

hts2024.eu

**HTS Modelling  
Workshop 2024**

June 10 – 13  
Parkhotel Bad Zurzach  
Switzerland

# Recent Advancements in BELFEM

**Christian Messe, Frédéric Sirois, Gregory Giard**  
**Jun 10th, 2024**

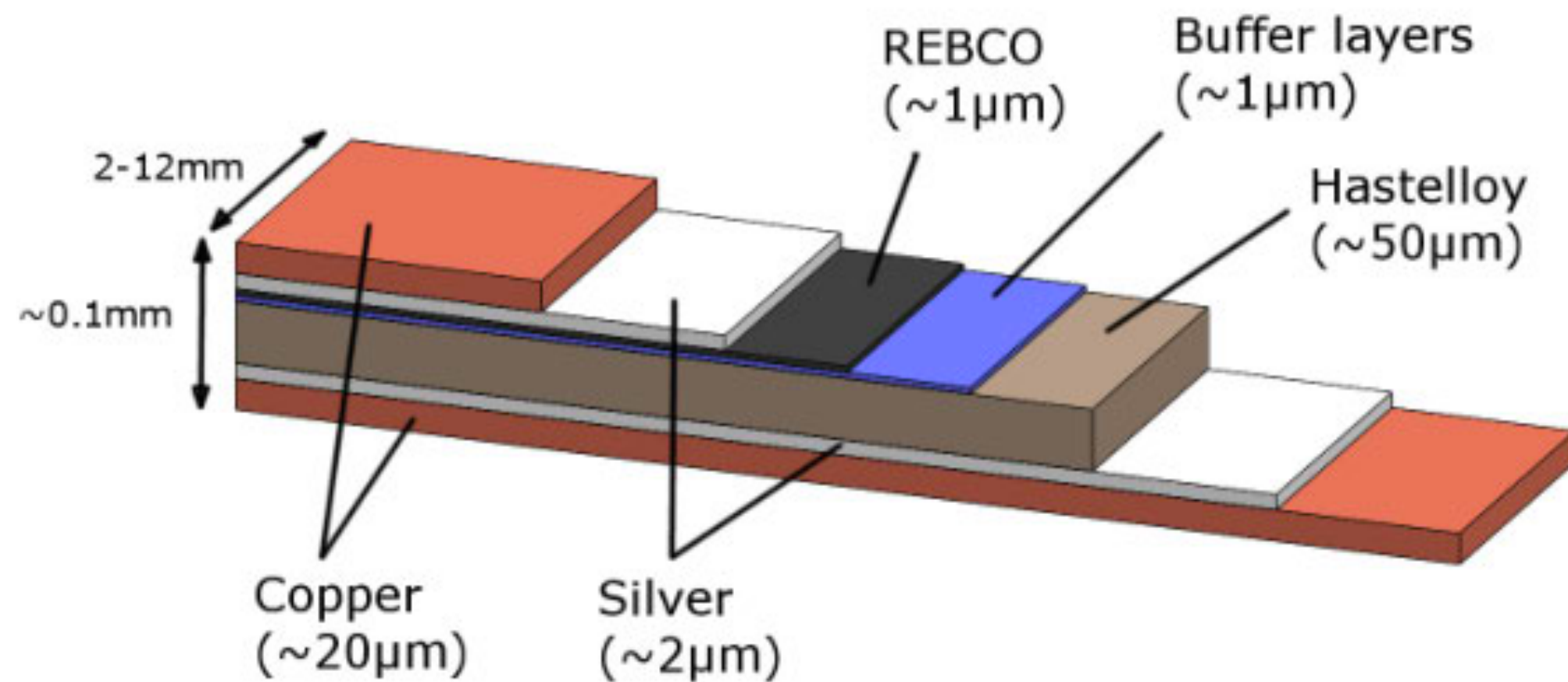
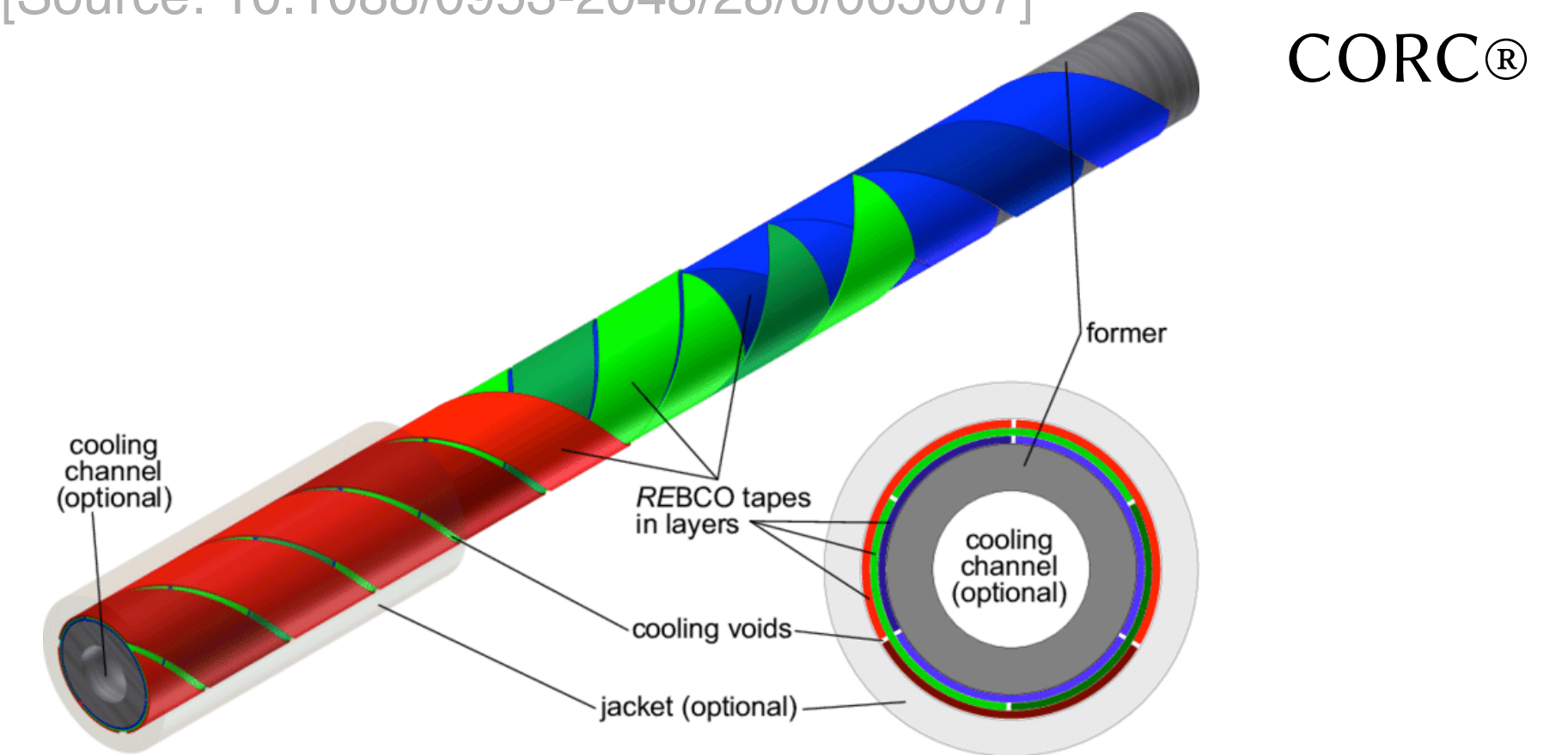


- **Motivation: HTS Tape and Cable Modeling**
- **Maxwell Recap and  $h$ - $\phi$  fundamentals**
- **Why develop a custom codebase?**
- **Homologies and Cohomologies**
- **Last Year's progress**
- **Current Efforts**
- **Summary and Outlook**
- **Thoughts on Material Modeling**

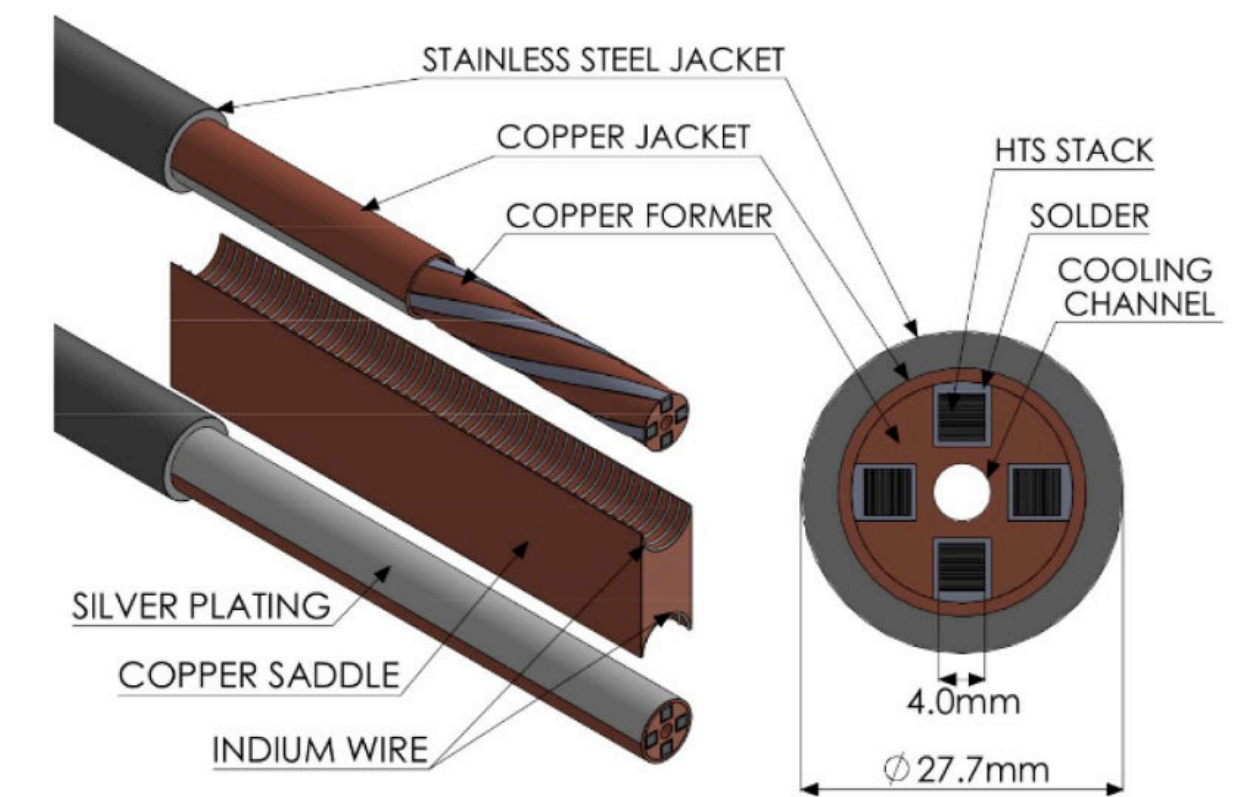
## want ability to model:

- **quasi-magnetodynamic modeling**
  - understand electromagnetic behavior of cables
- **coupled thermal modeling**
  - thermal behavior and physical coupling with EM
  - quench behavior
- **other phenomena**
  - current sharing
  - ...

[Source: 10.1088/0953-2048/28/6/065007]



VIPER



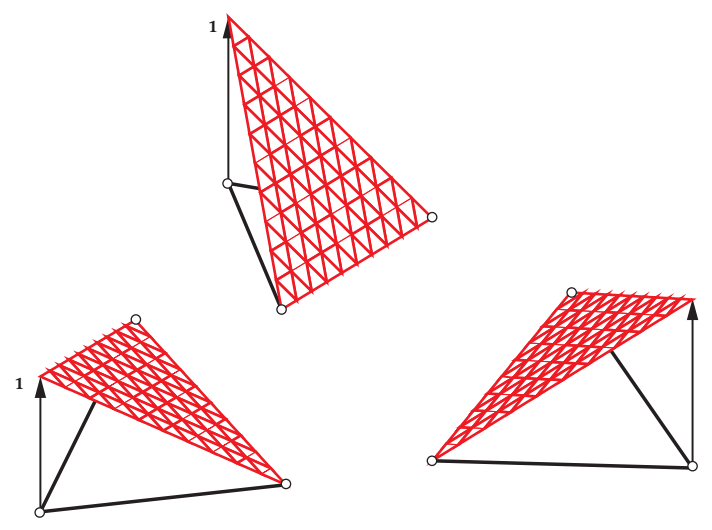
[Source: 10.1088/1361-6668/abb8c0]

## FEM Weak forms are based on different laws

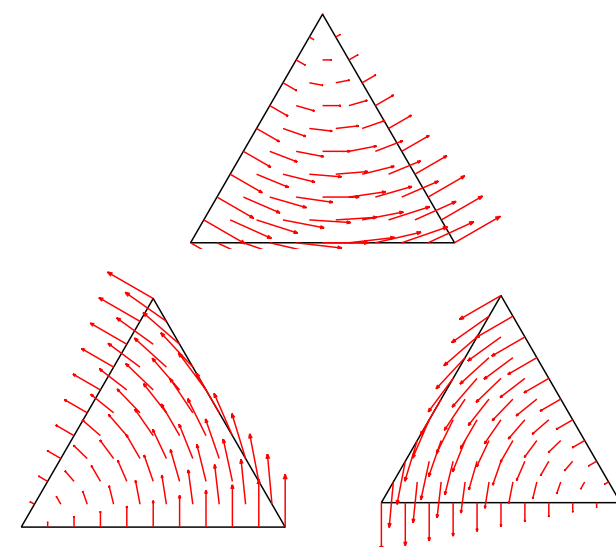
- h-formulation is based on Faraday's law
- a-formulation is based on Ampère's law
- $\phi$ -formulation can be based on either Faraday or Gauß

## Mixed formulations:

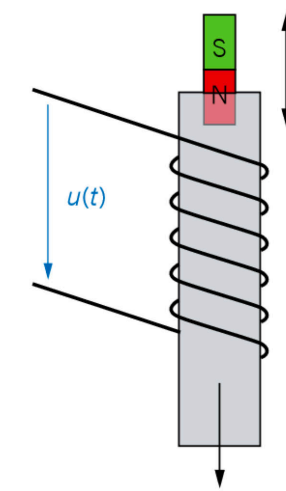
- there is no "universally best" formulation
- mixed formulations aim to combine benefits of individual formulations



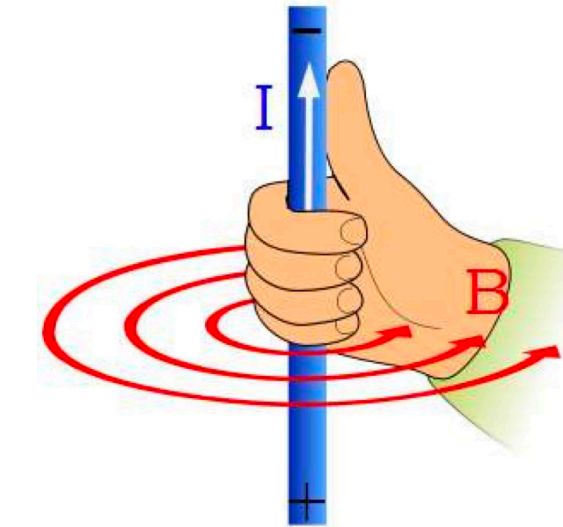
*Lagrange Elements*  
( $\phi$  formulation)



*Nédélec Elements*  
(h-formulation)



**Faraday's law**  
 $\nabla \times \mathbf{e} = -\dot{\mathbf{b}}$



**Ampère's law**  
 $\nabla \times \mathbf{h} = -\mathbf{j}$

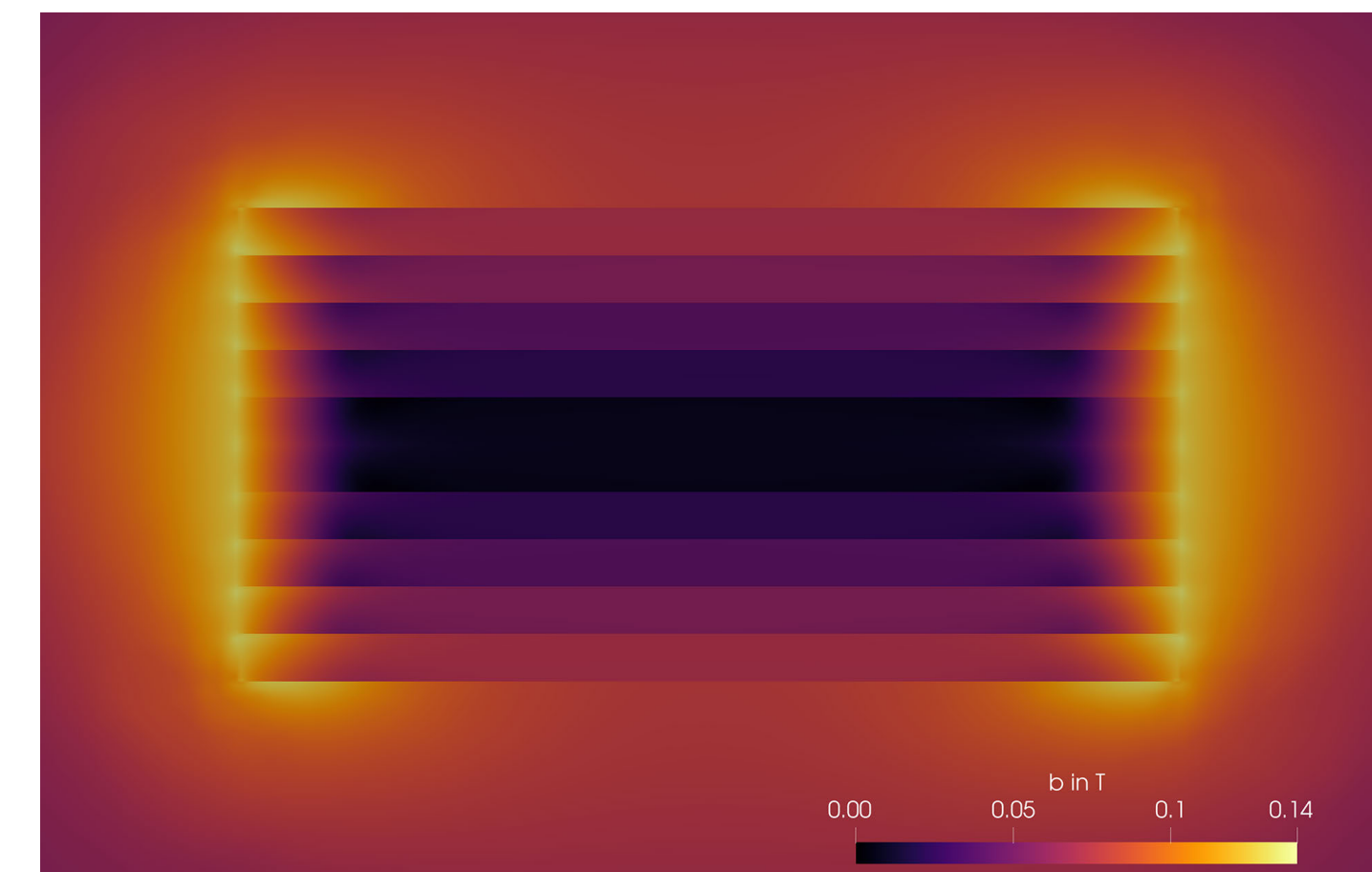
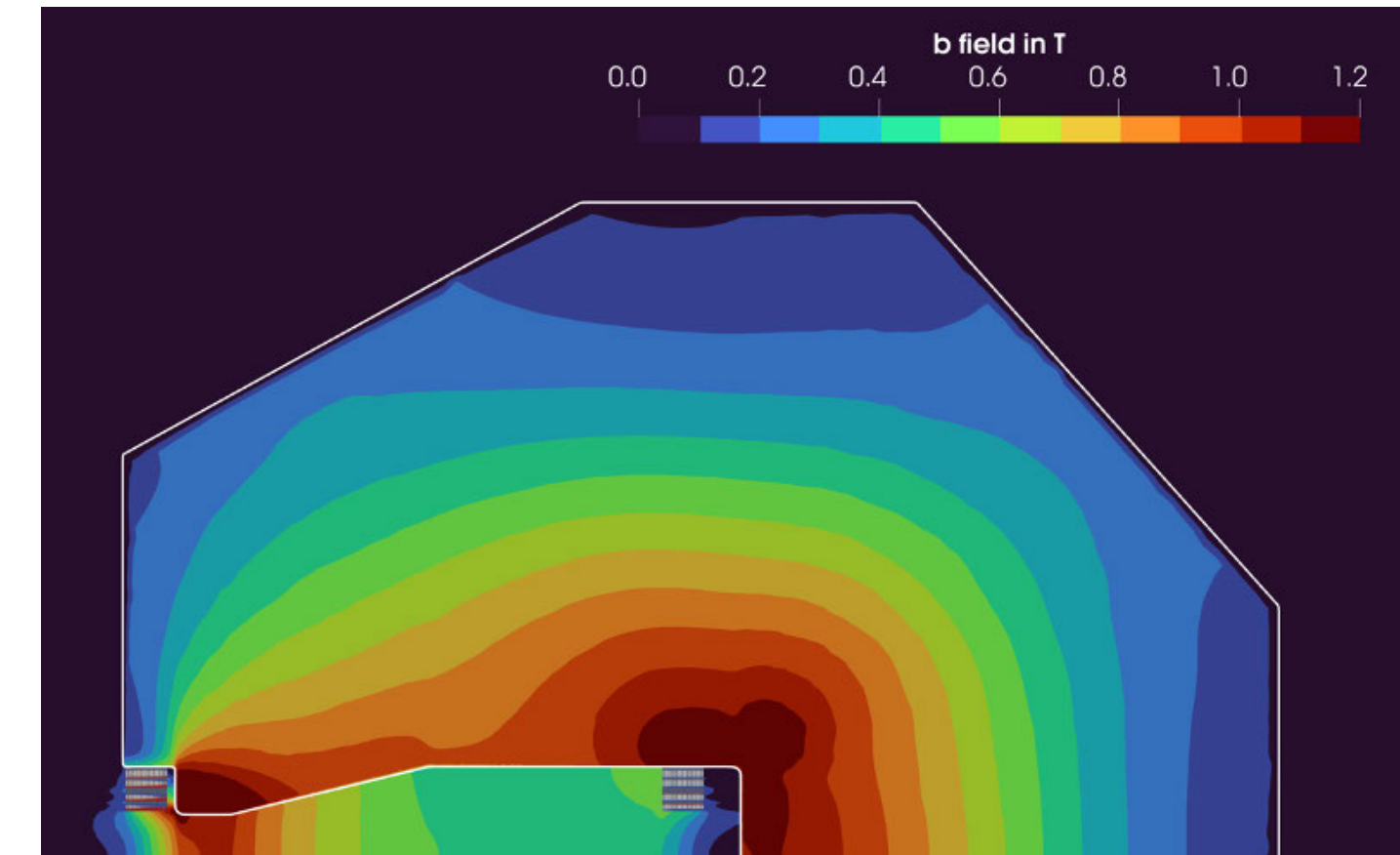
	Air / Vacuum	Conductor	Ferromagnetic Alloy
<b>Governing Equation</b>	$\nabla \times \mathbf{h} = \mathbf{0}$ Ampère-Maxwell	$\nabla \times \mathbf{h} = \mathbf{j}$ Ampère-Maxwell	$\nabla \times \mathbf{e} = \dot{\mathbf{b}}$ Faraday's Law
<b>Degree of Freedom</b>	$\mathbf{h} = -\nabla \phi$ Magnetic Scalar Potential	$\mathbf{h}$ Magnetic Field	$\mathbf{b} = \nabla \times \mathbf{a}$ Magnetic Vector Potential
<b>Transport Law</b>	none	$\mathbf{e} = \rho \cdot \mathbf{j}$ Ohm's Law	$\mathbf{h} = \mathbf{v} \cdot \mathbf{b}$ Magnetic Law
<b>Comment</b>	minimal number of dofs	need edge elements	simple material law implementation

## Model using Volume Elements

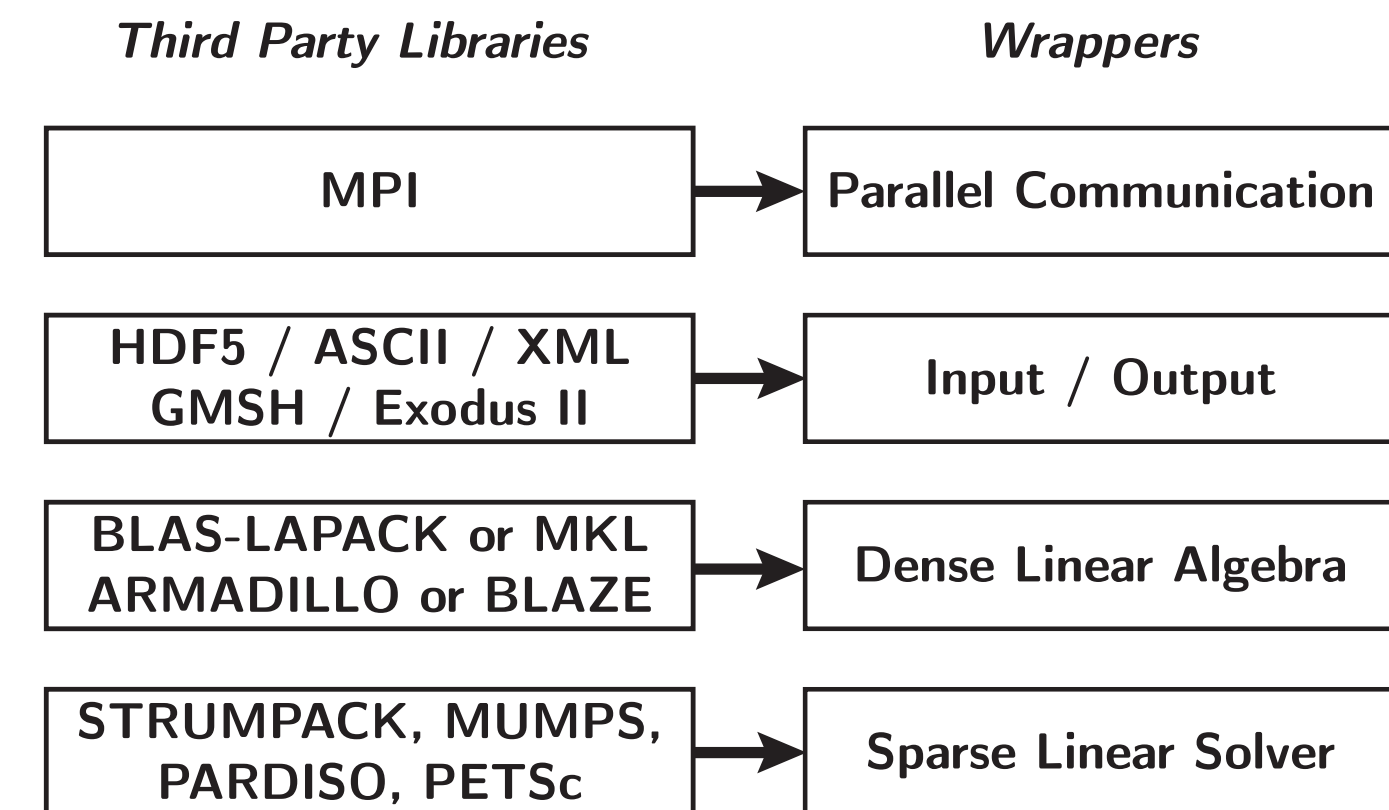
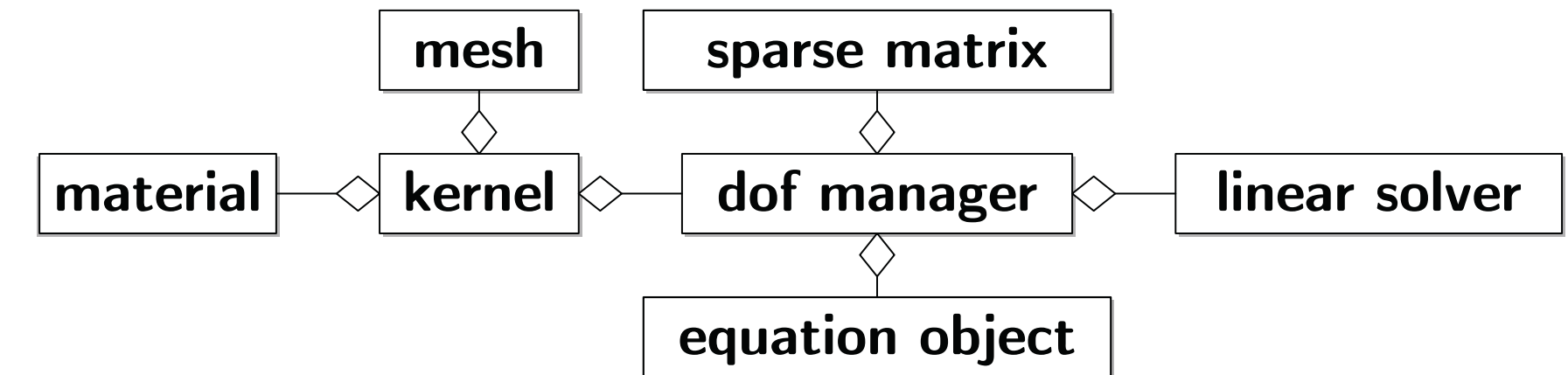
- got momentum in late 2010s to early 2020s
  - very robust formulation
  - significantly reduced degrees of freedom in non-conducting domains
- ➔ very high performance gain compared to pure h-formulation
- ➔ ideal for large 3D models!

## Model using Thin-Shell Elements

- first published in 2022 for mixed h- $\phi$
  - can resolve individual layers of HTS tapes
- ➔ ideal for quench investigation of cables



- **application needs: have thin-shell model that**
  - uses the  $h$ - $\phi$  formulation
  - supports thermal conduction and quenching
  - supports current sharing between overlapping tapes
  - supports convective cooling with LN2 and LHe
- **model development needs**
  - have full control over data structure
  - have full knowledge of underlying algorithms
- **utilization of community software**
  - use open source data formats (gmsht, hdf5, exodus-ii)
  - link to popular solver libraries such as PETSc, MKL, ...
- **in-house resource utilization**
  - work with STRUMPACK team to maximize solver performance  
<https://portal.nersc.gov/project/sparse/strumpack/>



# Homologies and Cohomologies

## Boundary Conditions

current is applied over Ampere's circuital law:

- homologies represent the loops that can be drawn around the conducting regions that fulfill Ampère's law
- only integral current "I [A]" needs to be known

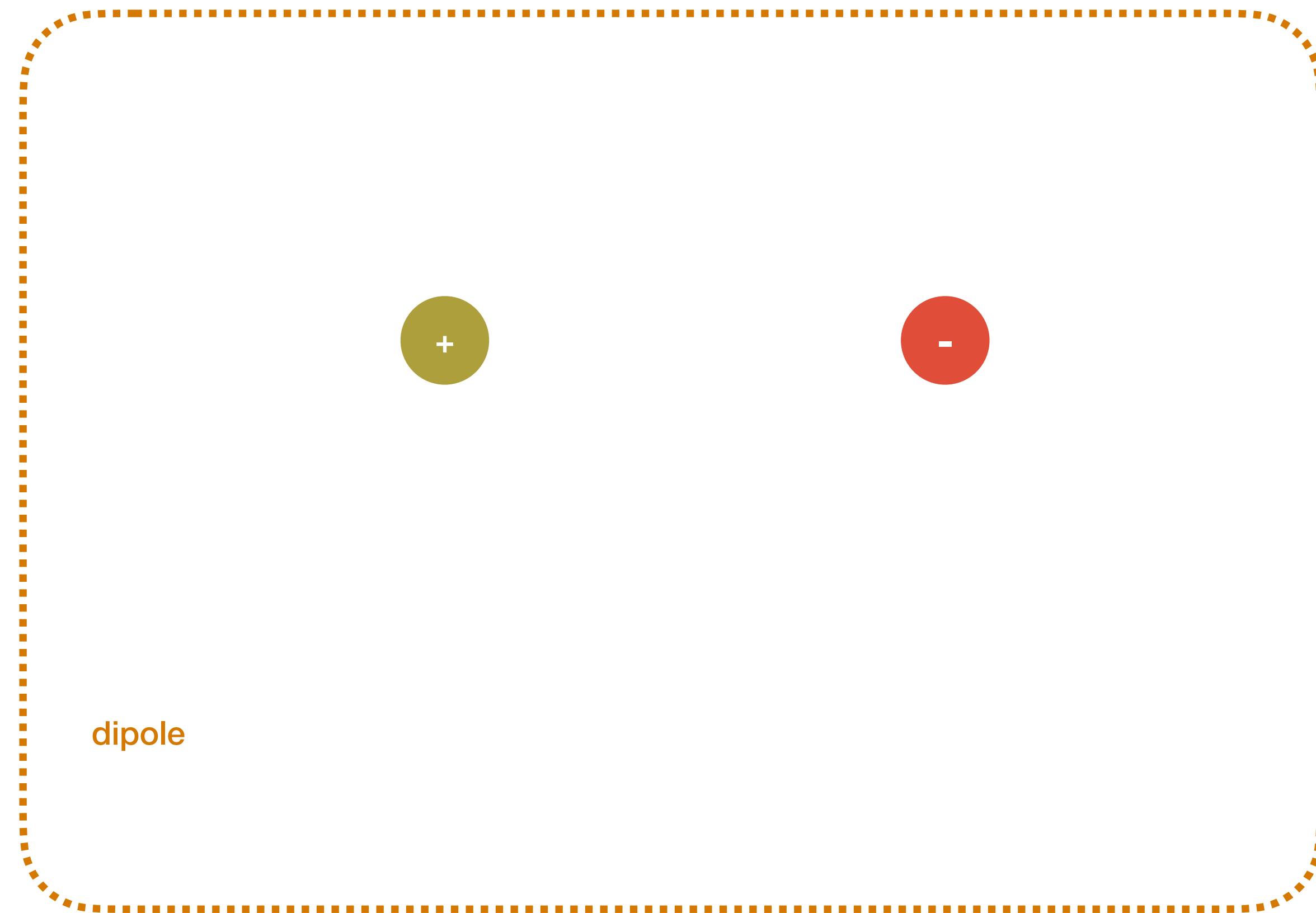


- cohomologies are cuts in the domain over which jumps in the magnetic potential  $\phi$  are imposed so that  $\Delta\phi=I$ .

- ➔ very elegant mathematics!
- ➔ homology definition not user friendly
- ➔ difficult to implement in commercial codes

Ampère's circuital law

$$\oint h \, dl = I$$





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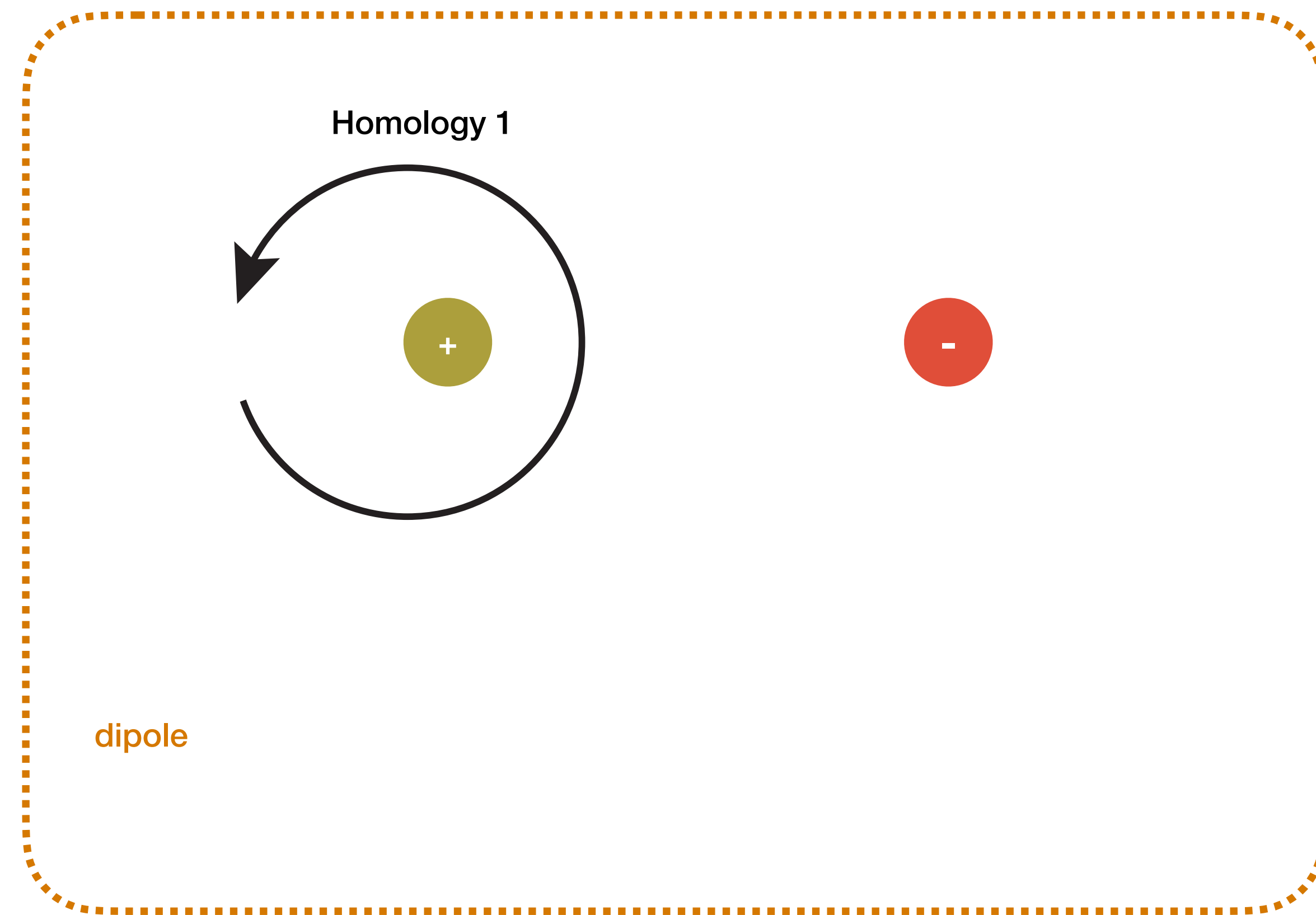


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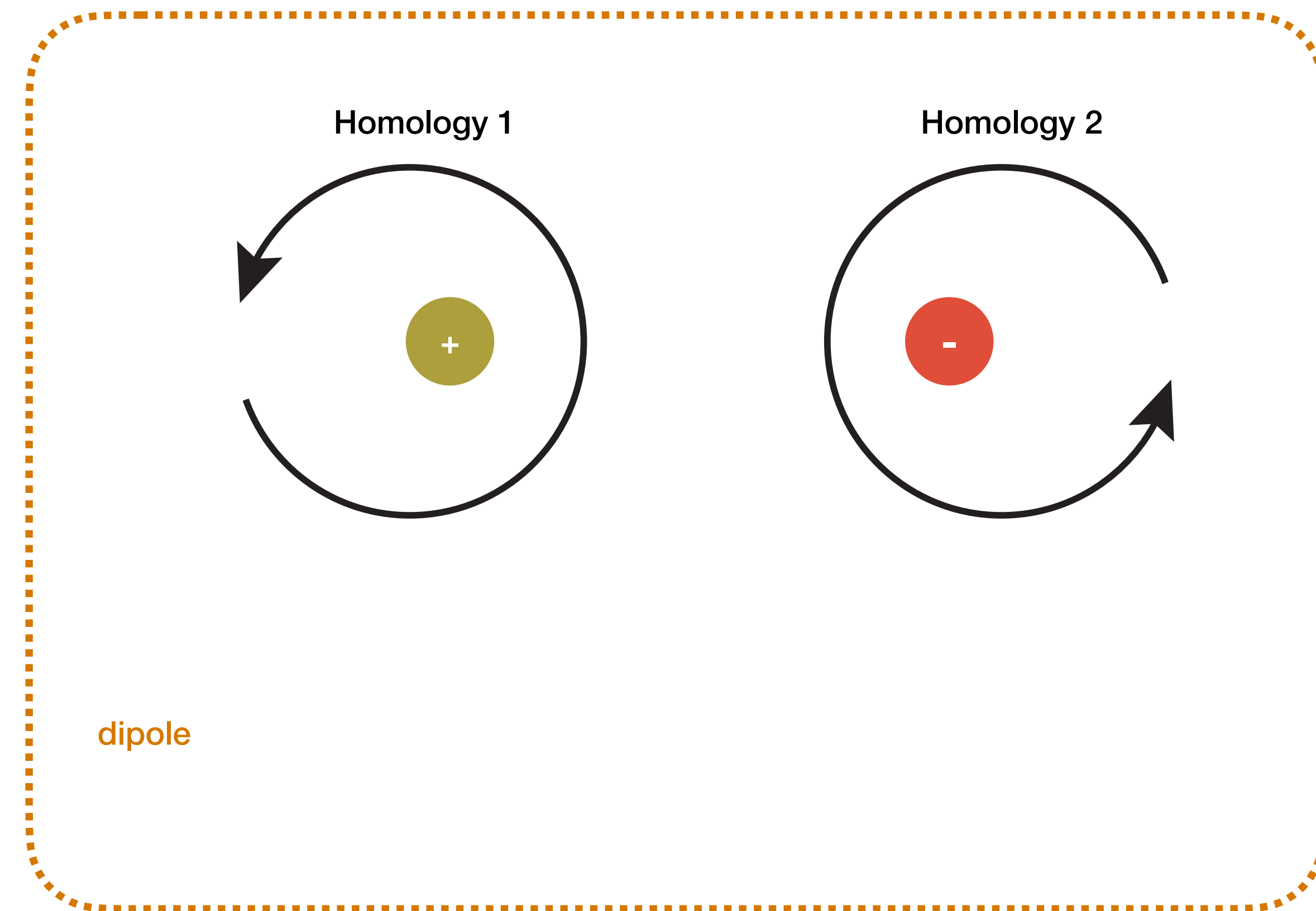


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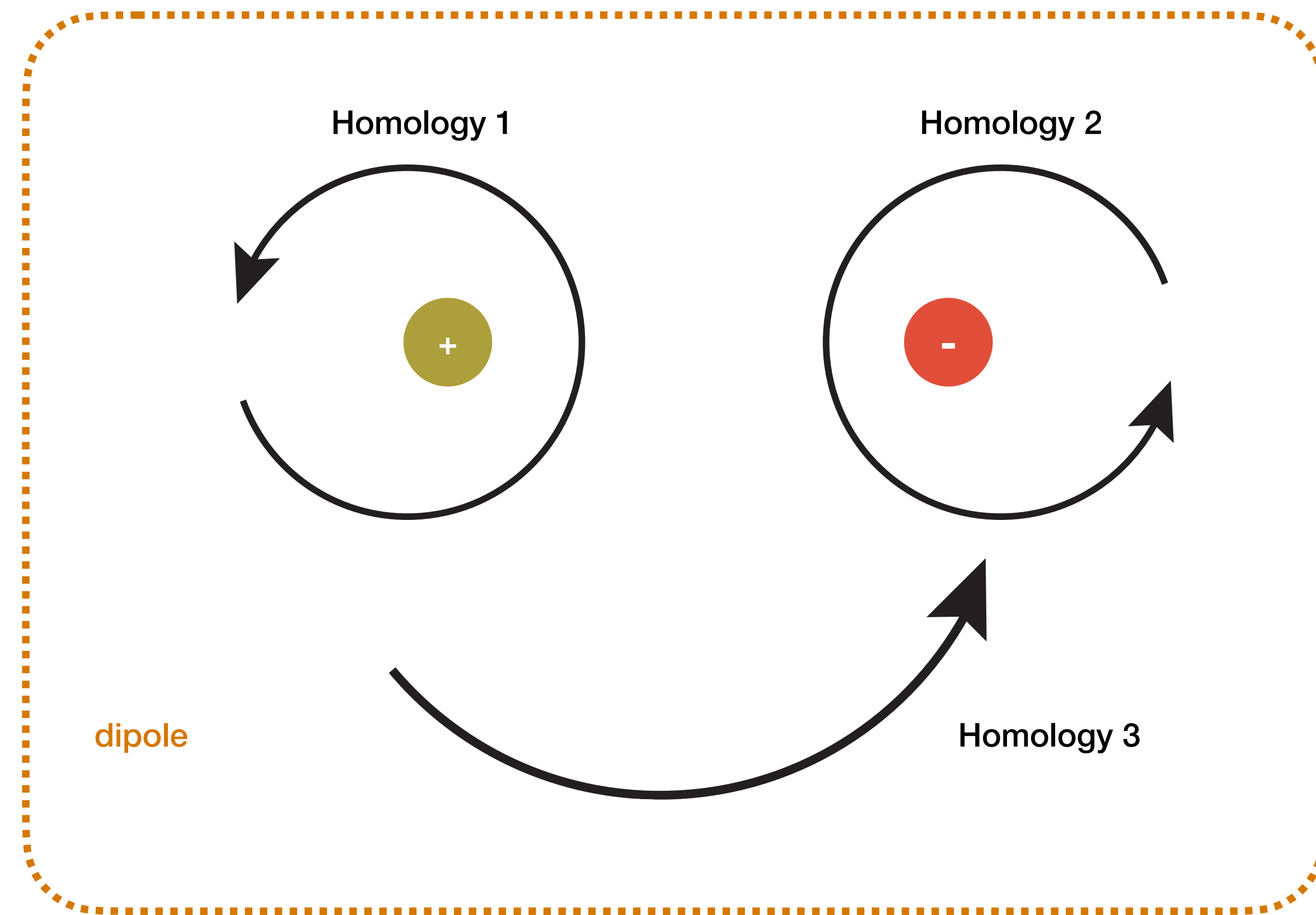


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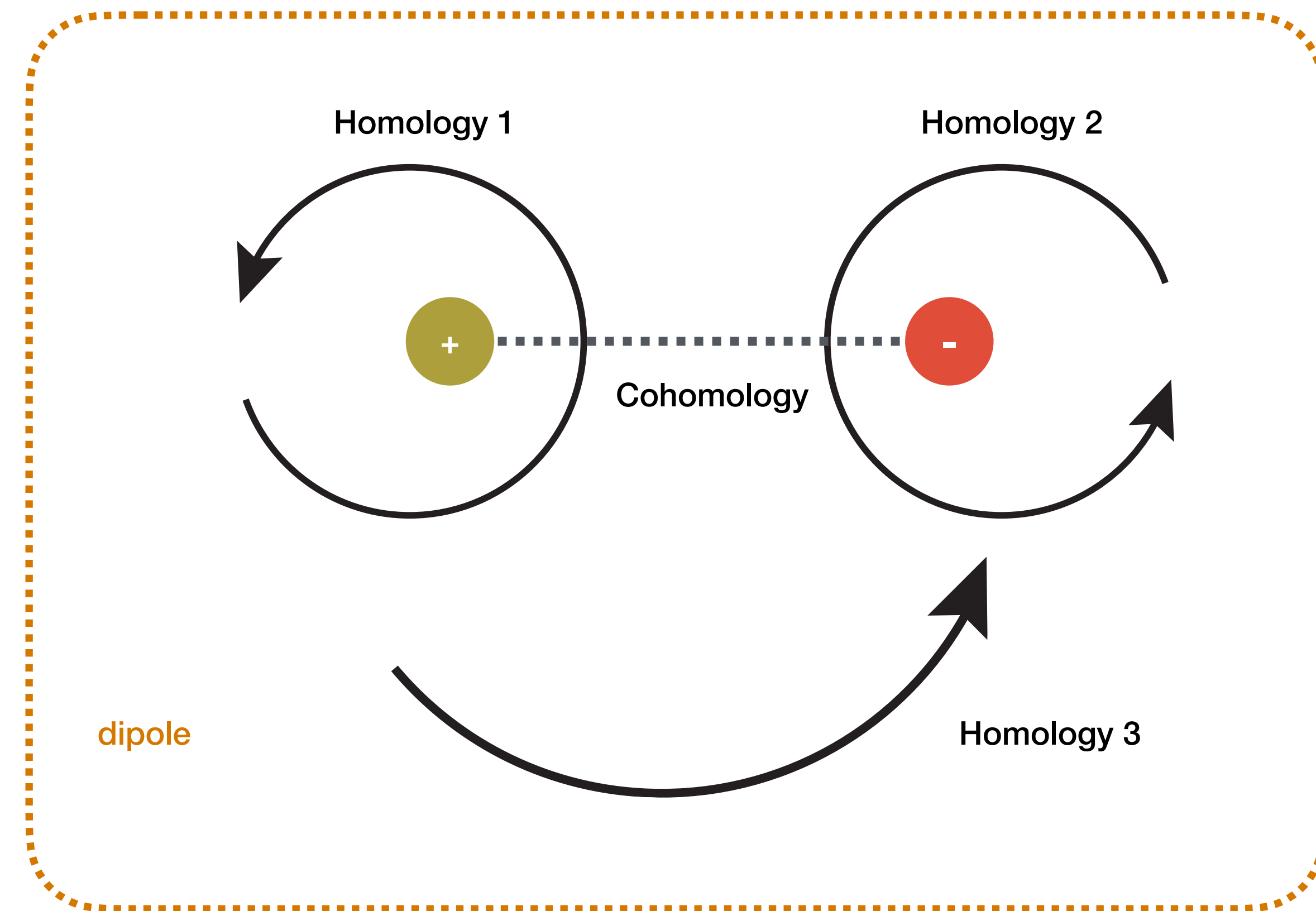


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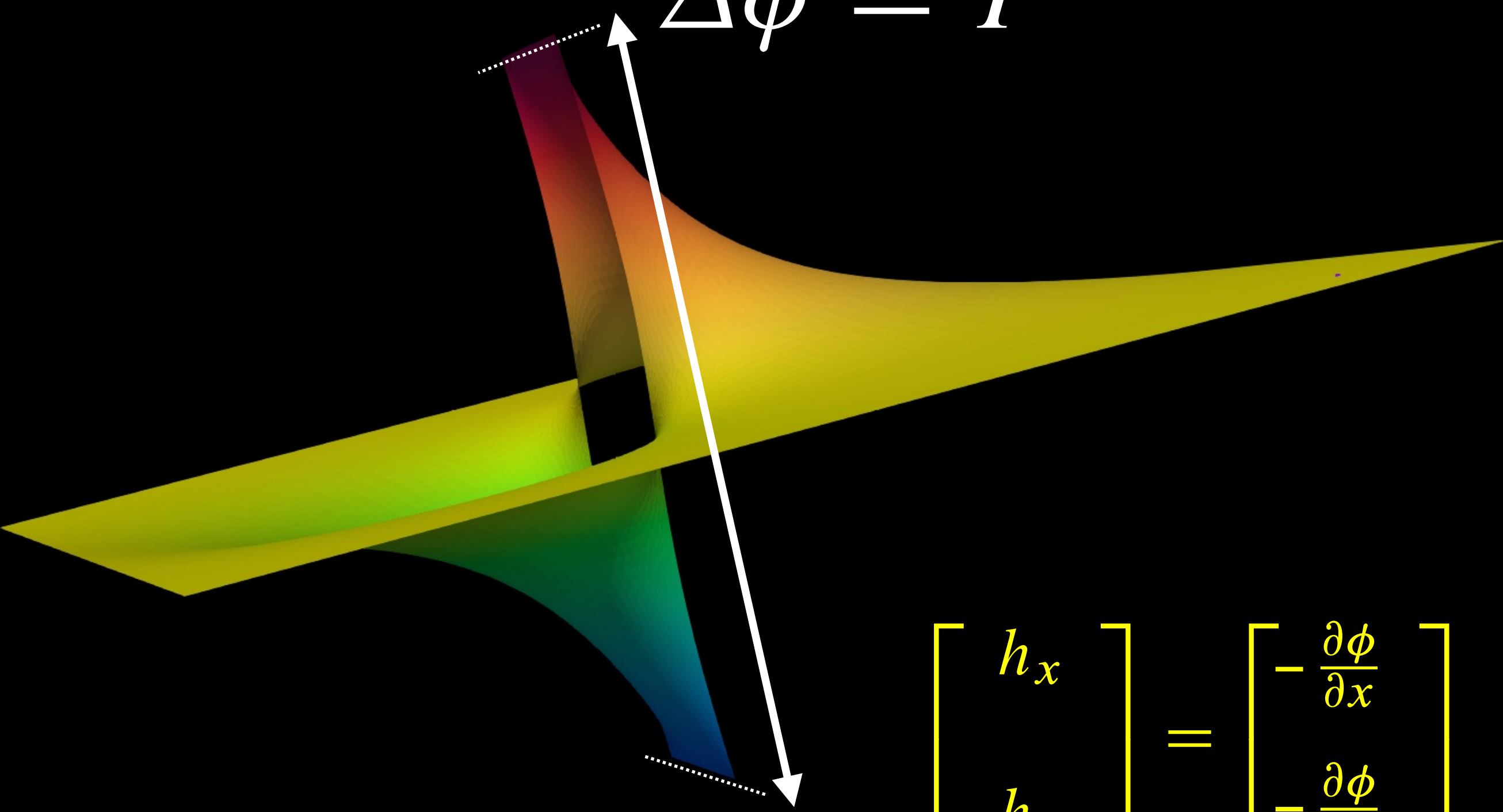
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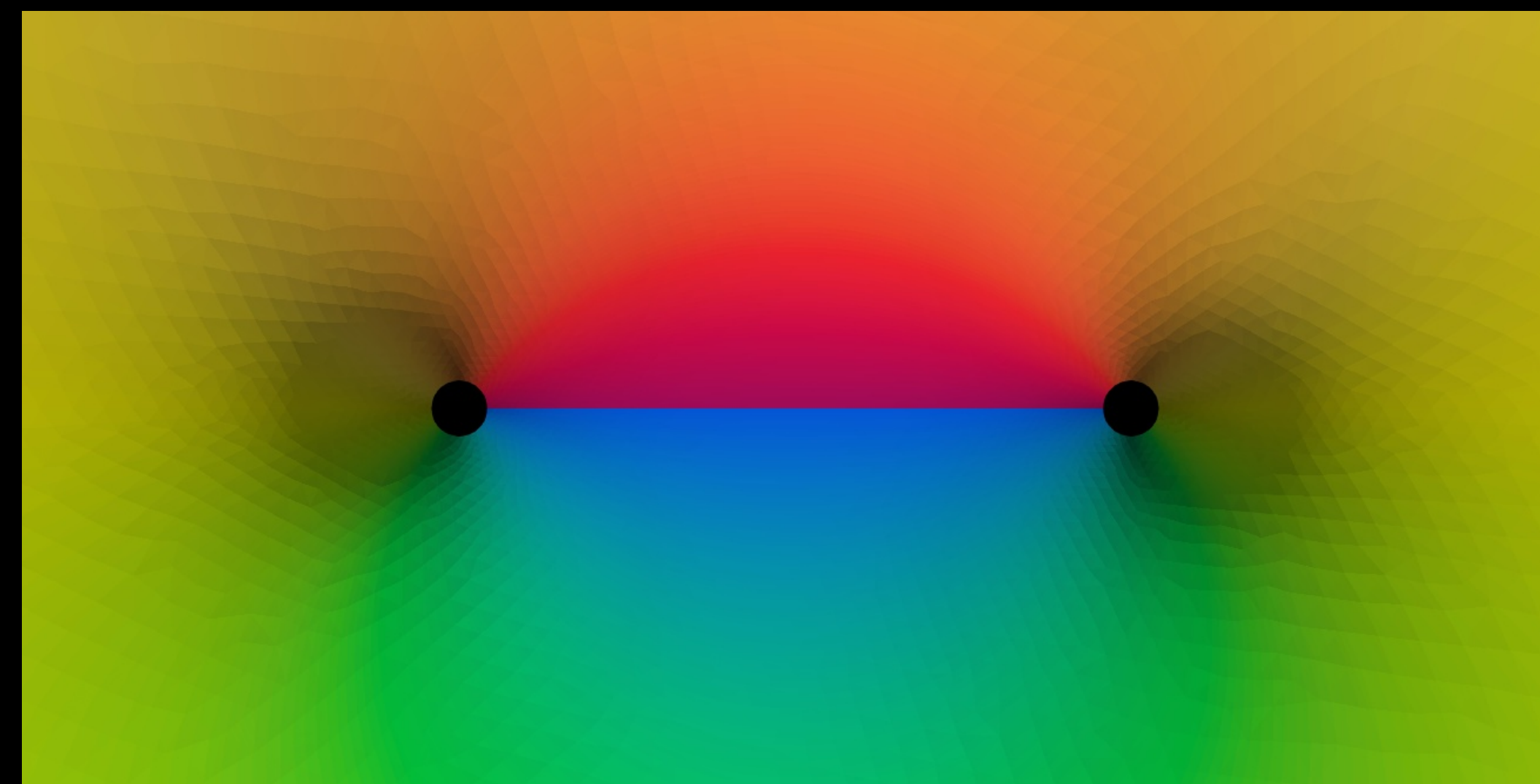
$$\oint \mathbf{h} \, dl = I$$



$$\Delta\phi = I$$

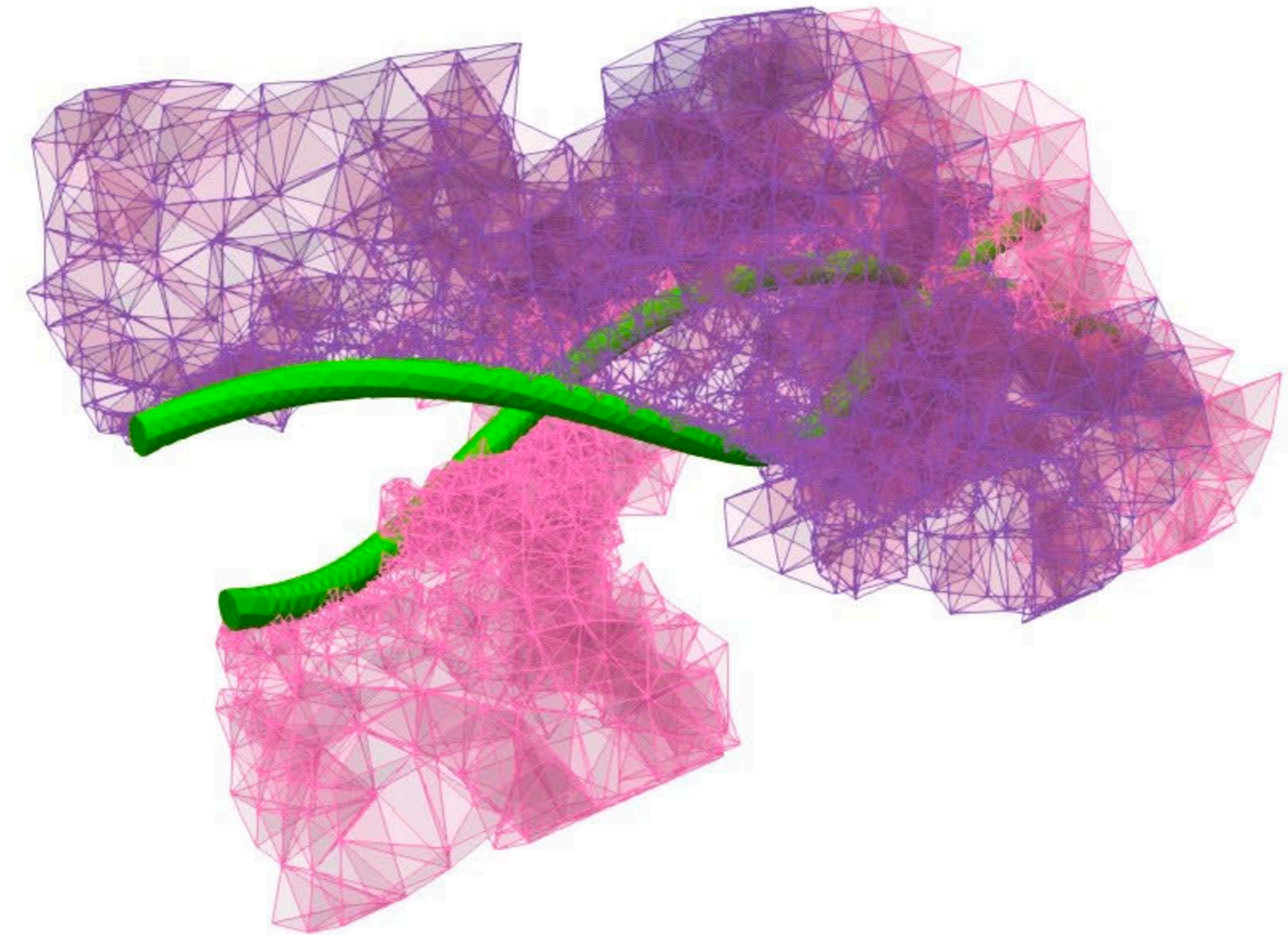


$$\begin{bmatrix} h_x \\ h_y \end{bmatrix} = \begin{bmatrix} -\frac{\partial\phi}{\partial x} \\ -\frac{\partial\phi}{\partial y} \end{bmatrix}$$



## PhD Student Gregory Giard (Polytechnique Montreal):

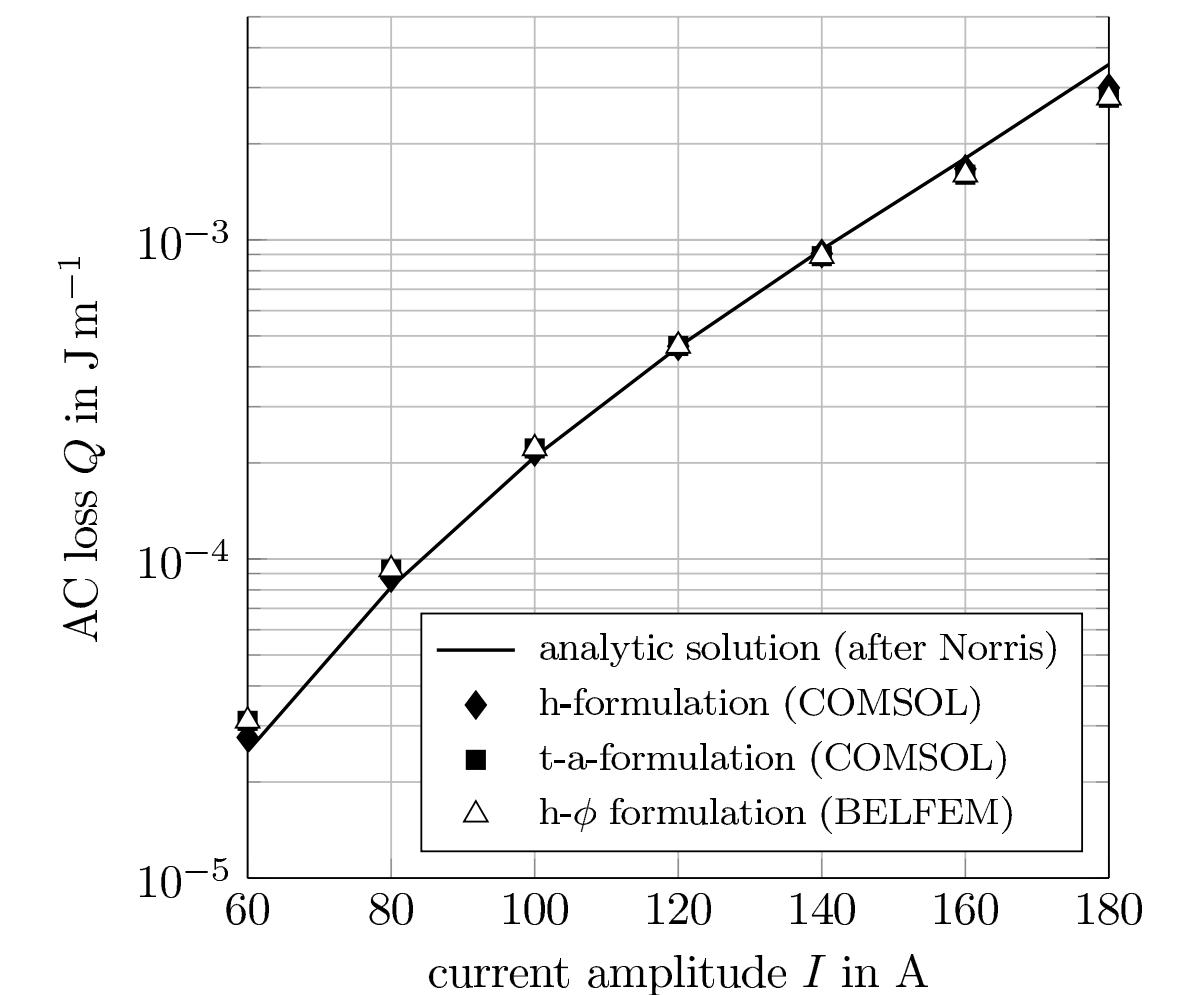
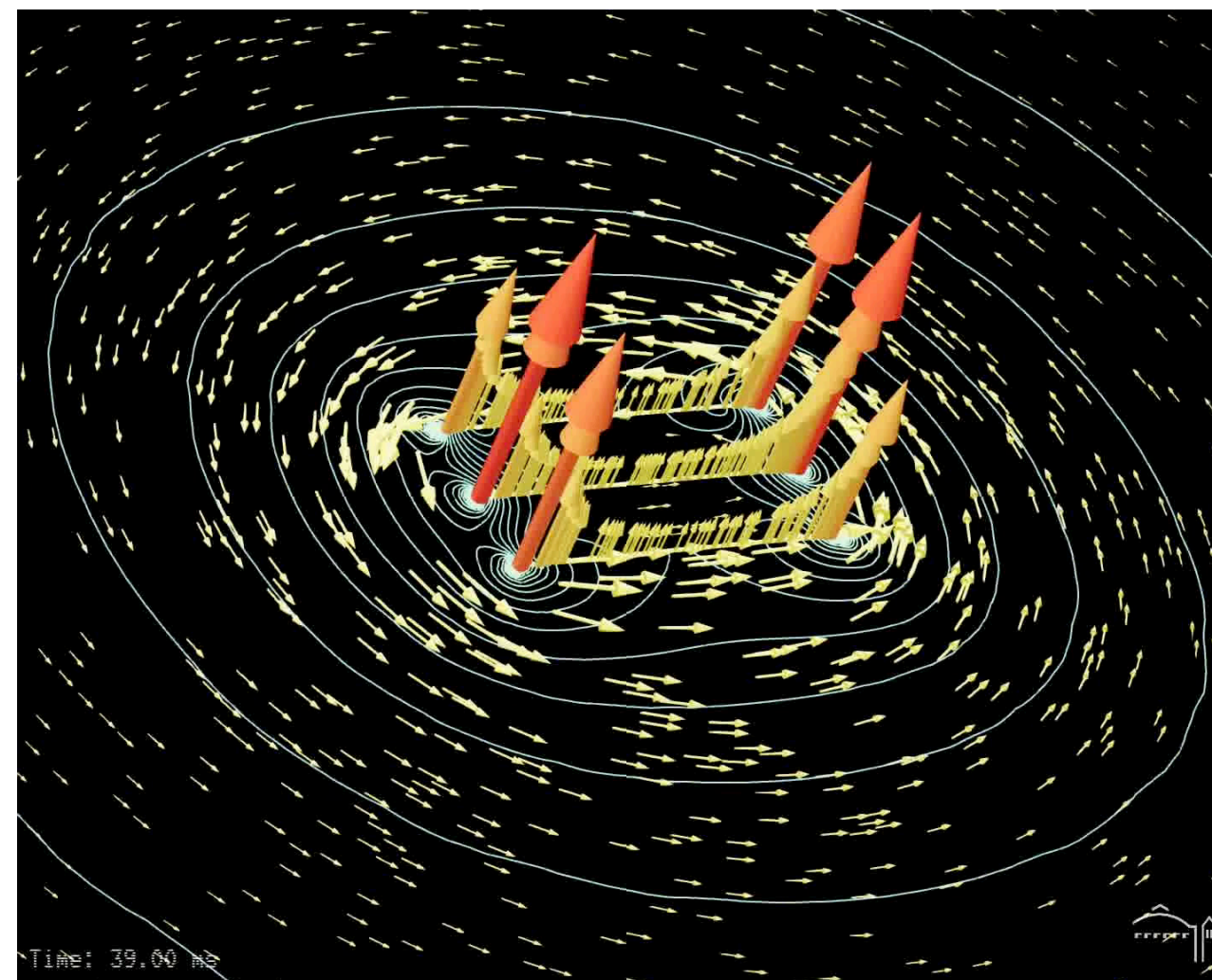
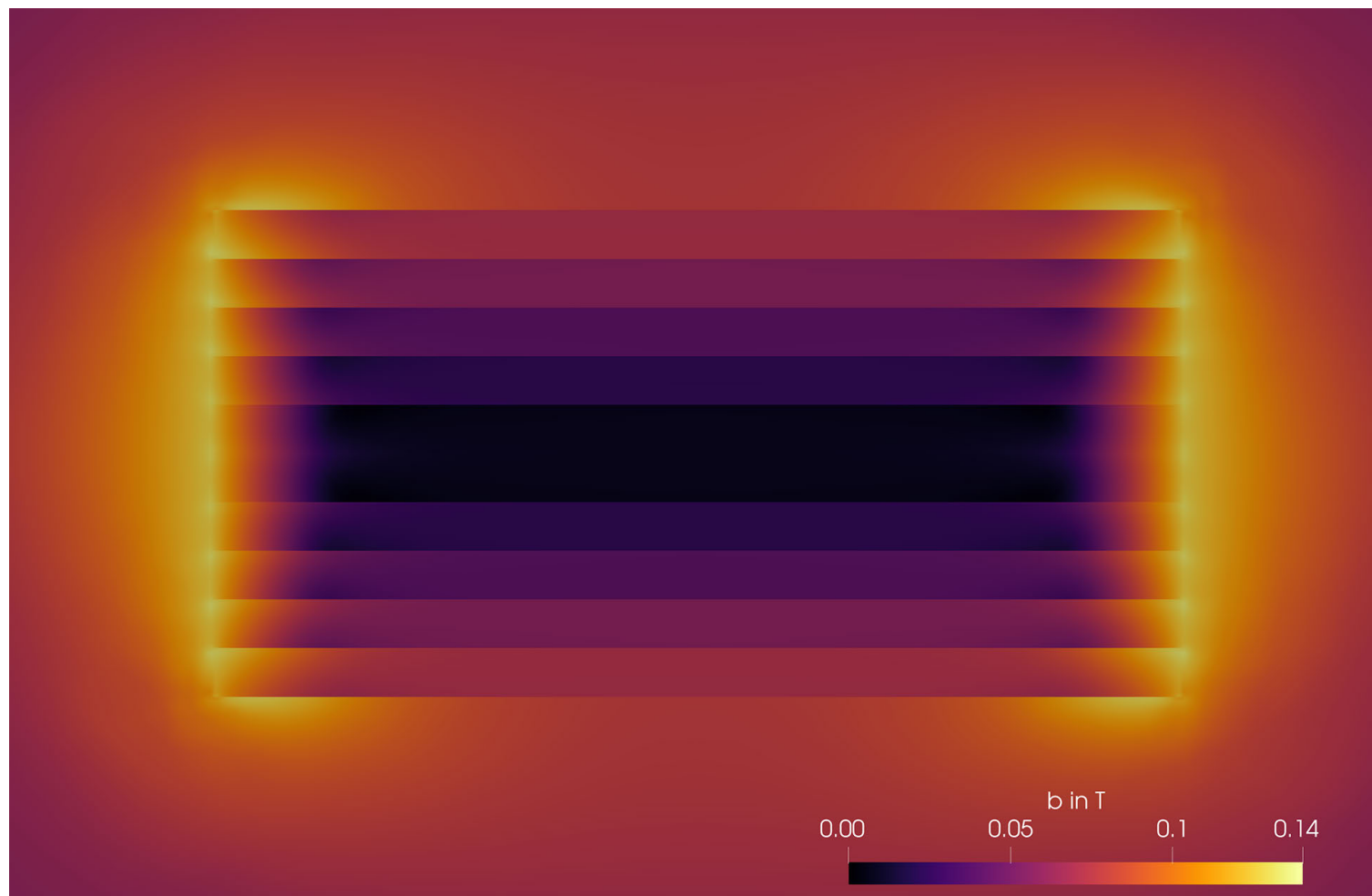
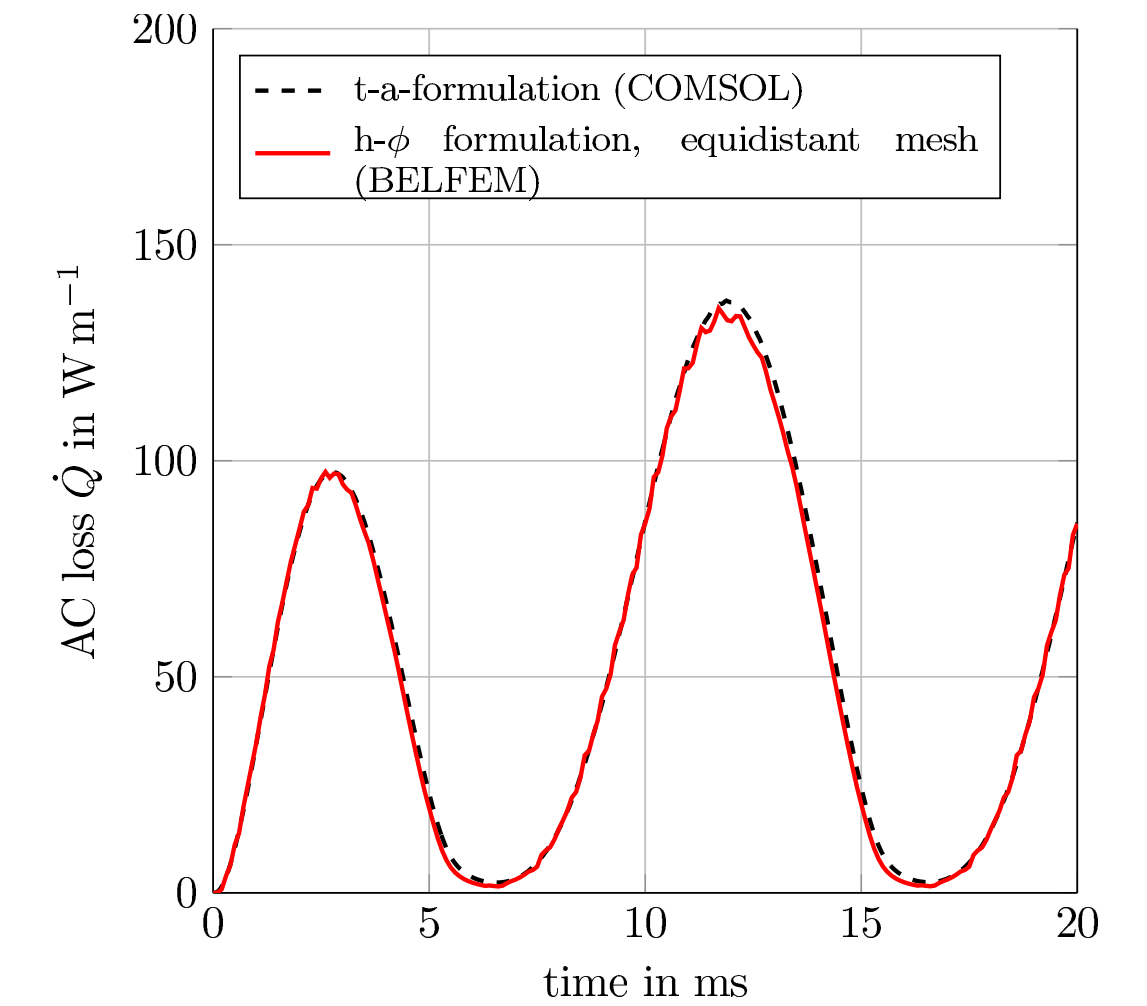
- visiting scholar at LBL from 01/23-06/23
- contribution to adaptive time stepping method
- development of 3D thermal conduction model
- implementing automated cohomology computation in 3D
- automated identification of cut orientations based on user provided currents (“the user shall not worry about cohomologies”)
- clean formulation of “thin” and “thick” cuts using “change of basis” for the former and XFEM for the latter.



# Current Status

## Published Paper in SuST 2023

- Christian Messe, Berkeley Lab
- Nico Riva, MIT
- Sofia Viarengo, Politecnico di Torino
- Gregory Giard & Frédéric Sirois, Polytechnique Montreal
- validated against analytical methods + COMSOL / GetDP
- first research promises faster and more detailed results than other established methods such as t-a





IOP Publishing  
 Supercond. Sci. Technol. 36 (2023) 114001 (12pp)  
 Superconductor Science and Technology  
<https://doi.org/10.1088/1361-6668/ac17f9>

## BELFEM: a special purpose finite element code for the magnetodynamic modeling of high-temperature superconducting tapes

Christian Messe<sup>1,\*</sup>, Nicolò Riva<sup>2</sup>, Sofia Viarengo<sup>3</sup>, Gregory Giard<sup>1</sup> and Frédéric Sirois<sup>4</sup>

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<sup>2</sup> Massachusetts Institute of Technology, Plasma Science and Fusion Center, Cambridge, MA, United States of America  
<sup>3</sup> MAHTEP Group, Dipartimento Energia 'Galileo Ferraris', Politecnico di Torino, Turin, Italy  
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**Abstract**  
 Predicting the performance and reliability of high-temperature superconducting (HTS) cables and magnets is a critical component of their research and development process. Novel mixed finite element formulations, particularly the  $h$ - $\phi$ -formulation with thin-shell simplification, present promising opportunities for more efficient simulations of larger geometries. To make these new methods accessible in a flexible tool, we are developing the Berkeley Lab Finite Element Framework (BELFEM). This paper provides an overview of the relevant formulations, discusses the current state of the art, and discusses the main aspects of the BELFEM code structure. We validate a first 2D thin-shell implementation in BELFEM against selected benchmarks computed in COMSOL Multiphysics and compare the performance of our code with a comparable formulation in GetDP. We also outline the next steps in the development process, paving the way for more advanced and robust modeling capabilities.

Keywords: HTS, FEM, modeling,  $h$ - $\phi$ -formulation, code development

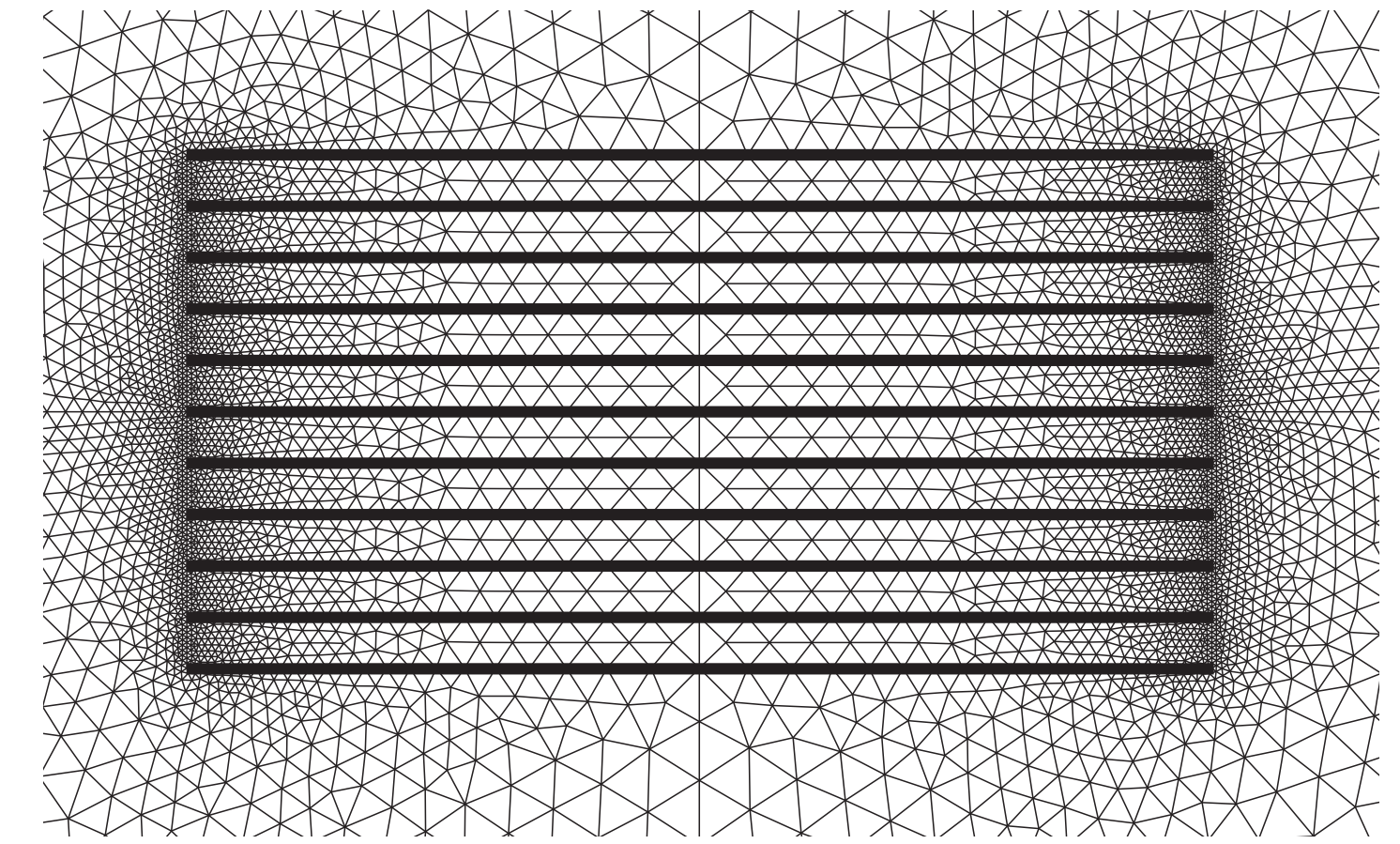
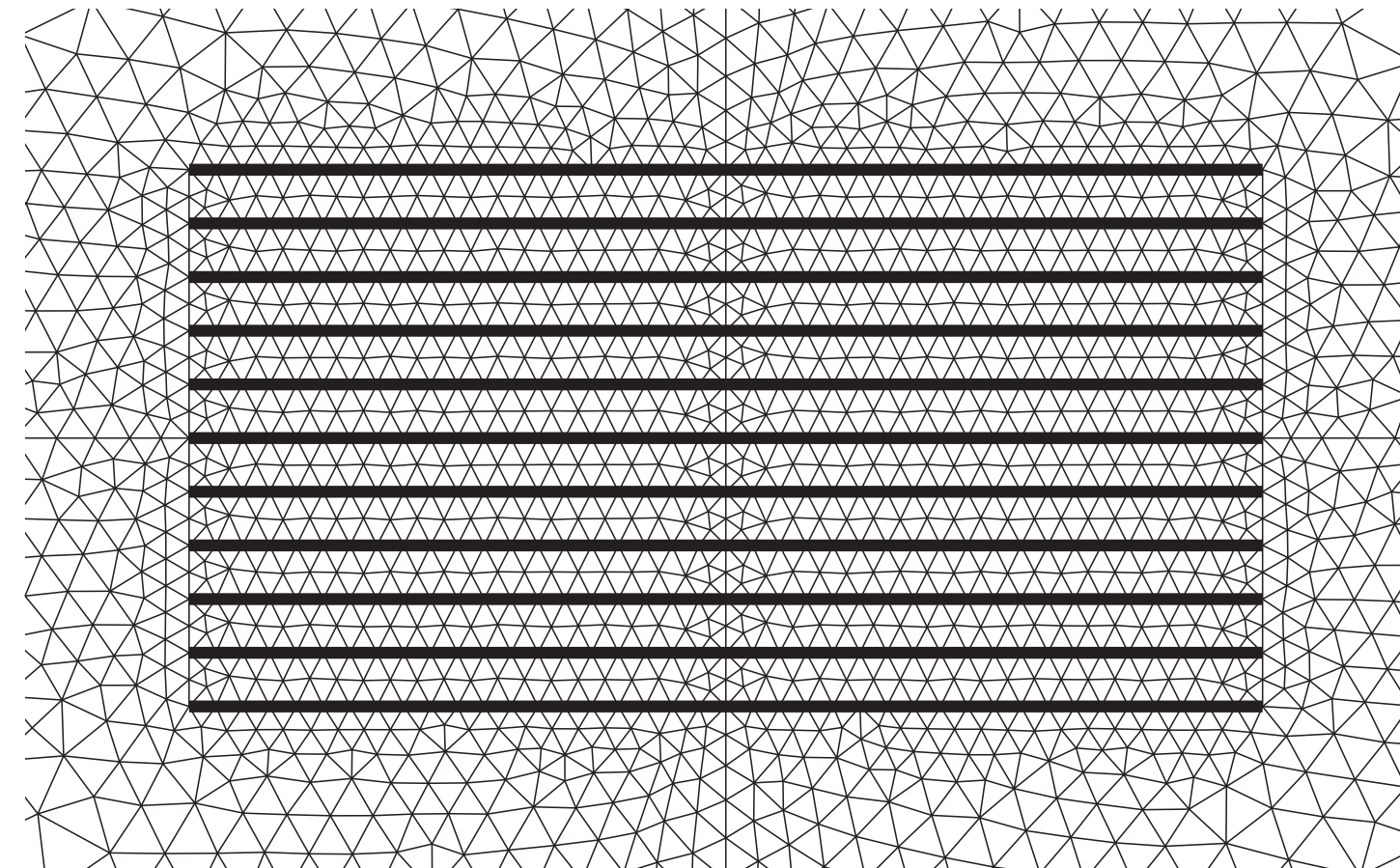
(Some figures may appear in colour only in the online journal)

### 1. Introduction

High-temperature superconducting (HTS) cables and magnets play a crucial role in nuclear fusion applications [1], particle accelerators [2], medical devices [3], and even spacecraft engines [4]. To ensure their safe operation, it is necessary to analyze and understand their electrodynamic and

thermal performance. Here, advanced finite element methods are essential and very powerful tools: not only do they solve the Maxwell's equations in their magnetodynamic form and the heat transfer equation that describe the physical behavior of these devices, they are also able to realistically model the highly nonlinear behavior of the used materials. The computational cost of these methods, however, remains an important challenge. In recent years, significant progress has been made in the development of so called *mixed formulations*, which promise to be more efficient than traditional finite element

\* Author to whom any correspondence should be addressed.



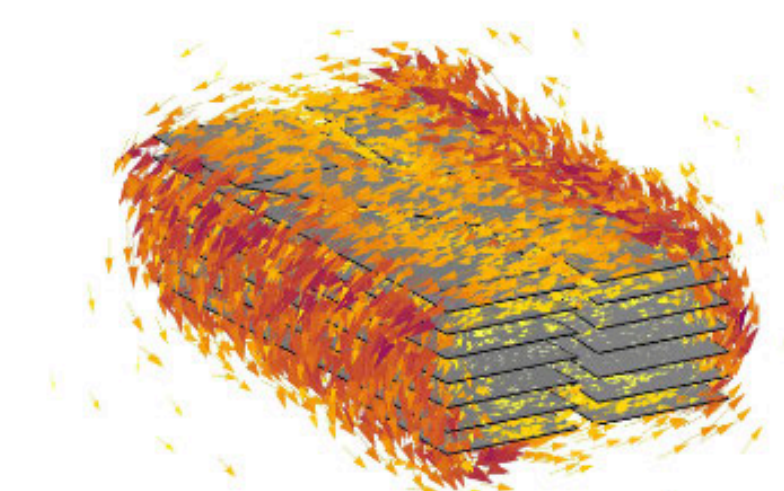
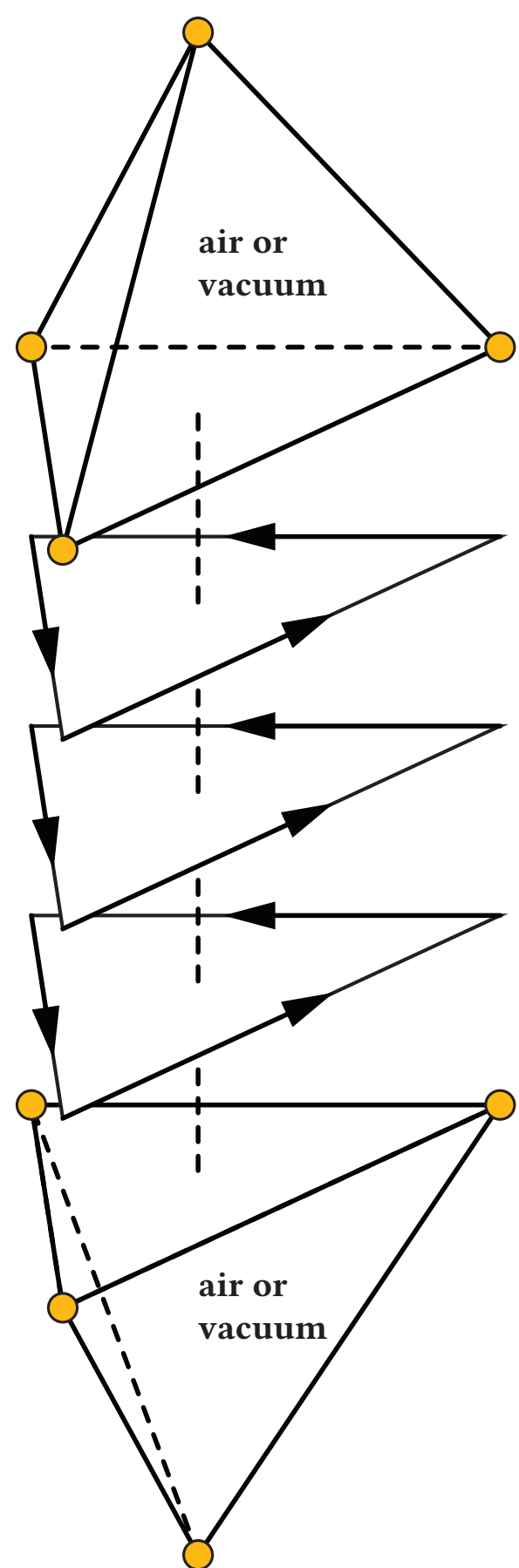
Code and Library	Constant time step		Adaptive time step	
	Coarse Mesh	Fine Mesh	Coarse Mesh	Fine Mesh
GetDP (MUMPS)	5:13	10:57	2:36	7:58
BELFEM (MUMPS)	1:56	7:15	0:23	1:09
BELFEM (STRUMPACK)	0:55	2:54	0:11	0:25

## Goal:

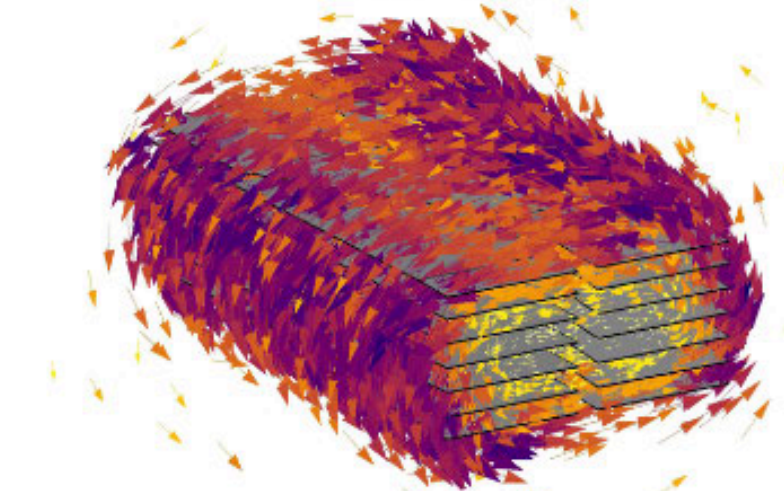
- model a thin shell tapes tack in 3D after Alves et al, 2022
- extend model to encompass solder and thermal model
- be able to do the coupled EM-Thermal quenching model by end of the year

## Roadmap:

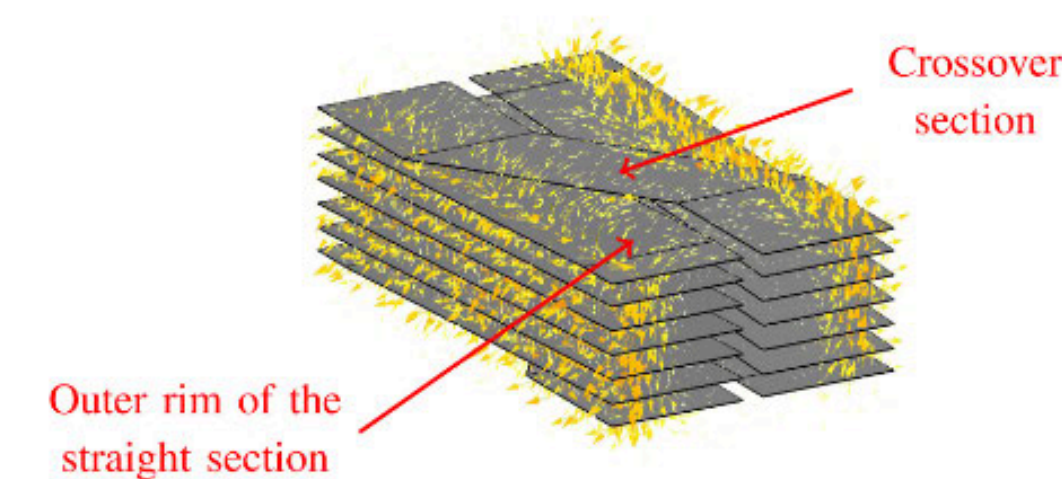
- overhaul data structure for simplified programming of weak governing equations
- improve degree of freedom management system
- first benchmark with 3D tapestack
- implement solder and thermal model
- benchmark involving quench
- address contact sharing (Spring 2025)



(a)  $t = T/8$



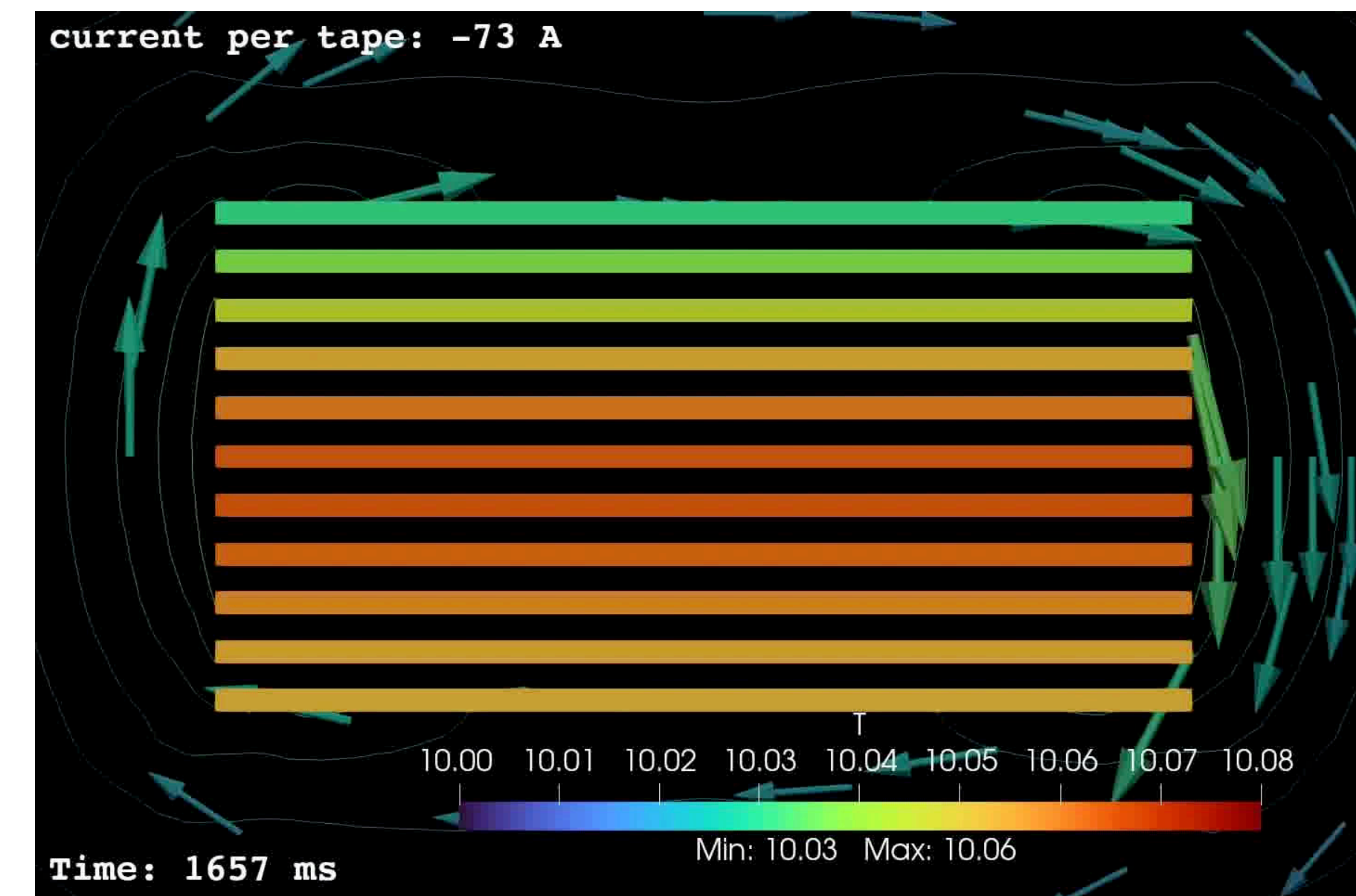
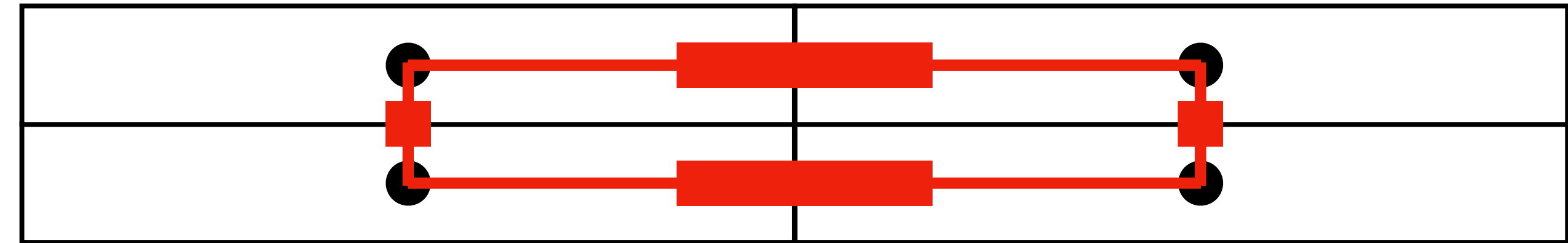
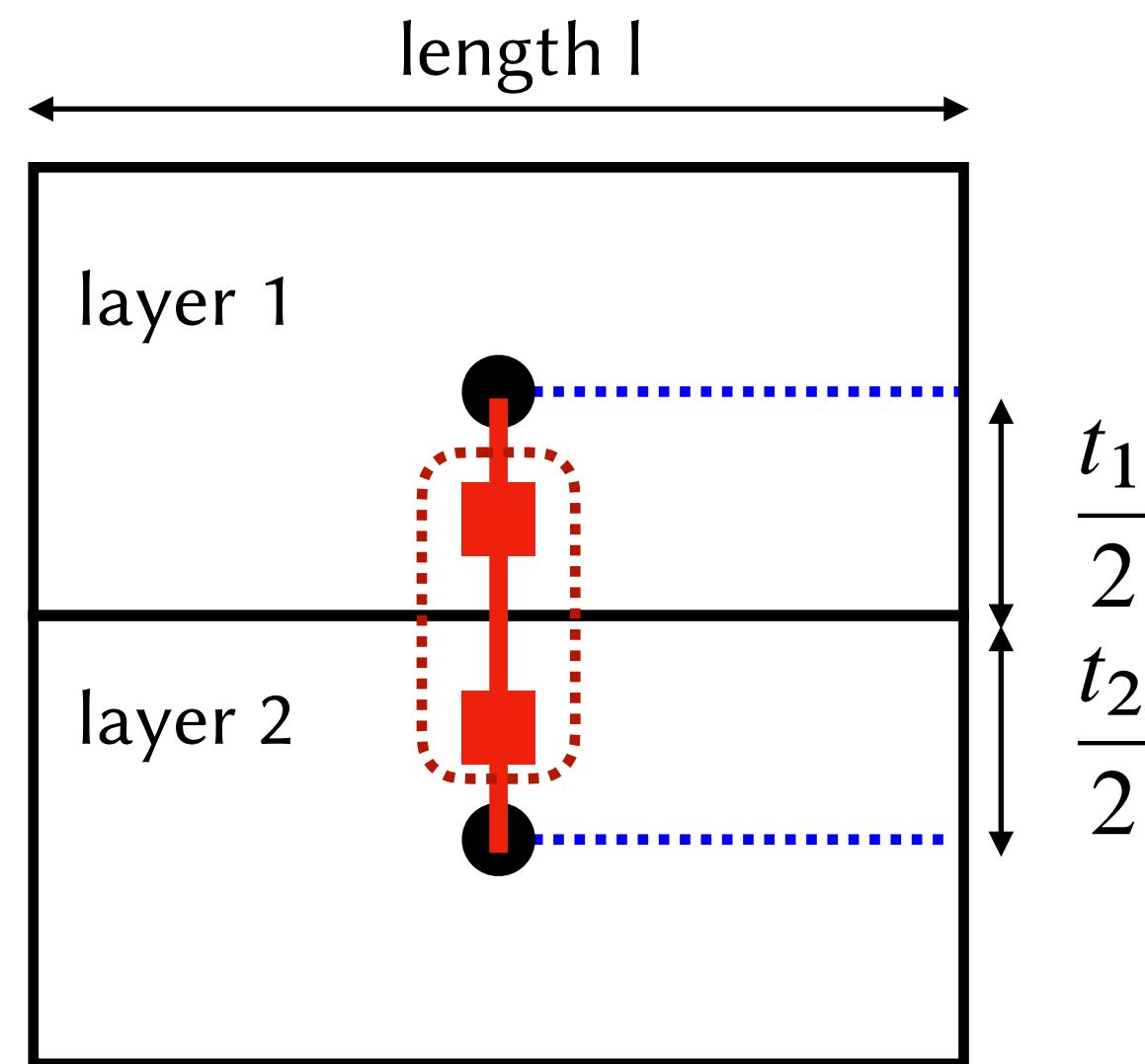
(b)  $t = T/4$



(c)  $t = T/2$

[ Alves et al, 10.1109/TASC.2022.3143076 ]

- linear discretization + mass lumping collapses finite-element method to resistor grid
- can be first order even if Maxwell is second order
- degrees of freedom sit on the edges
  - assumes constant temperature per layer per element
  - excellent numerical stability



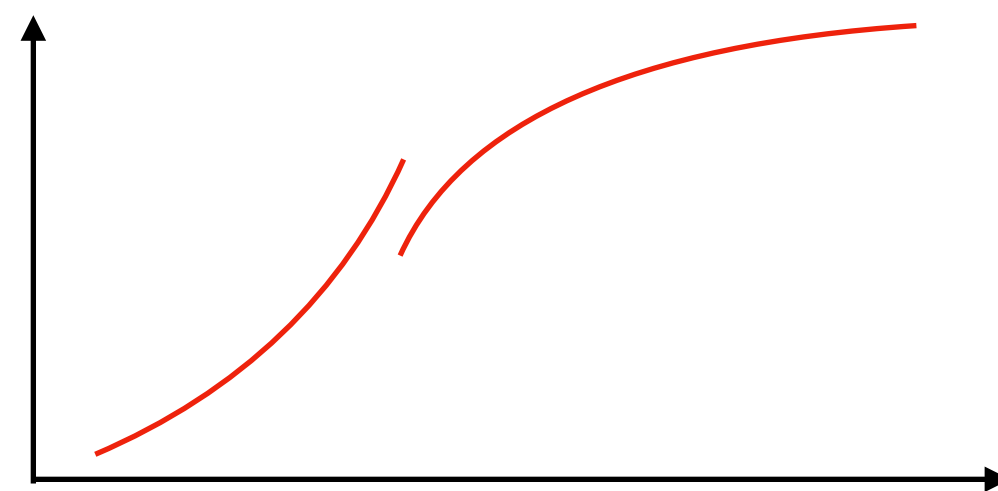


- developing a finite-element framework tailored to HTS cable & magnet development needs
- demonstrated proof of concept in 2D, currently working on 3D
- first performance tests very promising!
- Work in progress: Automated cohomology computation
- Work in progress: coupled thermal-EM 3D tape model for quenching (goal: winter 2024)
- Future Goals: Implement inter-tape current sharing ( ~ spring 2025)

# Thoughts on Material Modeling

- **High Nonlinearities for HTS Materials**

- material properties must be evaluated at every integration point
- nonlinearities require many iterations
- material curves must be smooth (consistent derivatives, continuous)
- avoid expensive functions such as exp or log



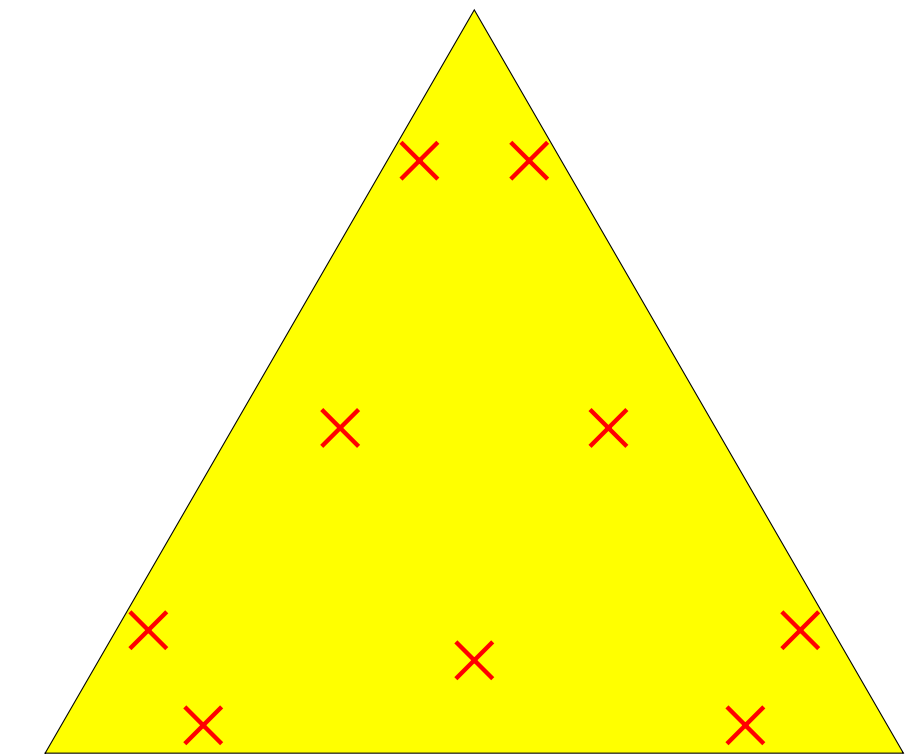
piecewise polynomials lead to convergence issues due to

- discontinuous material properties
- discontinuous derivatives

**CSE can be up to ~3 x faster!!!**

- **Boobytraps in modeling and coding**

- piecewise polynomials
- wasteful implementations
- validity range of functions



triangle integration points  
( 5th order )

**bad code: wasting multiplications:**

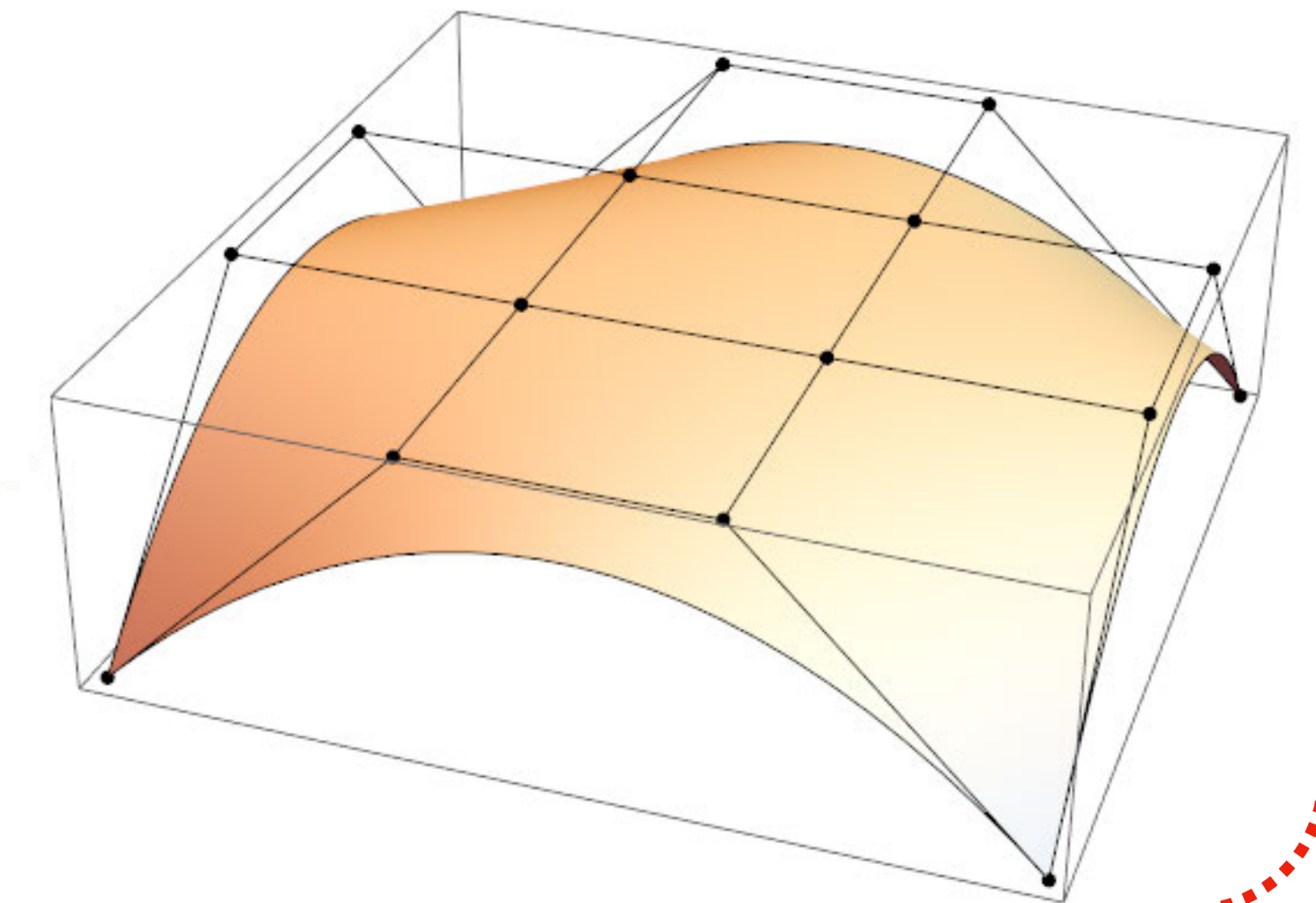
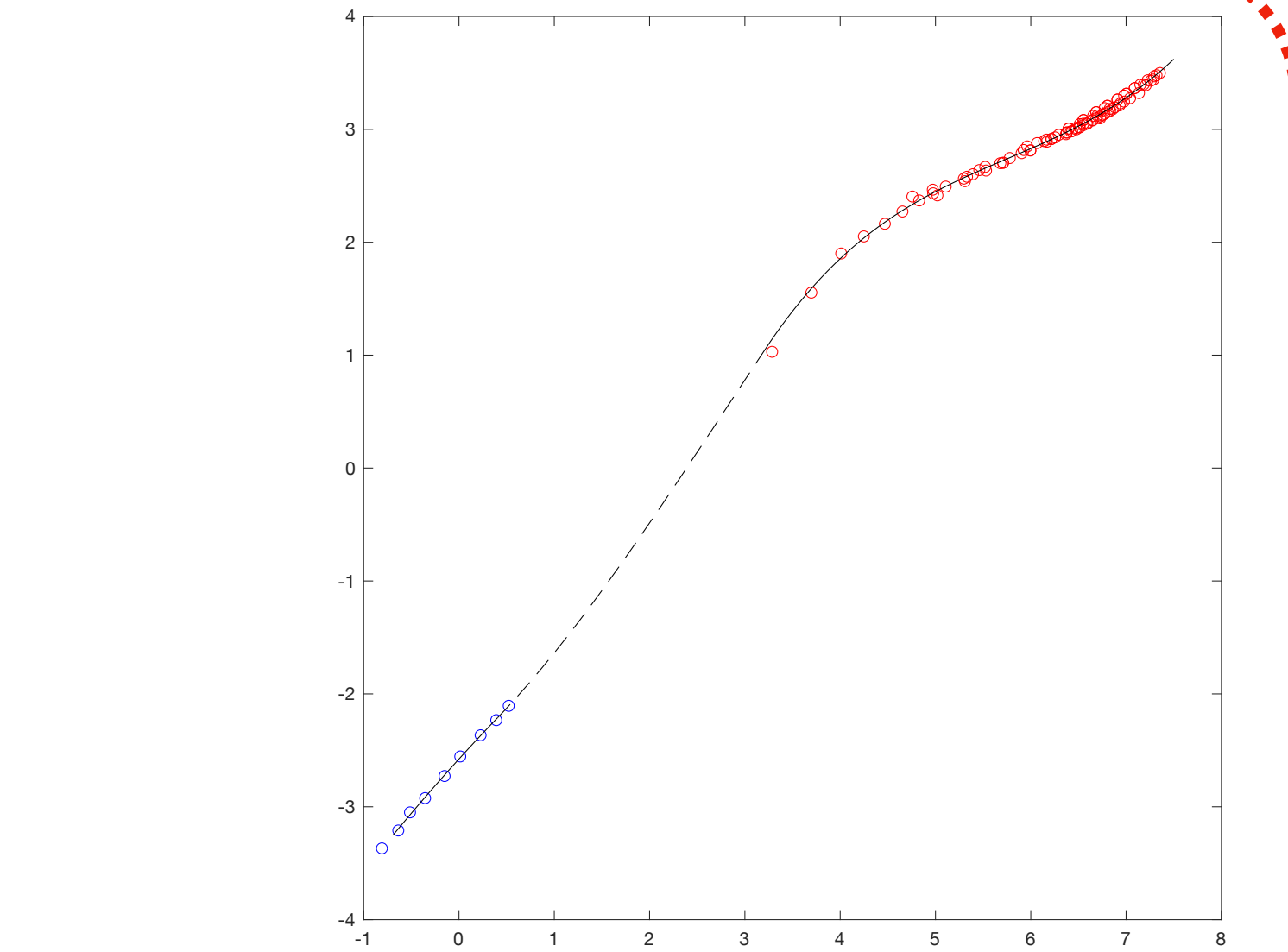
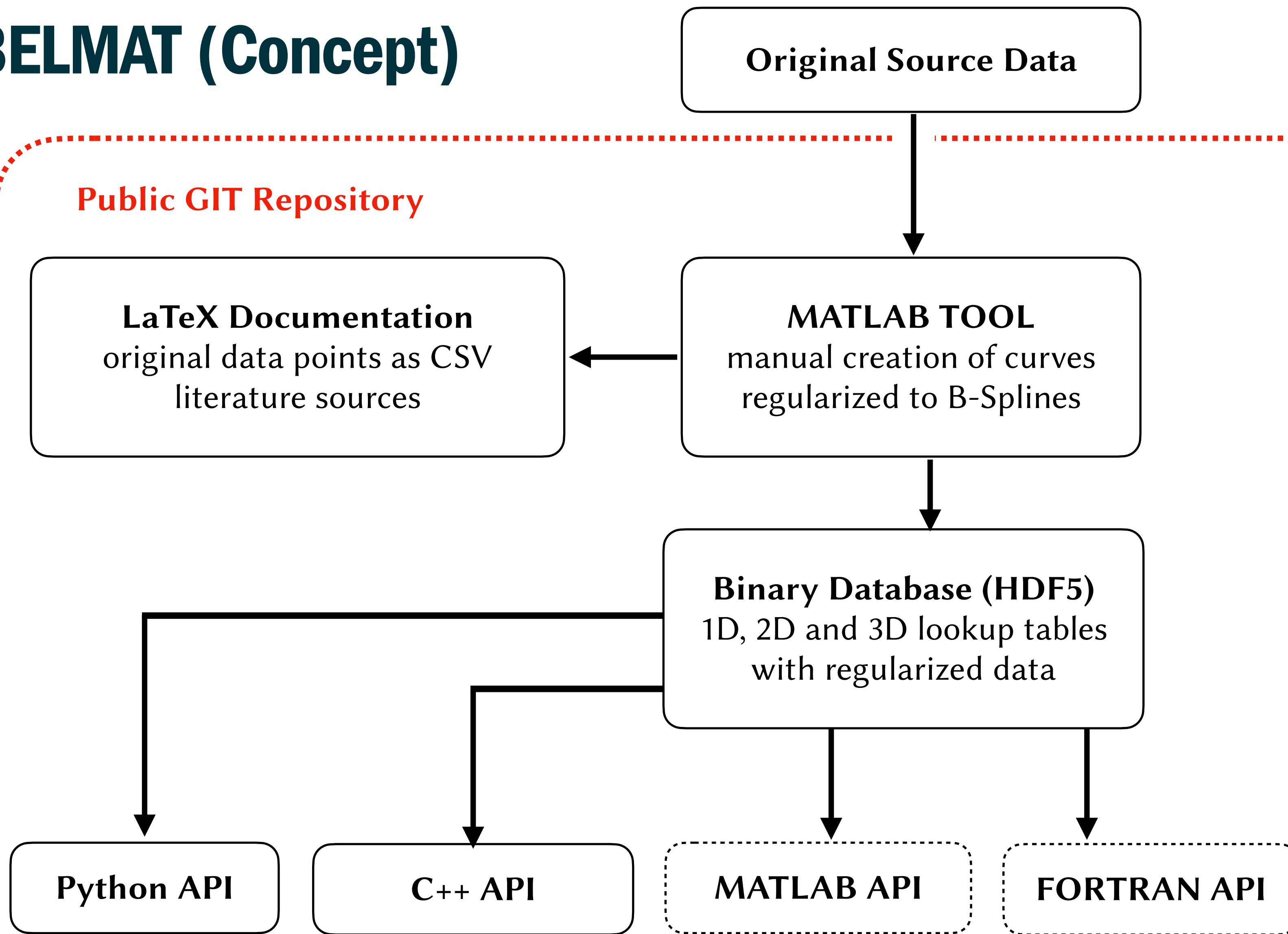
$$y = 10^{( a + b * \log(T) + c * \log(T)^2 + d * \log(T)^3 + \dots )}$$

**good code: Common Subexpression Elimination (CSE)**

$$\log T = \log(T)$$

$$y = 10^{( a + \log T * ( b + \log T * ( c + \log T * ( d + \dots ) ) ) )}$$

# BELMAT (Concept)



→ LET'S DEFINE A COMMUNITY WIDE STANDARD!

# Usecase Example

- implemented in SparseLizard: ASC 2022
- implemented in BELFEM: SUST 2023

## $H-\phi$ Formulation in Sparselizard Combined With Domain Decomposition Methods for Modeling Superconducting Tapes, Stacks, and Twisted Wires

N. Riva, A. Halbach, M. Lyly, C. Messe, J. Ruuskanen, and V. Lahtinen

**Abstract**—The growing interest in the modeling of superconductors has led to the development of effective numerical methods and software. One of the most utilized approaches for magnetoquasistatic simulations in applied superconductivity is the  $H-\phi$  formulation. However, due to the large number of degrees of freedom (DOFs) present when modeling large and complex systems (e.g. large coils for fusion applications, electrical machines, and medical applications) using the standard  $H-\phi$  formulation on a desktop machine becomes infeasible. The  $H-\phi$  formulation solves the Faraday's law formulated in terms of the magnetic field intensity  $H$  using edge elements in the whole modeling domain. For this reason, a very high resistivity is assumed for the non-conducting domains, leading to an ill-conditioned system matrix and therefore long computation times. In contrast, the  $H-\phi$  formulation uses the  $H$ -formulation in the conducting regions, and the  $\phi$  formulation (magnetic scalar potential) in the surrounding non-conducting domains, drastically reducing DOFs and computation time. In this work, we use the  $H-\phi$  formulation in 2D for the magnetothermal (AC losses and quench) analysis of stacks of REBCO tapes. The same approach is extended to a 3D case for the AC loss analysis of a twisted superconducting wire. All the results obtained by simulations in Sparselizard are compared with results obtained with COMSOL. Our custom tool allows us to distribute the simulations over hundreds of CPUs using domain decomposition methods, considerably reducing the simulation times without compromising accuracy.

**Index Terms**—HTS, REBCO, modeling, AC Loss, quench,  $H-\phi$ -formulation, cloud, DDM

**INTRODUCTION**  
WHEN modeling superconducting materials, the electrical resistivity is generally modeled using the power law constitutive relationship [1], which may include a complex critical current density dependence [2]. The highly nonlinear properties and strong anisotropic field dependence of the critical current density could lead to a very large computation time. Moreover, the high aspect ratio of the superconducting tapes (especially in the case of High-Temperature Superconductors (HTS)) leads to a large number of elements and degrees of freedom (DOFs).

This research is supported by Quantest, Type One Energy, PFC MIT, CPS and LLNL. M. Lyly acknowledges support from the Academy of Finland project 32487.

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N. Riva is with MIT Plasma Science and Fusion Center (MA, USA) (e-mail: nriva@mit.edu).

C. Messe and J. Ruuskanen are with Lawrence Berkeley National Laboratory (CA, USA) (e-mail: cmesse@lbl.gov).

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<sup>2</sup>Massachusetts Institute of Technology, Plasma Science and Fusion Center, Cambridge, MA, United States of America

<sup>3</sup>MAATEP Group, Dipartimento Energia "Guglielmo Ferraris", Politecnico di Torino, Turin, Italy

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**Keywords:** HTS, FEM, modeling,  $H-\phi$ -formulation, code development

(Some figures may appear in colour only in the online journal)

**1. Introduction**  
High-temperature superconducting (HTS) cables and magnets play a crucial role in nuclear fusion applications [1], particle accelerators [2], medical devices [3], and even spacecraft engines [4]. To ensure their safe operation, it is necessary to analyze and understand their electrodynamic and thermal performance. Here, advanced finite element methods are essential and very powerful tools: not only do they solve the Maxwell's equations in their magnetodynamic form and the heat transfer equation that describe the physical behavior of these devices, they are also able to realistically model the highly nonlinear behavior of the used materials. The computational cost of these methods, however, remains an important challenge. In recent years, significant progress has been made in the development of so called *mixed formulations*, which promise to be more efficient than traditional finite element

freedom (DOFs). A widely used method is the  $H$  formulation [3]. However, due to the large scale of systems such as electrical machines [4] and fusion devices [5], the computational limits are rapidly reached with the  $H$  formulation [6, 7]. Moreover, the use of the  $H$  formulation in nonconducting domains leads to unnecessary large number of DOFs due to the vectorial nature of the magnetic field intensity  $H$  and to numerical instabilities due to the imposed high resistivity to avoid eddy currents in such domains, leading to an ill-conditioned matrix. The development of approaches more efficient than the  $H$  formulation to be implemented in commercial and in-house software is of paramount importance to improve the computational efficiency of the models. Recently, several works have led to drastic improvements in computational efficiency using the  $A-H$  [8]–[10], the  $T-A$  (similar to the  $A-H$ ) [11]–[13], and the  $H-\phi$  formulations [14]–[17]. This paper aims at addressing the current challenges of 3D modeling 2G HTS using the  $H-\phi$  formulation combined with domain decomposition methods (DDM) [18], [19], enabling massive parallel computation and drastically reduced simulation time. The presented case studies are chosen to represent fusion-energy inspired industrially relevant cases in AC loss and quench modeling.

In section II, we briefly describe the formulation and its implementation in Sparselizard and we describe the utilized custom DDM tool. In section III, we present validating results using simple 2D models, and in section IV, we move on to more complex 3D models, demonstrating the virtues of our DDM-based tool. Finally, in section V, we draw conclusions.

**II.  $H-\phi$  FORMULATION AND IMPLEMENTATION**

**A. Formulation**  
The  $H-\phi$  formulation where current constraints are imposed using cohomology cuts is well-known in computational electromagnetics [20]–[23] and the mathematical main ideas in an electromagnetic context can be traced back to Kotega's curly works on making cuts for scalar potentials [24]. Eventually, it was brought to the context of superconductor AC loss simulations by Lahtinen, Stenwall *et al.* [14], [15]. The finite element formulation is obtained by developing the weak form of Faraday's law of induction and the Gauss law for the magnetic field. Ohm's law is used as transport law. The magnetic field strength  $H$  is discretized in the conducting regions of the computational domain with Nédélec

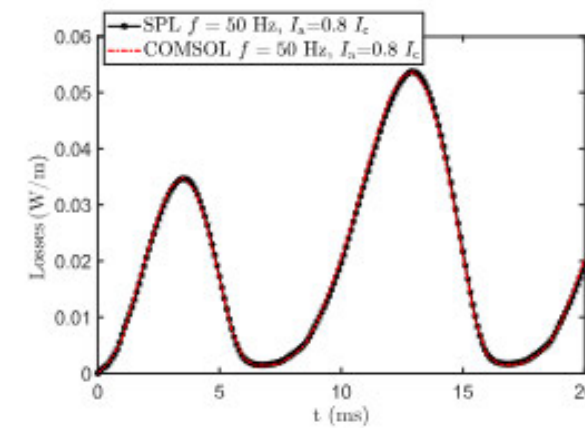


Fig. 5. Instantaneous losses computed on the entire assembly (NbTi wires + copper), with Sparselizard (continuous line + markers) and COMSOL (dashed line).

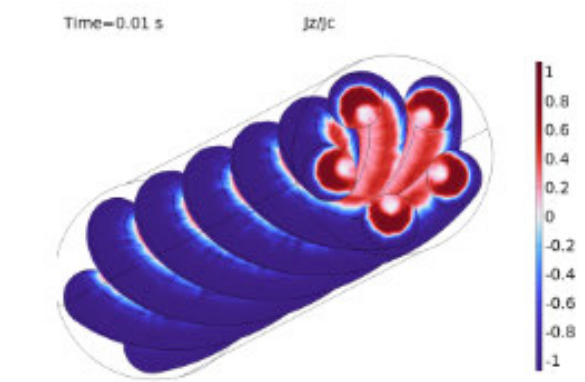


Fig. 6. Normalized current density at  $t = 10$  ms.

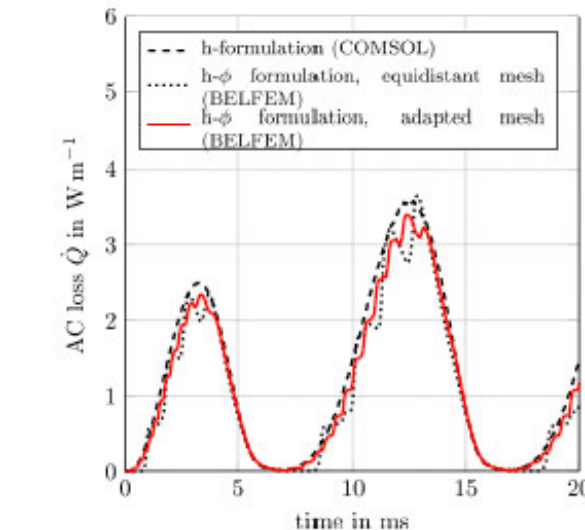


Figure 12. AC losses for the tape stack at  $I = 90$  A per tape at  $f = 50$  Hz.

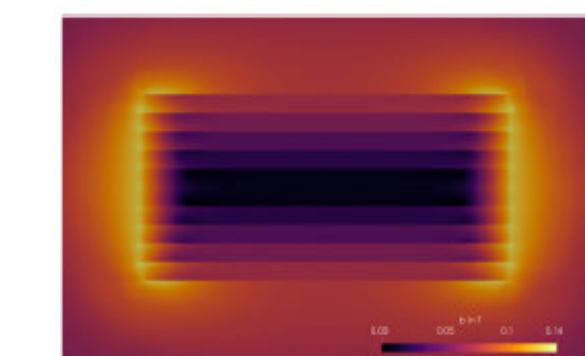


Figure 13. Magnetic flux density at 15 ms with  $I = 90$  A per tape at  $f = 50$  Hz.

```

main.cpp
/private/tmp/main.cpp
main
1 int main( int argc,
2 char * argv[] )
3 {
4 // Material selection.
5 std::string database = "materials.hdf5" ; // database file
6 std::string label = "copper" ; // material to compute
7 uint rrr = 50 ; // purity value
8
9 // create the material
10 belmat::material mat( database, label, rrr );
11
12 double B = 0.0 ; // magnetic flux density B
13 double T = 0.0 ; // temperature in K
14
15 std::cout << mat.label() << std::endl ;
16 std::cout << " density @298.15K: " << mat.density() << std::endl ;
17 while( T < 300 )
18 {
19 std::cout << " " << T ;
20
21 if( mat.has_cp() )
22 {
23 std::cout << " " << mat.cp( T ) ;
24 }
25 if( mat.has_k() )
26 {
27 std::cout << " " << mat.k( T, B ) ;
28 }
29 if( mat.has_rho() )
30 {
31 std::cout << " " << mat.rho( T, B ) ;
32 }
33 std::cout << std::endl ;
34 T += 5.0 ;
35 }

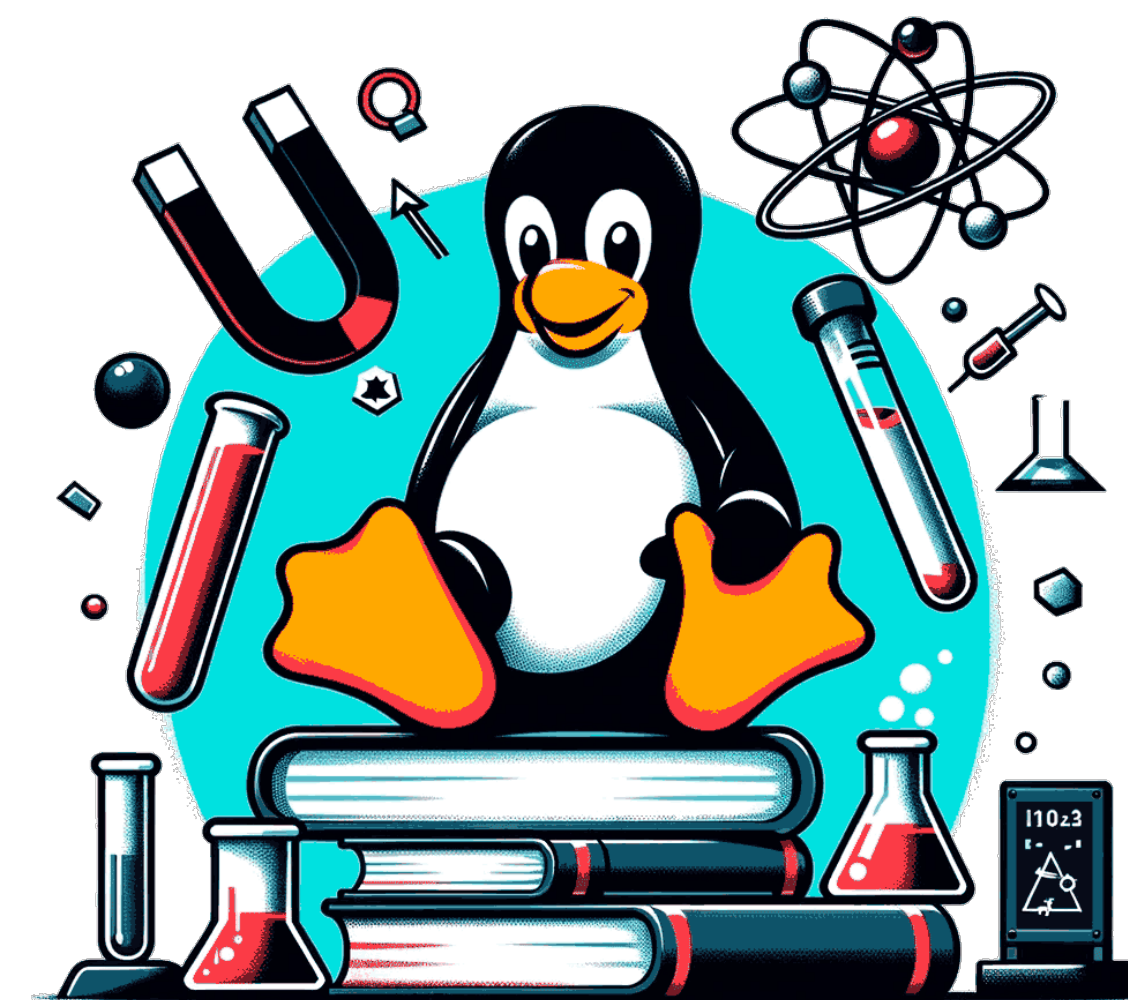
```



**One More Thing**

## RPM Repository for RedHat 9 / Alma Linux 9 / Rocky Linux 9:

- precompiled RPMs for numerical libraries such as:
  - PETSc
  - MUMPS
  - STRUMPACK
  - SuiteSparse
  - Nlopt
  - Sundials
- Features:
  - compiled with both GNU and Intel Compiler suites
  - Support for MPI, MKL and CUDA



<https://belfem.lbl.gov/scis.html>