hts2024.eu

**HTS Modelling Workshop 2024**

June 10 – 13
Parkhotel Bad Zurzach
Switzerland

# Recent Advancements in BELFEM

**Christian Messe, Frédéric Sirois, Gregory Giard**

**Jun 10th, 2024**

# Outline

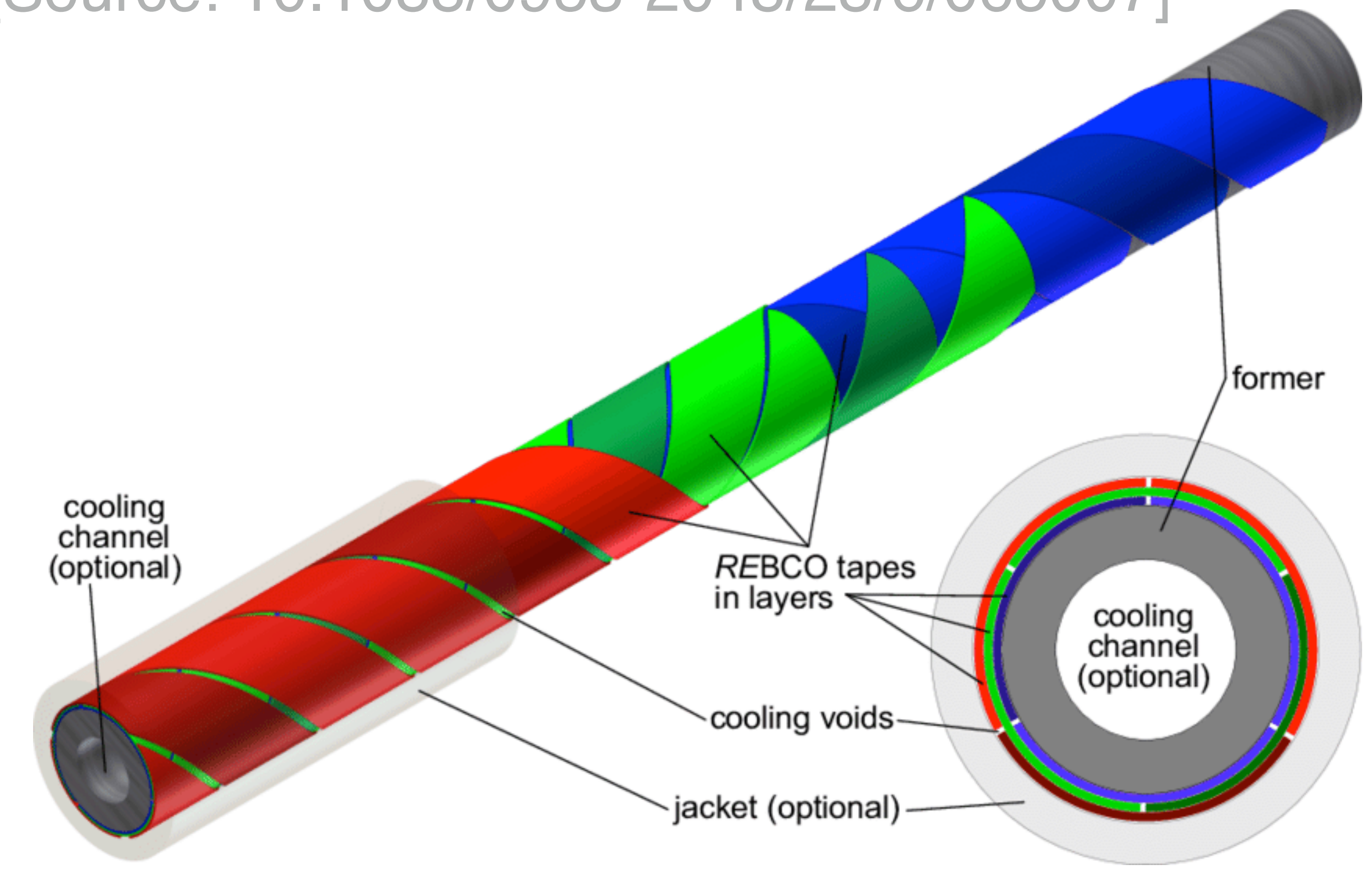
- Motivation: HTS Tape and Cable Modeling
- Maxwell Recap and h-ϕ fundamentals
- Why develop a custom codebase?
- Homologies and Cohomologies
- Last Year’s progress
- Current Efforts
- Summary and Outlook
- Thoughts on Material Modeling

# BELFEM: Motivation and Project Goals

## want ability to model:

- **quasi-magnetodynamic modeling**
  - understand electromagnetic behavior of cables
- **coupled thermal modeling**
  - thermal behavior and physical coupling with EM
  - quench behavior
- **other phenomena**
  - current sharing
  - …

[Source: 10.1088/0953-2048/28/6/065007]



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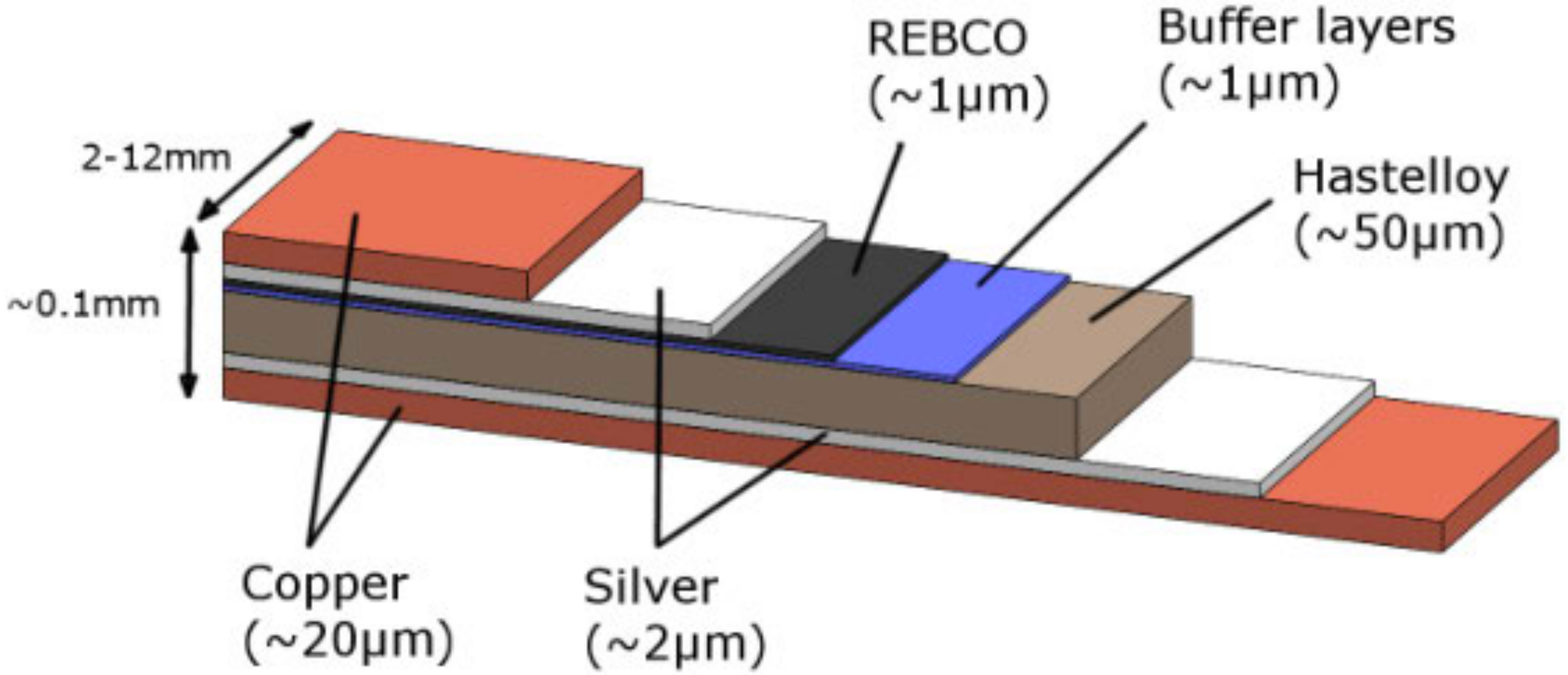
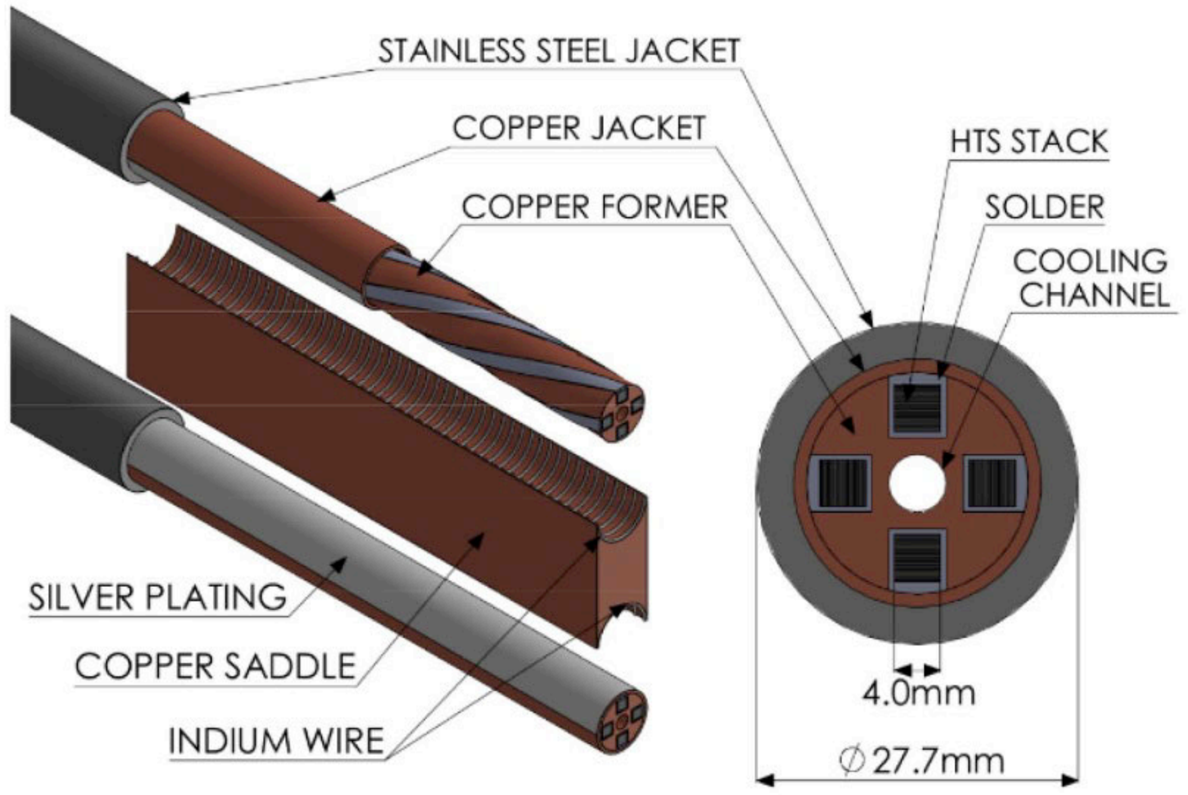




VIPER

[Source: 10.1088/1361-6668/abb8c0]

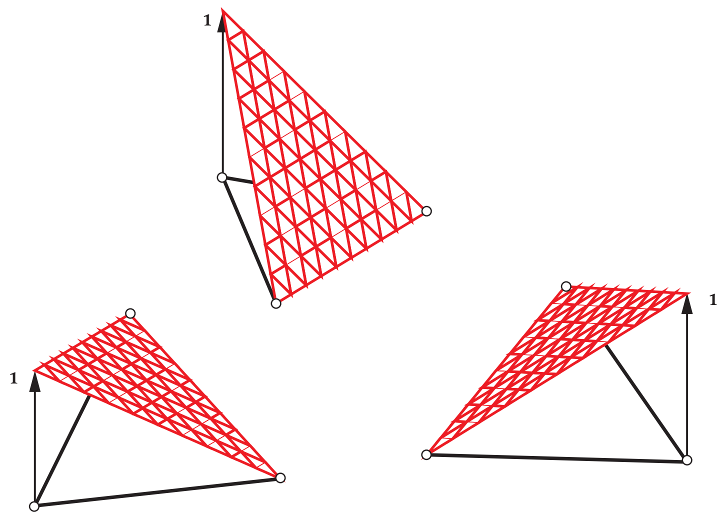
# Recap: Maxwell in FEM

**FEM Weak forms are based on different laws**

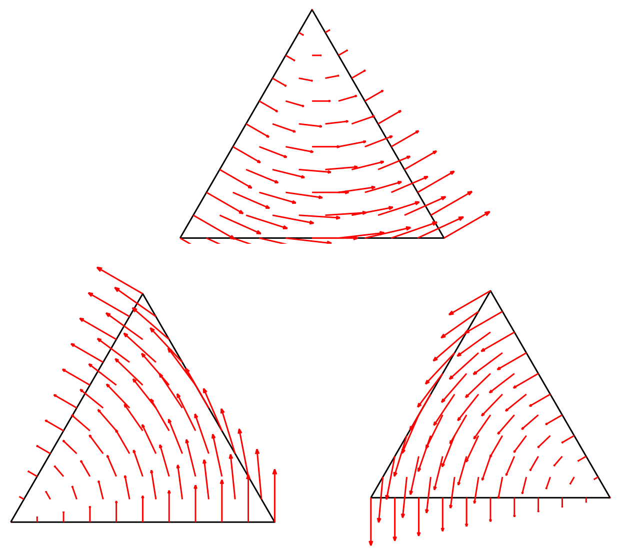
- h-formulation is based on Faraday's law
- a-formulation is based on Ampéré's law
- ϕ-formulation can be based on either Faraday or Gauß

**Mixed formulations:**

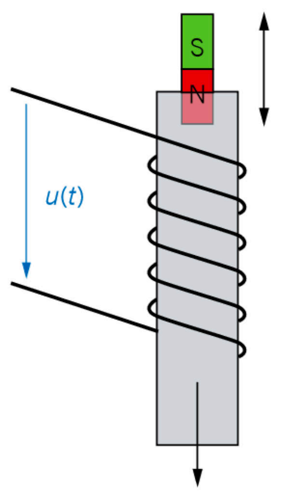
- there is no "universally best" formulation
- mixed formulations aim to combine benefits of individual formulations



*Lagrange Elements*
*($\phi$ formulation)*

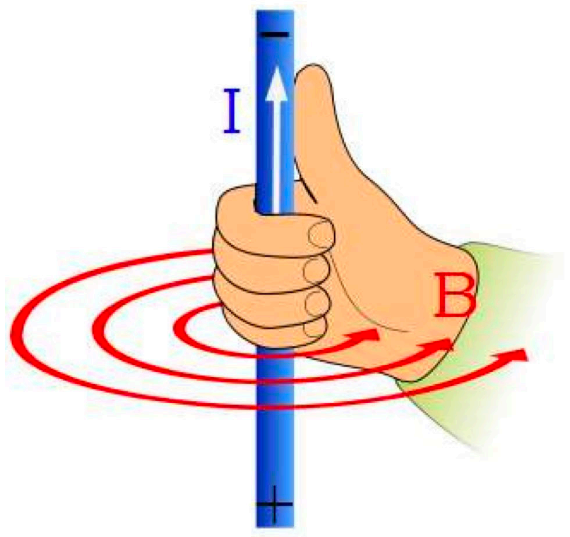


*Nédélec Elements*
*(h-formulation)*



**Faraday's law**

$$\nabla \times \boldsymbol{e} = -\dot{\boldsymbol{b}}$$

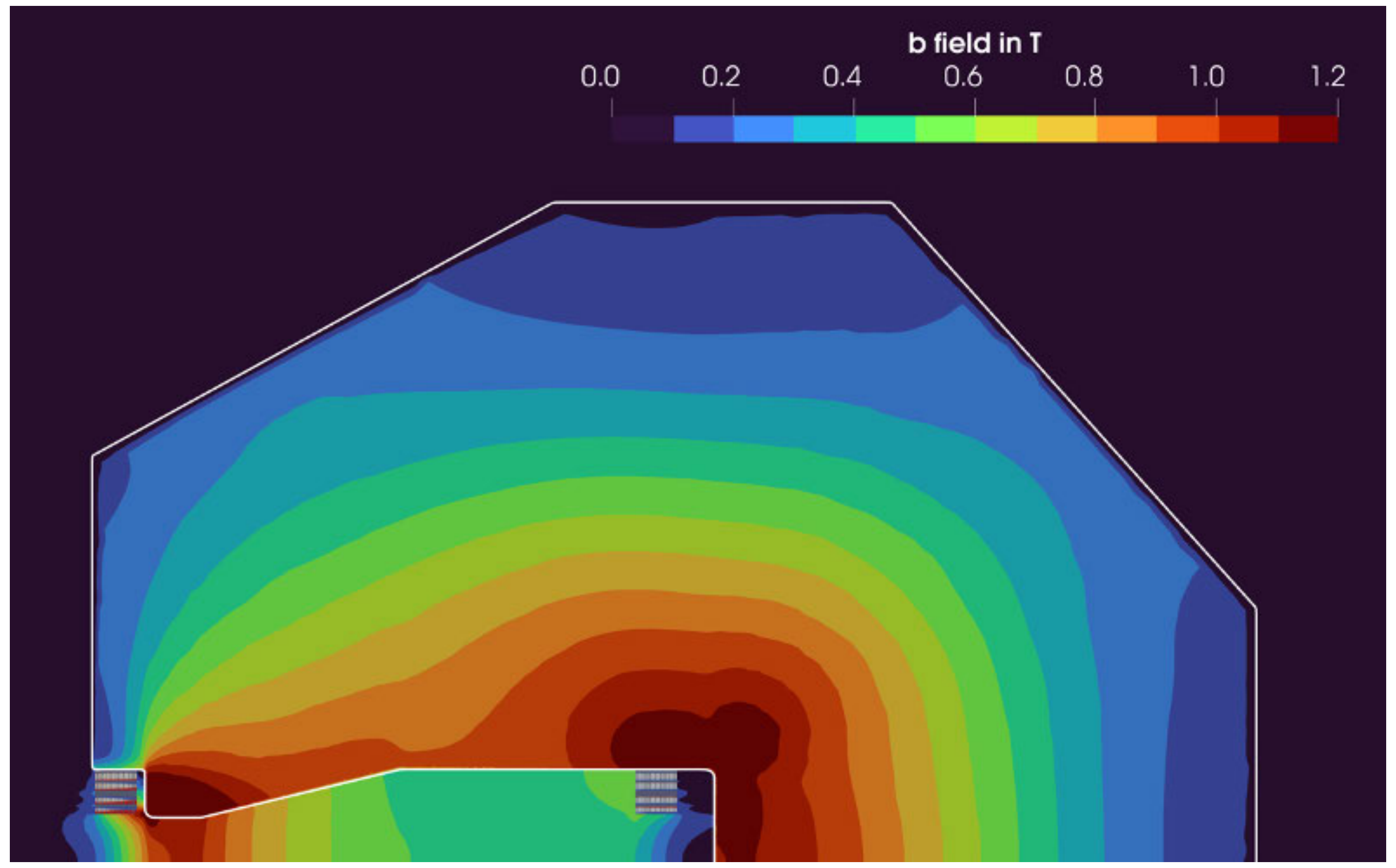


**Ampére's law**

$$\nabla \times \boldsymbol{h} = -\boldsymbol{j}$$

| | Air / Vacuum | Conductor | Ferromagnetic Alloy |
|---|---|---|---|
| **Governing Equation** | $\nabla \times \boldsymbol{h} = \boldsymbol{0}$<br>Ampére-Maxwell | $\nabla \times \boldsymbol{h} = \boldsymbol{j}$<br>Ampére-Maxwell | $\nabla \times \boldsymbol{e} = \dot{\boldsymbol{b}}$<br>Faraday's Law |
| **Degree of Freedom** | $\boldsymbol{h} = -\nabla\phi$<br>Magnetic Scalar Potential | $\boldsymbol{h}$<br>Magnetic Field | $\boldsymbol{b} = \nabla \times \boldsymbol{a}$<br>Magnetic Vector Potential |
| **Transport Law** | none | $\boldsymbol{e} = \boldsymbol{\rho} \cdot \boldsymbol{j}$<br>Ohm's Law | $\boldsymbol{h} = \boldsymbol{\nu} \cdot \boldsymbol{b}$<br>Magnetic Law |
| **Comment** | minimal number of dofs | need edge elements | simple material law implementation |

# H-ϕ formulation: Fundamentals

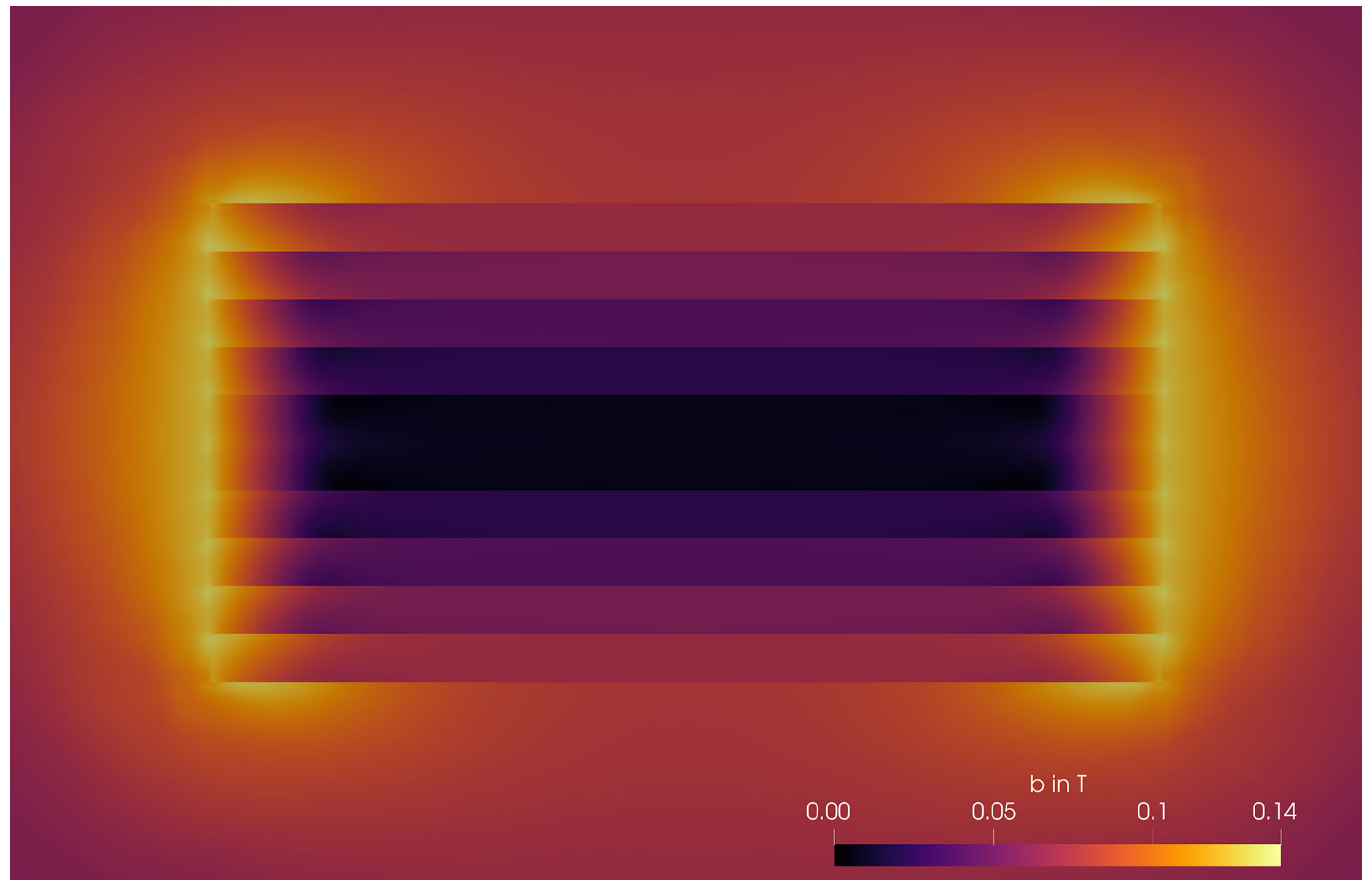


**Model using Volume Elements**

- got momentum in late 2010s to early 2020s
- very robust formulation
- significantly reduced degrees of freedom in non-conducting domains

➡ very high performance gain compared to pure h-formulation

➡ ideal for large 3D models!



**Model using Thin-Shell Elements**

- first published in 2022 for mixed h-ϕ
- can resolve individual layers of HTS tapes

➡ideal for quench investigation of cables

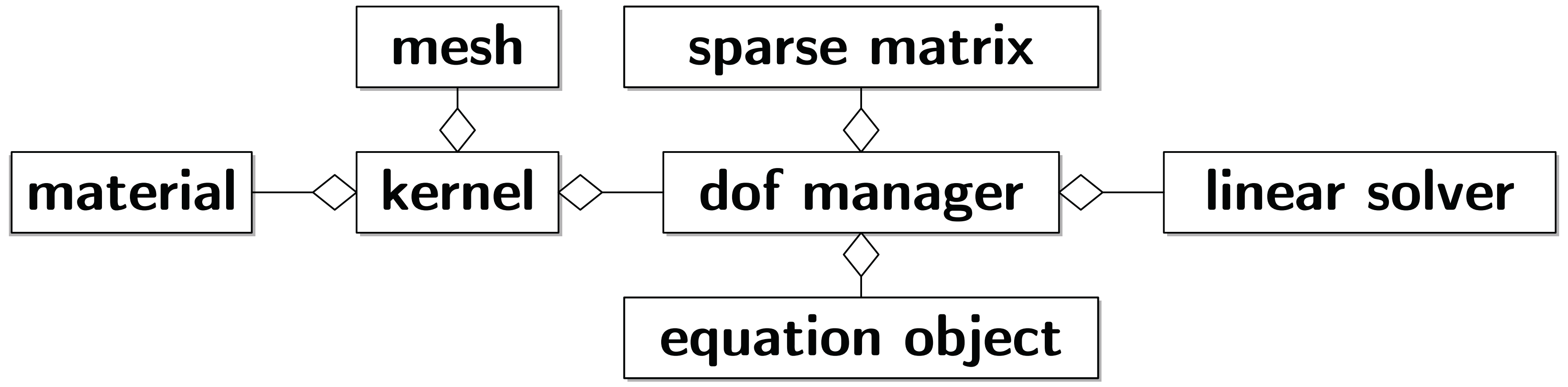
# Why Develop a Custom Codebase?

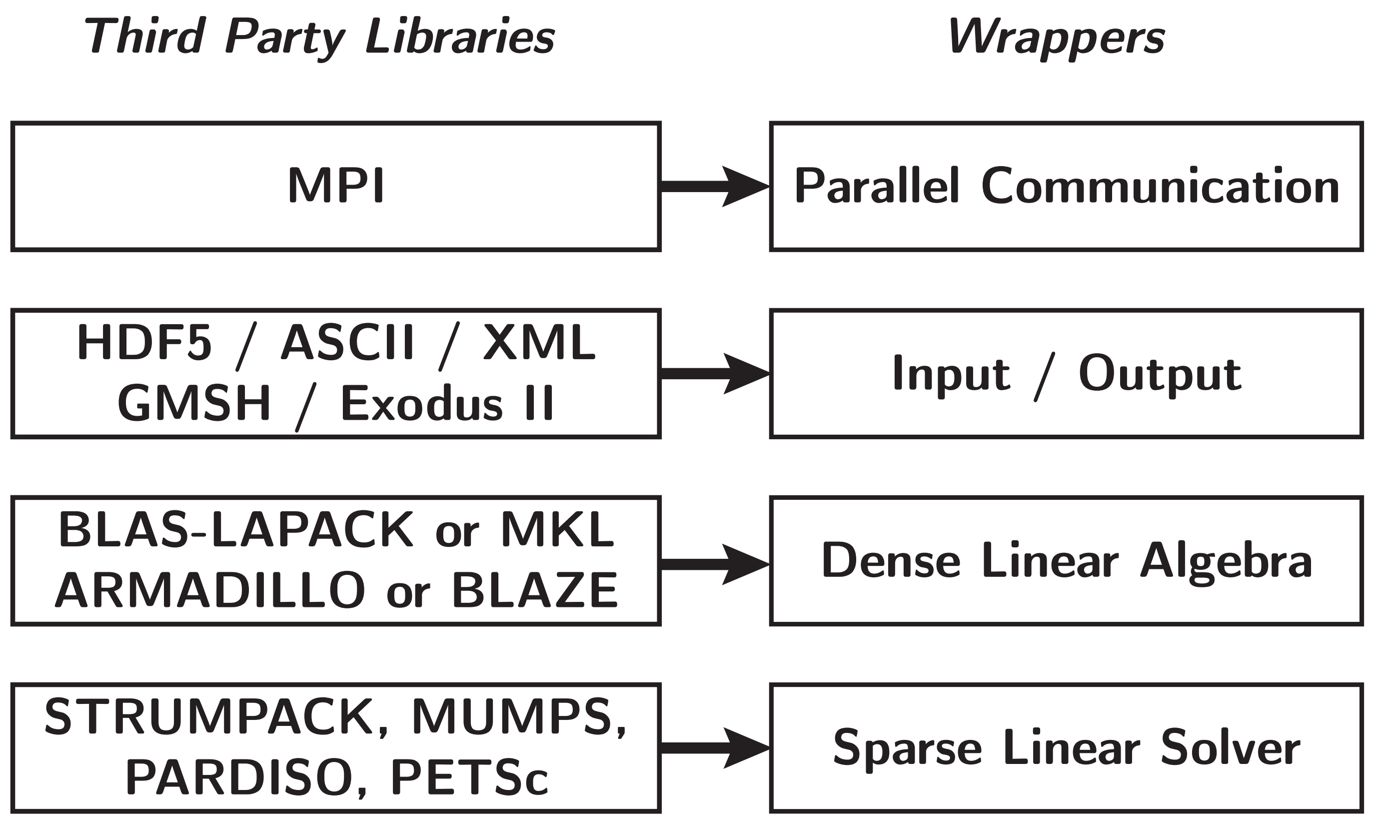
- **application needs: have thin-shell model that**
  - uses the h-ϕ formulation
  - supports thermal conduction and quenching
  - supports current sharing between overlapping tapes
  - supports convective cooling with LN2 and LHe

- **model development needs**
  - have full control over data structure
  - have full knowledge of underlying algorithms

- **utilization of community software**
  - use open source data formats (gmsh, hdf5, exodus-ii)
  - link to popular solver libraries such as PETSc, MKL, …

- **in-house resource utilization**
  - work with STRUMPACK team to maximize solver performance
    https://portal.nersc.gov/project/sparse/strumpack/




*Third Party Libraries* → *Wrappers*

- MPI → Parallel Communication
- HDF5 / ASCII / XML, GMSH / Exodus II → Input / Output
- BLAS-LAPACK or MKL, ARMADILLO or BLAZE → Dense Linear Algebra
- STRUMPACK, MUMPS, PARDISO, PETSc → Sparse Linear Solver

# Homologies and Cohomologies

# H-ϕ formulation: Homologies

## Boundary Conditions

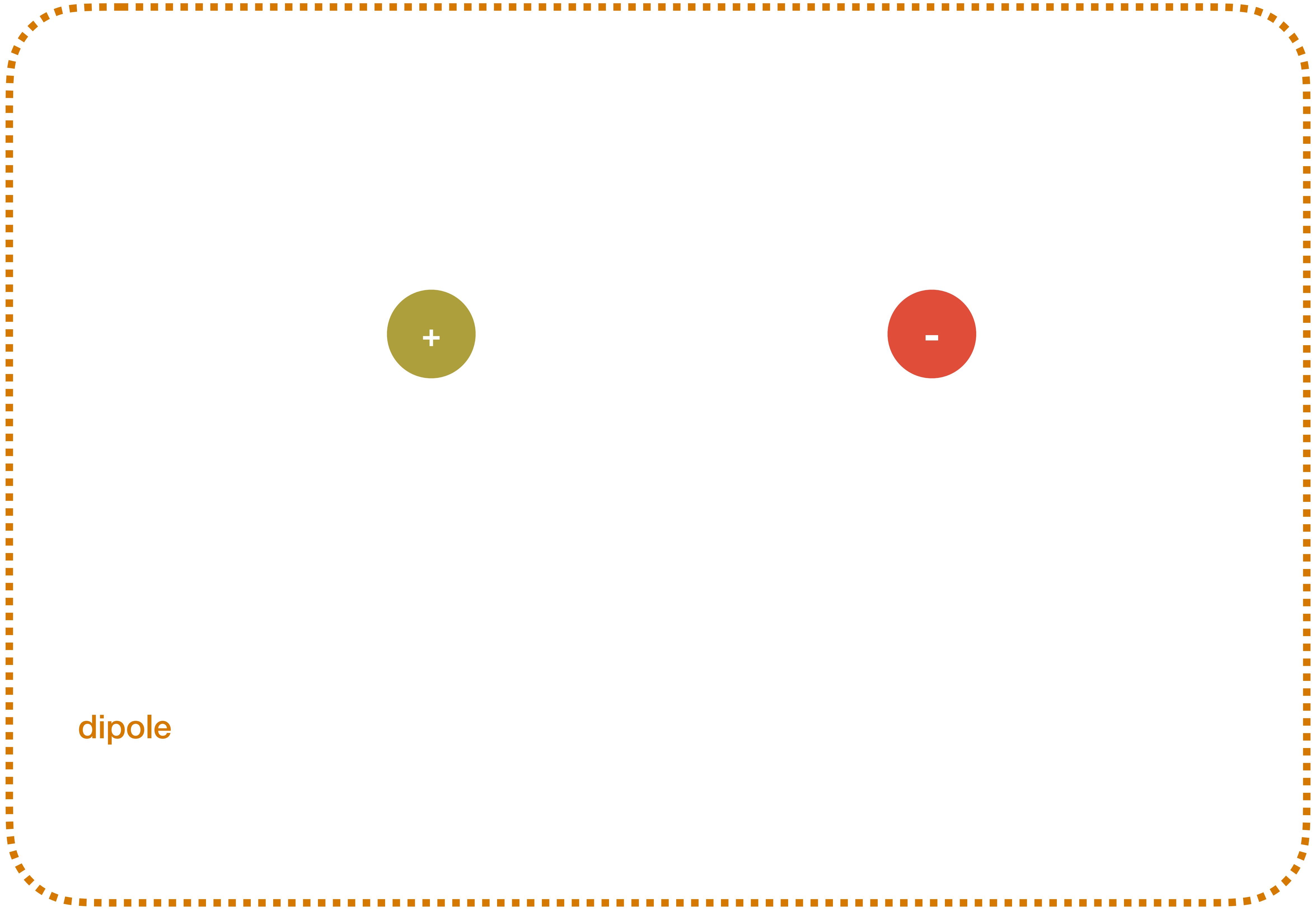
current is applied over Ampere's circuital law:

**Ampéré's circuital law** $\oint \boldsymbol{h}\,\mathrm{d}l = I$

- homologies represent the loops that can be drawn around the conducting regions that fulfill Ampéré's law
- only integral current "**I [A]**" needs to be known

- cohomologies are cuts in the domain over which jumps in the magnetic potential ϕ are imposed so that Δϕ=I.

➡ <u>very elegant mathematics!</u>
➡ <u>homology definition not user friendly</u>
➡ <u>difficult to implement in commercial codes</u>



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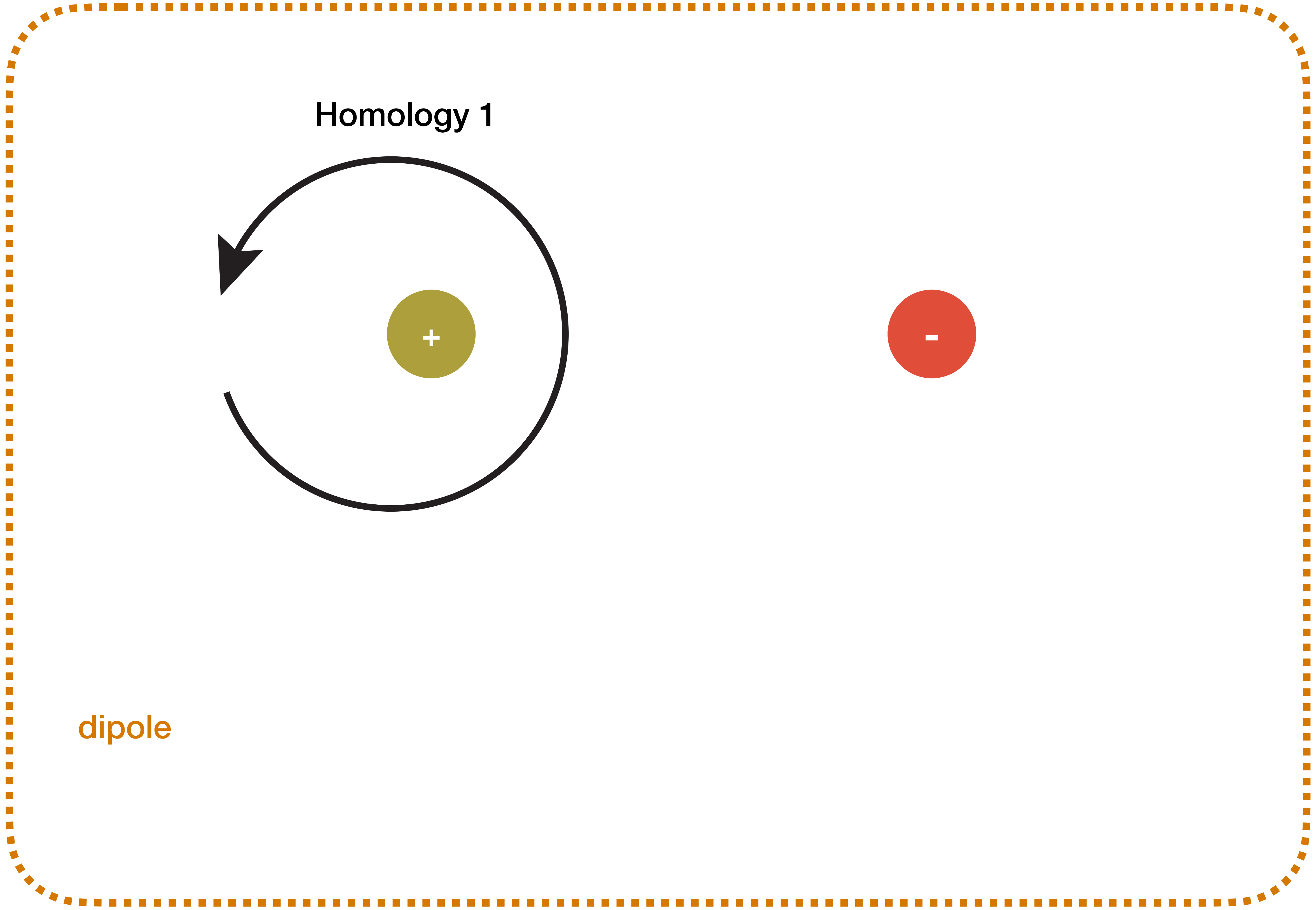

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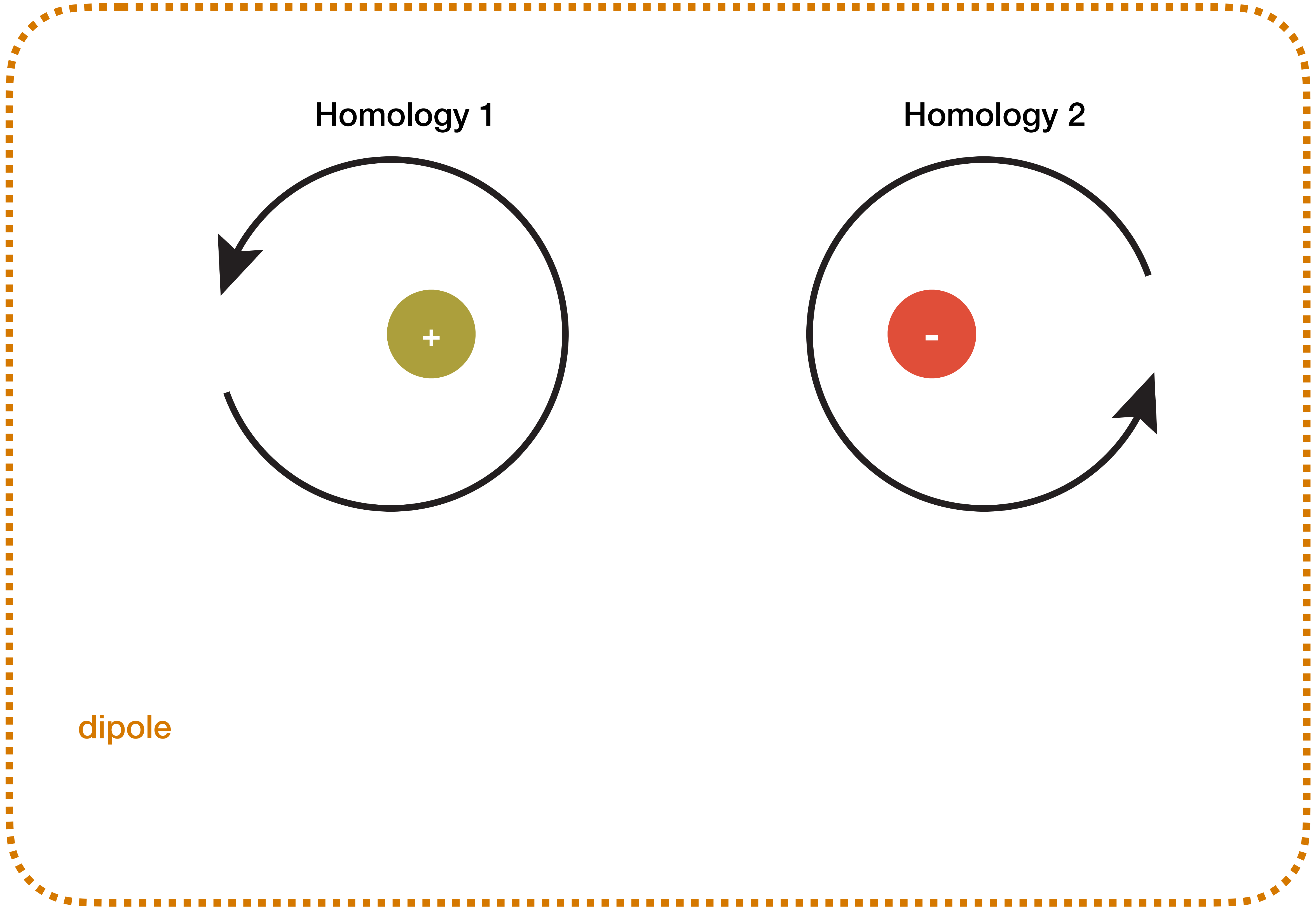

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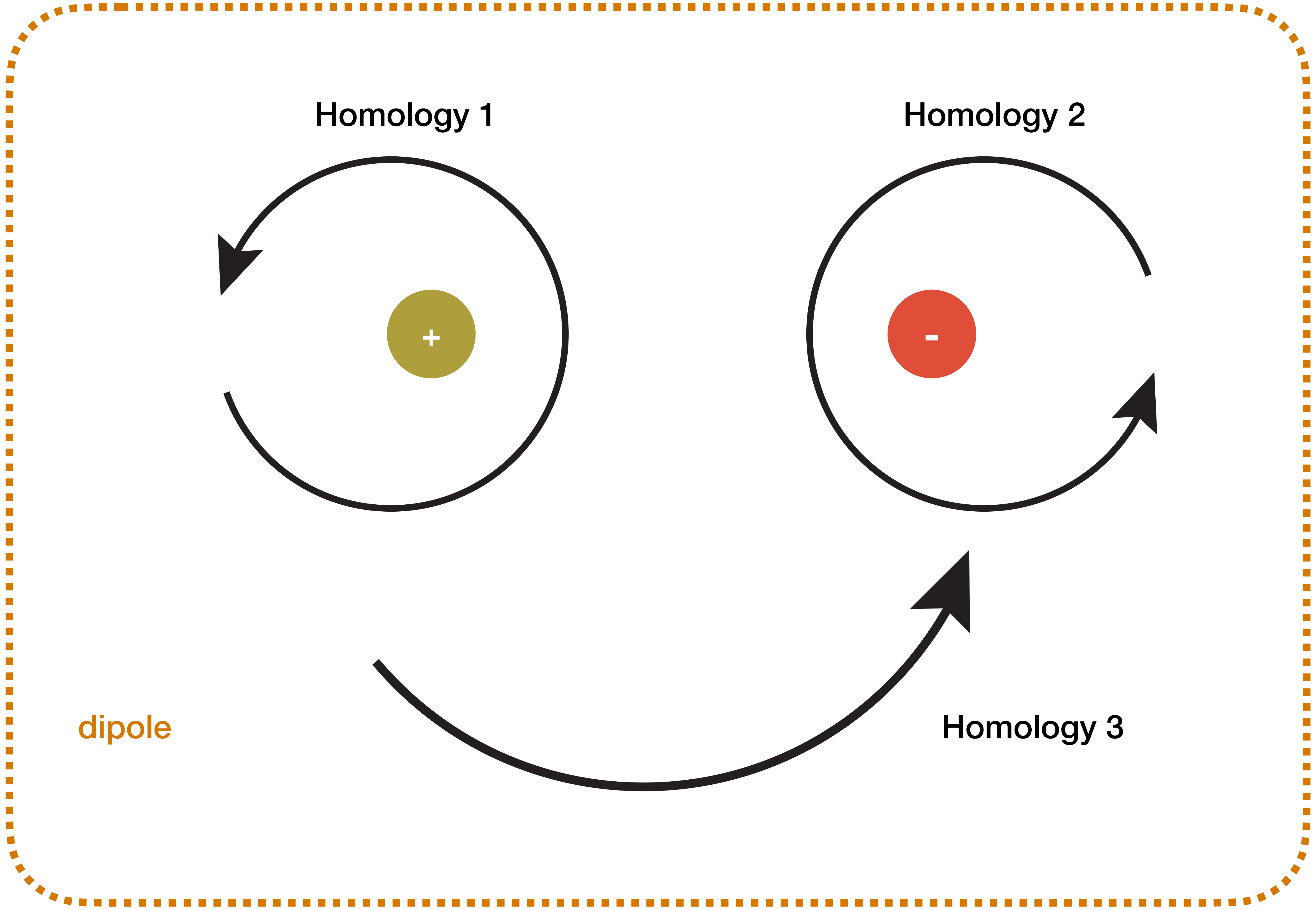

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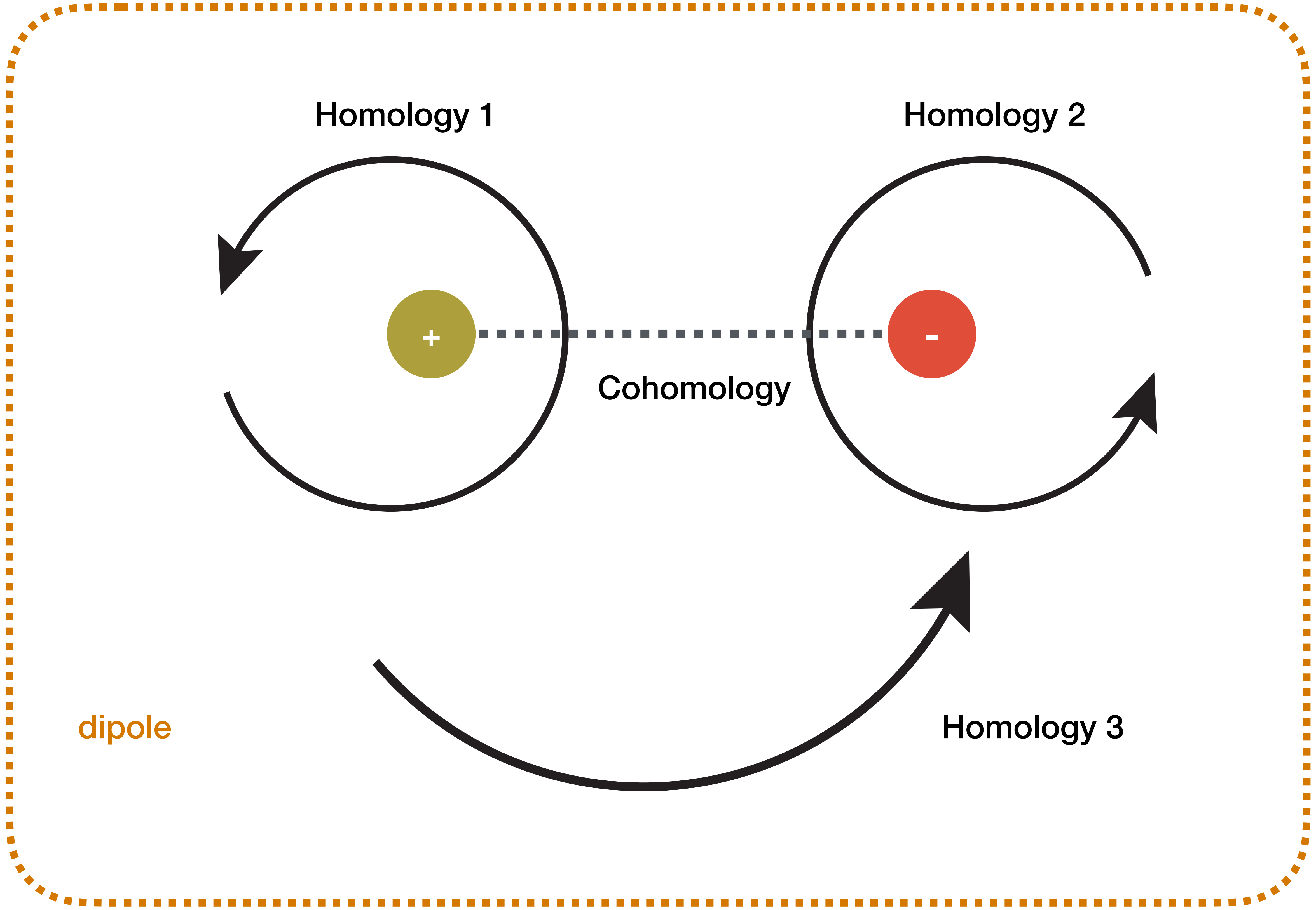
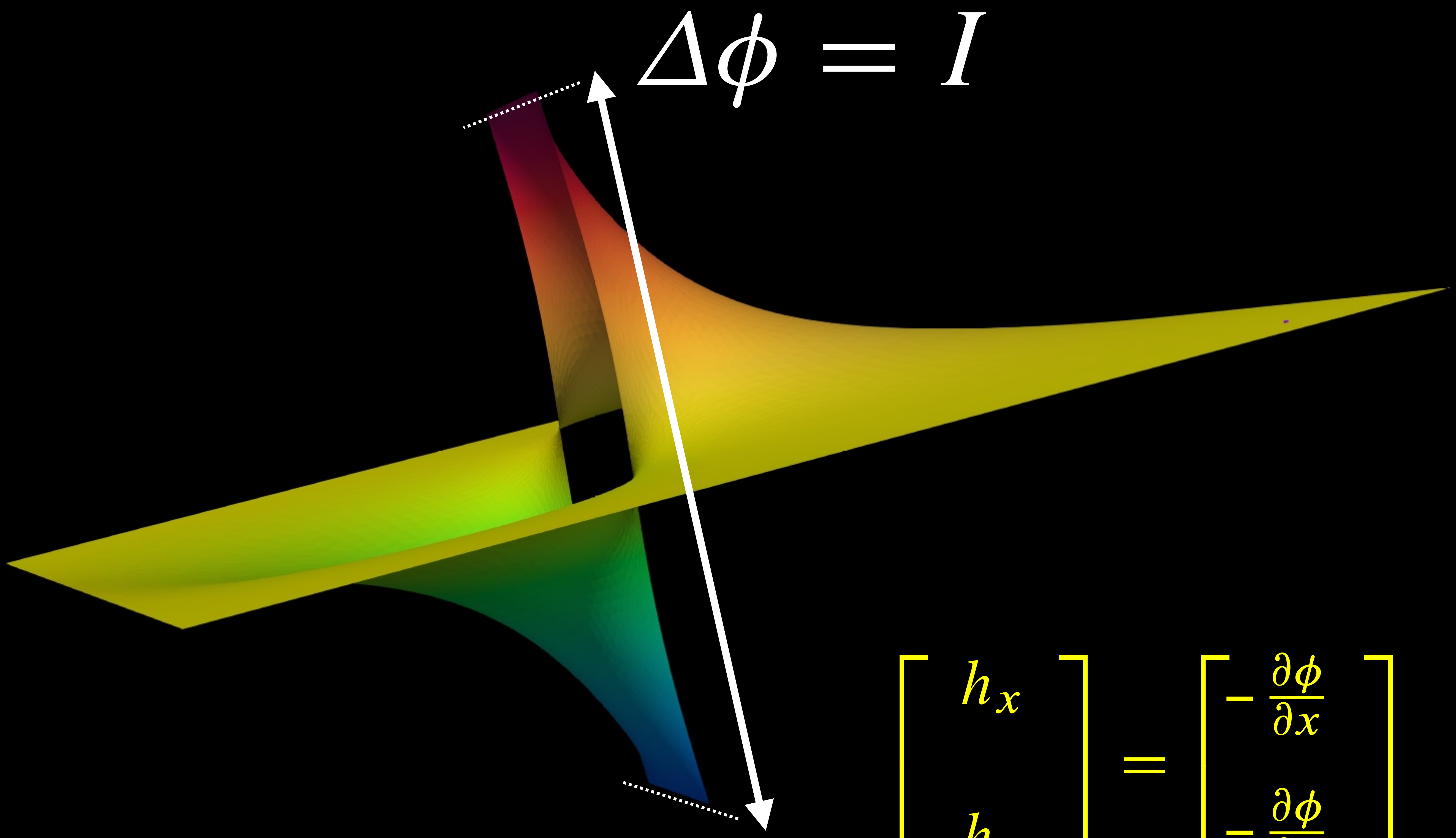
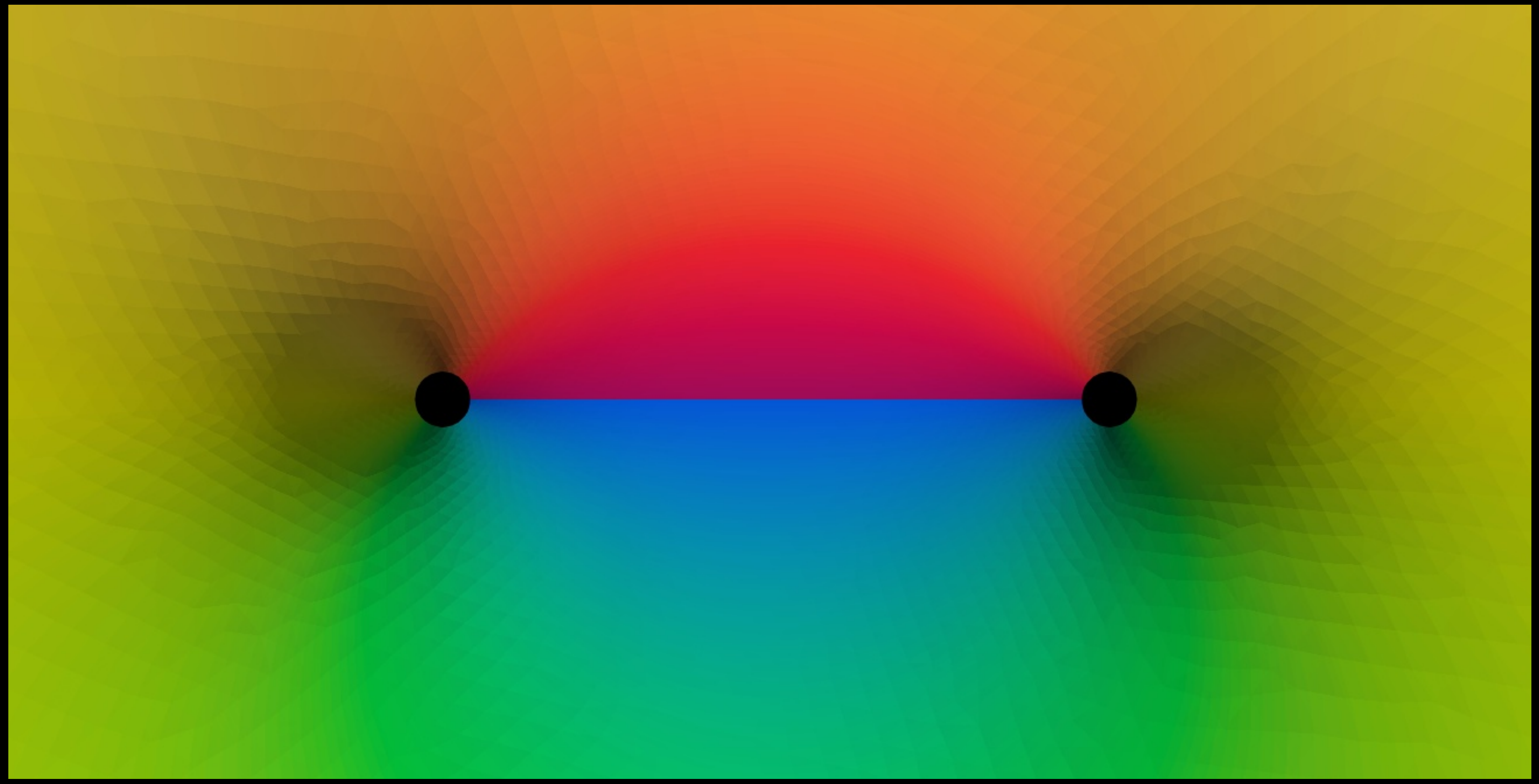


# H-ϕ formulation: Jump Visualization



$$\Delta\phi = I$$

$$\begin{bmatrix} h_x \\ h_y \end{bmatrix} = \begin{bmatrix} -\frac{\partial\phi}{\partial x} \\ -\frac{\partial\phi}{\partial y} \end{bmatrix}$$



# Cohomologies: Work in Progress

**PhD Student Gregory Giard (Polytechnique Montreal):**

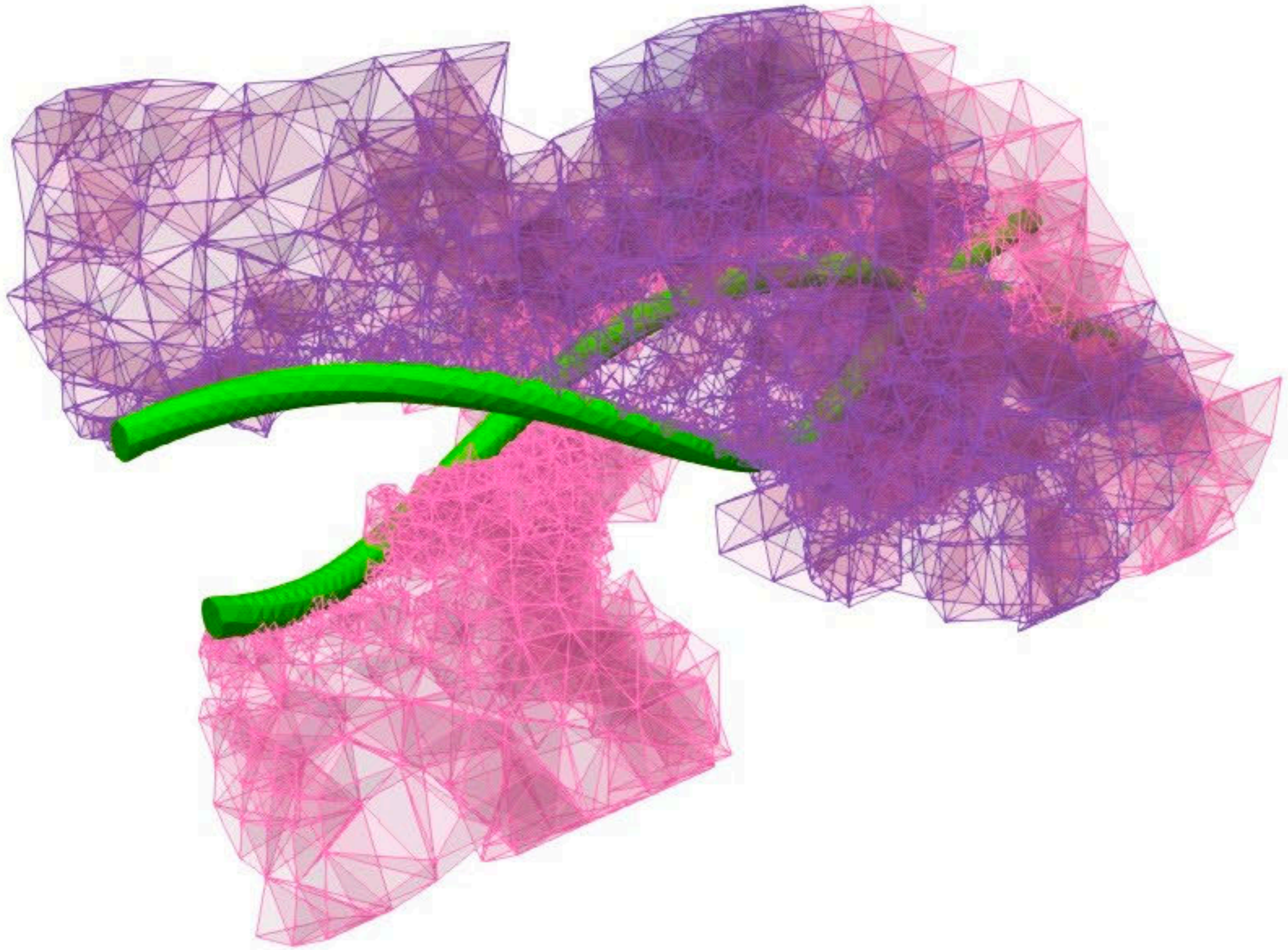
- visiting scholar at LBL from 01/23-06/23

- contribution to adaptive time stepping method
- development of 3D thermal conduction model

- implementing automated cohomology computation in 3D

- automated identification of cut orientations based on user provided currents ("the user shall not worry about cohomologies")

- clean formulation of "thin" and "thick" cuts using "change of basis" for the former and XFEM for the latter.



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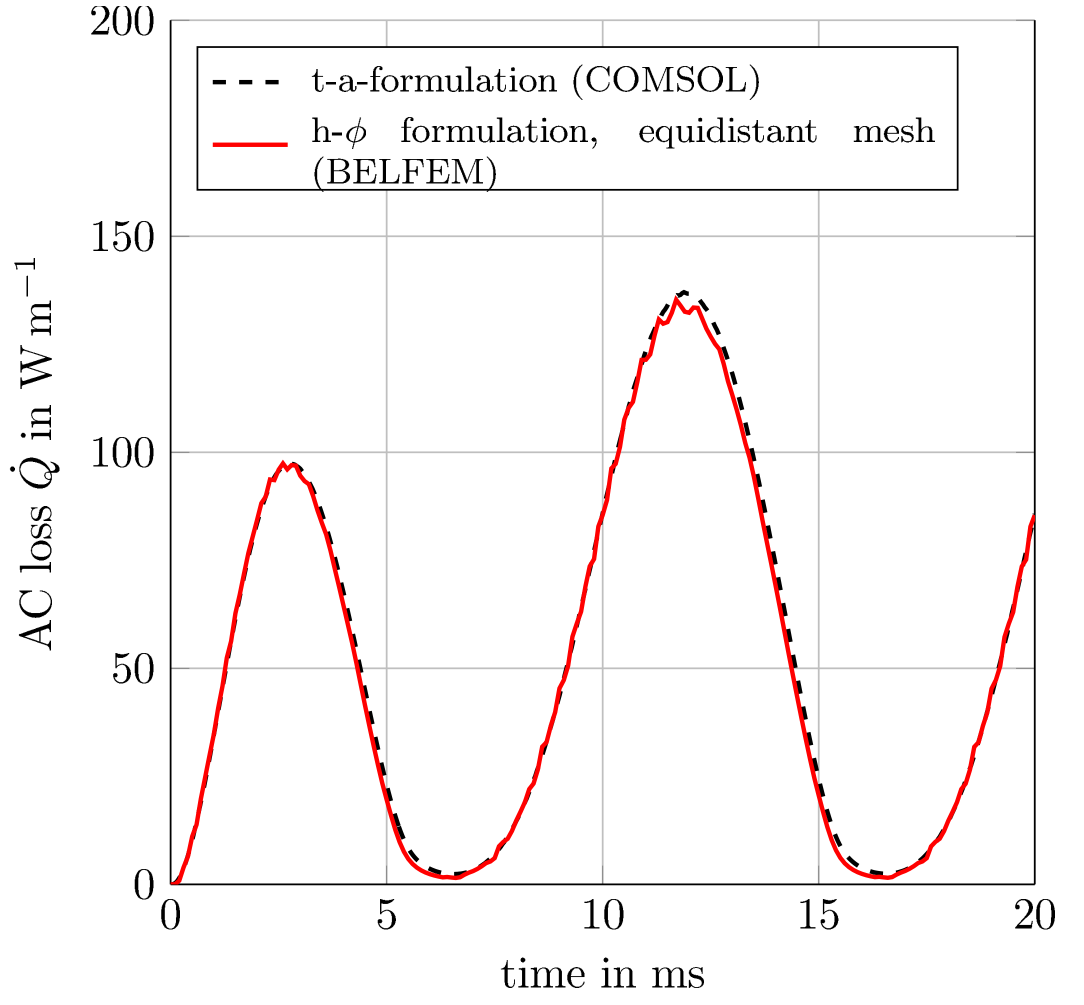
# Current Status

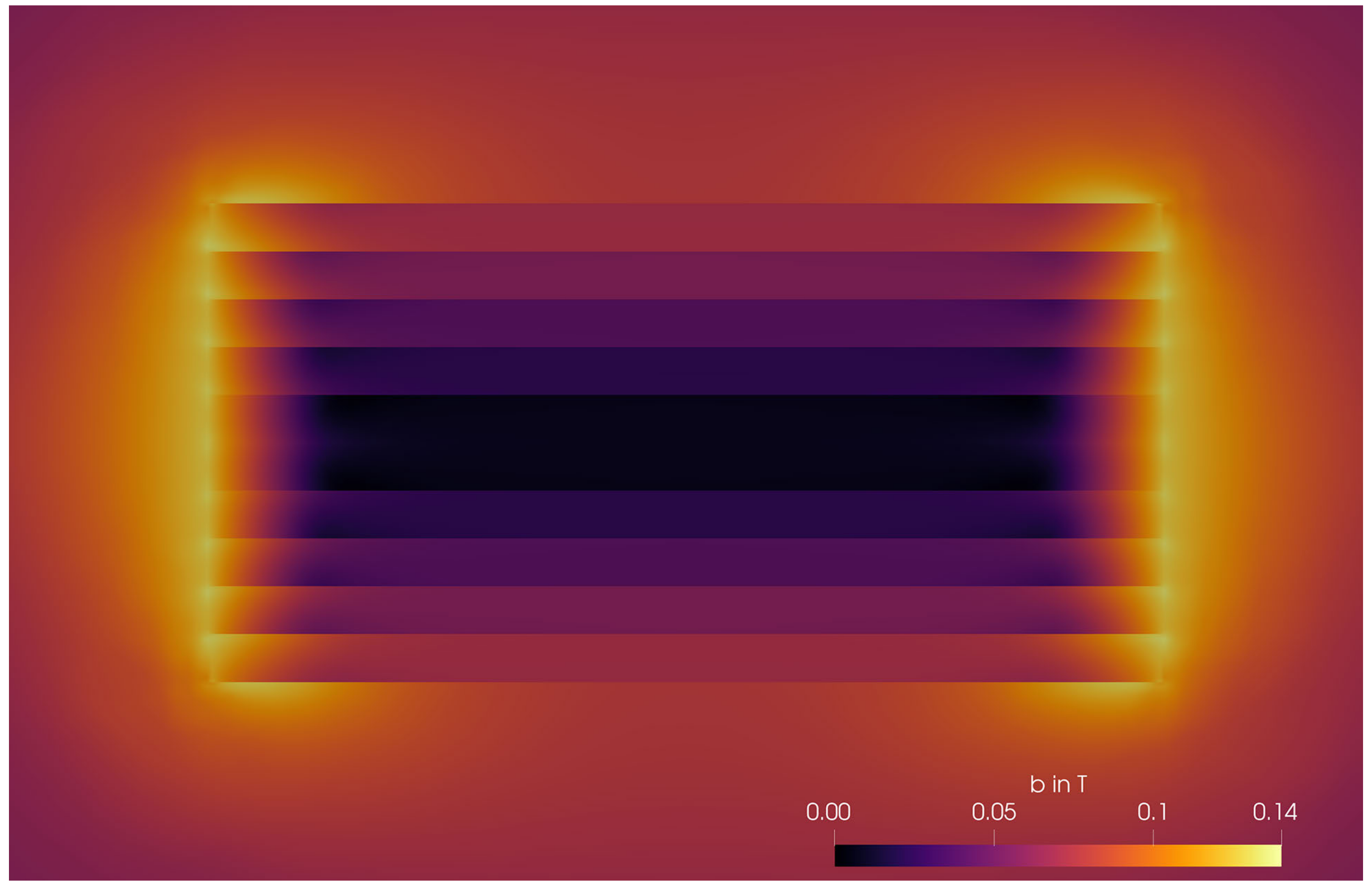
# Thin Shell Formulation: 2D Results 2023

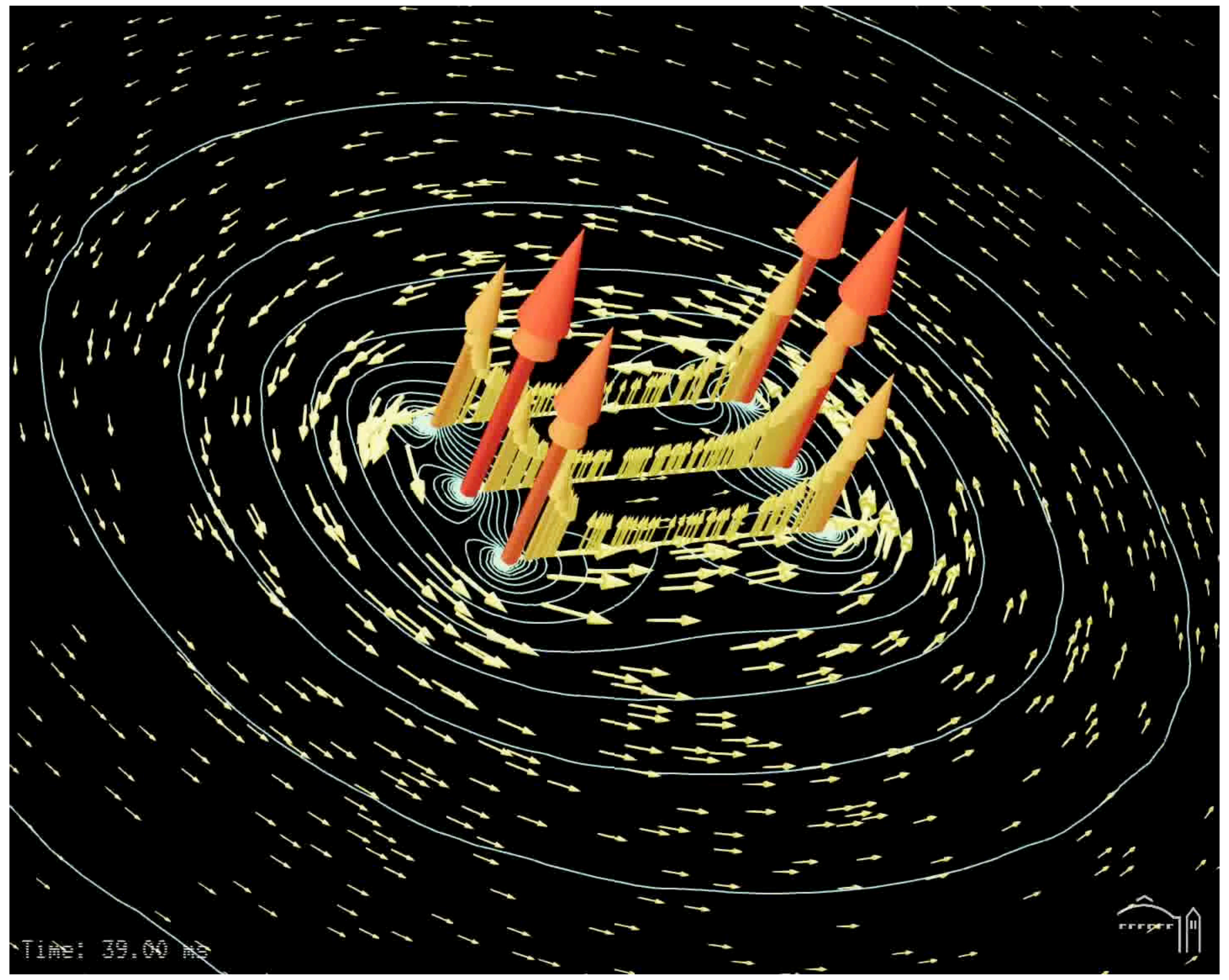
**Published Paper in SuST 2023**

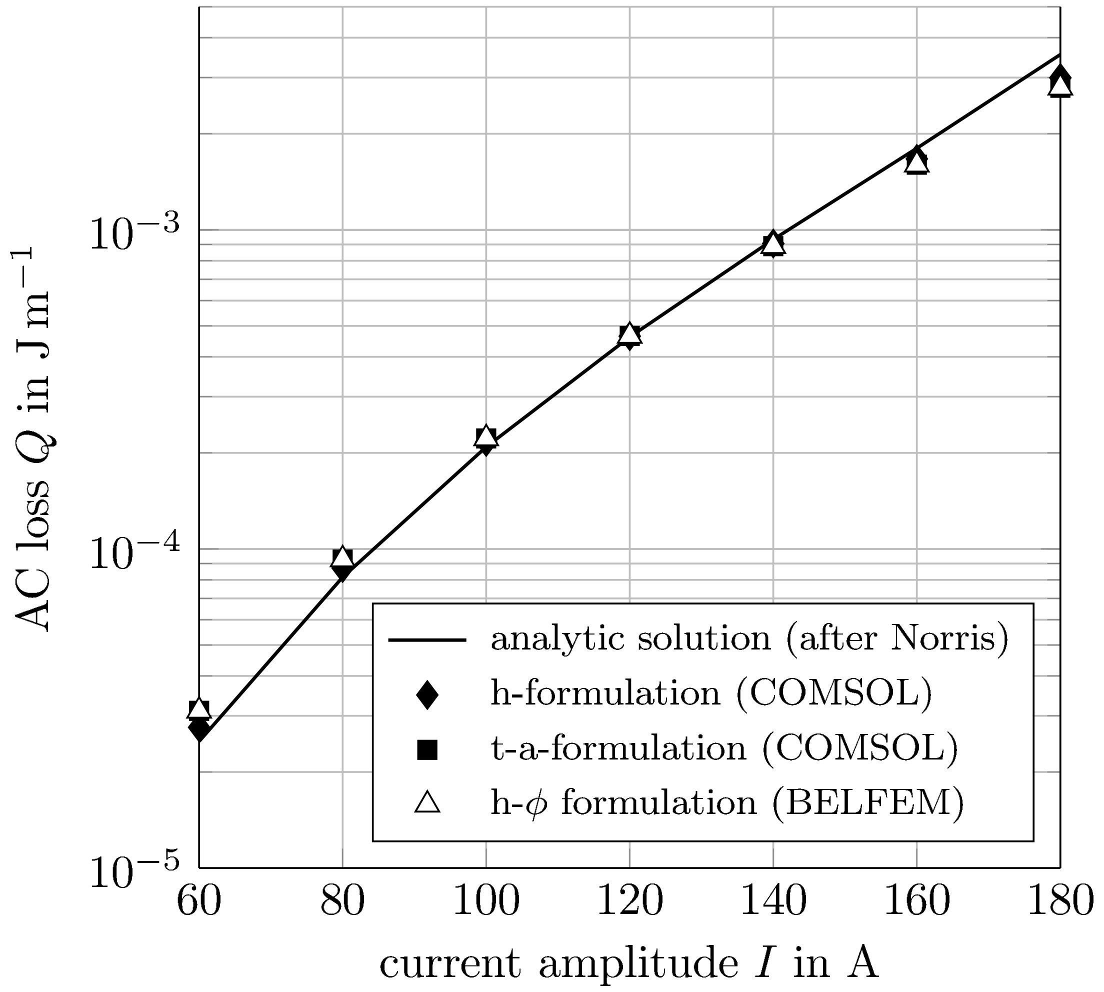
- Christian Messe, Berkeley Lab
- Nico Riva, MIT
- Sofia Viarengo, Politechnico di Torino
- Gregory Giard & Frédéric Sirois, Polytechnique Montreal

- validated against analytical methods + COMSOL / GetDP
- first research promises faster and more detailed results than other established methods such as t-a









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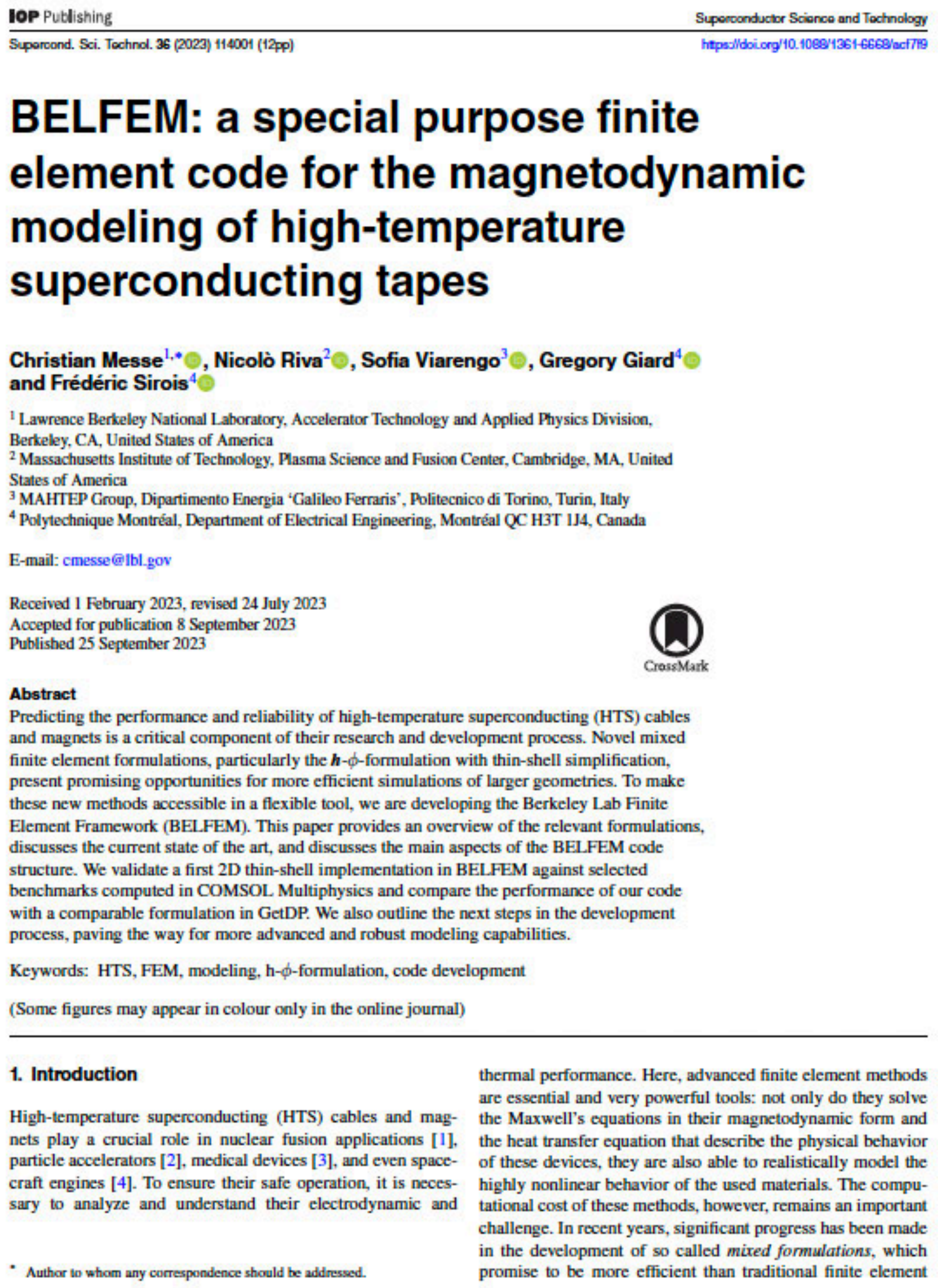

**IOP** Publishing

Superconductor Science and Technology

Supercond. Sci. Technol. 36 (2023) 114001 (12pp)

https://doi.org/10.1088/1361-6668/acf7f9

## BELFEM: a special purpose finite element code for the magnetodynamic modeling of high-temperature superconducting tapes

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[2] Massachusetts Institute of Technology, Plasma Science and Fusion Center, Cambridge, MA, United States of America
[3] MAHTEP Group, Dipartimento Energia 'Galileo Ferraris', Politecnico di Torino, Turin, Italy
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Accepted for publication 8 September 2023
Published 25 September 2023

**Abstract**

Predicting the performance and reliability of high-temperature superconducting (HTS) cables and magnets is a critical component of their research and development process. Novel mixed finite element formulations, particularly the $\boldsymbol{h}$-$\phi$-formulation with thin-shell simplification, present promising opportunities for more efficient simulations of larger geometries. To make these new methods accessible in a flexible tool, we are developing the Berkeley Lab Finite Element Framework (BELFEM). This paper provides an overview of the relevant formulations, discusses the current state of the art, and discusses the main aspects of the BELFEM code structure. We validate a first 2D thin-shell implementation in BELFEM against selected benchmarks computed in COMSOL Multiphysics and compare the performance of our code with a comparable formulation in GetDP. We also outline the next steps in the development process, paving the way for more advanced and robust modeling capabilities.

Keywords: HTS, FEM, modeling, h-$\phi$-formulation, code development

(Some figures may appear in colour only in the online journal)

### 1. Introduction

High-temperature superconducting (HTS) cables and magnets play a crucial role in nuclear fusion applications [1], particle accelerators [2], medical devices [3], and even spacecraft engines [4]. To ensure their safe operation, it is necessary to analyze and understand their electrodynamic and thermal performance. Here, advanced finite element methods are essential and very powerful tools: not only do they solve the Maxwell's equations in their magnetodynamic form and the heat transfer equation that describe the physical behavior of these devices, they are also able to realistically model the highly nonlinear behavior of the used materials. The computational cost of these methods, however, remains an important challenge. In recent years, significant progress has been made in the development of so called *mixed formulations*, which promise to be more efficient than traditional finite element

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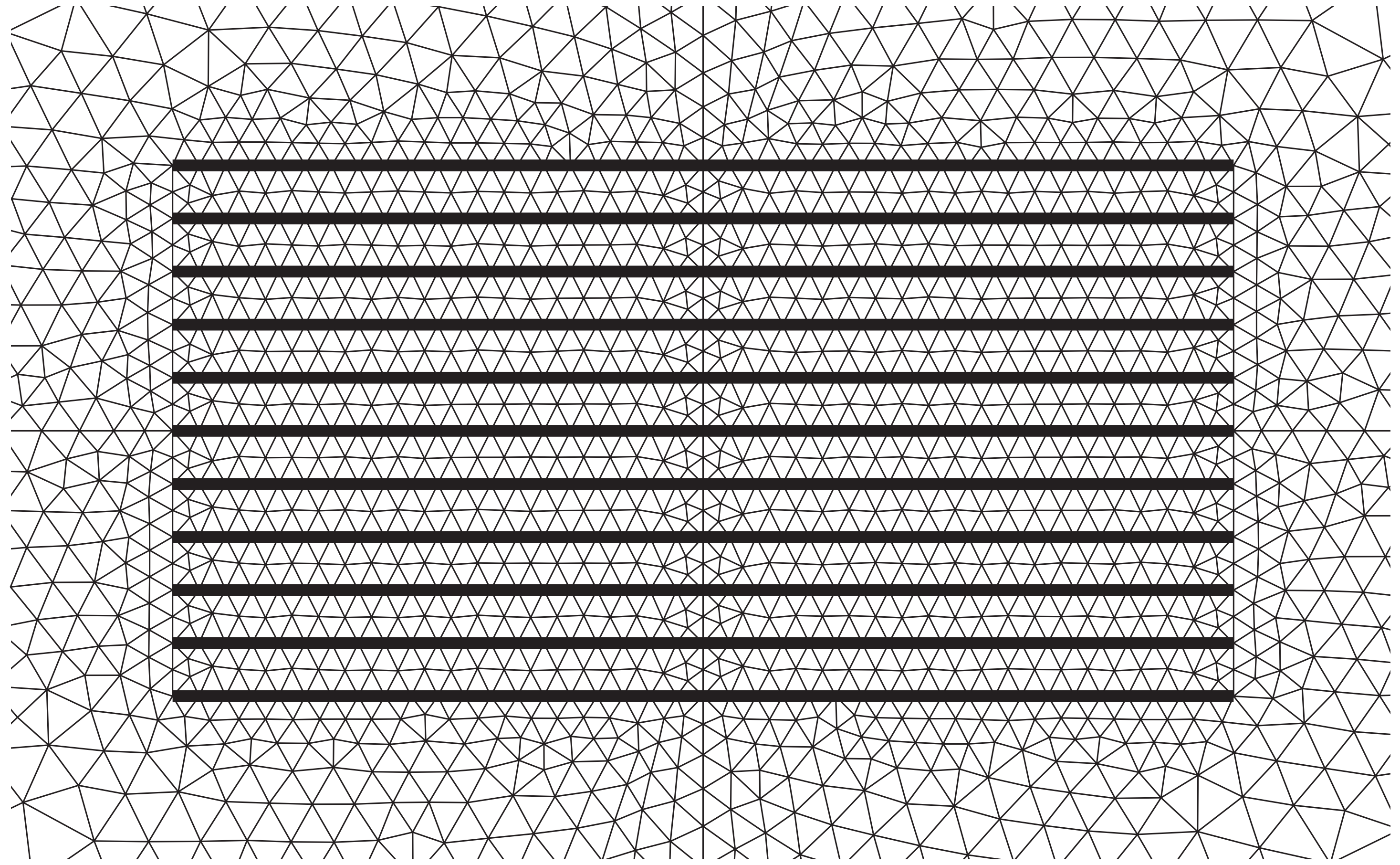
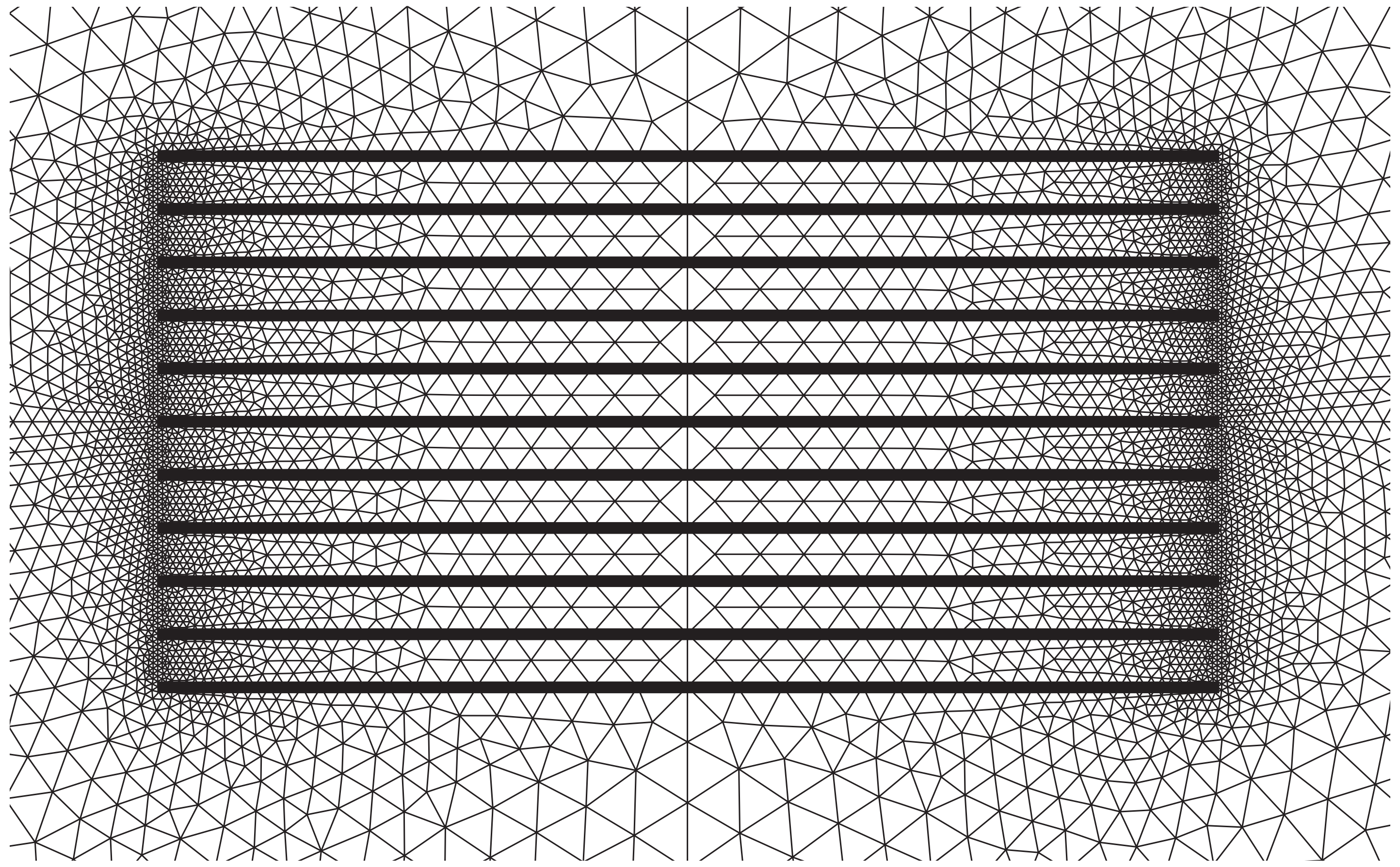
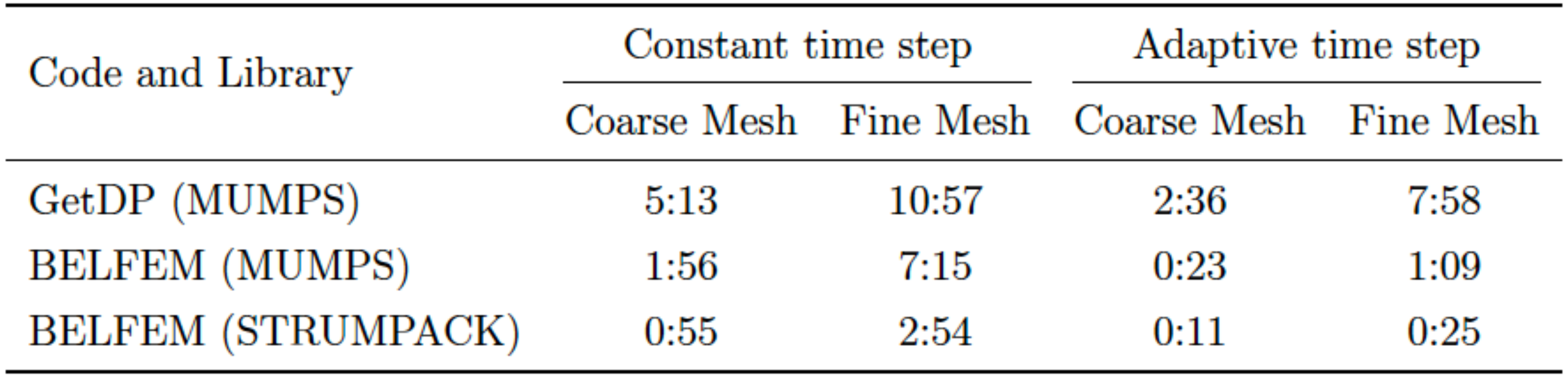
[*] Author to whom any correspondence should be addressed.

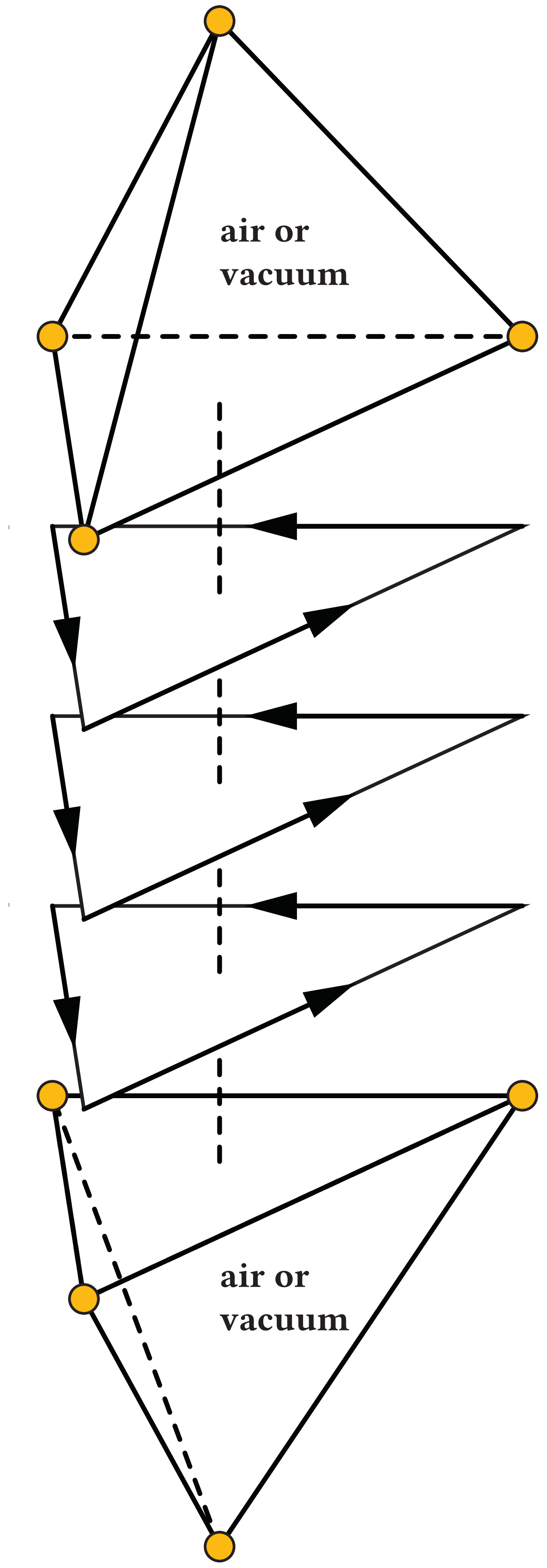






| Code and Library | Constant time step |  | Adaptive time step |  |
|---|---|---|---|---|
|  | Coarse Mesh | Fine Mesh | Coarse Mesh | Fine Mesh |
| GetDP (MUMPS) | 5:13 | 10:57 | 2:36 | 7:58 |
| BELFEM (MUMPS) | 1:56 | 7:15 | 0:23 | 1:09 |
| BELFEM (STRUMPACK) | 0:55 | 2:54 | 0:11 | 0:25 |

# Current Efforts

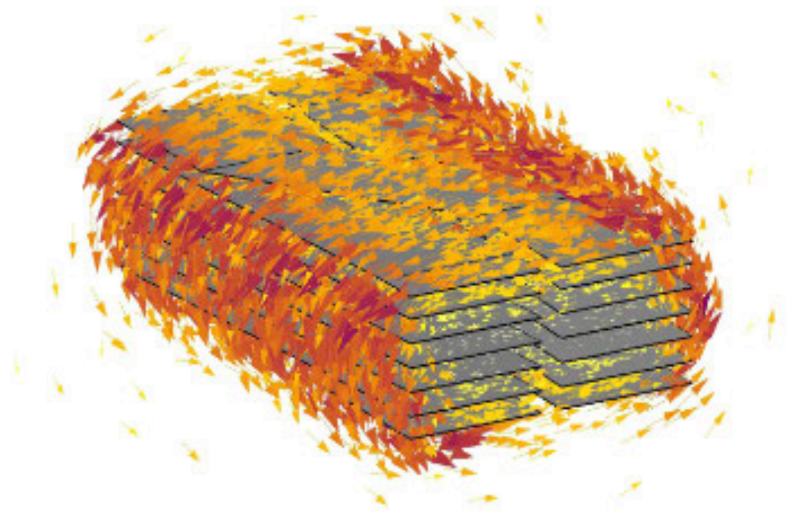


**Goal:**

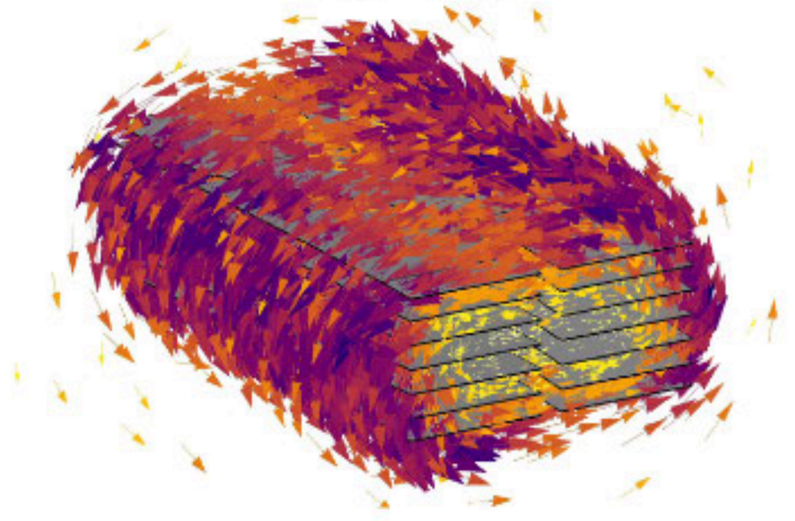
- model a thin shell tapes tack in 3D after Alves et Al, 2022
- extend model to encompass solder and thermal model
- be able to do the coupled EM-Thermal quenching model by end of the year

**Roadmap:**

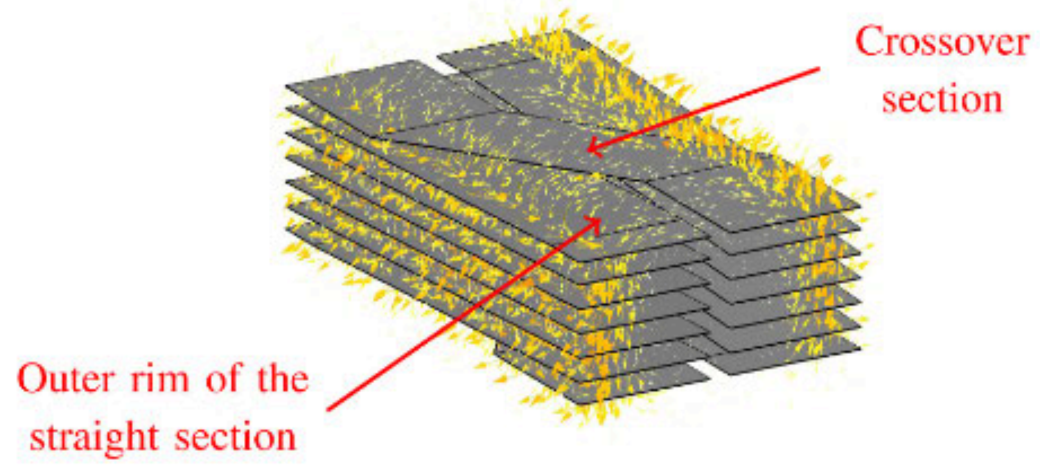
- overhaul data structure for simplified programming of weak governing equations
- improve degree of freedom management system
- first benchmark with 3D tapestack
- implement solder and thermal model
- benchmark involving quench
- address contact sharing (Spring 2025)



(a) $t = T/8$



(b) $t = T/4$

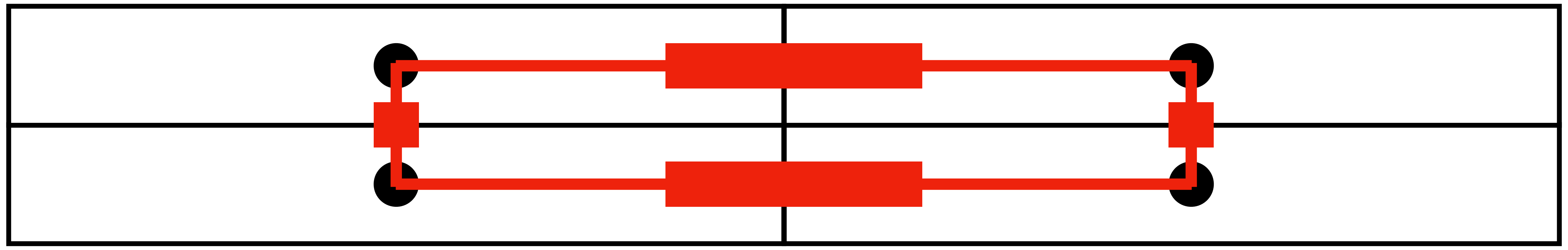


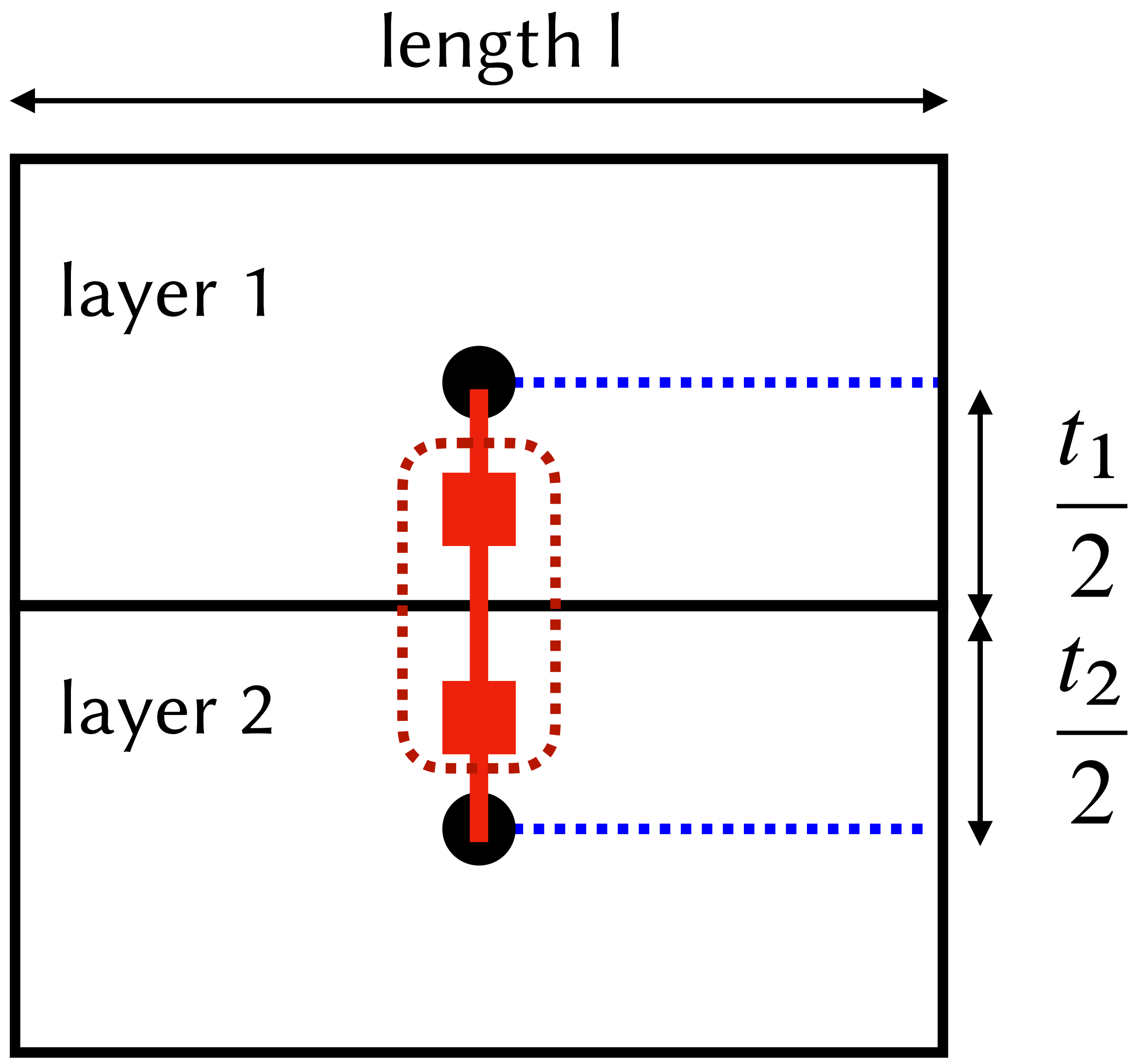
(c) $t = T/2$

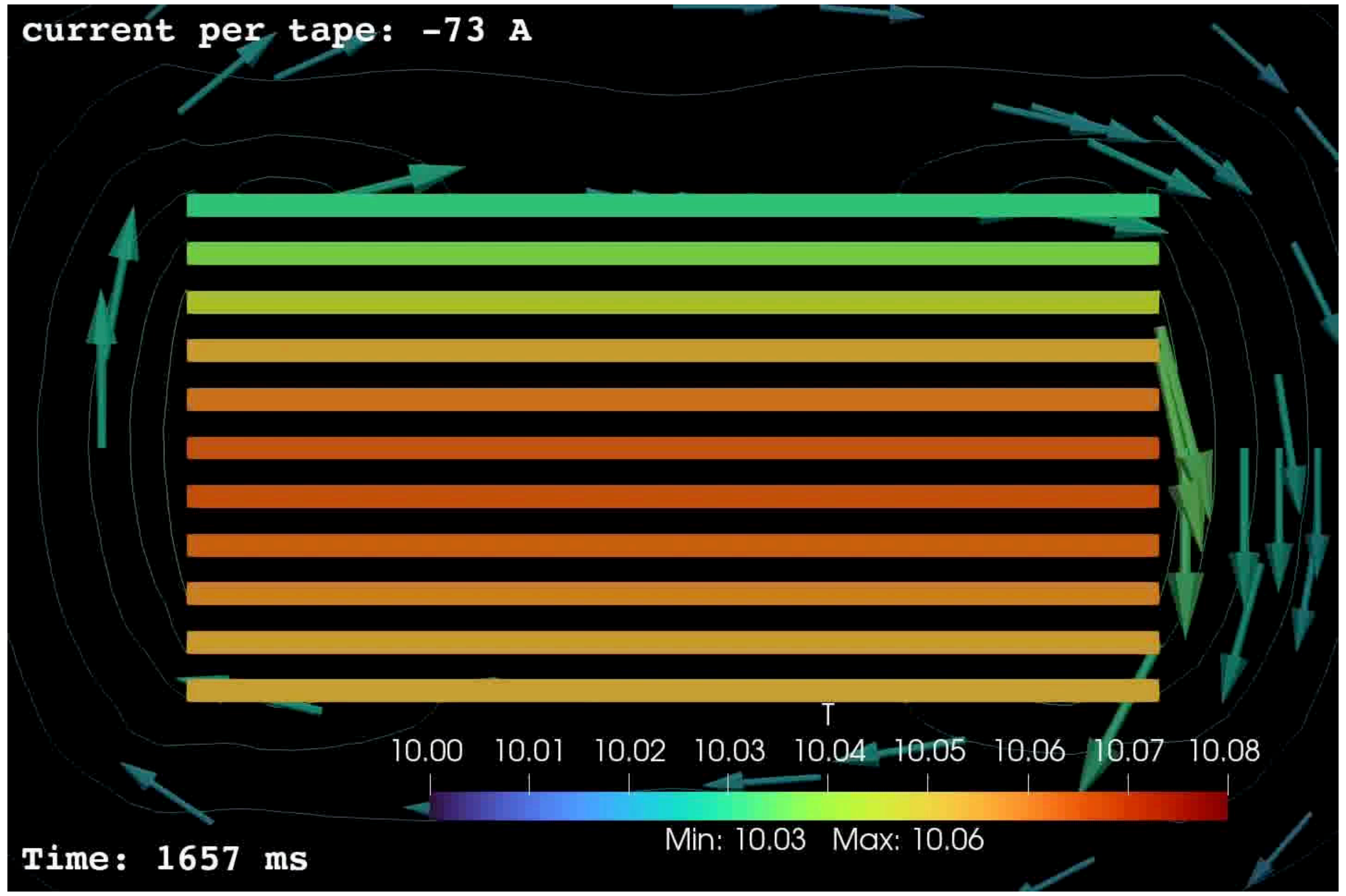
[ Alves et al, 10.1109/TASC.2022.3143076 ]

# Work in progress: thermal coupling

- linear discretization + mass lumping collapses finite-element method to resistor grid
- can be first order even if Maxwell is second order
- degrees of freedom sit on the edges
  ⟶ assumes constant temperature per layer per element
  ⟶ excellent numerical stability







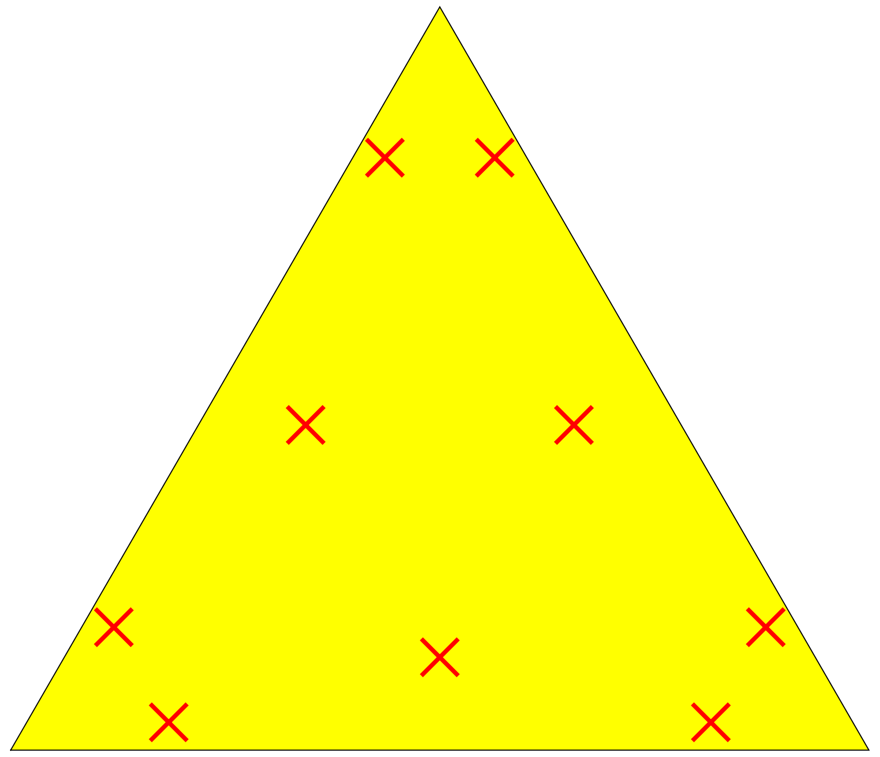
# Intermediate Summary

- developing a finite-element framework tailored to HTS cable & magnet development needs
- demonstrated proof of concept in 2D, currently working on 3D
- first performance tests very promising!
- Work in progress: Automated cohomology computation
- Work in progress: coupled thermal-EM 3D tape model for quenching (goal: winter 2024)
- Future Goals: Implement inter-tape current sharing ( ~ spring 2025)

# Thoughts on Material Modeling

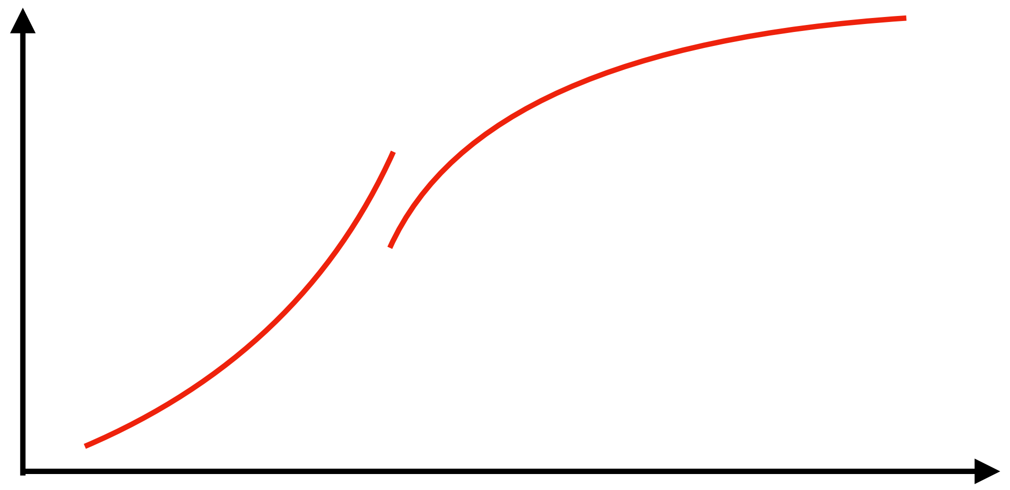
# Material Modeling

- **High Nonlinearities for HTS Materials**
  - material properties must be evaluated at every integration point
  - nonlinearities require many iterations
  - material curves must be smooth (consistent derivatives, continuous)
  - avoid expensive functions such as exp or log



triangle integration points
( 5th order )

- **Boobytraps in modeling and coding**
  - piecewise polynomials
  - wasteful implementations
  - validity range of functions



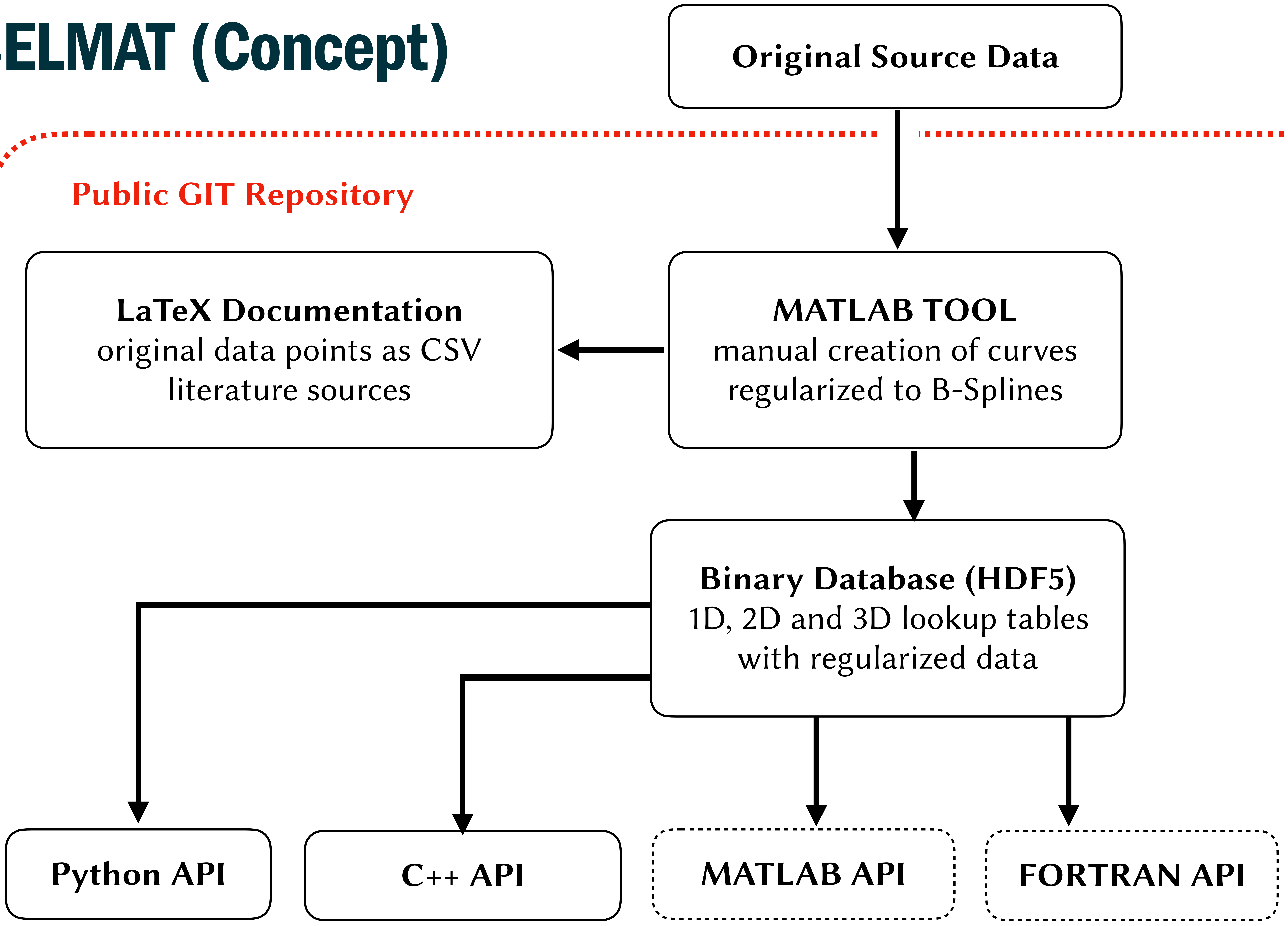
piecewise polynomials lead to convergence issues due to

- discontinuous material properties
- discontinuous derivatives

**CSE can be up to ~3 x faster!!!**

```
bad code:  wasting multiplications:

y = 10^( a + b ∗ log(T) + c ∗ log(T)^2 + d ∗ log(T)^3 + ...  )

good code: Common Subexpression Elemination (CSE)

logT = log(T)

y = 10^( a + logT ∗ ( b + logT ∗ ( c + logT ∗ ( d + ... ) ) ) ))
```

# BELMAT (Concept)



**Original Source Data**

Public GIT Repository

**LaTeX Documentation**
original data points as CSV
literature sources

**MATLAB TOOL**
manual creation of curves
regularized to B-Splines

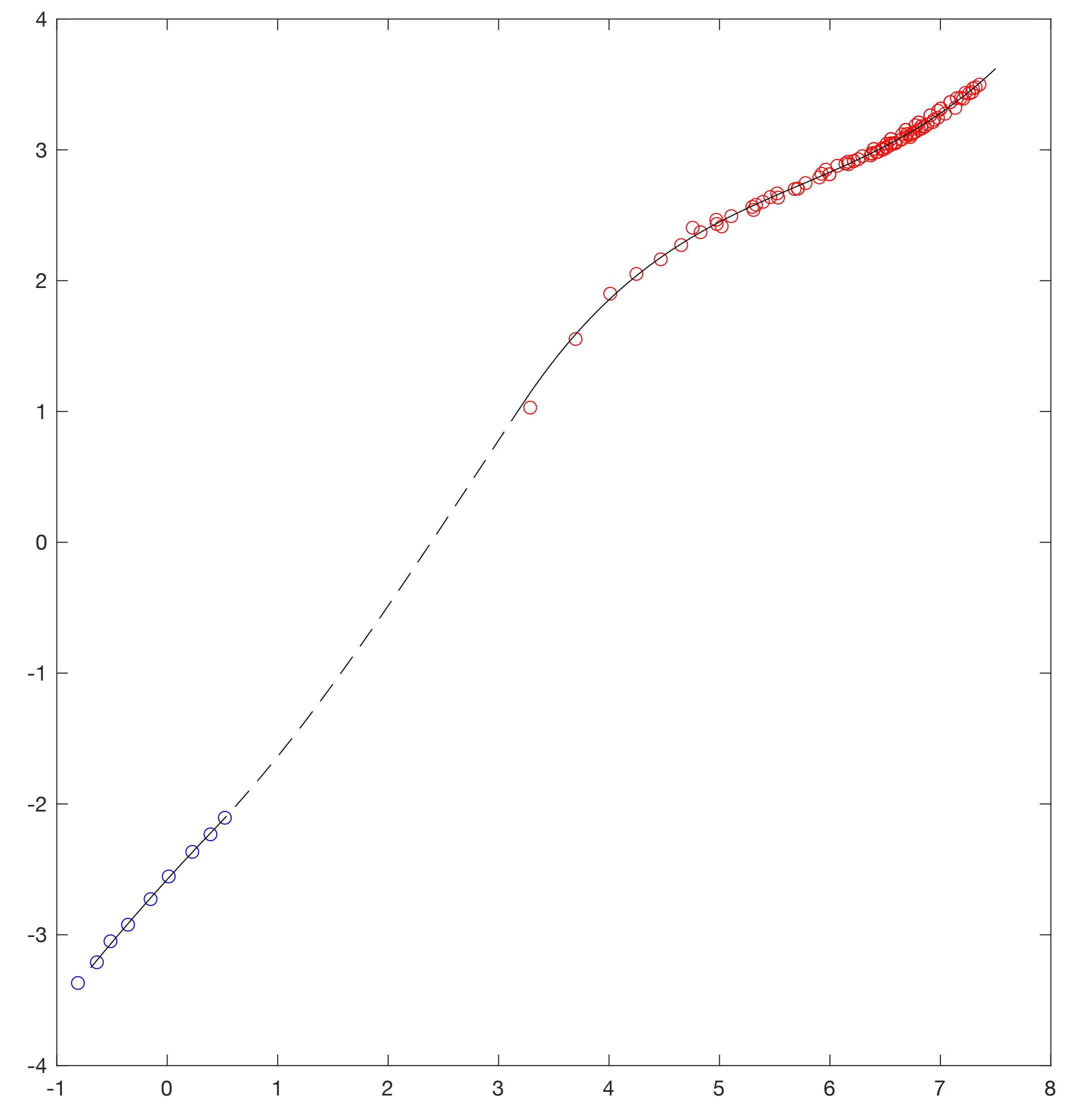
**Binary Database (HDF5)**
1D, 2D and 3D lookup tables
with regularized data

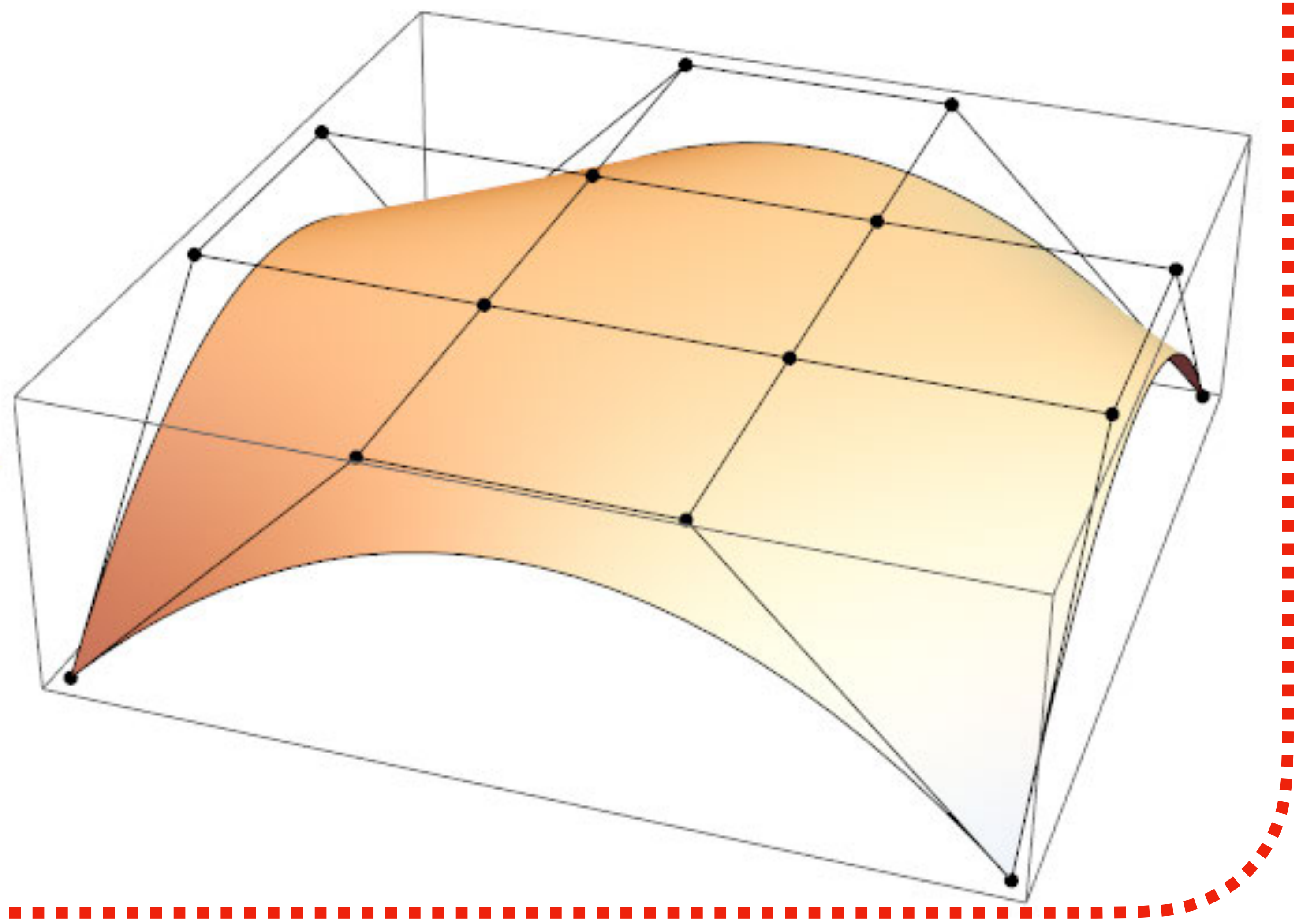
**Python API**

**C++ API**

**MATLAB API**

**FORTRAN API**





—> LET’S DEFINE A COMMUNITY WIDE STANDARD!

# Usecase Example

- **implemented in SparseLizard: ASC 2022**
- **implemented in BELFEM: SUST 2023**

$H$-$\phi$ Formulation in Sparselizard Combined With Domain Decomposition Methods for Modeling Superconducting Tapes, Stacks, and Twisted Wires

N. Riva, A. Halbach, M. Lyly, C. Messe, J. Ruuskanen, and V. Lahtinen

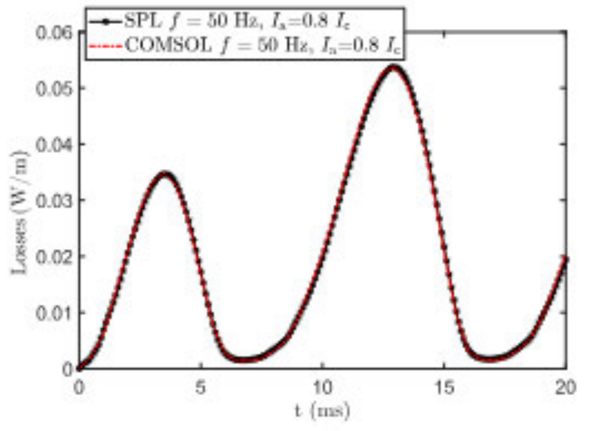


Fig. 5. Instantaneous losses computed on the entire assembly (NbTi wires + copper), with `Sparselizard` (continuous line + markers) and COMSOL (dashed line).

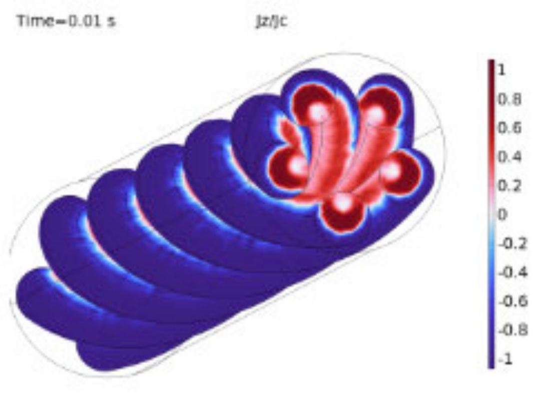
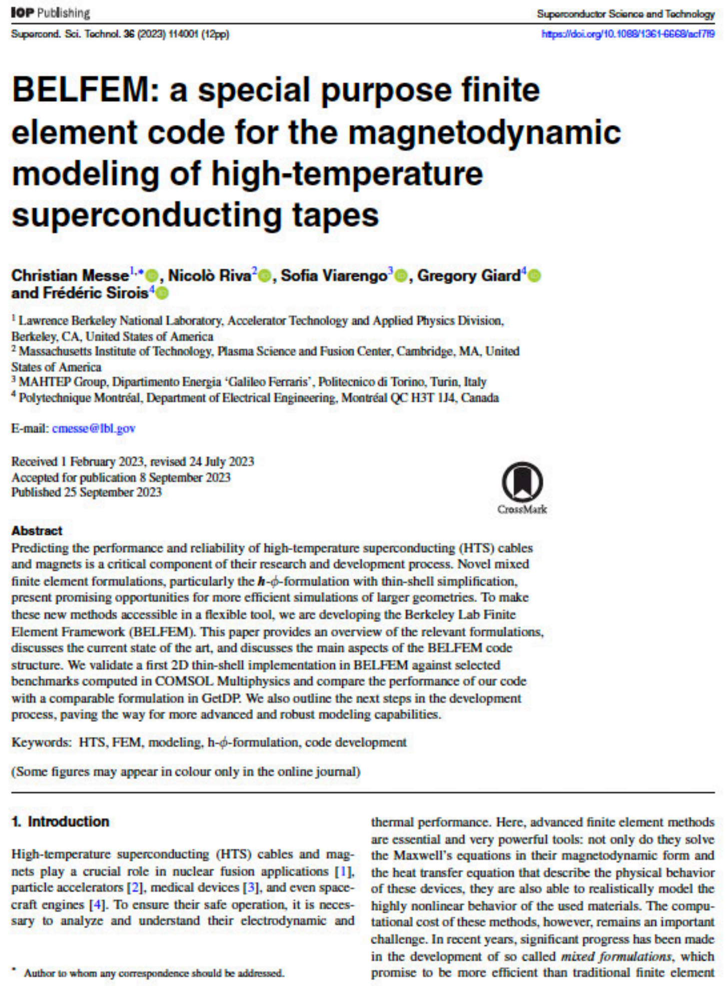


Fig. 6. Normalized current density at $t = 10$ ms.



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BELFEM: a special purpose finite element code for the magnetodynamic modeling of high-temperature superconducting tapes

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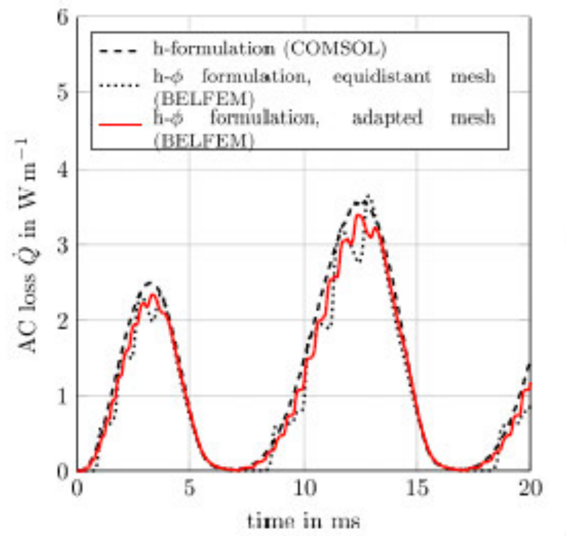


Figure 12. AC losses for the tape stack at $I = 90$ A per tape at $f = 50$ Hz.

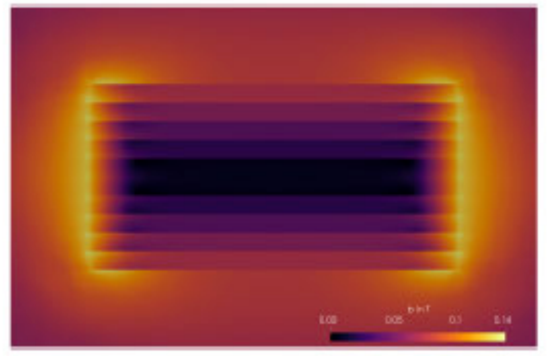
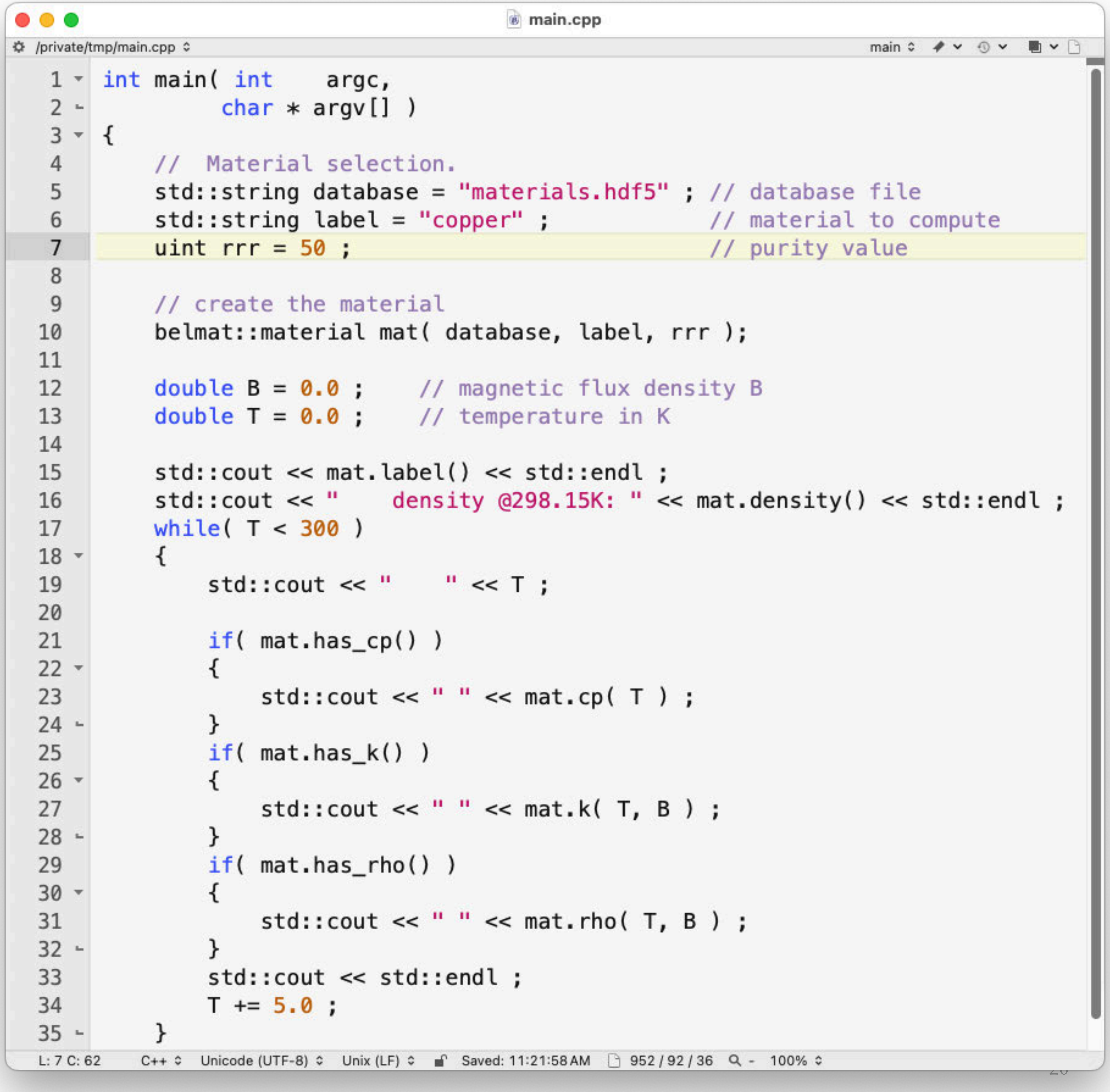


Figure 13. Magnetic flux density at 15 ms with $I = 90$ A per tape at $f = 50$ Hz.



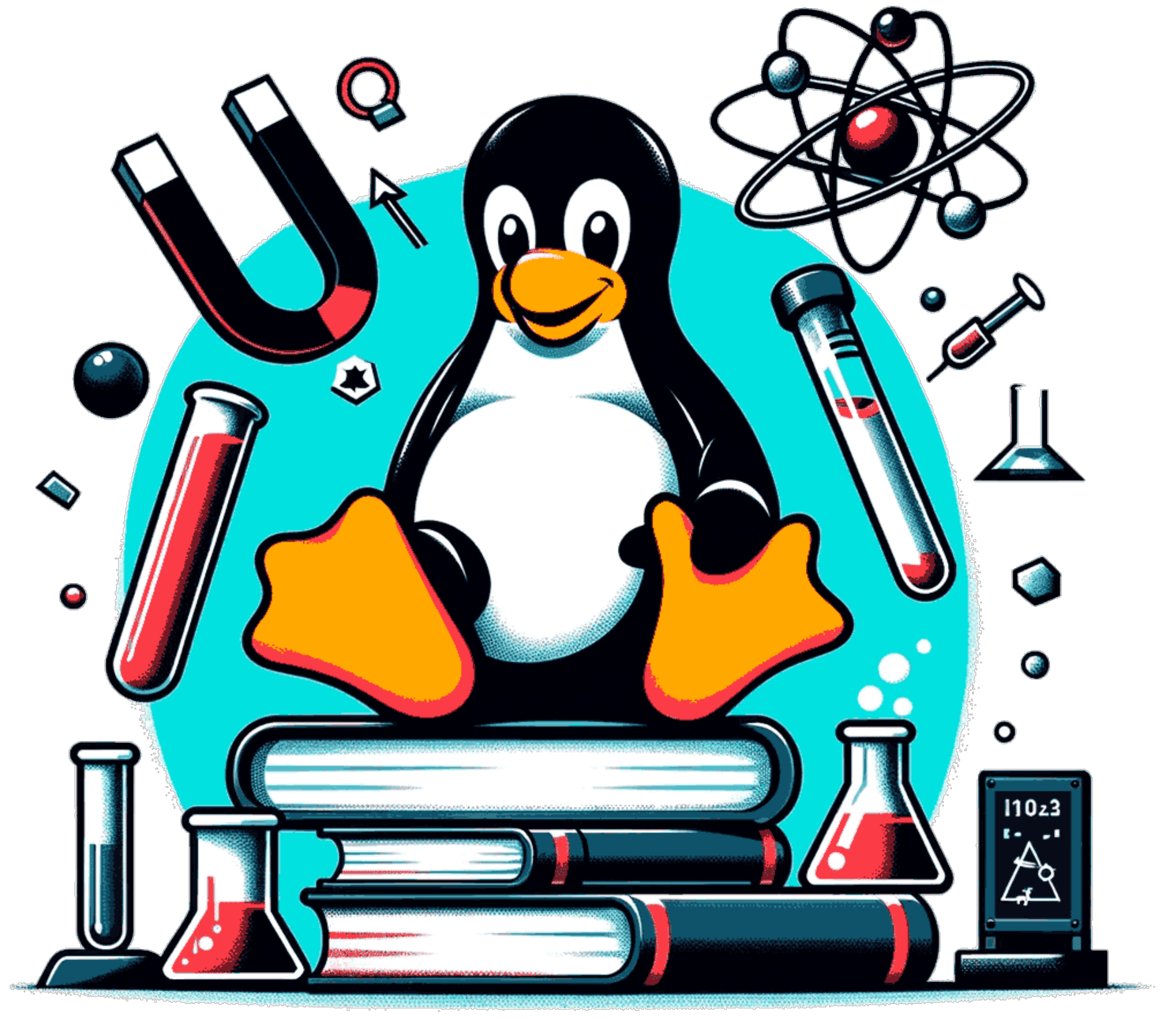
```cpp
int main( int    argc,
          char * argv[] )
{
    //  Material selection.
    std::string database = "materials.hdf5" ; // database file
    std::string label = "copper" ;            // material to compute
    uint rrr = 50 ;                           // purity value

    // create the material
    belmat::material mat( database, label, rrr );

    double B = 0.0 ;    // magnetic flux density B
    double T = 0.0 ;    // temperature in K

    std::cout << mat.label() << std::endl ;
    std::cout << "    density @298.15K: " << mat.density() << std::endl ;
    while( T < 300 )
    {
        std::cout << "    " << T ;

        if( mat.has_cp() )
        {
            std::cout << " " << mat.cp( T ) ;
        }
        if( mat.has_k() )
        {
            std::cout << " " << mat.k( T, B ) ;
        }
        if( mat.has_rho() )
        {
            std::cout << " " << mat.rho( T, B ) ;
        }
        std::cout << std::endl ;
        T += 5.0 ;
    }
```

# One More Thing

# Announcing the Scientific Core Libraries Repo

**RPM Repository for RedHat 9 / Alma Linux 9 / Rocky Linux 9:**

- precompiled RPMs for numerical libraries such as:
  - PETSc
  - MUMPS
  - STRUMPACK
  - SuiteSparse
  - Nlopt
  - Sundials
- Features:
  - compiled with both GNU and Intel Compiler suites
  - Support for MPI, MKL and CUDA



`https://belfem.lbl.gov/scls.html`