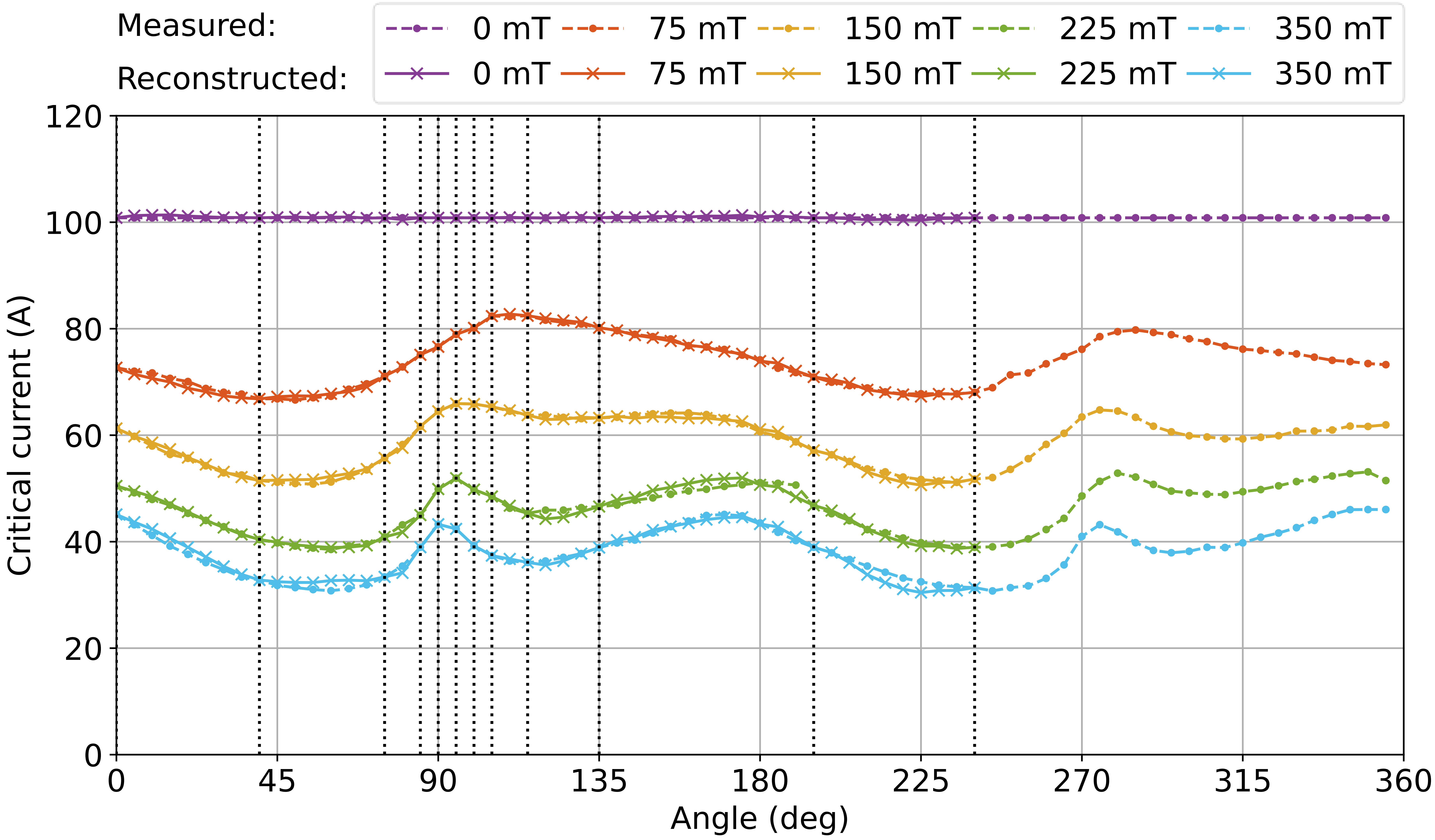
Compute once – save and load when needed!

Measure

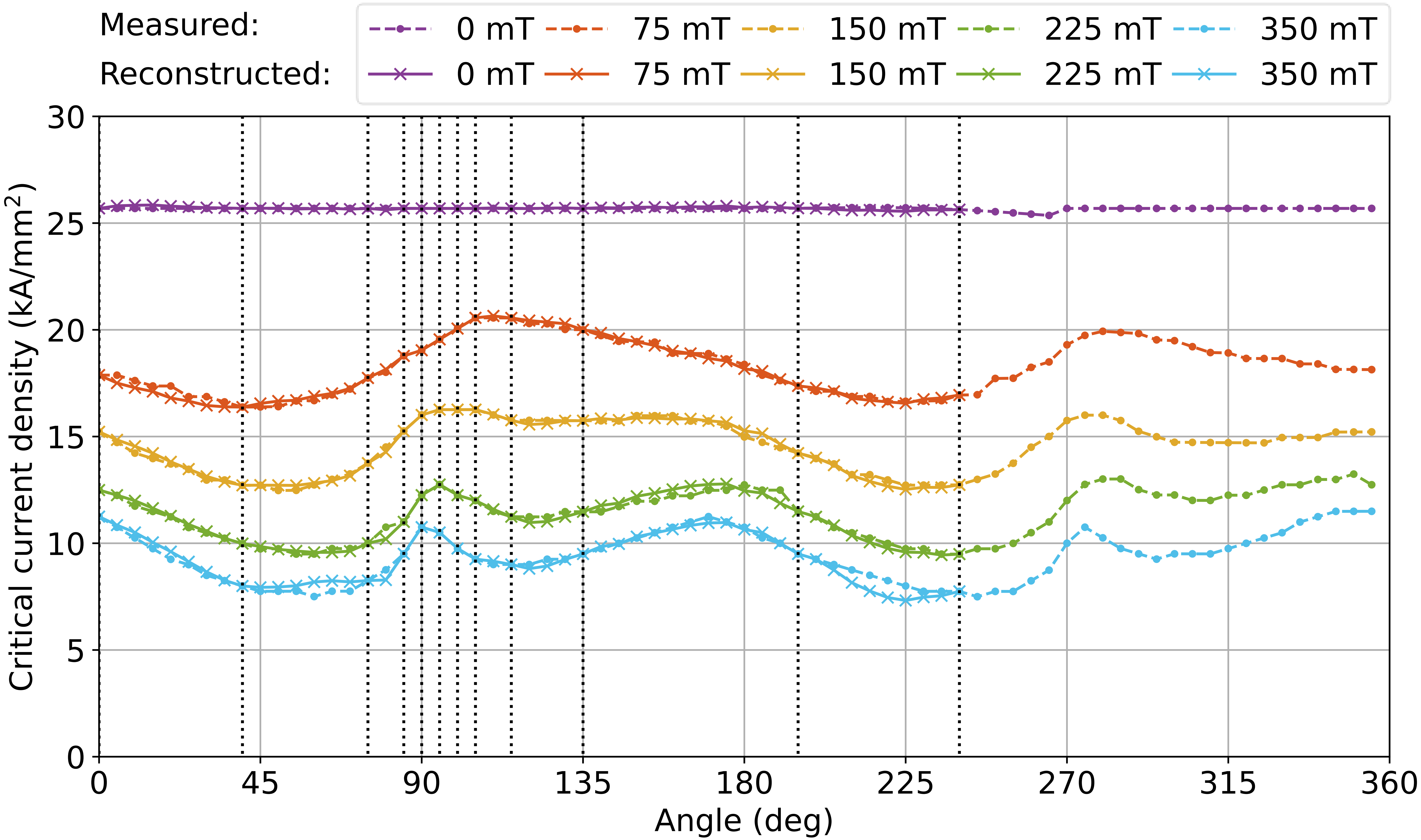
Compute every time

# Sparse measurements - results

## Critical current



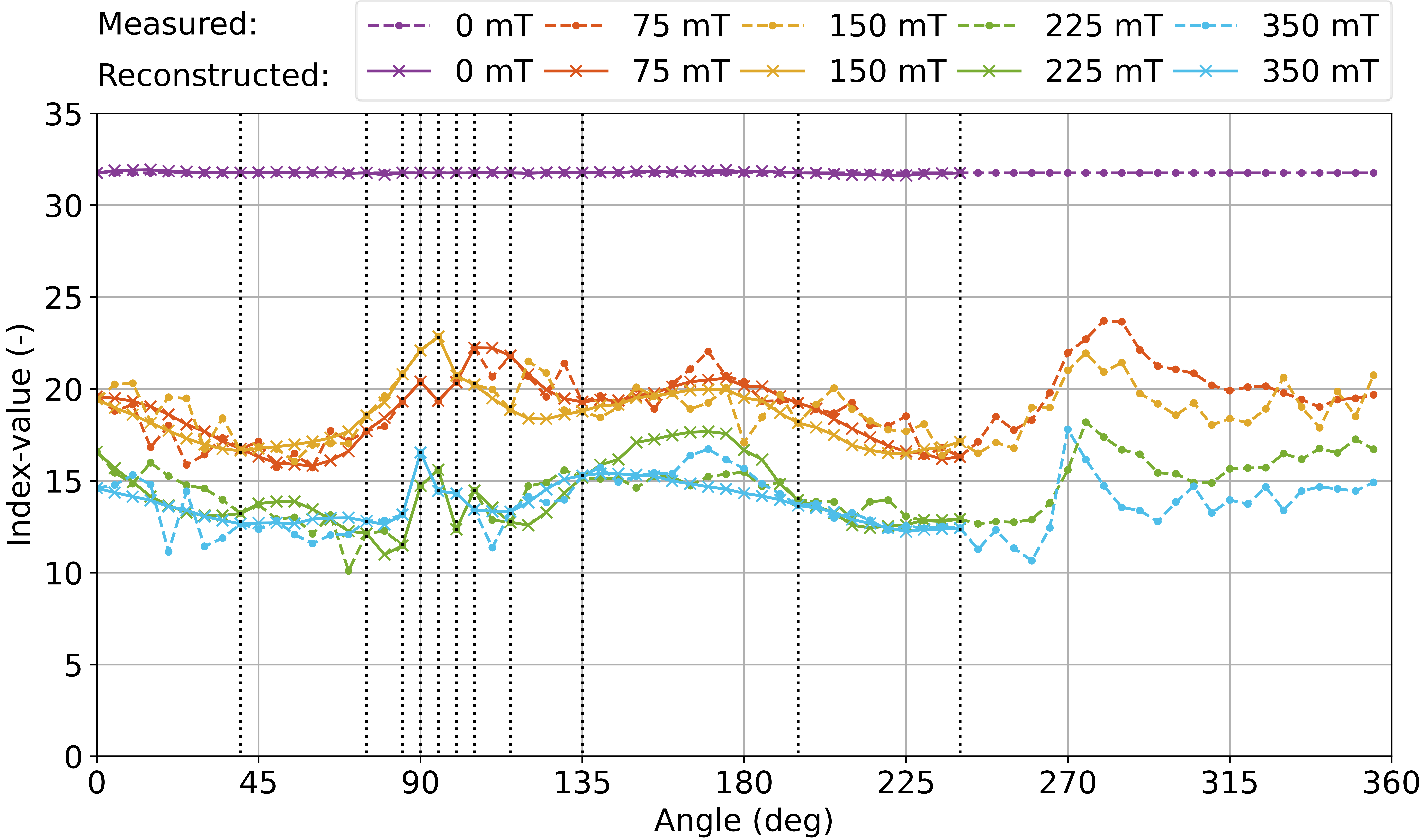
# Sparse measurements - results
## Critical current density




Current density obtained using method described in: Zermeño, V. M., Habelok, K., Stępień, M., & Grilli, F. (2017). A parameter-free method to extract the superconductor's Jc (B, θ) field-dependence from in-field current–voltage characteristics of high temperature superconductor tapes. *Superconductor Science and Technology*, *30*(3), 034001

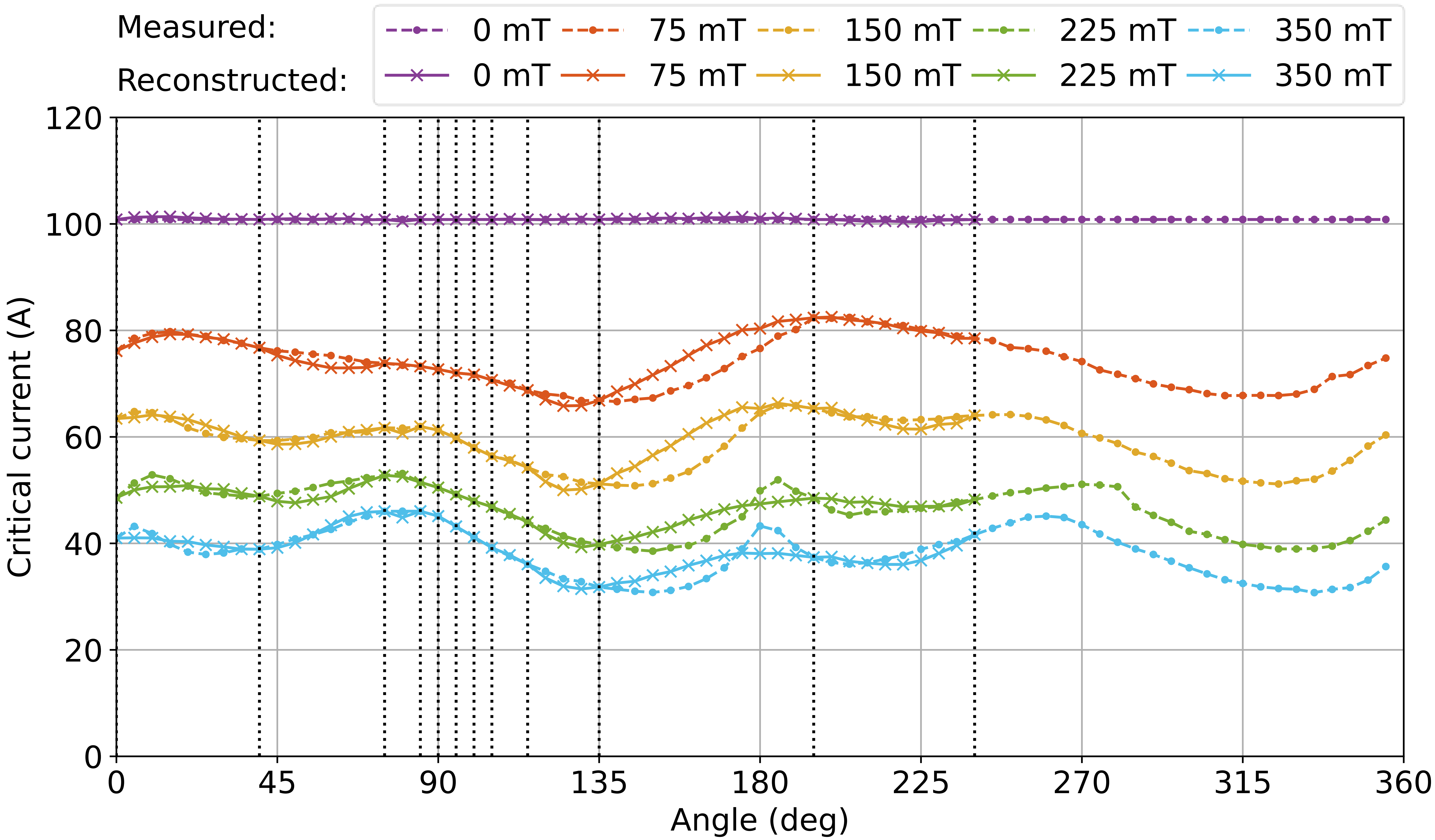
# Sparse measurements - results

## Index-value $n$



# Sparse measurements - results

## Critical current (shifted by 90°)



# Summary

- It is possible to reconstruct signal based only on few measurements.
- Function/vector basis can be tailored to the problem.
- Method is sensitive to signal shifts due to chosen basis.
- Span of reconstructed signal depends on span of snapshots and therefore basis.
- The model doesn't provide analytical description i.e. Kim-model: $\mathrm{I_c}(\theta, \mathbf{B}) = I_{c0}\left(1 + \frac{|\mathbf{B}|}{B_{c0}}\varepsilon(\theta)\right)^{-b}$
- Signals can be compared only by mode amplitudes: $\mathbf{a}$
- Model is viable for interpolation only.